Stimulated bremsstrahlung in inelastic electron-atom collisions

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Stimulated multiquantum emission and absorption in inelastic scattering of fast electrons by light nuclei in an external laser field are investigated. Partial cross sections are found for $1S\to 2S$ and $1S\to n$ transitions in a hydrogen-like atom. In the case of the transition $1S\to n=2$ for various intensities of laser radiation we have analyzed the contribution of stimulated channels to the total excitation cross section. In the low-frequency approximation we obtain for the total photoabsorption cross section analytical results which are universal and are valid for arbitrary one-electron models of the target atom. At nonrelativistic energies of the incident electrons, conditions of stimulted emission of quanta of the laser field are possible. Beginning with energies of a few MeV, however, this process cannot occur in the inelastic channels for any collision parameter.

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In the problem of plasma heating by laser fields and in study of the simultaneous action of electromagnetic radiation and of an electron beam on a neutral gas, a fundamental role is played by stimulated bremsstrahlung—the emission or absorption of several quanta induced by scattering of electrons (see Ref. 1, and also Refs. 2 and 3 and citations therein). Together with the direct observation of satellites in the scattered-electron energy spectrum, the question of greatest practical importance is the total photoabsorption and the possibility of obtaining negative absorption for anisotropic electron motion.[2] The corresponding theory, as far as we know, has been limited to discussion of exclusively elastic electron-atom collisions, in which no account has been taken of excitation or ionization of the target atom. As for the cross sections for the inelastic channels, the papers which have been published on this subject (see for example Refs. 4–6) do not contain a complete solution of the problem even for the hydrogen atom.

In the present work the stimulated photoabsorption of a low-frequency laser field is investigated with inclusion of all inelastic channels. It is shown that the contribution of these channels is in no way small, and therefore inelastic processes must be taken into account on a par with elastic scattering in calculation of the total photoabsorption cross section in light atoms. We have also obtained analytical results for the cross sections for impact excitation of arbitrary levels of a hydrogen-like atom with simultaneous emission or absorption of several quanta of the external field.

We shall discuss inelastic electron-atom collisions occurring in an external laser field in the framework of the semiclassical approach, in which the motion of the incident electron is described classically and the states of the electrons of the target are described quantum mechanics. We shall assume also that the external field is moderately intense and is nonresonant so that its influence on the states of the electrons in the atom can be neglected. In this case the important fact turns out to be that the actual trajectories of the relative motion are no longer, as usual, the straight lines $\mathbf{R}(t)=\mathbf{b}+\mathbf{v}t$ ($\mathbf{b}$ is the impact parameter and $\mathbf{v}$ is the velocity of the incident electron), but depend on the time in a more complicated way. The physical reasons for the change of $\mathbf{R}(t)$ have no effect on the probabilities and cross sections for stimulated transitions. In this sense the problems of collisions in a laser field or a magnetic field in the presence of spatial periodicity of the properties of the medium, under conditions of channeling in the crystal, and so forth, can be solved by a unified method.

The retarded potential of interaction of an electron of the target with a Coulomb center moving in an arbitrary classical trajectory is conveniently represented in the form of a Fourier integral:

$$\hat{V}(t)=-e\Phi(\mathbf{r},t)=-\frac{e}{(2\pi)^3}\int d^3q e^{i\mathbf{qr}}\Phi(\mathbf{q},t). \tag{1}$$

The Fourier component $\Phi(\mathbf{q},t)$ satisfies the equation

$$\frac{1}{c^2}\frac{\partial^2\Phi(\mathbf{q},t)}{\partial t^2}+q^2\Phi(\mathbf{q},t)=4\pi e\exp[-i\mathbf{qR}(t)]. \tag{2}$$

In a nonrelativistic laser field in which the variable velocity $\tilde{v}=eE/m\omega$ acquired by the electron is much less than the velocity of light c, the expression for the classical trajectory $\mathbf{R}(t)$ is described in the general case in the form

$$\mathbf{R}(t)=\mathbf{b}+\mathbf{v}t+\mathbf{a}_1\sin(\omega t+\varphi)-\mathbf{a}_2\cos(\omega t+\varphi). \tag{3}$$

Here $\mathbf{a}_{1,2}=e\mathbf{E}_{1,2}/m\omega^2$ are the amplitudes of oscillations in mutually perpendicular directions, ω is the frequency, and φ is the phase term determined by the initial conditions. For the trajectory (3) Eq. (2) has the following solution:

$$\Phi(\mathbf{q},t)=4\pi e\sum_{\nu=-\infty}^{\infty}J_\nu(N)\frac{\exp[i\nu(\alpha-\varphi)-i\mathbf{qb}-i\omega_\nu t]}{q^2-(\omega_\nu/c)^2}, \tag{4}$$

$$\omega_\nu=\mathbf{qv}+\nu\omega,\quad N=[(\mathbf{qa}_1)^2+(\mathbf{qa}_2)^2]^{1/2},$$
$$\cos\alpha=\mathbf{qa}_1/N,\quad \sin\alpha=\mathbf{qa}_2/N,$$

where $J_\nu(N)$ is a Bessel function.

