Jump discontinuity on the front of a rarefaction wave front in a plasma

A. V. Gurevich and A. P. Meshcherkin

P. N. Lebedev Physics Institute, USSR Academy of Sciences
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Hydrodynamic flow of a collisionless nonisothermal plasma is considered. It is shown that jump discontinuities are possible in such flows. The discontinuities have no finite width even though the applicable Poisson equations contain a parameter with the dimension of length, namely the Debye radius. Such a discontinuity can exist only under nonstationary conditions. It is shown that it is produced on the front of a rarefraction wave. The velocity of the discontinuity increases with the time t, and the jump of the ion density decreases in proportion to t^{-2}. The discontinuity has the features of a caustic for the ions, and has no direct analog in ordinary hydrodynamics.

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In ordinary hydrodynamics, a rarefaction wave is bounded by weak discontinuities (Ref. 1, §101). In a plasma, as will be shown below, a rarefaction wave has a jump discontinuity at the interface with vacuum. The present paper is devoted to an investigation of this phenomenon. It is important that the jump discontinuity appears in the hydrodynamics of a nonisothermal plasma when full account is taken of the Poisson equation, i.e., in a system having a parameter with the dimension of length (Debye radius) and higher derivatives. In ordinary hydrodynamics, when higher derivatives are taken into account, the jump discontinuities, as is well known, are always smeared out and acquire a finite width.[1,2]

1. JUMP DISCONTINUITY IN NONISOTHERMAL PLASMA

One-dimensional motion of a plasma with cold ions is described by the system of hydrodynamic equations jointly with the Poisson equation

$$\frac{\partial N}{\partial t}+\frac{\partial}{\partial x}(Nv)=0, \tag{1}$$

$$\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial x}+\frac{eZ}{M}\frac{\partial \varphi}{\partial x}=0, \tag{2}$$

$$\frac{\partial^2\varphi}{\partial x^2}=-4\pi e(NZ-N_e). \tag{3}$$

Here N and v are the density and hydrodynamic velocity of ions with charge eZ and mass M, N_e is the electron density, and φ is the potential of the electric field. The electrons move much faster than the ions, therefore in the lowest approximation in v/v_{T_e} [$v_{T_e}=(T_e/m)^{1/2}$ is the thermal velocity of the electrons] the electron distribution is quasistationary and is determined only by the potential $N_e=N_e(\varphi)$ of the electric field. In particular, in the case of an equilibrium distribution of the electrons in the field (Ref. 2)

$$N_e=N_0\exp(e\varphi/T_e). \tag{4}$$

A more general form of electron distribution, valid only at arbitrary one-dimensional motions of a collisionless plasma was considered in Ref. 3.

We consider the possible appearance in an nonisothermal plasma of a jump discontinuity (jumps of N, v, and φ) that moves with velocity u. From (1)–(3) we obtain the following boundary conditions on the discontinuity:

$$[(v-u)N]=0,\quad [(v-u)^2/2+e\varphi Z/M]=0,$$
$$\left[\frac{d^2\varphi}{dx^2}\right]=-4\pi e(Z[N]-[N_e(\varphi)]). \tag{5}$$

The square brackets denote here, as usual, the difference between the corresponding quantities taken on the left (–) and on the right (+) of the discontinuity.

It follows from the relations in (5) that the ion velocity v, the potential φ, the electric field intensity $E=-\partial\varphi/\partial x$, and the electron density N_e are always continuous. The ion density N, on the contrary, can have on the discontinuity (5) an arbitrary jump, while the respective derivatives $\partial E/\partial x$ and $\partial v/\partial x$ of the field intensity and of the velocity can have a jump determined by the jump of N:

$$[N]=C_0,\quad v_+=v_-=u, \tag{6a}$$

$$\left[\frac{\partial E}{\partial x}\right]=4\pi eZC_0,\quad \left[\left(\frac{\partial v}{\partial x}\right)^2\right]=\frac{4\pi e^2Z}{M}C_0, \tag{6b}$$

$$[\varphi]=[N_e]=[v]=[E]=0. \tag{6c}$$

The condition (6b) was obtained by differentiating Eq. (2).

It follows from (5) and (6) that there is no particle flux through the discontinuity surface, i.e., the considered jump discontinuity is not a hydrodynamic shock wave as defined by Landau and Lifshitz (Ref. 1, §81), but is a special kind of discontinuity that has no direct counterpart in ordinary hydrodynamics.

