Resonance transition radiation of relativistic charged particles in an ordered system of vacancion pores

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(Submitted 30 December 1980; resubmitted 28 August 1981)
Zh. Eksp. Teor. Fiz. **82**, 34–49 (January 1982)

The transition radiation emitted by relativistic charged particles on defects—vacancion pores— of the crystal lattice of a solid is considered. It is shown that in the case in which the pore dimension is smaller than the characteristic distance over which the field produced by the moving charge falls off the intensity of the transition radiation depends on the concentration and dimensions of these pores. The use of the transition radiation to investigate the vacancion porosity of solids is therefore suggested. The resonance transition radiation emitted in ordered one- and three-dimensional vacancion-pore structures, i.e., in pore chains and vacancion-pore lattices, is analyzed. It is shown that in the region of emitted-quantum frequencies $\hbar\omega \lesssim 10^2$ keV the transition radiation emitted in the experimentally observed pore lattices in such metals as Al and Ni is more intense than the bremsstrahlung. It is suggested that solids containing vacancion-pore lattices, and possessing a developed vacancion porosity can be used in relativistic-charged-particle detectors and as x-ray sources.

PACS numbers: 61.80.Mk, 61.70.Bv

1. INTRODUCTION

Upon the passage of a charged particle through the boundary between two media with different dielectric properties, there is emitted transition radiation whose properties were first studied by Ginzburg and Frank.[1] Resonance transition radiation is emitted during the passage of a uniformly moving charged particle through an inhomogeneous medium whose dielectric properties vary periodically.[2-5]

Besides the transition radiation, an additional emission channel is possible in a solid because of the periodic disposition of the atoms in the crystal lattice.[6-8] This radiation arises as a result of the diffraction of the field of the particle at the Bragg angles in the ordered system of atoms.

The properties of resonance transition radiation have been investigated in detail in a number of experiments (see, for example, Refs. 2, 9–12) on the flight of relativistic charged particles through various layered media, which are sets of periodically alternating laminae possessing different dielectric properties. The spacing of such one-dimensional periodic structures that are usually used in these experiments ranges from $L_l \approx 5\times 10^{-2}$ to $L_l \approx 1$ cm. Similar layered media are used to construct detectors for high-energy charged particles. Resonance transition radiation possesses threshold properties. An important parameter here is the layered medium's spacing, which characterizes the minimum particle energy starting from which the entire emission spectrum is regenerated. To broaden the energy range of the detectable particles in the region of lower energies, we need periodic structures with sufficiently small spacings.

Interesting structure formations, which arise under certain conditions in various solids, and whose dielectric properties are periodic with a sufficiently small periodicity dimension (of the order of hundreds of angstroms), are ordered structures of crystal-lattice defects. Foremost among these structures are the ordered structures of vacancion pores.[13-17]

It has now been experimentally established that the irradiation of various metals by fast particles (fast neutrons, ions, electrons) produces in them vacancion pores that in a number of cases form ordered structures of several types: one-dimensional (pore chains[13,14]), two-dimensional (pore walls[15]), and three-dimensional (vacancion-pore lattices[16,17]).

In each of these structures the size distribution of the pores is very narrow, and all the pores are located strictly at a definite distance from each other. The mean dimension of the pores and the order constant in such ordered structures depend on the conditions of the irradiation and the type of metal. Uusually the mean pore dimension $\bar{R}$ ranges from about 10 to 10^3 Å, while the order constant L for the various metals ranges from about 10^2 to 10^4 Å.

The most interesting ordered defect structure is the vacancion-pore lattice. It is produced in a definite temperature range starting from some radiation doses. All the pores in the pore lattice are practically of the same size. The mean dimension of the pores located at the nodal sites for the various metals ranges from 20 to 100 Å, while the pore-lattice constant ranges from 200 to 800 Å. The symmetry of the pore lattice coincides with that of the original crystal lattice of the metal. The pore lattice is a stable structure formation.[18,19] At present pore lattices have already been found in such metals as Mo, Ni, Al, Nb, Ta, and W, in the alloys TZM and $Mo_{0.95}Ti_{0.05}$, as well as in fluorite.[17] Evidently, pore lattices can be formed in other solids.

Ordered defect structures can also be produced under conditions other than those produced by irradiation. Thus, the diffusional decay of solid solutions leads to the formation of an ordered lattice of the precipitates of the new phase.[20] This lattice is also periodic in its

 0038-5646/82/010020-10$04.00

dielectric properties.

Another interesting phenomenon, which leads to the formation of one-dimensional ordered structures (vacancion-pore chains), is the "vacancion breakdown" of solids.[14] This phenomenon is observed in ionic crystals, in which the vacancion-pore chains are formed under conditions of high-temperature annealing of the pores by the action of a constant electric field. The pores in such chains may have a fairly extended cylindrical shape, and the axes of the pore chains always coincide with the direction of the electric field.

Ordered vacancion-pore structures differ essentially from the synthetic layered media usually used in experiments on transition radiation in a number of their properties. First, they possess fairly small periodicity dimensions $L_l/L_{\text{por}} \approx 10^3$ and, secondly, the ordered pore structures, being three-dimensional structure formations, consist of pores of finite linear dimensions. And, which is particularly important, the pore dimension can be significantly smaller than the distance L_E over which the electric field produced by the relativistic charged particle is nonzero. Therefore, in contrast to the emission in a one-dimensional layered medium, the intensity of the transition radiation emitted in a pore lattice will depend essentially on the transverse dimensions of the pores.

The investigation of the transition radiation emitted in ordered defect structures may prove to be useful in the determination of the general laws governing the formation of such ordered structures. Furthermore, the investigation of the transition radiation emitted on vacancion pores is of interest by itself. This radiation can be used to investigate the defect structure of solids. It may prove to be especially effective in the investigation of the surface layers of solids, which contain different defects: vacancion pores, cracks.

In the present paper we consider the transition radiation emitted by relativistic charged particles in ordered vacancion-pore structures. We first investigate the radiation emitted on one vacancion pore, and then determine the parameters of the radiation emitted in vacancion-pore chains and lattices. We also present the results of transition-radiation calculations performed for the experimentally observed vacancion-pore lattices in Al and Ni. The resonance transition radiation emitted in a pore lattice is compared with the bremsstrahlung emitted in the crystal.

2. TRANSITION RADIATION EMITTED BY RELATIVISTIC CHARGED PARTICLES ON A VACANCION PORE

Let us determine the radiation that is generated during the flight of a relativistic particle with charge e through one vacancion pore located in a transparent medium.

The moving charge produces an electromagnetic field in the medium. The scattering of this electromagnetic radiation on the vacancion pore results in the generation of the transition radiation of the relativistic charged particle on the pore. The description of this phenomenon with the framework of the kinematic theory of diffraction is similar to the description of the scattering of light by an inhomogeneous medium,[21] whose role is played in the present case by the vacancion pore.

Let us suppose that the permittivity of the medium containing the vacancion pore has the following form:

$$\varepsilon(\omega, \mathbf{r}) = \varepsilon^{(0)}(\omega) + \varepsilon^{(1)}(\omega, \mathbf{r}). \tag{2.1}$$

Here $\varepsilon^{(0)}(\omega)$ describes the permittivity of the homogeneous medium in the absence of the pore ($\varepsilon^{(0)} = \varepsilon_1$) and $\varepsilon^{(1)}(\omega, \mathbf{r})$ characterizes the deviation of the permittivity ε_2 of the vacancion pore from the permittivity ε_1 of the solid in which the pore is located.

