Stratification of a transverse field in many-valley semiconductors

M. Asche, Z. S. Gribnikov, V. M. Ivashchenko, H. Kostial, V. V. Mitin, and O. G. Sarbeĭ

Institute of Physics, Academy of Sciences of the Ukrainian SSR, Kiev
(Submitted 20 March 1981)
Zh. Eksp. Teor. Fiz. **81**, 1347–1361 (October 1981)

The current and the electric field distributions were determined for n-type Si samples oriented along the [110] axis. At temperatures below 50–55°K in a certain range of heating electric fields the results could be interpreted as splitting of a sample into layers with opposite directions of a transverse electric field parallel to the [$\bar{1}$10] axis. As a rule, two-layer and nearly one-layer distributions were formed, and the interlayer wall shifted on application of a weak magnetic field giving rise to an anomalous Hall effect in weak magnetic fields. This transverse stratification was due to the multivalued Sasaki effect predicted by Z. S. Gribnikov, V. A. Kochelap, and V. V. Mitin [Sov. Phys. JETP **32**, 991 (1971)]. A consequence of this effect was also a longitudinal "domainization" (formation of low- and high-field domains) near the lower critical field of the Sasaki effect (this occurred in a range of fields narrower than the range of existence of the Sasaki effect), and a region of saturation of the current due to such domainization. In the saturation region the stratification occurred only in a high-field domain. A numerical calculation carried out by the Monte Carlo method predicted the same range of existence of the multivalued Sasaki effect as that found experimentally.

PACS numbers: 72.20.My, 72.80.Cw, 72.20.Ht

1. INTRODUCTION

It has been found[1-5] that for certain directions of a heating electric field applied to a many-valley semiconductor a homogeneous distribution of this field becomes unstable in the presence of small fluctuations of the transverse component of the current. Under these conditions a sample splits into layers with a finite transverse field separated by interlayer (domain) walls parallel to the current. This stratification is a consequence of a multivalued distribution of electrons between the equivalent valleys of a semiconductor[1] predicted by Reik and Risken[6] in a calculation of the transverse anisotropy of the conductivity of n-type Ge (this is known as the multivalued Sasaki effect, which we shall denote by MSE). The effect resulting in an absolute negative transverse conductivity (Erlbach effect) predicted in Ref. 7 is only a consequence of the instability of the state with an equal distribution of electrons between the valleys in the MSE; a semiconductor then tends to assume a stable strongly anisotropic state.

In spite of repeated attempts, a reliable experimental confirmation of the MSE has not been provided up to 1980. The observations of Shyam and Kroemer[8] of switching of a transverse field in n-type Ge at room temperature under the MSE conditions could not be reproduced subsequently even in the same laboratory,[9] as reported by Gaylord and Rabson,[10] and they were also criticized convincingly from the theoretical point of view.[10,11] Moreover, an attempt to detect a negative transverse resistance of n-type Si at $T=77$°K was also unsuccessful.[12] Although some special features of the behavior of a transverse (relative to the heating field) current were observed[13] for n-type Ge at $T=20$°K, including regions of an "absolute" negative conductivity, an unfortunate design of the experiments made it difficult to provide even a qualitative reliable interpretation of the results.[14]

One of the manifestations of the MSE, in the form of stratification of a sample of n-Si at $T=27$°K, was observed reliably only in 1980 by Asche, Kostial, and Sarbeĭ,[15] although reports of a saturation region in the current-voltage characteristic attributed in particular to the MSE appeared somewhat earlier.[16]

We shall give below the results of a numerical Monte Carlo calculation of the homogeneous MSE as well as new results of an experimental investigation of the phenomena due to the MSE in n-type Si, and we shall compare these results with the calculated data.

2. QUALITATIVE DESCRIPTION OF STRATIFICATION

The investigated phenomenon can be explained most easily by considering a semiconductor with two equivalent valleys, which is bounded in the direction of the y axis (Fig. 1) and carries a current along the x axis. This current has two identical components:

$$i_x^{(1)}=i_x^{(2)}, \quad i_x^{(1,2)}=en_{1,2}\mu_{xx}^{(1,2)}E_x, \quad n_1=n_2, \quad \mu_{xx}^{(1)}=\mu_{xx}^{(2)},$$

due to electrons from different valleys. Moreover, electrons from valleys 1 and 2 carry equal but oppositely directed currents:

$$i_y^{(1)}=-i_y^{(2)}, \quad i_y^{(1,2)}=en_{1,2}\mu_{yx}^{(1,2)}E_x, \quad \mu_{yx}^{(1)}=-\mu_{yx}^{(2)},$$

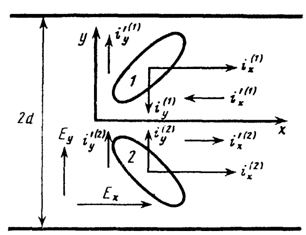

FIG. 1. Fields and partial currents in a two-valley semiconductor.

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so that the total current is $i_y = 0$.