We investigate now the conditions for the existence of such a discontinuity. To this end we determine the structure of the solution of Eqs. (1)–(3) in the vicinity of the discontinuity. We consider first stationary flow, i.e., we assume that N, v, and φ depend only on $\xi=x-ut$. Integrating then the equations (1) and (2),

$$(v(\xi)-u)N(\xi)=I_0,\quad (v(\xi)-u)^2/2+eZ\varphi(\xi)/M=v_0^2/2,$$

and using the conditions (6) on the discontinuity, we find that

$$I_0=0,\quad v(\xi)=u,\quad \varphi(\xi)=\text{const}.$$

It follows therefore that such a discontinuity is impos-

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sible under stationary conditions.

We consider therefore the vicinity of a nonstationary discontinuity (6) that moves with velocity $u(t)$. It is convenient to change from x and v to new variables ξ and v_1 connected with the moving discontinuity:

$$\xi=x-\int u\,dt, \quad v_1=v-u, \quad u=u(t). \tag{7}$$

Equations (1)-(3) are then rewritten in the form

$$\frac{\partial N}{\partial t}+v_1\frac{\partial N}{\partial \xi}+N\frac{\partial v_1}{\partial \xi}=0, \quad \frac{\partial v_1}{\partial t}+v_1\frac{\partial v_1}{\partial \xi}-\frac{eZ}{M}(E-E(0,t))=0,$$
$$\frac{\partial E}{\partial \xi}=4\pi e(ZN-N_e(\varphi)), \quad \frac{du}{dt}=\frac{eZ}{M}E(0,t), \quad E(\xi,t)=-\frac{\partial\varphi}{\partial\xi}. \tag{8}$$

It is natural to seek the solution of Eqs. (8) in the vicinity of the discontinuity point $\xi=0$ in the form of a series in powers of ξ. Taking (6) into account, we have

$$N=N_0(t)+N_1(t)\xi+\ldots, \quad v_1=v_1(t)\xi+v_2(t)\xi^2+\ldots,$$
$$E=E_0(t)+E_1(t)\xi+\ldots.$$

It follows then from (8) that

$$u(t)=\int E_0(t)\,dt, \quad v_1=-\frac{d\ln N_0(t)}{dt}, \quad E_1=4\pi e(ZN_0(t)-N_e(0,t)),$$

and the function $N_0(t)$ is defined by the equation

$$-N_0\frac{d^2N_0}{dt^2}+2\left(\frac{dN_0}{dt}\right)^2-\frac{4\pi e^2}{M}N_0^2(ZN_0-N_e(0,t))=0. \tag{9}$$

The solution of Eq. (9) depends essentially on the form of the function $N_e(0,t)=N_e[\varphi(0,t)]$ that describes the time variation of the electron density at the discontinuity point moving with velocity $u(t)$ (7). In particular, at $N_e(0,t)=\text{const}=N_{e0}$ we have

$$N_0(t)=N_{00}/(a\sin\tau+ZN_{00}/N_{e0}), \quad \tau=\Omega(t+t_0),$$
$$t_0=\Omega^{-1}\arcsin[(1-ZN_{00}/N_{e0})/a], \quad \Omega^2=4\pi e^2N_{e0}/M,$$

N_{00} is the value of N_0 at the initial instant $t=0$, and a is a dimensionless constant. It is seen that the density $N_0(t)$ becomes infinite at $\tau=\pi-\arcsin(ZN_{00}/aN_{e0})$.

If $N_e(0,t)=N_{e0}/(t/t_0+1)$, then Eq. (9) is satisfied by the quasineutral solution

$$N_0(t)=N_{e0}/Z(t/t_0+1).$$

In the case of a power-law dependence $N_e(t)=N_{e0}/t^{\alpha}$ we have $N_0(t)=N_{00}/t^{\alpha}$, and a nontrivial solution of Eq. (9) is possible when $\alpha=2$. Then

$$N_e(t)=N_{e0}/t^2, \; N_0(t)=N_{00}/t^2, \; N_{00}=N_{e0}/Z+2M/4\pi e^2Z,$$
$$v_1=-d\ln N_0/dt=2/t, \; E_1=2M/t^2. \tag{10}$$

It is seen that in this case the electron and ion densities are not equal on the discontinuity, and their ratio is a constant quantity independent of time.

The relations obtained are not contradictory. A jump discontinuity of the type considered can therefore exists in principle in nonstationary plasma flow.