Below in considering the transition radiation emitted on a pore we shall be interested in the emission of quanta with frequencies significantly higher than the characteristic atomic frequencies ω_0. In the region of frequencies $\omega \gg \omega_0$, the quantities $\varepsilon^{(0)}$ and $\varepsilon^{(1)}$ are equal to

$$\varepsilon^{(0)}(\omega) = 1 - \omega_0^2/\omega^2, \quad \omega_0^2 = 4\pi e^2 NZ/m_e, \tag{2.2}$$

$$\varepsilon^{(1)}(\omega, \mathbf{r}) = \chi(\omega)\theta(\mathbf{r}), \quad \chi(\omega) = \varepsilon_2 - \varepsilon_1 = \omega_0^2/\omega^2. \tag{2.3}$$

Here NZ is the mean electron density in the medium in which the pore is located, $\theta(\mathbf{r})$ is the pore-shaped function, equal to unity if the radius vector $\mathbf{r}$ is located inside the pore and to zero otherwise, and m_e is the electron mass.

Let us note that, although below we shall consider the transition radiation emitted only on the vacancion pores, the case in which the charged particle passes through an arbitrary inhomogeneity (e.g., the precipitate of a new phase) characterized by the permittivity ε_p can be considered in similar fashion. In this case the quantity $\chi(\omega)$ in the expression (2.3) should be replaced by the quantity

$$\chi(\omega) = \frac{\bar{\omega}_0^2}{\omega^2}, \quad \bar{\omega}_0^2 = \frac{4\pi e^2}{m_e}(NZ - N_p Z_p), \tag{2.4}$$

where $N_p Z_p$ is the mean electron density in the precipitate.

In the region of frequencies $\omega \gg \bar{\omega}_0$ we have $\varepsilon^{(0)} \gg \varepsilon^{(1)}$, and therefore in computing the transition radiation emitted on the inhomogeneities of the medium we can use perturbation theory,[2,21] which reduces in this case to the determination of the intensity of the radiation generated by the electromagnetic wave scattered on the inhomogeneity of the medium of permittivity $\varepsilon^{(1)}$.

Below we shall designate the quantities characterizing the incident electromagnetic wave by one prime (e.g., E', H') and the corresponding quantities for the scattered wave by two primes (e.g., E'', H'').

Let us find the radiation emitted by a particle on a pore in the case in which the relativistic charged particle traverses a vacancion pore in the form of a cylinder of radius $R_\perp$ and length $R_\parallel$, the particle moving with constant velocity $\mathbf{v}$ along the axis of this cylinder. As has already been noted, pores of a similar shape are formed under conditions of a "vacancion breakdown."[14] The electromagnetic field produced by a relativistic charged particle in a medium of permittivity ε_1 has the well-known form

$$\mathbf{E}'(\mathbf{k},\omega)=\frac{4i\pi e}{\varepsilon_1}\frac{\varepsilon_1\omega\mathbf{v}-\mathbf{k}c^2}{k^2c^2-\varepsilon_1\omega^2}\delta(\omega-\mathbf{kv}). \tag{2.5}$$

It can be seen from this expression that, for ultra-relativistic particles (i.e., for $v\approx c$), the electromagnetic field (2.5) is practically a transverse field of strength $E'_\perp$ ($E'_\perp/E'_\parallel\sim\gamma\gg1$). Let us orient the z axis along the axis of the cylindrical pore. Then the components E'_x and E'_y of the electric field (2.5) assume the form

$$E'_{x,y}=\frac{e\alpha e^{i(\omega/v)z}}{\pi v\varepsilon_1}\frac{x,y}{(x^2+y^2)^{1/2}}K_1(\alpha[x^2+y^2]^{1/2}), \tag{2.6}$$

where

$$\alpha=(\omega/v)(1-\varepsilon_1\beta^2)^{1/2},\quad \beta=v/c.$$

The field of the scattered electromagnetic wave is described with the aid of the induction $\mathbf{D}''_\omega$ ($\mathbf{D}''_\omega=\varepsilon^{(0)}\mathbf{E}''_\omega+\varepsilon^{(1)}\mathbf{E}'_\omega$), which satisfies the following equation:

$$\Delta\mathbf{D}_\omega''+\varepsilon_1\frac{\omega^2}{c^2}\mathbf{D}_\omega''=-\operatorname{rot}\operatorname{rot}\varepsilon^{(1)}\mathbf{E}_\omega'. \tag{2.7}$$

The solution to Eq. (2.7) can be written down with the aid of the retarded potentials, and at a sufficiently large distance R_0 from the pore has the form[17]

$$\mathbf{D}_\omega''=-\frac{e^{ik'R_0}}{4\pi R_0}[\mathbf{k}'\times[\mathbf{k}'\times\mathbf{G}]], \tag{2.8}$$

where

$$\mathbf{G}(\omega)=\int\varepsilon^{(1)}(\omega)\mathbf{E}'(\omega,\mathbf{r})e^{-i\mathbf{k}'\mathbf{r}}d^3r. \tag{2.9}$$

Here $\mathbf{k}'=\omega c^{-1}\varepsilon_1^{1/2}\mathbf{n}$ is the wave vector of the emitted quantum. Notice that Eq. (2.7) was derived in the perturbation-theory approximation for $\varepsilon^{(0)}\gg\varepsilon^{(1)}$ $[(\varepsilon_1-\varepsilon_2)/\varepsilon_1\ll1]$, and without allowance for the small terms of the order of $\varepsilon^{(1)}E''_\omega$; therefore, below we shall, for convenience, write in Eq. (2.7) the quantity $\varepsilon_2=1$ in place of the quantity ε_1, and, accordingly, the wave vector $\mathbf{k}'=\omega/c$.

Let us note one important characteristic of the electromagnetic field (2.8), (2.9): the characteristic distance over which the electromagnetic field produced by the relativistic charged particle in the medium falls off is, for $v\approx c$, equal to

$$L_E=\frac{c}{\omega}\left(1-\beta^2+\frac{\omega_0^2}{\omega^2}\right)^{-1/2}.$$

This distance determines the transverse dimensions of the region in which the incident electromagnetic wave excited by the particle traversing the medium undergoes significant scattering on the inhomogeneities of the medium. The dimensions of the vacancion pores are usually tens of angstroms, while the value of L_E, for example, for quanta of wavelength $\lambda=1$ Å and $\gamma=10^3$ is equal to $L_E\approx2\times10^2$ Å, and for vacancion pores with dimensions $\bar{R}\lesssim10^2$ Å the condition $L_E>R$ is fulfilled. Therefore, in contrast to one-dimensional layered media, the transition radiation emitted by relativistic particles on vacancion pores as the particles fly through the region $\rho\lesssim L_E$ (ρ is the distance from the trajectory of the particle) will depend essentially on the number of pores in this region and the transverse pore dimensions. This indicates that the transition radiation can be used to investigate crystal-lattice defects and, especially, the vacancion porosity of irradiated solids.