We shall assume that in addition to the field E_x there is also a weak fluctuation field E_y. This field can alter the average electron energy. Firstly, it cools electrons from the valley 1 and heats electrons from the valley 2 by consuming powers $E_y i_y^{(1)}$ and $E_y i_y^{(2)}$, as shown in Fig. 1; secondly, the fluctuation field alters the heating by the field E_x: electrons from the valley 1 are cooled slightly by the power $E_x i_x'^{(1)}$ (we shall use a prime to denote the components of the current due to the fluctuation field E_y:

$$i_x'^{(1,2)} = en_{1,2}\mu_{xy}^{(1,2)}E_y),$$

and the electrons from the valley 2 are heated by the power $E_x i_x'^{(2)}$; thirdly, the field E_y heats to the same extent the electrons in both valleys at the expense of the powers $E_y i_y'^{(1)}$ and $E_y i_y'^{(2)}$ (this is an effect of the second order of smallness because

$$i_y'^{(1,2)} = en_{1,2}\mu_{yy}^{(1,2)}E_y).$$

For the first two reasons the electrons in the second valley are heated more strongly than those in the first valley, and they are transferred to the first valley, so that the conductivity becomes anisotropic. There is little change[1)] in the current components i_x and i_y' but new components i_y and i_x' appear and these are due to the nondiagonal components of the conductivity tensor directed (Fig. 1) opposite to i_y' and i_x.

If the transfer of electrons from the hot to the cold valley does not depend strongly on the heating field, so that $|i_y| < |i_y'|$, fluctuations of the field E_y die out and there are no special effects. However, if the electron density in the hot valley decreases strongly on increase in the heating field, then at some critical field E_c the current $|i_y|$ can exceed $|i_y'|$. The fluctuation field E_y grows and a new nonequilibrium anisotropic state appears in the semiconductor in which the first valley is filled preferentially and a finite transverse electric field is established. Moreover, an increase in the current component i_x' reduces strongly the total current along the x axis so that an N-type region with a negative differential resistance can appear in the current-voltage characteristic.

Similarly, we can consider a fluctuation E_y of the opposite sign, which, subject to the condition $|i_y| > |i_y'|$, gives rise to an anisotropic state in which the second valley is filled preferentially with electrons. Under real conditions both these anisotropic states can coexist and, depending on the boundary conditions, doping inhomogenity, etc., a sample may split into layers with different directions of the transverse field. A theory of stratification of the field E_y is presented in Refs. 2–4.

In the simplest case, which occurs most frequently in the experiments described below, we are dealing with a two-layer distribution of the transverse field. As shown in Ref. 5, a weak magnetic field directed at right-angles to the plane of Fig. 1 displaces the interlayer wall toward one of the surfaces of the sample (by an amount which increases with the magnetic field); this results in the growth of the layer with the transverse field directed parallel to the Hall current and shrinking of the layer with the transverse field opposite to the Hall field. This behavior of the interlayer wall has the effect that a very large change in the transverse voltage occurs in very weak magnetic fields and outside this narrow range of magnetic fields the transverse voltage should increase with the magnetic field, as expected for the usual Hall effect. It should be stressed that steeper dependences of the transverse voltage than those observed in the usual Hall case occur not only in the range of existence of the MSE, but also close to this range. However, in the MSE range, where the steep dependence of the voltage on the magnetic is related to the interlayer displacement, the slope of the dependence is greater and it is practically unaffected by the heating field, whereas outside the MSE range the slope of the dependences of the transverse voltage on H decreases smoothly away from the MSE. Following Ref. 5, we shall regard the steep dependence in question as the anomalous Hall effect.

As mentioned earlier, the MSE appears only when the electron density in the hotter valleys decreases strongly, i.e., when there is a large increase in the probability of the intervalley scattering on increase in the electron energy. Among the various intervalley scattering mechanisms the one which has the necessary property is the scattering by intervalley phonons of energy $\hbar\omega_i$ at low temperatures when $kT \ll \hbar\omega_i$. The intervalley transition time in a heating field is governed by the spontaneous emission of intervalley phonons by electrons of energy exceeding $\hbar\omega_i$ whose number increases exponentially on increase in $\bar{\varepsilon}$ under weak heating conditions ($\bar{\varepsilon} \ll \hbar\omega_i$, where $\bar{\varepsilon}$ is the average energy of electrons). This circumstance ensures a rapid reduction in the intervalley relaxation time and in the electron density when the effective heating field in the hotter valley rises. Conversely, the intervalley impurity scattering and the intervalley electron-electron energy exchange tend to suppress the difference between the electron densiites and average energies in the different valleys, and make it more difficult to observe the MSE. Therefore, in studies of the MSE one should select the purest possible material and to carry out experiments at sufficiently low temperatures.