Let us compare this with ordinary hydrodynamics. Jump discontinuities occur in ideal hydrodynamics described by Euler's equations, which do not contain any characteristic spatial scales (Ref. 1, §81). When viscosity and dispersion are taken into account, parameters with the dimension of length do appear, and the Euler equations are correspondingly supplemented by terms containing higher derivatives: the Navier-Stokes, Korteweg-de Vries, and other equations. The discontinuous solutions vanish then and the discontinuities are smeared out. The viscosity describes the characteristic discontinuity smearing width (Ref. 1, §81). In the presence of spatial dispersion, oscillations can arise in the discontinuity region.[2]

Equations (1)-(3), which describe the flows of a strongly nonisothermal plasma, contain a parameter with the dimension of length (the Debye radius) and higher derivatives. They have the character of hydrodynamic equations with dispersion, but without dissipation, and reduce in the case of small perturbations to the Korteweg-de Vries equation. The Debye radius determines the characteristic scale of the oscillations that are excited in the plasma. It is natural to expect strong discontinuities to arise in this case only in the approximation of hydrodynamics of the Euler type, which does not contain the Debye radius. The corresponding equations can be obtained from the complete system by changing over to the quasineutral-plasma approximation and by special averaging over the oscillations (Whitham's method[4]). The possible existence and the process of formation of discontinuities (laminar shock waves according to Sagdeev's terminology[5]) in the solutions of the averaged equations was investigated by Pitaevskiĭ and one of us.[6] The solutions of the complete equations are in this case, of course, continuous; they have an oscillatory structure in the vicinity of the discontinuities.

We have seen above, however, that the complete system of the plasma-hydrodynamics equations admits in principle not only such oscillatory solutions, but also real jump discontinuities that have no finite width, despite the presence of higher derivatives and of a characteristic parameter with the dimension of length—the Debye radius. We shall prove below the existence of strong discontinuities of this type by a concrete construction of a stationary discontinuous solution on the front of a rarefaction wave.

2. RAREFACTION WAVE

Let the plasma fill at the initial instant $t=0$ the left half-space, $(x\to-\infty)$, with vacuum in the right half-space $(x\to+\infty)$. The initial interface between the plasma and the vacuum can be regarded as either abrupt or diffuse. In the course of time the plasma flows into the vacuum. The corresponding flow is called a rarefaction wave in ordinary gasdynamics. In an ideal gas with an abrupt initial interface with the vacuum, the rarefaction wave is self-similar, i.e., it is described by functions that depend only on the ratio x/t. The point $x=0$ corresponds to the position of an abrupt initial boundary at $t=0$ (see Ref. 1, §101). If the initial interface is diffuse, then the flow becomes rapidly close to self-similar. On the whole, a similar picture is observed also in the flow of a nonisothermal plasma.

To describe the rarefaction wave in a nonisothermal plasma it is convenient to change in Eqs. (1)-(3) from x and to the variables τ and ξ, which are close to self-similar:

$$\tau=x/t,\quad \xi=\ln(t/t_0). \tag{11}$$

Here t_0 is a certain characteristic time that will be defined later on. Equations (1)–(3) take in the variables (11) the form

$$\frac{\partial N}{\partial \xi}+(v-\tau)\frac{\partial N}{\partial \tau}+N\frac{\partial v}{\partial \tau}=0,\quad \frac{\partial v}{\partial \xi}+(v-\tau)\frac{\partial v}{\partial \tau}+\frac{eZ}{M}\frac{\partial \varphi}{\partial \tau}=0, \tag{12}$$

$$\frac{\exp(-2\xi)}{t_0^2}\frac{\partial^2\varphi}{\partial\tau^2}=-4\pi e(NZ-N_e(\varphi)).$$

It is natural to separate the self-similar solution, i.e., represent the functions N, v, and φ in the form

$$N=N_0(\tau)(1+n_1(\tau,\xi)),\ v=v_0(\tau)+v_1(\tau,\xi),\ \varphi=\varphi_0(\tau)+(T_e/e)\psi_1(\tau,\xi). \tag{13}$$

From (12) and (13) we have

$$(v_0-\tau)\frac{dN_0}{d\tau}+N_0\frac{dv_0}{d\tau}=0,\quad (v_0-\tau)\frac{dv_0}{d\tau}+\frac{eZ}{M}\frac{\partial\varphi_0}{\partial\tau}=0,\quad N_0Z=N_e(\varphi_0).$$

Hence, taking (4) into account, we obtain the known self-similar solution[3]

$$N_0(\tau)=N_0\exp(-\tau/s_0-1),\quad v_0(\tau)=\tau+s_0,$$
$$\varphi_0(\tau)=(T_e/e)(-\tau/s_0-1),\quad N_e=N_0Z,\quad s_0=(ZT_e/M)^{1/2}. \tag{14}$$

Equations (14) are valid only at $\tau\geqslant -s_0$; at $\tau<-s_0$ the plasma is not perturbed, i.e.,

$$N_0(\tau)\equiv N_0,\quad v_0(\tau)\equiv 0,\quad \varphi_0(\tau)\equiv 0.$$

At $\tau=-s_0$ the solution has a weak discontinuity.