For vacancion pores of cylindrical shape the components of the vector $\mathbf{G}$ with allowance for the expressions (2.3), (2.6), and (2.9) are equal to

$$G_{x,y}=\frac{4ie\alpha\chi We^{i\varkappa R_\parallel}}{v\varkappa q\varepsilon_1}\frac{\sin\varkappa R_\parallel}{\alpha^2+q^2}k_x',k_y', \tag{2.10}$$

where

$$W=\frac{q}{\alpha}+qR_\perp J_2(qR_\perp)K_1(\alpha R_\perp)-\alpha R_\perp J_1(qR_\perp)K_2(\alpha R_\perp),$$

$$\varkappa=\frac{\omega}{v}(1-\varepsilon_1^{1/2}\beta\cos\vartheta),\quad q=k'\sin\vartheta,$$

$J_1(x)$ and $J_2(x)$ are Bessel functions.

The intensity of the emission of quanta in the frequency and solid-angle intervals $d\omega$ and $d\Omega$, respectively, is equal to

$$dI_{\omega,n}=c\varepsilon_1^{1/2}|E_\omega''|^2R_0^2d\omega\,d\Omega. \tag{2.11}$$

Let us, taking account of the fact that the medium does not scatter the electromagnetic waves $E''_\omega=D''_\omega/\varepsilon^{(0)}$ at the observation point, substitute the expressions (2.8) and (2.10) into (2.11). As a result we obtain the following expression for the intensity of the radiation emitted by a relativistic particle during its flight through a vacancion pore:

$$\frac{dI_\omega}{d\omega\,d\vartheta}=\frac{2e^2v^2\chi^2}{\pi c^3\varepsilon_1^{1/2}}\frac{(1-\varepsilon_1\beta^2)W^2\cos^2\vartheta\sin\vartheta\sin^2\varkappa R_\parallel}{(1-\varepsilon_1\beta^2\cos^2\vartheta)^2(1-\beta\cos\vartheta)^2}. \tag{2.12}$$

Let us consider a few limiting cases.

In the case in which an ultrarelativistic particle with energy $E\gg mc^2$ traverses a vacancion pore of small dimension $\alpha R_\perp\ll1$, the radiation intensity is equal to

$$\frac{dI_\omega}{d\omega\,d\vartheta}=\frac{2e^2\chi^2}{\pi c\varepsilon_1^{1/2}(qR_\perp)^2}\times\frac{[qR_\perp+qR_\perp J_2(qR_\perp)-2J_1(qR_\perp)(1-{}^1/_4\alpha^2R_\perp^2)]^2\cos^2\vartheta\sin^3\vartheta\sin^2\varkappa R_\parallel}{(1-\varepsilon_1\beta^2\cos^2\vartheta)^2(1-\beta\cos\vartheta)^2}. \tag{2.13}$$

The intensity of the pencil radiation ($\vartheta\ll1$) emitted by ultrarelativistic particles on small-sized pores ($R_\perp\approx R_\parallel=R$, $qR\ll1$, $\varkappa R\ll1$) is equal to

$$\frac{dI_\omega}{d\omega\,d\vartheta}\approx\frac{1}{8\pi}\frac{e^2\omega_0^4}{c^7}\omega^2R^6\vartheta^3.$$

Notice that the radiation intensity in this case increases with increasing pore dimension ($dI_\omega/d\omega\propto R^6$), and practically does not depend on the particle energy.

In the case of the traversal of large pores, i.e., for $\alpha R_\perp\gg1$, the intensity of the radiation emitted by a relativistic particle into the small solid angle with $\vartheta\ll1$ is, according to the expression (2.12), equal to

$$\frac{dI_\omega}{d\omega\,d\vartheta}=\frac{8e^2\chi^2}{\pi c}\frac{\vartheta^3\sin^2\left[\frac{\omega R_\parallel}{2c}(1-\beta^2+\vartheta^2)\right]}{(1-\beta^2+\chi+\vartheta^2)^2(1-\beta^2+\vartheta^2)^2}.$$

This expression coincides with the expression for the intensity of the transition radiation emitted in a plate of thickness $L=2R_\parallel$ (Ref. 2).

In the region of frequencies and emission angles where the radiation intensity $\vartheta \ll 1-\beta^2 \ll \chi$ depends on the particle energy, i.e., $dI_\omega/d\omega \sim E^4$, while $dI_\omega/d\omega \sim E^2$ at $1-\beta^2 \sim \vartheta^2$.

The dependence of the intensity of the transition radiation emitted on large vacancion pores (i.e., on vacancion pores for which $\alpha R_\perp \gg 1$) on the particle energy indicates that materials containing vacancion pores can be used in relativistic-charged-particle detectors.

Let us compare the transition radiation emitted on a vacancion pore having a sufficiently small longitudinal dimension, i.e., for which $\varkappa R_\parallel \ll 1$, with the diffraction radiation generated when a charged particle flies through an aperture in a thin ($l \sim \lambda$) screen.[2] In Ref. 2 the diffraction radiation is described with the aid of an approximate computational method based on the Huygens principle. In that analysis the field of the emission from the aperture is the result of the superposition of secondary fields emanating from the points of the aperture, and has at a sufficiently large distance from the aperture the form [cf. the expressions (2.8) and (2.9)]

$$E''_{dif} = \frac{ke^{ikR_0}}{2\pi i R_0} \int E'(\omega, r) e^{-ik\rho} dS, \tag{2.14}$$

where $k=\omega/c$.

In this expression, in contrast to (2.8), E' is the field produced by the moving charge in a vacuum, and not in a medium with permittivity ε_1. It has the form of (2.6) with $\alpha=\omega/v\gamma$. Moreover, in the analysis of the diffraction radiation emitted during the flight through the aperture the screen essentially differs in its optical properties from the medium in which the vacancion pore is located, and which is traversed by the charged particle. Thus, it is assumed in the analysis of the diffraction radiation[2] that the screen is ideally conductive ($\varepsilon_2=\infty$) and absolutely opaque. On the other hand, the investigation of the transition radiation emitted on a pore located in a medium has been performed for a transparent medium (screen) with allowance for the polarization of the material of which it is made.

Let us compare the intensity of the transition radiation (I_{tr}) emitted on a vacancion pore with a small longitudinal dimension $R_\parallel \approx \lambda$ ($\varkappa R_\parallel \ll 1$) with the intensity of the diffraction radiation (I_{dif}) emitted on an aperture.[2] For pores and apertures having small transverse dimensions $\alpha R_\perp \ll 1$ ($\lambda \ll R_\perp$), according to the expressions (2.8) and (2.14), the field of the transition radiation emitted on a pore (E''_{tr}) is weaker than the field of the diffraction radiation emitted on an aperture (E''_{dif}) by a factor of χ (i.e., $E''_{tr}=\chi E''_{dif}$) and, accordingly, the emission intensities are connected by the relation $I_{tr} \sim \chi^2 I_{dif}$. This difference in the intensities is due to the difference in the optical properties of the medium and the screen in the two investigations. For pores and apertures having large transverse dimensions $R_\perp \sim \lambda\gamma$, the deviation from each other of the transition—and diffraction—radiation fields is due not only to the difference in the optical properties of the screens in the two approaches, but also to the fact that in (2.8) the field E' produced by a charged particle in a medium with permittivity ε_2 differs from the field E' in (2.14), which is the field produced by the charged particle in a vacuum. For the radiation emitted at small angles $\vartheta \sim \gamma^{-1} \ll 1$ in the frequency band $\omega \ll \omega_0\gamma$, the transition-radiation field is weaker than the diffraction-radiation field by a factor of γ^{-2}, i.e., $E''_{dif} \sim \gamma^2 E''_{tr}$, so that the radiation intensity $I_{dif} \sim \gamma^4 I_{tr}$.