3. SAMPLES AND MEASUREMENT METHOD

In all our measurements we used phosphorus-doped silicon of resistivity $\rho_{300^\circ K} \approx 100\ \Omega \cdot$ cm from which samples $l = 0.7$–1.3 cm long were cut along the [110] directions. Probes for the determination of the transverse field were deposited on the side $(\bar{1}10)$ surfaces and the distance between them was varied within the range 0.16–0.3 cm; this distance was usually 2–2.5 times greater than the third dimension. Many samples also had additional controlled probes on the (001) side surfaces, which made it possible to find the distribution of the transverse field in the $[\bar{1}10]$ direction. The current and probe contacts were usually deposited by alloying with Au–Sb in vacuum, and the current contacts occupied the whole end surfaces, whereas the size of the probe contacts was 0.02–0.03 cm. The current flowing in the potential probe circuit represented 5% of the total current through the sample.

The measurements were carried out in the temperature range 20–77°K. The most detailed investigations were made at 27°K and all the results reported below were obtained at this temperature. The phenomena due to the MSE were observed at temperatures up to 50–55°K; there was no manifestation of the MSE above 55°K.

A heating electric field was created by rectangular voltage pulses of ~10^{-4} sec duration, when the steady-state conditions were established in all the circuits; this was important because slow changes in the transverse voltage were observed in the probe measurements.

Three types of measurements were carried out on all the samples. Firstly, the current-voltage characteristics were determined in the [110] direction. Secondly, the dependences of the potentials of the various probes and of the transverse voltage on the heating electric field were obtained. Thirdly, a study was made of the influence of a magnetic field directed along the [001] axis on all these characteristics. Moreover, some samples were subjected to deformation by the application of a uniaxial pressure via the current contacts and directed along the current. All the measurements mentioned above were carried out on such compressed samples.

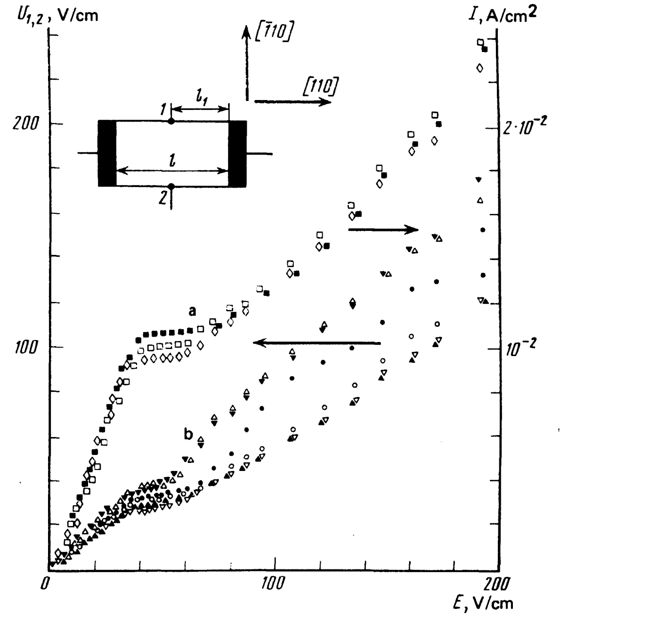

FIG. 2. Current (a) and probe potentials relative to the anode, reduced to the distance from the anode, $U_{1,2} = V_{1,2}/l_1$ (b) plotted as a function of the average applied electric field E in the absence of a magnetic field and in magnetic fields ± 0.06 T: ■), ●), ○) $B = 0$; □), △), ▲) $B = 0.06$ T; ◇), ▽), ▼) $B = -0.06$ T; ○), ▽), △) probe 1; ●) ▼), ▲) probe 2; ■), ◇), □), current-voltage characteristics.

4. EXPERIMENTAL RESULTS AND QUALITATIVE DISCUSSION

The current-voltage characteristics and the influence of a magnetic field on these characteristics were the same for all the investigated samples. Typical characteristics are shown in Fig. 2. It is worth noting a more or less pronounced region of saturation of the current, whose onset point at $E_c = V_c/l$ can vary somewhat from sample to sample (usually within the range 40–50 V/cm). A magnetic field has a much stronger influence on the current in the saturation region than outside the region, and in the range $V > V_c$ the current depends on the sign (polarity) of the magnetic field.

The general behavior of the potential on the probes located on (110) surfaces in the presence of a longitudinal electric field and a magnetic field is also similar for the majority of the samples; this behavior is also illustrated in Fig. 2. There is a typical region of a slower rise of the probe potential in the range $V > V_c$ (amounting to near-saturation for some of the samples) and the extent of this region is always less than that of the saturation region of the current-voltage characteristics. This slow rise changes to a faster rise of the potential of one (if there is a potential difference between the probes) or both probes, which continues until the end of the saturation region in the current-voltage characteristics. Beyond this region there is a change in the slope of the curves which then becomes practically the same as the slope in the range $V < V_c$. In the case of some of the samples there are two regions of slow rise of the probe potential in the saturation region of the current-voltage characteristic (Fig. 3).