Using (4), we now rewrite Eqs. (12) for n_1, v_1, and ψ_1 in the form

$$\frac{\partial n_1}{\partial\xi}-\frac{v_1}{s_0}(1+n_1)+(s_0+v_1)\frac{\partial n_1}{\partial\tau}+(1+n_1)\frac{\partial v_1}{\partial\tau}=0,$$
$$\frac{\partial v_1}{\partial\xi}+(s_0+v_1)\frac{\partial v_1}{\partial\tau}+v_1+s_0^2\frac{\partial\psi_1}{\partial\tau}=0,$$
$$\frac{\partial^2\psi_1}{\partial\tau^2}=-\frac{\varepsilon}{s_0^2}\exp\left(-\frac{\tau-2s_0\xi}{s_0}\right)\{1+n_1-\exp\psi_1\}, \tag{15}$$
$$\varepsilon=4\pi e^2N_{e0}M^{-1}\exp(-1)t_0^2.$$

It is seen that the variables τ and ξ enter in the right-hand side of Eqs. (15) only in the combination $\tau-2s_0\xi$. It is natural to seek a particular solution of (15) in a corresponding form. We then arrive in place of (15) at the equations

$$(V_1-1)\frac{dn_1}{dy}+(n_1+1)\left(\frac{dV_1}{dy}-V_1\right)=0,\quad (V_1-1)\frac{dV_1}{dy}+V_1+\frac{d\psi_1}{dy}=0,$$
$$\frac{d^2\psi_1}{dy^2}+\varepsilon\exp(-y)(n_1+1-\exp(\psi_1))=0,\quad V_1=\frac{v_1}{s_0},\quad y=\frac{\tau-2s_0\xi}{s_0}. \tag{16}$$

These equations describe a nonlinear wave traveling along τ. Since the functions n_1, v_1, and ψ_1 of interest to us constitute a strong perturbation of the self-similar solution only in the vicinity of a wave front moving towards positive values of τ, it follows that at sufficiently large negative y there should be satisfied the condition

$$n_1\to 0,\ V_1\to 0,\ \psi_1\to 0\quad \text{as}\quad y\to-\infty. \tag{17}$$

It is important that the first two equations in (16) have a singularity at $V_1=1$. Let $V_1=1$ at the point $y=y_k$. From the requirement that there be no singularity in the solutions at $y=y_k$ we obtain the following expansion in the vicinity of y_k:

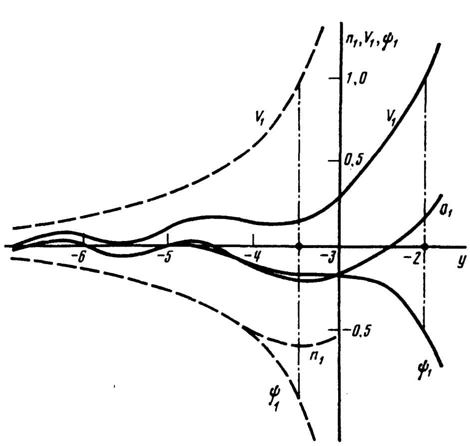

FIG. 1. Plots of n_1, V_1, and ψ_1 vs y, obtained by numerically integrating Eqs. (16) and (17), at parameter values $y_k=-3.46$ and $\varepsilon=4.0$ (solid line) and $y_k=-2.0$ and $\varepsilon=4.0$ (dashed line). The dash-dot line shows the position of the jump discontinuity.

$$n=n_0+\frac{6-\nu_0}{6+\nu_0}(y-y_k)+\frac{\nu_0(-\nu_0^2+24\nu_0+36)}{(6+\nu_0)^2(\nu_0+12)}(y-y_k)^2,$$
$$V_1=1+(y-y_k)+\frac{\nu_0}{6+\nu_0}(y-y_k)^2,\quad \psi_1=\psi_0-(y-y_k)-(y-y_k)^2,$$
$$\frac{d\psi_1}{dy}=-1-2(y-y_k)-\frac{4\nu_0}{6+\nu_0}(y-y_k)^2,$$
$$\nu_0=\varepsilon\exp(-y_0+\psi_0),\quad n_0=\ln[(\nu_0+2)\exp y_0/\varepsilon],$$
$$n=\ln(n_1+1).$$

It is seen that a solution without singularities, arriving at the point y_k, is determined by two constants, ψ_0 and ε. Choosing ψ_0 to satisfy the condition (17) on the left of the singular point, we continue next the solution into the region $y\geqslant y_k$. It is shown in Fig. 1 for $y_k=-3.46$, $\varepsilon=4.0$ and for $y_k=-2.0$, $\varepsilon=4.0$.