To conclude this section, let us formulate the conditions of applicability of the expression (2.12) for the intensity of the transition radiation emitted on a pore. The transition radiation emitted on a vacancion pore was considered above with the use of perturbation theory, and, besides the requirement that $\varepsilon^{(0)} \gg \varepsilon^{(1)}$, for the results obtained here to be applicable, it is also necessary that the next perturbation-theory approximation be smaller than the preceding one. For pencil radiation emitted on pores of small dimension, i.e., on pores for which $\alpha R_\perp \ll 1$, $qR_\perp \ll 1$, $R_\perp \approx R_\parallel = R$, we obtain by comparing the expressions for E' and E'' the following condition for the applicability of the expression (2.12):

$$\omega c^{-1}(\omega_0/c)^2 R^3 \vartheta \ll 1.$$

It can be seen from this relation that the main results obtained here for the transition radiation emitted on vacancion pores are well applicable in the description of the pencil radiation.

3. TRANSITION RADIATION EMITTED ON A CHAIN OF VACANCION PORES

Let us now proceed to consider the radiation that arises during the flight of a relativistic charged particle through a chain of vacancion pores. As has already been noted, ordered vacancion-pore chains can be produced artificially in various solids as a result of the irradiation of them by fast-particle fluxes[13] and also as a result of a "vacancion breakdown."[14] The vacancion breakdown effect occurs in ionic crystals, in which the ordered pore chains form under the action of an electric field. The formation of the pore chains occurs as a result of the motion of the pores under the action of the external electric field; therefore, the axes of the pore chains coincide with the direction of the electric field. This, apparently, may significantly facilitate the performance of experiments in which it is necessary to produce ordered charged-particle motion along the axis of the pore chain.

Let us consider an ordered vacancion-pore chain consisting of N pores, and having a spacing equal to L. All the pores in the chain have a cylindrical shape and linear dimensions $R_\perp$ and $R_\parallel$. Let a relativistic charged particle transverse the pore chain, moving along the axis of the chain with velocity v. Let us orient the z axis along the pore chain. Then, for $\omega \gg \omega_0$, the permittivity $\varepsilon(\omega, \mathbf{r})$ of the medium containing the vacancion pore chain can be written in a form similar to (2.1), in which the quantity $\varepsilon^{(1)}(\omega, \mathbf{r})$ is, in the cylindrical system of coordinates (ρ, z), equal to

$$\varepsilon^{(1)}(\omega, \mathbf{r}) = \chi(\omega) \sum_{n=0}^{N-1} \theta(R_\perp - \rho)\theta(z-nL)\theta(2R_\parallel + nL - z),$$

where

$$\theta(x)=\begin{cases}1, & x>0\\ 0, & x<0\end{cases}.$$

The transition radiation that arises during the flight of a relativistic charged particle through a vacancion pore chain is due to the scattering of the electromagnetic field (2.6) produced by the moving charged particle not by one pore, but by the N pores of the pore chain.

To determine the intensity of the radiation emitted on a pore chain, let us compute the quantity $\mathbf{G}$, which, for a pore chain consisting of N pores, is denoted below by $\mathbf{G}^{(N)}$. The calculations lead to the following result for this quantity:

$$\mathbf{G}^{(N)}=\mathbf{G}^{(1)}\sum_{n=0}^{N-1}e^{i\varkappa nL}. \tag{3.1}$$

Here the quantity $\mathbf{G}^{(1)}$ corresponds to the value of $\mathbf{G}$ for one pore, i.e., is given by (2.10).

Performing the computations in much the same way as was done in the preceding section, we find with the use of (2.11), (2.8), and (3.1) the following expression for the intensity of the transition radiation emitted on a pore chain:

$$\frac{dI_\omega^{(N)}}{d\omega\, d\vartheta}=\frac{\sin^2(\varkappa NL/2)}{\sin^2(\varkappa L/2)}\frac{dI_\omega^{(1)}}{d\omega\, d\vartheta}. \tag{3.2}$$

Here $dI_\omega^{(1)}/d\omega d\vartheta$ is the intensity of the transition radiation emitted on one pore [see the expression (2.13)].

It can be seen from the expression (3.2) that when the resonance condition

$$\varkappa L=(\omega L/v)(1-\varepsilon_1^{1/2}\beta\cos\vartheta)=2\pi n,\quad n=\pm1,2,\dots \tag{3.3}$$

is satisfied, the radiation intensity increases rapidly with increasing number of pores:

$$dI_\omega^{(N)}/d\omega\, d\vartheta\sim N^2 dI_\omega^{(1)}/d\omega\, d\vartheta. \tag{3.4}$$

The resonance conditions (3.3) determine the emission angle of the resonance quanta.

An increase in the intensity of the radiation emitted during the flight of a relativistic particle through a pore chain in comparison with the same quantity for one pore can also be achieved upon the fulfillment of the following condition:

$$\frac{N\varkappa L}{2}\approx\frac{N}{4}\frac{L}{\lambda}(1-\beta^2+\vartheta^2+\chi)\ll1,$$

which is fulfilled in the case of ultrarelativistic particles ($v\approx c$), which emit pencil radiation ($\vartheta\ll1$). In this case the radiation intensity is, as in (3.4), proportional to the square of the number of pores.

Let us now analyze a few limiting cases.

Let a relativistic charged particle traverse a chain of pores having an extended shape with a small transverse dimension, i.e., for which $R_\parallel\gg R_\perp$, $\alpha R_\perp\ll1$, $\varkappa R_\parallel\sim1$. The intensity of the pencil radiation ($qR\ll1$) in this case is equal to

$$\frac{dI_\omega^{(N)}}{d\omega\, d\vartheta}=\frac{e^2\omega_0^4R_\perp^4}{2\pi c^5}\frac{\vartheta^3\sin^2(\varkappa R_\parallel)}{(1-\beta^2+\vartheta^2+\chi)^2}\frac{\sin^2(N\varkappa L/2)}{\sin^2(\varkappa L/2)}.$$

In the case in which the particle traverses pores of large transverse dimension $\alpha R_\perp\gg1$, the expression (3.2) obtained within the framework of perturbation theory for the intensity of the radiation emitted on a pore chain goes over into the expression obtained earlier (see Ref. 2) for the intensity of the transition radiation emitted on a pile of plates each of which is of thickness $R_\parallel$.

Let us now determine the total radiation emitted by the relativistic particle during its flight through the pore chain. To do this, let us integrate the expression (3.2) over the angles with the aid of the well-known relation[3]

$$\frac{\sin^2(N\varkappa L/2)}{\sin^2(\varkappa L/2)}=2\pi N\sum_{n=-\infty}^{\infty}\delta(\varkappa L-2\pi n).$$

The general expression for the total radiation intensity has quite an unwieldy form; therefore, let us consider the case—the most interesting one—in which the radiation emitted on the pore chain has a high-directional ($\vartheta\ll1$) character, and all the pores in the pore chain have a small transverse dimension $\alpha R_\perp\ll1$.