This behavior of the potentials demonstrates clearly that a static domain of a high electric field, which is created by a field $E_x = E_c$ and which displaces a low-field domain, travels between the cathode and the probes when the voltage is sufficiently high. The occurrence of two regions of a slow rise of the potential (Fig. 3) shows that two high-field domains can coexist in a sample: one of them travels between the cathode and the probes and for some reason it ceases to grow before occupying the whole of this region, and

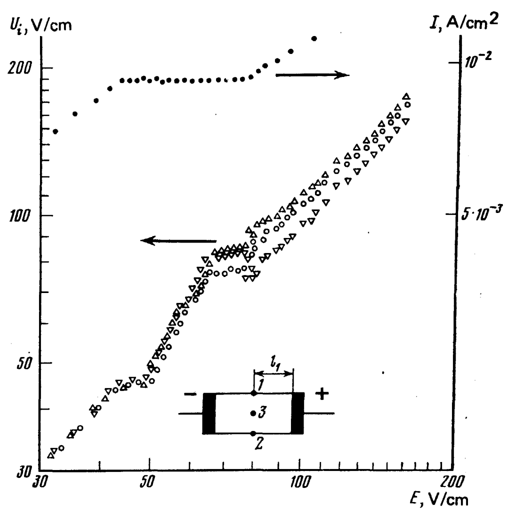

FIG. 3. Same as Fig. 2 but in the absence of a magnetic field: ●) current voltage characteristic; △) probe; ▽) probe 2; ○) probe 3.

a second domain travels between the anode and the probes. It follows from Figs. 2 and 3 that the field intensities in the low- and high-field domains are approximately 40 and 70–80 V/cm, respectively.

This longitudinal "domainization" of a sample gives rise to the saturation region in the current-voltage characteristics and it is related, as shown below, to an N-type negative differential conductivity at the critical point (E_c) of the appearance of the MSE.

As mentioned above, the current-voltage characteristics and the general behavior of the probe potentials were the same for all the samples, but the dependence of the transverse potential difference on the longitudinal electric and magnetic fields differed considerably from sample to sample. In general, it was possible to identify three types of such dependences.

The most frequently encountered case is illustrated in Fig. 4a, which applies to the same sample as in Fig. 2. In $H=0$, this sample exhibits a more or less abrupt appearance of a transverse potential difference in the saturation region of the current-voltage characteristic, whish is followed by a rapid rise of this potential on increase in the longitudinal voltage and a further approximate saturation, as well as an anomalously large and asymmetric change in this potential on reversal of the sign of the magnetic field.

The limiting case of this behavior is exhibited by the second type of sample, in which the averge transverse field near the point of saturation in $H=0$ is close to the average longitudinal field,[2)] whereas the magnetic field hardly alters the transverse potential difference for one direction of H and reverses it for the opposite direction of the magnetic field. An example of such behavior is given by Fig. 1 in Ref. 15.

Finally, the third type of dependence observed rarely is demonstrated in Fig. 4b. In this case throughout the investigated range of longitudinal voltages the transverse potential difference in $H=0$ is slight or nonexistent, and the magnetic field produces a practically symmetric (relative to the sign of H) anomalously high Hall emf.

All these three types of behavior of the transverse

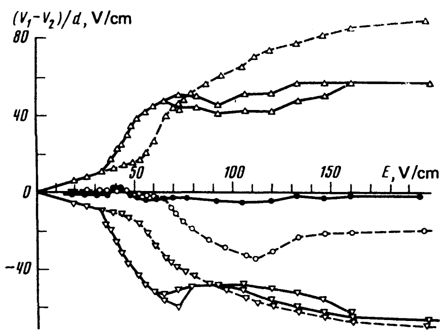

FIG. 4. Average field between the probes 1 and 2 plotted as a function of E in the absence of a magnetic field and fields of ± 0.06 T: O) $B=0$; △) $B=0.06$ T; ▽) $B=-0.06$ T. The dashed curves represent case a and the continuous curves correspond to case b (see text).

potential difference can be explained by assuming that transverse (in the simplest case, two-layer) structures appear—in addition to longitudinal domainization—at the critical MSE point.[2-4] If a sample is oriented exactly along the [110] direction, if it is sufficiently homogeneous, and if the boundary conditions are symmetric, then the interlayer wall should occupy the central position in $H=0$ (Ref. 3) and there should be no transverse potential difference, in spite of the existence of the transverse field in each of the layers. A similar situation occurs also in the samples exhibiting the dependences of the type shown in Fig. 4b. A magnetic field displaces the interlayer (domain) wall symmetrically to one or the other surface of the sample and this is the cause of the anomalously high Hall emf.[5]

When the sample is not oriented exactly or the conditions on the surfaces are asymmetric, the interlayer wall should not occupy the central position and a transverse voltage appears in the presence of the MSE even when $H=0$. This is the case illustrated in Fig. 4a. The interlayer wall is still far from the surface and the magnetic field alters the transverse voltage anomalously strongly in each direction. If the domain wall in $H=0$ is located already close to one of the surfaces, we observe the second of the above types of behavior of the transverse potential difference.