The jump-discontinuity conditions can be satisfied at the singular point $y=y_k$. Indeed, the total flow velocity, as follows from (13), (14), and (16), is

$$v=\tau+s_0+s_0V_1(y), \tag{18}$$

and the plasma velocity u at a given y is, according to (11) and (16),

$$u=dx/dt=s_0y+2s_0\xi+2s_0=s_0[2+y+2\ln(t/t_0)],\quad x=tys_0+2s_0\xi t. \tag{19}$$

It follows from (18) and (19) that the relation

$$u=v \tag{20}$$

is satisfied at $y=y_k$, where the value of y_k is defined by the condition

$$V_1(y_k)=1.$$

Consequently, the conditions (6) are satisfied at the point $y=y_k$ and the solution admits of the jump discontinuity shown by the dash-dot curve in Fig. 1. The value of y_k determines the position of the wave front. The solution constructed to the left of the discontinuity satisfies Eqs. (1)–(3). On the right of the discontinuity we have $N\equiv 0$ and consequently Eqs. (1) and (2) are likewise satisfied here identically. The conditions (6) are satisfied on the discontinuity. The potential and the electric field intensity at the point $y=y_k$ are continuous. To the right of the discontinuity, the solution of the

Poisson equation (3) is continued uniquely with account taken of the specified values of φ and E and of the specified jump $[\partial \boldsymbol{E}/\partial x]$ on the discontinuity (6).

The structure of the solution in the vicinity of the discontinuity is shown in Fig. 1. The value of N_e at the discontinuity point, as is clear from (11) and (14)–(16), decreases with time in proportion to t^{-2}. The ratio of the ion and electron densities is constant here in accord with (10). The velocity of the ion front (of the discontinuity), as seen from (19), increases with time in proportion to $2\ln(t/t_0)$.

The condition (17) for the transition to the self-similar solution as $y\to-\infty$ is satisfied in the general case because of the onset of a weak discontinuity: the functions n_1, v_1, and ψ_1 vanish at a certain value $y=y_1$ at which they have nonzero derivatives

$$n_1(y_1)=V_1(y_1)=\psi_1(y_1)=0,\quad \frac{dn_1}{dy}(y_1)=\frac{dV_1}{dy}(y_1)=\frac{d\psi_1}{dy}(y_1)=p_1>0.$$

To investigate the structure of the weak discontinuity of the self-similar solution, we must consider small perturbations of N, v, and φ. In this case the system of equations (1)–(3) reduces to the Korteweg-de Vries equation.[2] The structure of a weak discontinuity in the Korteweg-de Vries equation was considered earlier in Ref. 7, where it was shown that oscillations can arise in the region of the weak discontinuity, and the region broadens slowly with time, $\Delta x\sim t^{1/3}$, and remains local in terms of the self-similar variables $\tau=x/t$.

We now integrate numerically Eqs. (1)–(3) for the problem of plasma outflow into vacuum. We shall assume that there are no jump discontinuities at the instant $t=0$, and the plasma occupies a half-space with a diffuse boundary, for example,

$$N_0(x)=\begin{cases}N_0 & x<-a,\\ N_0(1-x/a), & -a<x<0,\\ 0, & x>0.\end{cases}\tag{22}$$

The result of the numerical integration of Eqs. (1)–(3) with initial conditions (22) is shown in Fig. 3. It is seen that a jump discontinuity after a finite time t_k

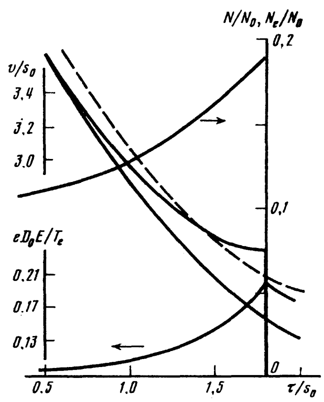

FIG. 2. Structure of complete solution of the system of equations (12) in the vicinity of the jump discontinuity. The dashed line shows the self-similar solution for the particle density $D_0=(T_e/4\pi e^2N_{e0})^{1/2}$.