According to (3.3), the emission of quanta at small angles will occur when the following condition is fulfilled:

$$2\pi nc/\omega_0L\ll1,$$

the maximum emission angle being then equal to $\vartheta_{max}\approx2\pi nc/\omega_0L$.

The frequency distribution of the radiation intensity in this case has the form

$$dI_\omega^{(N)}=\frac{e^2\omega_0^4R_\perp^4N}{4c^4L}\sum_n\frac{d\omega'}{\omega'}[2\omega'-\omega'^2(1-\beta^2)-p_n^2]\sin^2\frac{2\pi R_\parallel}{L}n, \tag{3.5}$$

where $\omega'=\omega/\omega_n$, $\omega_n=2\pi nv/L$, $p_n=\omega_0/\omega_n$. It can be seen from this expression that the distribution of the radiation intensity is a superposition of harmonics with different values of n.

The range of emitted frequencies is determined by the condition $|\cos\vartheta|\ll1$, which leads, when (3.3) is taken into account, to the following inequality:

$$1+\varepsilon_1^{1/2}\beta\geqslant\omega_n/\omega\geqslant1-\varepsilon_1^{1/2}\beta. \tag{3.6}$$

The first part of this inequality for ultrarelativistic particles, for which $\gamma^2=E/mc^2\gg1$, $\gamma^2\gg p_n^2$, determines the minimum (ω_{min}) and maximum (ω_{max}) radiation frequencies:

$$\omega_{min}\approx\frac{1}{2}\omega_np_n^2,\quad \omega_{max}\approx2\omega_n\gamma^2,$$

between which lies the entire emission spectrum of the quanta. Let us note that the condition for the positiveness of the radiation intensity (3.5) leads to the same frequency range.

It can be seen from the expression (3.5) that the radiation intensity for each harmonic at the emission frequency, which is equal to

$$\omega\approx\omega_n\gamma^2,$$

has the maximum value

$$\frac{dI_{\omega,n}^{(N)}}{d\omega}=\frac{e^2\omega_0^4R_\perp^4N}{4c^4L}\frac{1}{\omega_n}\left(1-\frac{p_n^2}{\gamma^2}\right)\sin^2\frac{2\pi R_\parallel}{L}n.$$

It can be seen from this expression that the maximum value of the intensity for each harmonic increases with increasing particle energy E, and that it approaches a limiting value at high energies (i.e., when $\gamma^2 \gg p_n^2$).

An important characteristic of resonance transition radiation is the threshold energy $E_p^{(n)}$, starting from which the harmonic with the number n appears in the emission spectrum. This quantity is determined from the condition (3.6), and is equal to

$$E_p^{(n)}/mc^2=\omega_0/\omega_n=\omega_0 L/2\pi cn.$$

The fact that $E_p^{(n)} \sim L$ is very important in connection with the performance of experiments on resonance emission on a pore chain. The value of L for a pore chain is fairly small ($L \approx 10^2 - 10^4$ Å), and therefore the entire radiation spectrum is regenerated starting from energies $E_p^{(1)}=(\omega_0 L/2\pi c)mc^2$. For example, for $L=5\times 10^2$ Å and $\hbar\omega_0=30$ eV, the quantity $E_p^{(1)}\approx 1$ MeV.

Let us now find the total number of quanta emitted by a relativistic charged particle as it passes through a unit length of a pore chain. For this purpose, let us divide the quantity (3.5) by the energy $\hbar\omega$ of a quantum and the total length NL of the chain and integrate the resulting expression over the frequencies. The maximum value of the total number of quanta emitted from a unit length of the pore chain is approximately equal to

$$m_{\text{por}} \approx \frac{1}{2\pi\cdot 137}\frac{\omega_0^4 R_\perp^4}{c^4}\frac{1}{L}\sum_{n=1}^{n_{max}}\frac{1}{n}\ln\left(\frac{4\pi c}{\omega_0 L}\gamma n\right)\sin^2\frac{2\pi R_\parallel}{L}n, \qquad (3.7)$$

where $n_{\max}\approx \omega_0 L/2\pi c$.

Let us, for comparison, point out that in the case of the transition radiation emitted on a pile of plates with spacing L_l the maximum number of quanta emitted from a unit length is equal to[2]

$$m_l \approx \frac{4}{\pi\cdot 137}\frac{1}{L_l}\sum_{n=1}^{n_{max}}\frac{1}{n}\ln(2\pi\gamma_0 n). \qquad (3.8)$$

The ratio of the quantities (3.7) and (3.8) is equal to

$$\frac{m_{\text{por}}}{m_l}\approx 10^{-1}\frac{\omega_0^4 R_\perp^4}{c^4}\frac{L_l}{L_{\text{por}}}. \qquad (3.9)$$

For the characteristic values $L_{\text{por}}=5\times 10^2$ Å, $R_\perp=50$ Å, and $\hbar\omega_0=60$ eV of the parameters of the ordered vacancion-pore chain that forms in a solid, and for the spacing $L_l=5\times 10^6$ Å of the layered medium, the gamma-quantum yield ratio (3.9) is equal to $m_{\text{por}}/m_l\approx 10^3$.

In the case of pores of a large dimension $\alpha R_\perp \gg 1$ the expression (3.2) obtained for the radiation intensity within the framework of perturbation theory goes over into the corresponding expression for the intensity of the radiation emitted by a relativistic charged particle in a layered medium.[2,3] All the quantities characterizing the transition radiation in this case are considered in detail in Refs. 2 and 3.

To conclude this section, let us make a few remarks about the effect of a deviation from the ordered disposition of the pores in the chain and the smearing of the narrow size distribution of the pores in the pore chain on the intensity of the resonance radiation emitted in this ordered structure. As is well known,[2,5] the characteristic distance over which the transition radiation is produced in a medium with permittivity ε is the coherence length $L_c\approx c/\omega(1-\varepsilon\beta^2+\vartheta^2)^{-1}$. It can be shown in much the same way as is done in Refs. 2 and 3 that, if there are distortions in the periodicity of the ordered pore chain, so that the pores are shifted from site positions in the ordered structure through distances $\sim(\overline{\delta L^2})^{1/2}$, and if the size distribution of the pores has a width $\sim(\overline{\delta R^2})^{1/2}$, then all the results obtained for the resonance transition radiation will not change provided the following conditions are fulfilled:

$$(\overline{\delta R^2})^{1/2}\ll L_c, \quad (\overline{\delta L^2})^{1/2}\ll L_c.$$

In the case of fairly large shifts of the pores from the nodal sites, i.e., for $(\overline{\delta L^2})^{1/2}\gg L_c$, the transition radiation is emitted on N independent pores, and the intensity of the radiation emitted on a pore chain is the sum of the radiation intensities for the individual pores each considered separately [cf. (3.5)]:

$$dI_\omega^{(N)}/d\omega\, d\vartheta \sim N dI_\omega^{(1)}/d\omega\, d\vartheta.$$

The occurrence in a system of randomly disposed pores of transition radiation with intensity proportional to the pore concentration, and dependent on the pore dimensions, can be used to investigate the vacancion porosity of solids.