In the first order of perturbation theory in $\hat{V}(t)$ the amplitude of excitation with transition of an atomic electron from a stationary state $|i\rangle$ to a state $|f\rangle\neq|i\rangle$

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has the form

$$\mathfrak{M}_{fi}(\mathbf{b})=-\frac{i}{\pi}\left(\frac{e^2}{\hbar v}\right)\int d^3q e^{-i\mathbf{qb}}\langle f|e^{i\mathbf{qr}}|i\rangle\cdot$$
$$\sum_{\nu=-\infty}^{\infty} J_\nu(N)\delta\left(\frac{\omega_f-\omega_i-\omega_\nu}{v}\right)\frac{\exp[i\nu(\alpha-\varphi)]}{q^2-(\omega_\nu/c)^2}. \qquad (5)$$

The cross sections for the inelastic channels, averaged over φ, are sums of the partial cross sections corresponding to absorption (or emission) of ν quanta of the external field:

$$\sigma_{fi}=\frac{1}{2\pi}\int_0^{2\pi} d\varphi \int d^2b|\mathfrak{M}_{fi}(\mathbf{b})|^2=\sum_{\nu=-\infty}^{\infty}\sigma_{fi}^{\nu}, \qquad (6)$$

$$\sigma_{fi}^{\nu}=4\left(\frac{e^2}{\hbar v}\right)^2\int\frac{d^3qJ_\nu^2(N)}{[q^2-(\omega_\nu/c)^2]^2}\delta\left(\frac{\omega_f-\omega_i-\omega_\nu}{v}\right)|\langle f|e^{i\mathbf{qr}}|i\rangle|^2. \qquad (7)$$

In the analogous problem of elastic scattering the semiclassical approach used above is inapplicable. However, a nonrelativistic quantum calculation again leads to an expression of the type (7), in which the argument of the delta function contains an additional term $\hbar q^2/2mv$, and instead of the square of the modulus of the form factor there is the usual term $[Z-\langle i\,|e^{i\mathbf{qr}}\,|i\rangle]^2$ (Z is the charge of the nucleus of the target atom).

We shall give the expression for the partial cross sections (7) for inelastic transitions in a hydrogen-like atom (with an effective nuclear charge Z_a) in the limit $v\ll c$:

$$\sigma_{1S\to n}^{\nu}=(\pi a^2)\frac{2^7}{n^3}\left(\frac{e^2}{\hbar v}\right)^2\sum_{k=0}^{n-2}\frac{(2/n)^k}{(k+4)}\left(\frac{\partial}{x\partial x}\right)^{k+4}$$
$$\times\left[\binom{n-2}{k}+\frac{(n-1)(n-2)}{3n^2(k+5)}\binom{n-3}{k}\left(\frac{\partial}{x\partial x}\right)\right]\frac{F(x)}{x^2}\Bigg|_{x=1+1/n}, \qquad (8)$$

where n is the principal quantum number of the shell excited and $a=\hbar^2/Z_a me^2$. In the case of longitudinal oscillations of the incident electron in the external field ($\mathbf{a}_1\equiv\mathbf{a}_0\|\mathbf{v};\mathbf{a}_2=0$) the function $F(x)$ in (8) is determined from the formula

$$F_\|(x)=J_{|\nu|}^2(N_0)\ln\beta, \qquad (9)$$

where we have used the notation

$$\beta=\left(1+\frac{x^2}{Q^2}\right)^{1/2},\quad 2Q=\xi\left(1-\frac{1}{n^2}-\nu\frac{\hbar\omega}{Z_a^2\mathrm{Ry}}\right),$$
$$N_0=\frac{a_0}{a}|Q|,\quad \xi=\frac{\hbar}{mav},\quad \mathrm{Ry}=\frac{me^4}{2\hbar^2},$$

and ξ is the ratio of the velocity of the atomic electron in the initial state to the velocity of the incident electron. For motion along a helical trajectory ($\mathbf{v}\cdot\mathbf{a}_{1,2}=\mathbf{a}_1\cdot\mathbf{a}_2=0$, $a_{1,2}=a_0$)

$$F_\perp(x)=I_{|\nu|}(N_0)K_{|\nu|}(N_0)-I_{|\nu|}(\beta N_0)K_{|\nu|}(\beta N_0), \qquad (10)$$

where $I_n(Z)$ and $K_n(z)$ are Bessel functions of imaginary argument.