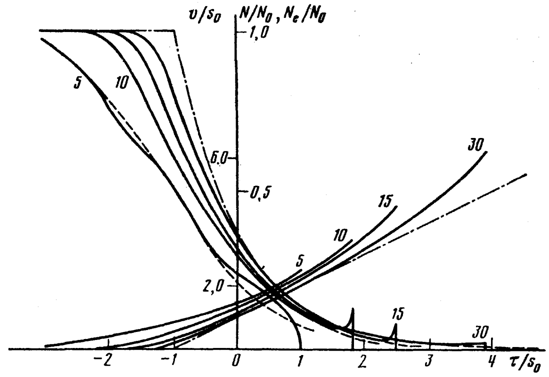

FIG. 3. Ion and electron densities N and N_e and the ion velocity v as functions of τ/s_0, obtained by numerically integrating Eqs. (1)–(3) with the initial condition (22). The dash-dot line shows the solution of the self-similar problem. The numbers on the curves are the values of the parameter $t\Omega_0$.

$\approx 7/\Omega$, $\Omega=(4\pi e^2N_0/M)^{1/2}$ is produced on the leading front of the rarefaction wave. There are no ions to the right of the discontinuity: $N=0$. To the left of the discontinuity the solution approaches rapidly the quasineutral self-similar distribution (14). The front velocity—the discontinuity velocity—increases logarithmically with time (cf. Ref. 8, Fig. 4) in accord with (19). The constant t_0 determined from the numerical solution is $t_0\approx 3.3/\Omega$, and $\varepsilon\approx 4.0$.[1)] The electron density near the discontinuity decreases with time in proportion to t^{-2}. The ion velocity v on the discontinuity is equal to the front velocity u. The condition (21), as seen from Fig. 3, is also well satisfied. Thus, the main features of the asymptotic analytic solution determined above agree well with the results of the numerical calculation.

We examine now in greater detail the discontinuity-formation process. At the initial instant $t=0$ the ion density in the vicinity of the front decreases in accordance with the linear law (22). The distribution of the electron density N_e, of the ion density N, and of the electric field intensity E is then of the form shown in Fig. 4a. The electrons lead the ions and produce ahead of the front a negative charge that holds back the electrons and accelerates the ions. It is important that in this case the electric field intensity is not monotonic: $E(x)$ has a maximum at the point x_m at which the ion and electron densities become equal (Fig. 4a). It is this feature which leads subsequently to a gradual steepening and to the onset of a strong discontinuity on the leading front of the rarefaction wave.

Indeed, it follows from (2) that the total change of the ion velocity is

$$\frac{dv}{dt}=\frac{eE}{M},\quad \frac{d}{dt}=\frac{\partial}{\partial t}+v\frac{\partial}{\partial x}.$$

Consequently, the ion velocity v increases fastest near the point x_m, where the field E is a maximum. On the other hand at points $x>x_m$ the velocity v increases more slowly with time. But this means that the ions from the region of x_m gradually catch up with the ions that were in front at the initial instant. Therefore the wave front becomes ever steeper with time. This is seen from