The existence of transverse fields directed oppositely in the left- and right-hand parts (layers) of a sample was demonstrated also by direct measurements, the results of which can be seen in Fig. 3. In the range of average longitudinal fields 50–70 V/cm the potential of a probe 3 located at the center of a (001) face of the sample was found to be higher (the potentials in Fig. 3 are plotted relative to the anode!) than the potentials of probes 1 and 2, which were approximately identical. This demonstrated that in fields of this kind there was a two-layer structure with an approximately central position of the interlayer wall and with transverse fields directed from the central line to the boundaries of the sample (S^+S^- structure[2-4]). In a field exceeding 75 V/cm, when a high-field domain occupied already the whole sample, the interlayer wall was displaced strongly toward the surface with the probe 2.

A careful comparison of Figs. 2 and 4a, and also of Figs. 1 and 2 in Ref. 15, with the current-voltage characteristic and the transverse potential shown in Fig. 4b demonstrates that the field E_c in which saturation begins in the current-voltage characteristic is not always identical with the field in which an anomalous Hall emf and/or a transverse voltage appears in $H=0$. For example, in Figs. 1 and 2 of Ref. 15, we find that E_c is approximately 43 V/cm; in roughly the same field we observe an abrupt change in the transverse voltage in $H=0$ and also an anomalously high Hall emf. However, in these figures the critical field is $E_c \approx 40$ V/cm and the average field corresponding to the appearance of the anomalous Hall emf (Fig. 4a) is 55 V/cm, whereas the transverse field appears in $H=0$ when 65 V/cm. In Fig. 4b the field E_c is approximately equal to the field of the onset of the anomalous

Hall effect.

We can explain these observations by noting that the formation of transverse structures which accompanies the splitting into longitudinal domains is possible only in that part of a sample where a high-field domain is located. Therefore, the anomalous Hall emf or the transverse voltage in $H=0$ may be observed between the probes only if a high-field domain exists near the probes.

A theory of the longitudinal domainization under the MSE conditions has not yet been developed and we do not know where and under what conditions is a high-field domain nucleated. In any case, we can draw the conclusion from the above result that in samples whose properties are illustrated in Fig. 4b here and in Figs. 1 and 2 in Ref. 15 a longitudinal high-field domain is nucleated near the probes, whereas in samples whose properties are presented in Figs. 2 and 3 such a domain is created near the cathode (or anode) and it reaches the probes only as a result of propagation. It should be noted that for the specific sample whose properties are shown in Fig. 2 there is stratification in the probe plane already in average longitudinal fields of 55–65 V/cm (this is indicated by anomalous Hall emf), but the interlayer wall occupies a near-central position, whereas in a field of 65 V/cm (when a longitudinal high-field domain occupies already the whole sample) this interlayer wall shifts to the boundary of the sample.

The dependence of the current through the sample on the sign (polarity) on the magnetic field shows that the MSE does occur in the sample (Fig. 2) in average fields 40–55 V/cm although there is no detectable voltage on the probes. In a homogeneous heating field such a dependence can appear as a result of electron heating if it has a component in the Hall direction.[17] Consequently, in the absence of the MSE there should be no magnetoresistance which is odd with respect to H in the case under consideration. In fact, in fields below E_c (Fig. 2) and at 77°K, when there is no MSE, the current is independent of the sign of H. True, such a dependence should be also absent when the MSE occurs and the interlayer wall occupies the ideal position in the central plane of the sample inside a domain exhibiting the MSE. Clearly, this condition is difficult to satisfy at least near the ends of a sample, where the transverse MSE field is short-circuited by the contacts and the slightest inhomogeneity displaces the interlayer wall. Therefore, all the samples exhibit a magnetoresistance which is odd in respect of H and which appears at the point E_c.

Figure 5 shows the dependences of the transverse voltage on the magnetic field obtained for one of the samples subjected to various longitudinal electric fields. We can see that in low longitudinal fields these dependences give the usual Hall emf, rising linearly with H. Beginning from ~50 V/cm and up to 300–400 V/cm, these dependences consists of two parts: a region of an anomalously rapid variation with the magnetic field in the case of low magnetic fields and a region of a gentler slope in stronger fields. It is worth noting that the slopes of the curves in low magnetic fields

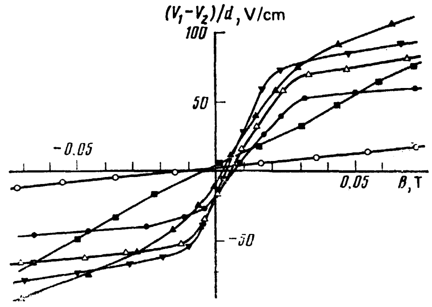

FIG. 5. Average field between the probes 1 and 2 plotted as a function of the magnetic field directed along the [001] axis, obtained for different values of E: ○) 36.2 V/cm; ●) 88.1 V/cm; △) 110 V/cm; ▼) 147 V/cm; ▲) 220 V/cm; ■) 433 V/cm.

almost coincide for all the longitudinal electric fields in the range 50–200 V/cm and there is only a change in the point of transition from the steep to the gentle dependence, whereas in high fields the reduction in the slope increases when the longitudinal field is increased.