4. TRANSITION RADIATION PRODUCED BY FLIGHT OF A RELATIVISTIC CHARGED PARTICLE THROUGH A VACANCION-PORE LATTICE

Let us determine the radiation that arises during the flight of relativistic charged particles through a vacancion-pore lattice.

Let us represent the permittivity $\varepsilon(\omega,\mathbf{r})$ of the solid containing the pore lattice in the frequency region $\omega \gg \omega_0$ in the form of a sum of the two quantities: $\varepsilon^{(0)}(\omega)$ and $\varepsilon^{(1)}(\omega,\mathbf{r})$ [see the expression (2.1)]. Here, as before, the quantity $\varepsilon^{(0)}(\omega)$ characterizes the permittivity of the solid in the absence of pores, i.e., is given by (2.2), while the quantity $\varepsilon^{(1)}(\omega,\mathbf{r})$ describes the permittivity of the ordered pore system, and is equal to

$$\varepsilon^{(1)}(\omega,\mathbf{r})=\chi(\omega)\bar\theta(\mathbf{r}),$$

where $\bar\theta(\mathbf{r})$ is a function characterizing the position of the pores in the solid, being equal to one if the radius vector $\mathbf{r}$ lies inside any pore and zero otherwise.

The permittivity $\varepsilon(\omega,\mathbf{r})$ of a solid containing a vacancion-pore lattice is a periodic function of the coordinates, i.e.,

$$\varepsilon(\omega,\mathbf{r})=\varepsilon(\omega,\mathbf{r}+\mathbf{s}), \qquad (4.1)$$

where $\mathbf{s}$ is an arbitrary translation vector in the pore lattice.

Let us, using the periodicity property (4.1), expand the spatially inhomogeneous component $\varepsilon^{(1)}(\omega,\mathbf{r})$ of the permittivity $\varepsilon(\omega,\mathbf{r})$ in a Fourier series in the reciprocal pore lattice vectors $\mathbf{q}$:

$$\varepsilon^{(1)}(\omega,\mathbf{r})=\sum_{\mathbf{q}}\varepsilon_{\mathbf{q}}e^{i\mathbf{q}\mathbf{r}}, \quad \varepsilon_{\mathbf{q}}=\frac{1}{V}\int\varepsilon^{(1)}(\omega,\mathbf{r})e^{-i\mathbf{q}\mathbf{r}}d^3r.$$

Here the reciprocal pore lattice vector $\mathbf{q}$ has the components $2\pi l/L$, $2\pi m/L$, and $2\pi n/L$, where l, m, and n are the arbitrary whole numbers, L is the length of the unit-cube edge in the pore lattice, and V is the volume of the unit cell of the pore lattice ($V \approx L^3$).

Let us find the intensity of the transition radiation emitted during the motion of a relativistic charged particle through the pore lattice. The calculations for $\omega \gg \omega_0$, $\varepsilon^{(0)} \gg \varepsilon^{(1)}$ yield the following expression for the radiation intensity per unit particle-path length in the pore lattice for the quanta emitted in the frequency range $d\omega$ into the solid angle $d\Omega$ (see also Ref. 2):

$$\frac{dI_\omega}{d\omega\, d\Omega}=\frac{e^2\omega^2}{2\pi\varepsilon_1^{5/2}cv}\sum_{\mathbf{q}}\varepsilon_{\mathbf{q}}^2\left|\left[\mathbf{k}\left(\frac{\omega\varepsilon_1\mathbf{v}}{c^2}+\mathbf{q}\right)\right]\right|^2\frac{\delta[\omega-(\mathbf{k}-\mathbf{q})\mathbf{v}]}{[(\mathbf{k}-\mathbf{q})^2-\varepsilon_1\omega^2/c^2]^2}. \tag{4.2}$$

Here $\mathbf{k}=\omega c^{-1}\varepsilon_1^{1/2}\mathbf{n}$ is the wave vector of the emitted quantum.

Notice that the transition radiation emitted in an ordered system of precipitates[20] can be investigated with the aid of the expression (4.5). For this purpose the quantity $\chi(\omega)$ should be taken to be the difference between the permittivities of the matrix of the solid and the material of which the precipitate is made [see the expression (2.4)]. The maximum value of the intensity (4.2) will occur in a pore lattice for which the quantity $|\chi(\omega)|$ has its greatest value.

It can be seen from the expression (4.2) that the transition radiation possesses resonance properties when the following condition is fulfilled:

$$\omega-\mathbf{k}\mathbf{v}=-\mathbf{q}\mathbf{v}. \tag{4.3}$$

This condition determines the emission angle ϑ (i.e., the angle between $\mathbf{k}$ and $\mathbf{v}$) at which resonance transition radiation occurs in the pore lattice.

Below we shall consider in greater detail the most interesting case, when the charged particle traverses the vacancion pore lattice in a direction parallel to one of the axes of the unit cube (i.e., the case in which $\mathbf{v} \parallel \mathbf{z}$).

The orientation of the axis of the pore lattice can easily be established in the experiments that can be performed on transition radiation emission in a pore lattice, using, for example, electron microscopy, or any other method that allows the investigation of the microscopic structure of the parent crystal lattice of the solid. Knowing the orientation of the pore lattice, we can direct the relativistic particle at any angle to the chosen axis in that lattice. Then, since for $\omega \gg \omega_0$, we have $\varepsilon_1 < 1$, the relation (4.3) will be satisfied only in the case in which $\mathbf{q}\cdot\mathbf{v} = -2\pi n v/L < 0$ ($n > 0$). The condition (4.3) in this case assumes the form

$$\cos\vartheta=c/v\varepsilon_1^{1/2}-2\pi nc/\omega\varepsilon_1^{1/2}L. \tag{4.4}$$

It can be seen from this relation that, when the condition $2\pi nc/\omega_0 L \ll 1$ is fulfilled, the radiation emitted by ultrarelativistic particles ($v \approx c$) is a pencil emission occurring in a narrow cone with maximum apex angle $\vartheta_{\max} \approx 2\pi nc/\omega_0 L$.

To determine the radiation intensity, we must find the Fourier components $\varepsilon_{\mathbf{q}}$ of the spatially inhomogeneous component of the permittivity $\varepsilon(\omega, \mathbf{r})$. As has already been noted, the symmetry of the vacancion pore lattice coincides with that of the parent crystal lattice of the solid in which the ordered pore structure is formed. Therefore, in such metals as Al and Ni, in which pore lattices have thus far been detected, the pores form the face-centered cubic (fcc) lattice, while in Mo, Nb, Ta, and W the pores form the body-centered cubic (bcc) lattice. Let us find the quantities $\varepsilon_{\mathbf{q}}$ for these two types of pore lattices. To do this, let us assume that each pore has the cubic shape with a cube-edge length equal to $2R$. The calculations yield for the quantity $\varepsilon_{\mathbf{q}}$ for the bcc pore lattice the expression

$$\varepsilon_{\mathbf{q}}=\frac{\chi}{\pi^3}\frac{1+\cos\pi(l+m-n)}{lmn}\sin\frac{2\pi R}{L}l\sin\frac{2\pi R}{L}m\sin\frac{2\pi R}{L}n. \tag{4.5}$$

For the fcc pore lattice the Fourier transform $\varepsilon_{\mathbf{q}}$ has the form

$$\varepsilon_{\mathbf{q}}=\frac{\chi}{\pi^3 lmn}\left\{\left[\cos\pi(l+m)\cos\frac{\pi Rn}{L}+\cos\pi(n-m)\cos\frac{\pi Rl}{L}\right.\right.$$
$$\left.+\cos\pi(l-n)\cos\frac{\pi Rm}{L}\right]2\sin\frac{\pi Rl}{L}\sin\frac{\pi Rm}{L}\sin\frac{\pi Rn}{L}$$
$$\left.+\sin\frac{2\pi R}{L}l\sin\frac{2\pi R}{L}m\sin\frac{2\pi R}{L}n\right\}. \tag{4.6}$$

It can be seen from the expressions (4.2), (4.5), and (4.6) that the emission spectra of charged particles in the pore lattices are superpositions of various harmonics having the numbers l, m, and n, the radiation intensities being largely determined by the contributions of the first harmonics.