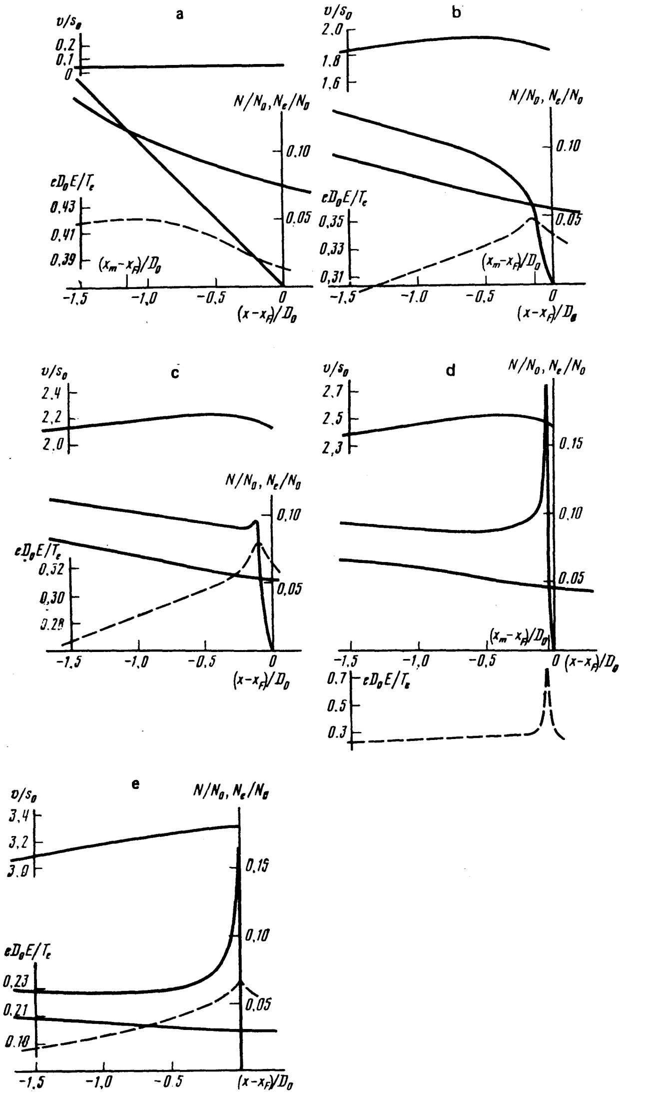

FIG. 4. Ion and electron densities N and N_e, the ion velocity v, and the electric field intensity E (dashed line) vs $(x-x_F)/D_0$ at various instants of time [$t=0.1/\Omega_0$ (a), $t=5/\Omega_0$ (b), $t=6/\Omega_0$ (c), $t=7/\Omega_0$ (d), $t=10/\Omega_0$ (e)] near the wave front, obtained by numerically integrating Eqs. (1)–(3) with initial condition (22). At the point x_m the ion and electron densities are equal. The scale on the $E(x)$ curve of Fig. 4d differs from the others by a factor 10—the electric field near the toppling point increases sharply prior to the toppling of the wave.

Figs. 4b, c, d, which show the time variation, obtained from the numerical calculation of the leading front having the form 4a at the initial instant.

At the instant $t=t_k\approx 7/\Omega$ the profile of the wave topples over and an abrupt jump of the ion density—a jump discontinuity—is formed on the leading front x_F of the rarefaction wave. The point x_m where the field is a maximum catches up with the leading-front point x_F (see Fig. 2); there is a substantial jump of the derivative dE/dx at $x=x_F$ (6)

$$x_m(t_k)=x_F(t_k).$$

This means that the ion velocity increases most strongly with time on the wave front x_F. The velocity behind the front increases more slowly, so that the steepening stops at $t\gtrsim t_k$. There are no ions ahead of the front, and the electric field is weaker here: $E<E(x_F)$. The ions therefore cannot move ahead of the front, so that the jump discontinuity on the wave front is not smeared out and is preserved with further motion of the plasma (Figs. 2 and 4e).

We note that Eqs. (1) and (2) are valid for cold ions—the kinetic straggle of the ion velocity is completely neglected in them. As shown in Ref. 9, the effective ion temperature decreases rapidly in the direction of the plasma expansion (the x axis):

$$T_i^{ef}\sim T_{i0}\exp(-2\tau/s_0).$$

Using (11) and (16), we obtain in the region of the rarefaction-wave front

$$T_i^{ef}\sim T_{i0}(t_0/t)^4.$$

Allowance for the kinetics can therefore not lead to a considerable smearing of the front (see Ref. 3).

The situation can change when plasma oscillations are excited. The oscillations are not one-dimensional and lead to scattering of the ions, and hence to an increase of their effective temperature along the x axis. Ion collisions can have a similar effect. This should lead to a broadening of the leading front of the rarefaction wave. It appears that effects of this type were observed by Eselevich and Faĭnshteĭn in a laboratory-plasma experiment.[10]

The rarefaction wave plays an essential role in ionospheric aerodynamics—it describes flow of plasma around bodies.[3,11] In particular, the structure of the perturbed region behind a half-plane around which a rarefield plasma flows with supersonic velocity v_0

$$v_0\gg s_0, \tag{23}$$

is described in the quasineutral approximation by the self-similar solution (14). When the complete Poisson equation is taken into account, as seen above, this solution is valid only over limited distances—up to the region of the jump discontinuity. Using the results obtained here, it is easy to verify that the boundary x_F of the leading front is defined under condition (23) by the expression

$$x_F=2z\frac{s_0}{v_0}\ln\left(\frac{s_0}{v_0}\frac{z}{D_0}\alpha\right), \tag{24}$$

$$D_0=(T_e/4\pi e^2N_{e0})^{1/2},\quad s_0=(T_e/M)^{1/2},\quad \alpha=0.3.$$

Here, as usual,[3,11] z is the coordinate along the plasma flux advancing towards the half-plane, and x is the coordinate in a direction perpendicular to the flow and is reckoned from the edge of the half-plane (the plasma moves along the z axis and spills over behind the half-plane along the x axis). Equation (24) is identical with Eqs. (11) and (19), the $x_F(z)$ curve is shown in Fig. 5 by the solid line. The ion density experiences a jump discontinuity at $x=x_F(z)$. At $x<x_F(z)$, near the discontinuity, the ion density is noticeably higher than the electron densities (11), (13), and (14) (see Fig. 3). This region is hatched in Fig. 5. It goes over next rapidly into the quasineutral zone described by the self-similar solution (14).[3,11]