The partial cross sections for the transition $1S\to 2S$ in the same approximation have the form

$$\sigma_\|^{\nu}=(\pi a^2)\frac{2^7}{5}\left(\frac{e^2}{\hbar v}\right)^2\frac{J_{|\nu|}^2(N_0)}{(Q^2+9/4)^5}, \qquad (11)$$

$$\sigma_\perp^{\nu}=\frac{(\pi a^2)}{15}\left(\frac{e^2}{\hbar v}\right)^2\left(-\frac{\partial}{x\partial x}\right)^5 I_{|\nu|}\left(x\frac{a_0}{a}\right)K_{|\nu|}\left(x\frac{a_0}{a}\right)\Big|_{x=(Q^2+9/4)^{1/2}}. \qquad (12)$$

We shall show how in inelastic scattering the appearance of satellites in the total cross section depends on the intensity of the laser field. In the figure we have given the results of a calculation of their relative contribution for the excitation channel $1S\to n=2$ in the case of linear longitudinal polarization. The parameters are chosen such that $\xi=0.25$ and $\hbar\omega/Z_a^2$ Ry $=0.1$. Curves 1–3 correspond to values of a_0/a equal to 5, 10, and 15. The cross section for the excitation $1S\to n=2$ in the absence of an external field is

$$\sigma_0\approx(\pi a^2)\frac{2^{19}}{3^{10}}\left(\frac{e^2}{\hbar v}\right)^2\ln\left(\frac{8}{3\xi}\right). \qquad (13)$$

Note that for the channel $1S\to n=2$ the sum of the partial cross sections $\sum_\nu\sigma_\|^\nu$ is practically equal to σ_0, and therefore it does not make sense to consider the question of any appreciable change in the transition probability in the presence of a low-frequency laser field. The only thing of interest here is the effective photoabsorption cross section $\sigma^{\mathrm{abs}}=\sum_\nu\nu\sigma_\|^\nu$, which is rather small for the choice of parameters of curve 1 in the figure, but for curves 2 and 3 it is respectively $\sigma^{\mathrm{abs}}\approx-0.1\sigma_0$ and $\sigma^{\mathrm{abs}}\approx-0.2\sigma_0$. The situation in which $\sigma^{\mathrm{abs}}\ll\sigma_0$ for very small ξ and low frequencies ω is most typical not only for the transition $1S\to n=2$, but also in general for any inelastic channel. It is just this circumstance which permits us to carry out a general investigation of photoabsorption for an arbitrary target, without restricting outselves to the model of a hydrogen-like atom.

The combined cross section for stimulated photoabsorption in all inelastic channels

$$\sigma_{\mathrm{inel}}^{\mathrm{abs}}=\sum_{f\neq i}\sum_{\nu=1}^{\infty}\nu(\sigma_{fi}^{\nu}-\sigma_{fi}^{-\nu}) \qquad (14)$$

determines the absorption coefficient of the laser field α_{inel}. Thus, for a linearly polarized wave with amplitudes E_0 with concentrations of incident electrons and atoms n_e and n_a, the inelastic part of the absorption coefficient is

$$\alpha_{\mathrm{inel}}=\frac{8\pi n_e n_a}{E_0^2}\frac{v}{c}\hbar\omega\sigma_{\mathrm{inel}}^{\mathrm{abs}}. \qquad (15)$$

In view of energy conservation, which is expressed

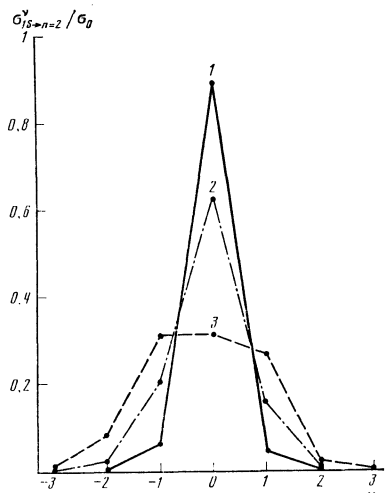

FIG. 1.