The condition $|\cos\vartheta| \leq 1$ with allowance for (4.4) determines the emitted-frequency range for each harmonic number considered separately. It should be noted that the emitted-frequency range in the case of the pencil radiation ($\vartheta \ll 1$) and the range for the radiation emitted into the rear hemisphere ($\vartheta \gtrsim 1$) differ somewhat from each other.[2]

Thus, in the case of the pencil radiation the emitted-frequency range is equal to

$$\frac{2\pi nv(1+\beta)}{L}\gamma^2 \geqslant \omega \geqslant \frac{L\omega_0^2}{2\pi nc}, \tag{4.7}$$

where $\gamma = (1-\beta^2)^{-1/2}$.

The wavelength of the pencil radiation depends on the emission angle ϑ and the emitted-harmonic number n. For $|\chi|\gamma^2 \ll 1$ the wavelength of the radiation is equal to $\lambda = L(1+\vartheta^2\gamma^2)/2\gamma^2 n$.

For $2\pi nc/\omega_0 L \gtrsim 1$ the angular radiation is not a pencil beam, and is directed into the rear hemisphere. The emitted-frequency range in this case is specified as follows:

$$\frac{2\pi nv(1+\beta)}{L}\gamma^2 \geqslant \omega \geqslant \frac{2\pi nv}{(1+\beta)L}. \tag{4.8}$$

In the metals Ni and Nb the observed pore-lattice constants are respectively equal to $L = 660$ and 750 Å (Ref. 17), and the first harmonics ($n = 1-3$) satisfy the relation $2\pi nc/\omega_0 L < 1$, which indicates that the radiation emitted by charged particles in these metals should have a high-directional character with an emitted-frequency range given by (4.7). As an example, let us point out that, in the case of the motion of relativistic electrons with energy $E=500$ MeV through a pore lattice of lattice constant $L=700$ Å in a solid with $\hbar\omega_0=59$ eV, the emitted-frequency range for the harmonics with number n is $35n$ MeV $\gtrsim \hbar\omega \gtrsim 99n^{-1}$ eV.

In the other metals (Al, Mo, Ta, and W) in which pore lattices have found to occur, the observed pore-lattice constants are somewhat smaller [L ranges from about 200 to 300 Å (Ref. 17)], and in these metals the relation $2\pi mc/\omega_0 L \gtrsim 1$ obtains even for the first harmonics. The radiation in these metals should be emitted into the rear hemisphere, and the emitted-frequency range is given by the relation (4.8). For example, for the radiation emitted by relativistic electrons with energy $E=500$ MeV in the case in which $L=200$ Å the emitted-frequency range is equal to $35n$ MeV $\gtrsim \hbar\omega \gtrsim 31n$ eV.

Let us now determine the intensity $dI_\omega/d\omega$ of the radiation emitted by a relativistic particle moving in a pore lattice along one of the unit-cube edges. For this purpose, let us integrate the expression (4.2) over the solid angle Ω. As a result of the computations we obtain

$$\frac{dI_\omega}{d\omega}=\frac{e^2\omega}{2\varepsilon_1{}^3 v^2}\sum_{\mathbf{q}}\varepsilon_{\mathbf{q}}{}^2\left\{\frac{(1-2w)\varepsilon_1\beta^2/2w+(1+g)w}{[(1-(1+g)w)^2+g(1-\varepsilon_1\beta^2)]^{1/2}}\right.$$
$$\left.-\frac{\varepsilon_1{}^2\beta^4[1-2w+(1+g)w][(1-\varepsilon_1\beta^2)+4w^2(1-(1+g)w/\varepsilon_1\beta^2)^2]}{4w^2[(1-(1+g)w)^2+g(1-\varepsilon_1\beta^2)]^{3/2}}-1\right\}. \quad (4.9)$$

Here $w=\omega_n/2\omega$, $\omega_n=2\pi nv/L$, and $g=(q_x^2+q_y^2)/q_z^2$.

In the region of high quantum-emission frequencies where $\omega_n/\omega \ll 1$, $|\chi|\gamma^2 \gg 1$, and for small pore dimensions $R \ll L$, the radiation intensity (4.9) varies according to the following law:

$$dI_\omega/d\omega \sim e^2\omega_0{}^4 R^6/4c^3\omega^2 L^5.$$

In this frequency range the radiation intensity practically does not depend on the particle energy.

At higher frequencies, i.e., for $\omega_n/\omega \ll 1$ and $|\chi|\gamma^2 \ll 1$, the radiation intensity depends weakly on the relativistic-particle energy:

$$\frac{dI_\omega}{d\omega}\approx\frac{e^2\omega^2}{4c^3}\sum_{\mathbf{q}}\frac{\varepsilon_{\mathbf{q}}{}^2}{q_z}\left(2-\frac{1}{\gamma^2}\frac{\omega}{\omega_n}\right).$$

As has already been noted, an important characteristic of the transition radiation emitted in an ordered vacancion-pore structure is the presence of a threshold effect for the energy of the moving particle, i.e., the existence of a particle energy starting from which there appear in the radiation spectrum (4.9) harmonics having the ordinal numbers n. Starting from the energy $E_p^{(1)}$ the harmonic with the number $n=1$ appears in the radiation spectrum, and the entire spectrum is reestablished. The radition intensity reaches saturation, having then the highest possible value. The threshold energy $E_p^{(1)}$ is equal to

$$E_p^{(1)}=\frac{\omega_0 L}{2\pi c}mc^2.$$

The fact that $E_p^{(1)}\sim L$ is very important for the performance of experiments on transition-radiation emission in pore lattices. The point is that the value of the threshold energy $E_p^{(1)}$ in a pore lattice having a sufficiently small spacing (i.e., with $L\approx 10^2-10^3$ Å) is significantly lower than the value of the same quantity for the transition radiation emitted in the normally used layered media. Let us, for comparison, point out that, for layered media with $L_l \approx 10^{-3}$ cm, $E_p^{(1)} \gtrsim 10^2$ MeV. On the other hand, in a pore lattice (in, for example, Mo) with spacing $L=220$ Å (Ref. 17) the threshold energy for relativistic electrons has the value $E_p^{(1)}=0.54$ MeV. In other experimentally observed pore lattices[17] $E_p^{(1)}\approx 1$ MeV; therefore, resonance radiation can appear in a pore lattice at significantly lower electron energies: $E \gtrsim 1$ MeV.