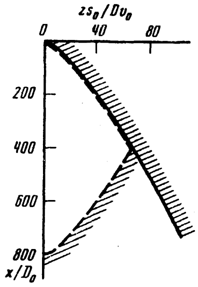

FIG. 5. Picture of flow around a half-plane (solid line) and around a plate of width $2R_0$ (dashed lines), both perpendicular to the flow of a rarefied supersonic plasma. The hatches mark the region where the ion density is noticeably higher than the electron density. $R_0 = 400D_0$.

The dashed line in Fig. 5 shows the picture of the flow of the plasma around a flat plate of width $2R_0$ perpendicular to the flow. The plasma then spills over on both sides. In that region behind the body which is bounded by the jump discontinuities (24) and

$$x_{F2} = 2R_0 - 2z\frac{s_0}{v_0}\ln\left(\frac{s_0}{v_0}\frac{z}{D_0}\alpha\right),$$

there are no ions. This is the "region of maximum rarefaction" in the definition of Refs. 11 and 12. It is seen now that it has a sharp and not a diffuse boundary. Behind the body, the collision of the plasma streams that spill over from the two direction near the axis $x = R_0$ can lead to excitation of ion-sound waves. The Landau absorption for this wave is mainly by the electrons. It is possible that it is this absorption which causes the rise of the electron temperature in the wake behind the body, as observed in the ionosphere by Samir *et al.*[13]

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[1)]We note that the values of the constants t_0 and ε vary with the character of the boundary conditions as $x \to +\infty$ and with the form of the function $N_e(\varphi)$ in Eq. (3), i.e. with the form of the electron distribution function (see Refs. 3 and 8).

[1]L. D. Landau and E. M. Lifshitz, Gidrodinamika sploshnykh sred (Hydrodynamics of Continuous Media), Gostekhizdat, 1953, §§81, 101.

[2]V. I. Karpman, Nelineĭnye volny v dispergiruyushcikh sredakh (Nonlinear Waves in Dispersive Media), Nauka, 1973 §§ 13–15.

[3]A. V. Gurevich and L. P. Pitaevskiĭ, in: Voprosy teorii plazmy (Problems of Plasma Theory), M. A. Leontovich, ed., No. 10, Atomizdat, 1980, p. 3.

[4]G. B. Whitham, Proc. Roy. Soc. **A20**, 798 (1965).

[5]R. Z. Sagdeev, in: Voprosy teorii plazmy (Problems in Plasma Theory), M. A. Leontovich, ed. No. 4, Atomizdat 1964, p. 20.

[6]A. V. Gurevich and L. P. Pitaevskiĭ, Zh. Eksp. Teor. Fiz. **65**, 590 (1973) [Sov. Phys. JETP **38**, 291 (1974)].

[7]A. V. Gurevich and L. P. Pitaevskii, in: Proc. 12th Internat. Conf on Phenomena in Ionized Gases, Vol. 1, Eindhoven, 1975, p. 273.

[8]A. V. Gurevich and A. P. Meshcherkin, Zh. Eksp. Teor. Fiz. **80**, 1810 (1981) [Sov. Phys. JETP **53**, 937 (1981)].

[9]A. V. Gurevich, L. V. Pariĭskaya, and L. P. Pitaevskiĭ, *ibid.* **49**, 647 (1765) [**21**, 449 (1966)].

[10]V. G. Eselevich and V. G. Faĭnshteĭn, *ibid.* **79**, 870 (1980) [**52**, 441 (1980)].

[11]A. V. Gurevich, L. P. Pitaevskiĭ, and V. V. Smirnova, Usp. Fiz. Nauk **99**, 3 (1969) [Sov. Phys. Usp. **12**, 595 (1970)].

[12]Ya. L. Al'pert, A. V. Gurevich, and L. P. Pitaevskiĭ, Iskusstvennye sputniki v razrezhennoĭ plazme (Artificial Satellites in a Rarefied Plasma), Nauka, 1964.

[13]U. Samir, R. Gordon, L. Brad, and R. Theis, J. Geophys. Res. **84**, 513 (1979). B. E. Troy, E. J. Meier, and U. Samir, *ibid.* **80**, 993 (1975).

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