by the delta function in Eq. (7), the momentum transfer $\hbar q$ and the argument of the Bessel function N depend on the emitted (or absorbed) energy $\nu\hbar\omega$. The correction to the argument of the Bessel function due to the presence of $\nu\omega$ can be assumed to be small if

$$\left(\frac{\tilde{v}}{v}\frac{I}{\hbar\omega}+1\right)^{2}\left(\frac{\hbar\omega}{I}\right)^{\nu}\ll 1, \tag{16}$$

where I is the average value of the excitation energy, which is approximately equal to the ionization potential of the target atom. When the inequality (16) is satisfied it is sufficient to expand the function $J_\nu^2(N)$ in the integrals of σ_{fi}^{ν} in series in powers of $\nu\omega$ and to retain only the first two terms of the expansion. The main contribution to $\sigma_{\rm inel}^{\rm abs}$ from zero-order terms in this expansion is from small $\mathbf{q}$. Summation over ν is accomplished by means of the relation

$$\sum_{k=1}^{\infty} k^2 J_k^2(z)=(z/2)^2. \tag{17}$$

For the condition

$$\gamma^2(\hbar\omega/I)\ll 1 \tag{18}$$

$[\gamma=(1-v^2/c^2)^{-1/2}$ is the Lorentz factor$]$ after a number of manipulations for evaluation of the zero-order terms one uses the well known sum rule

$$\sum_f \hbar(\omega_f-\omega_i)\,|\langle f|e^{i\mathbf{qr}}|i\rangle|^2=Z\hbar^2q^2/2m. \tag{19}$$

The second term in $\sigma_{\rm inel}^{\rm abs}$ turns out to be directly related to the effective bremsstrahlung of an electron in the field of an atom in the absence of laser field:

$$\varkappa=\sum_f \hbar(\omega_f-\omega_i)\sigma_{fi}. \tag{20}$$

The final result for the inelastic part of the cross section for stimulated photoabsorption in lowest order in ω is written in the following form:

$$\sigma_{\rm inel}^{\rm abs}\approx Z(\pi a_0^2)\gamma^4\left(\frac{e^2}{\hbar v}\right)^4\left(\frac{\hbar\omega}{\rm Ry}\right)-(3a_0^2-a_1^2-a_2^2)\frac{\omega\varkappa}{2\hbar v^2}, \tag{21}$$

$$a_0^2=(\mathbf{a}_1\mathbf{v}/v)^2+(\mathbf{a}_2\mathbf{v}/v)^2. \tag{22}$$

In a large region of relativistic energies of the incident electrons (beginning with a few MeV) over the entire interval of collision parameters the sign of the total photoabsorption cross section $\sigma_{\rm inel}^{\rm abs}$ is positive, since even in the ultrarelativistic limit ($\gamma\gg 1$) the negative term in Eq. (21) depends only weakly on γ and contains it only in the argument of the logarithm,

$$\varkappa\approx\frac{2\pi e^4}{mc^2}\ln\left(\frac{m^2c^4}{I^2}\gamma^3\right). \tag{23}$$

The main difference of photoabsorption in the nonrelativistic region is the dependence of its sign on the details of the collision process. For example, in the case of a linearly polarized laser wave when the angle between $\mathbf{a}_0$ and $\mathbf{v}$ is θ, the total cross section with inclusion of both inelastic and elastic channels is

$$\sigma^{\rm abs}\approx(\pi a_0^2)\left(\frac{e^2}{\hbar v}\right)^4\left(\frac{\hbar\omega}{\rm Ry}\right)\Big[(Z+Z^2)\cos^2\theta - \left(Z+\frac{Z^2}{2}\right)(3\cos^2\theta-1)\ln\left(\frac{mv^2}{I}\right)\Big]. \tag{24}$$

A similar formula has already been discussed in study of photoabsorption in scattering by bare ions.[2] Taking into account the electrons of the target leads to qualitatively the same conclusions. The absorption coefficient can become negative for small θ, but on averaging over the directions of $\mathbf{v}/v$ this effect drops out completely. The contribution of individual channels to the magnitude of the negative absorption is given by a factor $(Z+Z^2/2)$, from which it follows that inelastic collisions play a substantial role in it for $Z=1$ and 2, and are extremely important all the way up to $Z\approx 10$.

In the calculation of stimulated photoabsorption in collisions with multielectron atoms, in all of the results we must understand Z to mean the number of effectively excited electrons of the target, i.e., those for which $\xi\lesssim 1$.

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