As pointed out earlier, the steep dependences in low magnetic fields can be regarded as the anomalous Hall effect considered in Ref. 5 and consisting of a displacement of an S^+S^- wall on change in H from one $(\bar{1}10)$ surface of the sample to the other. When the surface is reached, the anomalous Hall effect disappears. From this point of view we should regard the change in the slope of the dependence of the transverse voltage from H in longitudinal fields exceeding 200 V/cm as a shift from the range of existence of the MSE to the range of single-valued behavior of the transverse voltage close to the MSE and characterized by two different slopes of the dependence in question.[5] It follows that the upper limit of the range of existence of the MSE expressed in terms of the longitudinal field is ~200 V°cm.

The displacement of an S^+S^- wall by a magnetic field could also be demonstrated directly by measuring the magnetic-field dependences not only of the total transverse voltage but also of the partial voltages detected by two pairs of additional probes located in the same transverse plane on a (001) surface as the probes 1 and 2. The results of such measurements are plotted in Fig. 6. When the interlayer wall passes through the midpoint between the probes, the potential difference between them should dispapear. We can see

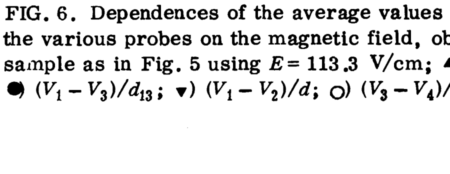

FIG. 6. Dependences of the average values of the fields between the various probes on the magnetic field, obtained for the same sample as in Fig. 5 using $E=113.3$ V/cm; ▲) $(V_4-V_2)/d_{42}$; ●) $(V_1-V_3)/d_{13}$; ▼) $(V_1-V_2)/d$; ○) $(V_3-V_4)/d_{34}$.

that in a field $+H$ the interlayer wall is shifted from an almost central position between the probes 3 and 4 to the probe 1 and in a field ~80 Oe it occupies the central position between the probes 1 and 3, whereas in fields with the reverse sign and approximately the same intensity it is located half-way between the probes 2 and 4.[3]

Similar measurements showed that some of the samples exhibited not two-layer distributions of the transverse field, but structures with a larger number of layers. This could be due to various inhomogeneities disturbing strongly the ideal effect. The importance of the inhomogeneities was supported also by other features of the behavior of the transverse voltage, such as the abrupt changes in the probe voltages on increase in the magnetic and longitudinal fields (Figs. 4a and 4b), the existence of two or three amplitudes of the transverse voltage pulses displayed on the screen of an oscilloscope (inset in Fig. 6), and occasional small oscillations of the transverse voltage and total current through the samples near certain average values.

In the interpretation of the experimental results we are in fact assuming that the two-valley model described in Sec. 2 is valid in the situation under discussion. The question arises what is the role of the third pair of valleys located on the [001] axis? The answer is simple in the case of the transverse voltage; if the [100] and [010] valleys are equivalent, the presence of this pair of valleys cannot give rise to any transverse field. It follows from the theory of Ref. 1 that in the presence of such valleys the criterion for the appearance of the MSE becomes somewhat more stringent, but there is no change in the qualitative pattern.

It is not as clear that the saturation of the current-voltage characteristics and the longitudinal domainization of a sample are again not due to the existence of the valleys located on the [001] axis. A stronger heating of electrons in this pair of valleys for a current along the [110] axis and the rapid transfer of electrons to the other two pairs of valleys with a larger effective mass along the current coulg generally result in an N-type negative differential conductivity and also in domainization similar to that reported in Ref. 18. In fact, a qualitative analysis made in Ref. 19 demonstrates, like the results of a numerical calculation, that a negative differential resistance of such origin is possible only in the presence of the MSE and should appear in a narrower range of currents and fields against the background of a negative differential conductivity of the MSE type.

This problem was solved experimentally by recording the current-voltage characteristics of a sample compressed along the [110] axis when electrons from the [001] valleys were transferred to the other two pairs of valleys. An estimate of the energy splitting obtained employing the usual values of the deformation potential constants gave a value of about 4 kT under a pressure of 2500 kG/cm^2. Under these conditions and in moderately strong longitudinal fields, when saturation was observed in the current-voltage characteristics, practically all the electrons from the pair of valleys on the [001] axis were transferred to the other two pairs and the semiconductor became of the two-valley type.

Figure 7 shows the current-voltage characteristics for the same sample as in Fig. 2, but obtained under different pressures. Clearly, the strongly pronounced saturation regions were retained up to ~3400 kG/cm^2, i.e., at pressures considerably greater than 2500 kG/cm^2 and almost three times greater than necessary for suppressing the saturation in the current-voltage characteristics when this saturation was of non-MSE origin.[4] The critical field E_c was practically independent of the pressure and the observed narrowing of the saturation regions was due to a reduction in the field at which they terminated. These results provided a strong argument in support of the hypothesis that the MSE was responsible for the saturation regions observed in the current-voltage characteristics recorded in our experiments.