Let us now determine the number dm of quanta emitted by a relativistic particle as it travels a unit length in a vacancion-pore lattice. Let us perform the numerical computation for the radiation emitted by relativistic electrons in the experimentally observed[17] vacancion-pore lattices of the two metals Al and Ni.

In Fig. 1 we present the results of the numerical calculation of the number dm of quanta emitted in the frequency interval $d\omega$ for Al having a pore lattice constant $L=700$ Å and $R=250$ Å (the curve 1) and for Ni with $L=620$ Å and $R=125$ Å (the curve 2) in the case in which the electron energy $E=50$ MeV.

The results of the calculations show that the number of quanta emitted over a unit length in a pore lattice practically does not depend on the electron energy when $E \gg E_p^{(1)}$. At the same time, the number of emitted quanta essentially depends on the lattice constant L and the mean radius $\bar{R}$ of the pores. According to (4.9), the highest yield of emitted quanta should occur in a pore lattice in which the pores have fairly large dimensions, i.e., for which $\bar{R}\approx L$.

Such pore lattices can be produced with large radiation doses under conditions of continuous pore swelling that conform to the stability conditions for the pore lattice.[18,19]

In Fig. 2 we present the results of the calculation of the number dm of quanta emitted in the pore lattice of Al for different pore radii ($R_1=100$ Å, $R_2=150$ Å, and $R_3=200$ Å) and a fixed pore-lattice constant $L=700$ Å. All the calculations were performed for an electron

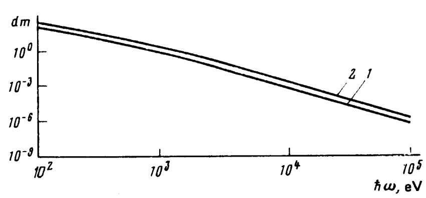

FIG. 1. Resonance transition radiation spectrum dm (number of quanta/electron · keV) for the experimentally observed pore lattices of the two metals Al and Ni. The energy of the radiation quanta is plotted along the abscissa axis. The curve 1 pertains to the pore lattice in Al (Ref. 17) for which $L=200$ Å and $R=250$ Å; the curve 2 corresponds to the results of the calculation for the pore lattice in Ni (Ref. 17) for which $L=620$ Å and $R=125$ Å. The calculations were performed for an electron energy $E=50$ MeV.

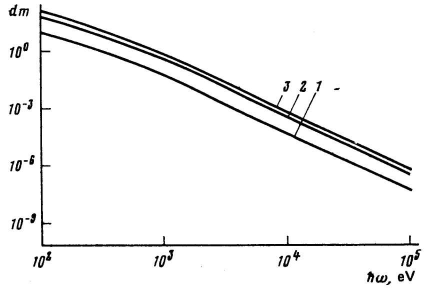

FIG. 2. Resonance transition radiation spectrum dm (number of quanta/electron · keV) for the pore lattice in Al as a function of the dimension of the pores in the lattice. The energy of the emitted quanta is plotted along the abscissa axis; the number of quanta, along the ordinate axis. The calculations were performed with a fixed pore-lattice constant $L = 700$ Å and different pore radiii: $R_1 = 100$ Å (the curve 1), $R_2 = 150$ Å (the curve 2), and $R_3 = 200$ Å (the curve 3). The electron energy $E = 200$ MeV.

energy of $E = 200$ MeV. It can be seen that the yield of emitted quanta increases as the vacancion porosity and the mean pore dimension increase.

Let us compare the number of quanta emitted during resonance emission in the pore lattice with the yield obtained in the bremsstrahlung emitted in the crystal. For electron energies $E \ll E_0 = (mc^2)^2 E_s^{-1} \omega_0 L_{\text{rad}}/8c$ (usually, for metals $E_0 \approx 1$ GeV) and in the region of quantum-emission frequencies $\hbar\omega \lesssim \hbar\omega_c = \hbar\omega_0\gamma$, the number of emitted quanta per unit length as determined by the bremsstrahlung obtained with allowance for the polarization of the medium is equal to[2]

$$dm_{\text{br}} = \frac{4}{3} \frac{1}{1+\omega_0^2\gamma^2/\omega^2} \frac{d\omega}{L_{\text{rad}}\omega}. \tag{4.10}$$

In Fig. 3 we compare the differential yield for the quanta emitted in the pore lattice and the yield (4.10) due to the bremsstrahlung of the relativistic particles in the crystal. The calculations were performed for electrons with energy $E = 200$ MeV moving in the pore lattice of Al with $L = 700$ Å for different pore dimensions: $R_1 = 100$ Å, $R_2 = 150$ Å, and $R_3 = 200$ Å. It can be seen from these results that, in the region of frequencies $\hbar\omega \lesssim 10^2$ keV, the quantum yield due to the transition radiation is higher than the yield due to the bremsstrahlung.

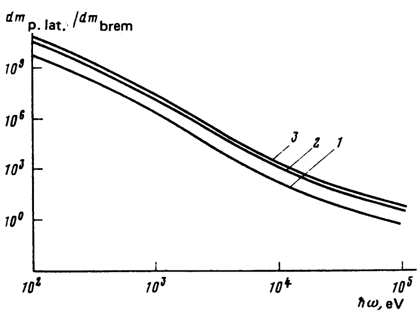

FIG. 3. Comparison of the computed yield of emitted quanta in resonance transition emission in the pore lattice in Al with the computed yield due to the bremsstrahlung for different dimensions of the pores in the lattice. The energy of the emitted quanta is plotted along the abscissa axis; the ratio of the number $dm_{\text{p.lat}}$ of emitted quanta in the resonance transition emission in the pore lattice to the number dm_{brem}, (4.10), of quanta emitted as a result of the bremsstrahlung. The curves 1, 2, and 3 pertain to different pore lattices in Al, in which $L = 700$ Å and the pore radii are respectively equal to $R_1 = 100$ Å (the curve 1), $R_2 = 150$ Å (the curve 2), and $R_3 = 200$ Å (the curve 3). The electron energy $E = 200$ MeV.

In conclusion, let us note that the present investigation of transition radiation emission on vacancion pores has been carried out for transparent media, and therefore the results obtained are valid for solids whose thickness $\mathscr{L}$ is smaller than the characteristic absorption lengths μ^{-1}. (For example, in Al, $\mu^{-1} = 15$ μm for $\hbar\omega = 10$ keV, while $\mu^{-1} \approx 1$ cm for $\hbar\omega = 50$ keV.)

We can conclude on the basis of the above-performed analysis and the data obtained on transition-radiation emission in ordered pore structures that materials containing pore lattices, and possessing a developed porosity can be used for the construction of charged-particle detectors and as x-ray sources. In this case it is desirable to use materials with small Z in order to decrease the quantum-absorption effects, since the absorption cross section in the x-ray frequency region, as determined by the photoelectric effect, is[2] $\sigma \sim Z^4$. Furthermore, the vacancion porosity of solids can be investigated with the aid of transition radiation emitted on pores. This method can apparently be especially effective in the study of the defect structure of the surface layers of solids.

In conclusion, I express my profound gratitude to Academician M. A. Leontovich for drawing my attention to many of the questions touched upon in the present paper, for numerous useful discussions of this paper, and for a number of valuable pieces of advice.

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Translated by A. K. Agyei