In considering the reasons for the disappearance of the parts of the saturation regions of the current-voltage characteristics at high pressures we must note two circumstances. Firstly, the pressure on a sample (Fig. 1) was not directed exactly along the [110] axis, as indicated by the appearance of a transverse piezovoltage in the experiments carried out in a weak heating field (up to 3 V/cm). In this case the compression of a sample not only gave rise to an energy gap between the [001] pair of valleys and the other two pairs, but also to a splitting between the [100] and [010] pairs. Even in the absence of the MSE, the [100] or [010] pair of valleys was filled more with electrons and a negative differential conductivity due to the MSE decreased (at higher pressures it disappeared completely). This mechanism was supported by the fact that the saturation region of the current-voltage characteristics could be restored in a magnetic field of one polarity. Secondly, an increase

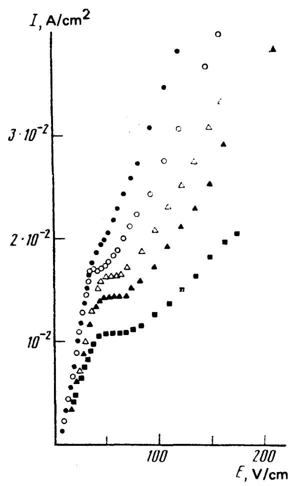

FIG. 7. Current voltage characteristics of the same sample as in Fig. 2 obtained for different pressures P along the current: ■) 0 kG/cm^2; ▲) 1500 kG/cm^2; △) 2230 kG/cm^2; ○) 3400 kG/cm^2; 4700 kG/cm^2.

in the pressure increased the superlinearity of the current-voltage characteristics far from the saturation region in weak fields, which was due an increase in the degree of ionization of electrons at donor impurities on application of a heating field.[21] Hence, we concluded that the pressure not only increased the density of thermal-equilibrium electrons,[21] but also accelerated an increase in the number of these electrons on increase in the field in the region under consideration, which again reduced the negative differential conductivity regions.

5. COMPARISON OF THE RESULTS OF MEASUREMENTS WITH NUMERICAL THEORETICAL ESTIMATES OF CRITICAL MSE FIELDS

In the preceding section the experimental results were compared with the qualitative results of Refs. 1–5. In the present section we shall make a quantitative comparison with the results of a numerical calculation of the critical fields of the appearance and disappearance of the MSE in n-type silicon. This comparison not only provides an additional proof of the correct treatment of the experimental results, but it also makes it possible to estimate the contribution of "weak" intervalley f-scattering mechanisms in silicon, which are the scattering involving impurities and the forbidden (by the selection rules) scattering involving low-energy f phonons.

The formulas of the general theory of the MSE[1,14] for a longitudinal electric field E_x along the [110] direction readily yield the following equation for the determination of the transverse field:

$$\vartheta=-a\frac{\Phi(E_1)-\Phi(E_2)}{\Phi(E_1)+\Phi(E_2)+(1+a)\Phi(E_3)}, \tag{1}$$

where $E_{1,2}=E_x(1+\vartheta^2\pm 2a\vartheta)^{1/2}$ and $E_3=E_x(1+a)(1+\vartheta)^{1/2}$ are the toal effective fields in the valleys of different pairs; $\vartheta=E_y/E_x$; the quantity a represents the mobility anisotropy in one valley and it is given by $a=(\mu_\perp-\mu_\parallel)/(\mu_\perp+\mu_\parallel)$; the function Φ describes a universal (for all the valleys) dependence of the product of the mobility and the time for electron transfers out of a valley on the heating field:

$$\Phi(E_\alpha)=\mu(E_\alpha)\tau_i(E_\alpha), \tag{2}$$

so that in $\mu(E_\alpha)$ the anisotropic factor $\hat{\mu}^{(\alpha)}=\hat{a}^{(\alpha)}\mu(E_\alpha)$ is already separated. The possibility of this separation is related to the assumed effective anisotropy of the carrier scattering.

The number and nature of solutions of equations of the (1) type are governed by the nature of the dependence of Φ on the heating field; they are analyzed qualitatively in Ref. 14 and the results of this analysis are used in Sec. 4. A quantitative solution of Eq. (4) can only be obtained numerically. The problem then splits into two stages: in the first stage the function $\Phi(E)$ is calculated, and in the second stage the dependence of the field E_y on E_x is found from Eq. (1).

We calculated $\mu(E)$ and $\tau_i(E)$ by the standard procedure for determining the distribution function by the Monte Carlo method.[22] In this calculation we allowed for the scattering by intravalley acoustic and intervalley g and f phonons, whose energies in silicon are well known (see the review in Ref. 23, from where these and other properties were taken). The constants of the interaction of electrons with these phonons were assumed to be the same as in Ref. 24.

The interaction of electrons with the f_1 phonons is limited by the fact that it is much weaker than the interaction with the g_1 phonons, and it is usually ignored in the analysis of the experimental results. However, at the temperature of our experiments (27°K) it is the f_1 scattering that can be the principal mechanism of the intervalley redistribution of electrons. Therefore, it was included in our calculations and the constant D_{f_1} was considered as a free parameter obeying the condition of smallness of f_1 compared with the g_1 scattering:

$$\omega_{g_1}^{1/2}D_{f_1}^2\ll\omega_{f_1}^{1/2}D_{g_1}^2.$$

As in Ref. 25, the scattering by intravalley acoustic phonons was assumed to be isotropic.[5)] This assumption was fairly rough and, until the limits of its validity are established, the calculations should be regarded as of the model type. One should also point out that these calculations ignored the intravalley scattering by ionized and neutral impurities, since its contribution even in nonheating fields did not exceed 25% under the experimental conditions. Other details of the calculations and the full results were given in Ref. 19.

Figure 8 shows the field dependence of the drift velocity v_D at $T=27$°K, calculated without allowance for the influence of the f_1 scattering on the distribution function as well as that obtained from the experimental data of Ref. 21 for the same samples on which the above experiments were carried out; we can use that the agreement between the experimental and calculated data is fully satisfactory.

The same figure gives the dependences of the reciprocal of the intervalley relaxation time τ_i^{-1} representing the transfer of carriers from any one valley to four valleys with different axes (the constant of the interaction with the f_1 phonons was assumed to be 0.15×10^{18} eV/cm). It is clear from Fig. 8 that in weak

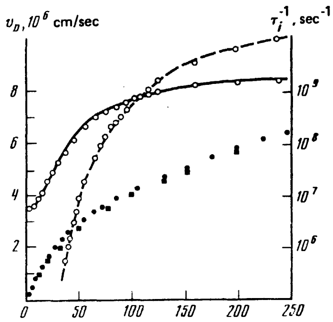

FIG. 8. Field dependences of the drift velocity (●—calculation, ■—experiment) and of the reciprocal intervalley transition time for the f_1 (continuous curve) and f_2 (dashed curve) phonons in n-type Si at 27°K.

fields (up to 100 V/cm) the intervalley transitions are dominated the the f_1 scattering, whereas in high fields they are dominated by the f_2 scattering. Naturally, the fields in which the scattering (relaxation) times become equal can be altered by a suitable selection of the f_1-scattering constant.

The results of calculations of the MSE for various values of this constant show that the lower critical field is always within the range 15–25 V/cm, whereas the experimental value was never less than 40 V/cm.

A critical MSE field closer to the experimental value was obtained by introducing additional intervalley scattering with a relaxation time τ_0 independent of the heating field. Such scattering could be due to, for example, donor impurities whose influence on the intervalley transitions in n-type Ge at low temperatures is a well known effect.[26] The existence of such a mechanism in n-type Si is supported by the absence of saturation of the current along the [110] direction in the case of sufficiently heavily doped samples,[16] and the slight changes in the critical field from sample to sample in our experiments. For $\tau_0^{-1} \sim 1.6\times10^7$ sec^{-1} and $D_{f_1} < 9\times10^6$ V/cm the critical field E_c is of the order of 40 V/cm. The upper critical field of the MSE is also close to the experimental value.

6. CONCLUSIONS

The results of our numerical calculations and experiments confirm convincingly that the MSE effects investigated theoretically earlier[1-5] do occur in n-type Si at low temperatures. The high sensitivity of these effects to the intervalley scattering mechanisms can clearly be used in studies of these mechanisms.

The coexistence in a given sample of longitudinal domainization and transverse stratificaton relative to the electric field raises the problem of theoretical investigation of the shape and structure of domain and interlayer walls, conditions for the creation of domains, their stability, etc. From the experimental point of view it would be interesting also to search for crystals in which the MSE may exist at high temperatures.

[1)]We shall assume that the electron mobility is not greatly dependent on the heating, which is in agreement with the available data for Ge and Si, i.e., we shall assume that even if the heating of the valleys is different, we still have $\mu_{xx}^{(1)} \approx \mu_{xx}^{(2)}$, $\mu_{yy}^{(1)} \approx \mu_{yy}^{(2)}$, $\mu_{xy}^{(1)} \approx -\mu_{xy}^{(2)}$.

[2)]The average field which we shall use here and later is defined simply as the ratio of the potential difference between two points to the distance between these points.

[3)]The potentials on the probes located on the opposite (001) surface behave exactly similarly.

[4)]If the saturation of the current-voltage characteristics is due to the transfer of electrons from a ballast pair of valleys, then it should indeed disappear at low pressures not exceeding 1200 kG/cm^2. This follows from Ref. 20, reporting a study of the current-voltage characteristics with the current directed along the [100] axis. In this case the transfer to the heavy pair of valleys was maximal. The pressure corresponding to an energy splitting of 4 kT ($T = 27$°K) was 1250 kG/cm^2 along this direction and the saturation regions disappeared at ~ 580 kG/cm^2.

[5)]The scattering anisotropy is one of the conditions of validity of the theory of Ref. 1, because it is assumed there that the transport coefficients of electrons in a valley depend only on the heating power.

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Translated by A. Tybulewicz