

# Fluctuation levels and cyclotron resonance in a random potential

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Potential fluctuations lead to the appearance of electronic energy levels lying below the bottom of the conduction band. The density of such levels in a strong magnetic field and the cyclotron-absorption-line shape due to the electrons occupying these levels are derived.

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Light uncharged impurities in semiconductors can be described by a random potential. The density of states in such a potential has a tail, i.e., it exponentially falls off in the region of large negative energies.<sup>1-3</sup> The dominant contribution to the density of states at these energies is made by the levels that occur in the exponentially rare deep potential wells.

A magnetic field has a strong effect on the magnitude of the density of states. The weak-field induced change in the exponent in the energy dependence of the density of states is found with the aid of perturbation theory. A strong magnetic field modifies the dependence of the exponent on energy. The pre-exponential factor is computed exactly in this case.

In a film or a thin surface layer, a normal magnetic field makes the spectrum discrete and infinitely multiply degenerate. Impurities smear out this spectrum. Exponential tails occur near each Landau level at both positive and negative energy values. Impurities smear out the cyclotron resonance line. At low temperatures and low electron concentrations the cyclotron resonance line splits up into a large number of sublevels. The position and shape of each sublevel's line are found.

## I. THE DENSITY OF STATES

The Hamiltonians of electrons in a magnetic field and in the field of impurities have the form

$$H = - \sum_{\alpha} \frac{1}{2m_{\alpha}} \left( \frac{\partial}{\partial \mathbf{r}_{\alpha}} - i \frac{e}{c} \mathbf{A}_{\alpha} \right)^2 + V(\mathbf{r}), \quad (1)$$

where the random potential  $V(\mathbf{r})$  is distributed according to the Gaussian law:

$$W(V) \sim \exp \left( - \int V^2 d^3\mathbf{r} / 2\gamma \right). \quad (2)$$

Let us reduce the Hamiltonian (1) to the isotropic form by making the change of variables

$$r_{\alpha} \rightarrow r_{\alpha} m_{\alpha}^{-1/2}, \quad A_{\alpha} = \tilde{A}_{\alpha} m_{\alpha}^{1/2}, \quad \gamma \rightarrow \tilde{\gamma} \prod_{\alpha} m_{\alpha}^{-1/2}. \quad (3)$$

This change of variables does not change the form of the distribution (2). It is convenient to choose the vector potential  $\tilde{\mathbf{A}}$  in the new variables in the gauge  $ec^{-1}\tilde{\mathbf{A}} = \frac{1}{2}\tilde{\boldsymbol{\omega}} \times \mathbf{r}$ , where the  $\tilde{\boldsymbol{\omega}}_{\alpha}$  are the components of the vector  $\boldsymbol{\omega}$  along the principal axes of the mass ellipsoid; they are connected with the magnetic-field components  $H_{\alpha}$

by the relation

$$eH_i = c\omega_i (m_i m_k)^{1/2}, \quad \omega_{\alpha} = eH_{\alpha} / c (m_{\beta} m_{\gamma})^{1/2},$$

where  $\alpha \neq \beta \neq \gamma$ . The modulus of the vector  $\boldsymbol{\omega}$  is equal to the cyclotron-resonance frequency. In the new variables the Hamiltonian (1) has the form

$$H = -1/2 \mathcal{D}^2 + V, \quad \mathcal{D} = \nabla + i[\boldsymbol{\omega} \times \mathbf{r}] / 2. \quad (4)$$

The density of states at the energy  $E$  is given by the continuum integral:

$$\rho(E) = \int \text{Sp} \delta(E - H) W(V) DV. \quad (5)$$

$\rho(E)$  is determined with exponential accuracy by the maximum of the exponent in the formula (2) with the additional condition that  $H\psi = E\psi$ . Performing a  $V$  variation, we obtain<sup>1</sup>

$$V = -|\varphi|^2, \quad (6)$$

where  $\varphi$  is found by solving the linear equation

$$1/2(\mathcal{D}^2\varphi) + \varphi^3 = E\varphi. \quad (7)$$

To compute the pre-exponential factor in the density of states, we must expand the exponent of the exponential function in a series in the small deviations of the random potential from the extremal value given by the formula (6):

$$V - |\varphi|^2 = \sum a_n V_n. \quad (8)$$

Here we choose

$$V_0 = |\varphi|^2 J_0^{-1/2}, \quad V_1 = \nabla |\varphi|^2 J_1^{-1/2},$$

where  $J_0$  and  $J_1$  are the normalization integrals:

$$J_0 = \int |\varphi|^4 d^3\mathbf{r}, \quad J_1 = \int (\nabla_{\alpha} |\varphi|^2)^2 d^3\mathbf{r}. \quad (9)$$

Further, let us use the method proposed in Ref. 3. The dominant contribution to the formula (5) is made by the smallest eigenvalue of the Hamiltonian  $H$ . Computing this eigenvalue in first order in  $a_0$  and second order in  $a_n$ , we have

$$\rho(E) = \left( \int e^A da_n \right)^{-1} \int \delta(a_0 K J_0^{1/2} + K a_n V_n \varphi (E - H)^{-1} \varphi V_n a_n) e^A \prod da_n, \quad (10)$$

where

$$K^{-1} = \int \varphi^2 d^3\mathbf{r}, \quad A = - \left( J_0 + 2J_0^{1/2} a_0 + \sum a_n^2 \right) / 2\tilde{\gamma}.$$

Let us perform the integration with respect to  $a_0$  with the aid of the  $\delta$  function; the integration with respect

to  $a_1$  is equivalent to integration with respect to the coordinate origin, and yields to within the Jacobian the volume  $L^d$  of the system. The  $a_n$  ( $n > 1$ ) Gaussian integral is equal to the product of the eigenvalues of the quadratic form ( $M$ ). An important change in the calculations as compared to the zero magnetic field case consists in the following. The eigenfunctions of the Hamiltonian with a magnetic field cannot be chosen to be real; therefore, the operator  $H$  is Hermitian, but not symmetric, and the coefficients  $a_n$  are real. Consequently, the quadratic form should be symmetrized before the evaluation of the integral. Evaluating the integral, we obtain

$$\rho(E) = \exp(-J_0/2\gamma) L^d (2\pi\gamma)^{-(d+1)/2} J_0^{-1/2} K^{-1} \prod_a J_{i_a}^{1/2} m_a^{1/2} D^{-1/2}, \quad (11)$$

where  $D = \Pi \lambda_L$ , the  $\lambda_L$  being the eigenvalues of the operator

$$M = 1 - \varphi[(E-H)^{-1} + (E-H^*)^{-1}] \varphi. \quad (12)$$

$D$  is the product of all the  $\lambda_L$ , except the first ones, which are equal to infinity and zero, and correspond to the functions  $V_0$  and  $V_1$ .

### 1. Weak fields

The integral  $J_0$  entering into the exponent of the formula (11) depends on the energy as follows:

$$J_0 = E^{2-d/2} f(\omega/E).$$

The function  $f$  can be found in the limits of small and large arguments. In weak fields, in which the cyclotron resonance frequency  $\omega$  is low compared to the energy  $E$ , Eq. (7) can be solved with the aid of perturbation theory. As a result, we obtain

$$J_0 = \int \varphi^4 d^d r = \int \varphi_0^4 d^d r + \int [\omega r]^2 \varphi_0^2 d^d r. \quad (13)$$

Determining  $\varphi_0$  by numerically solving Eq. (7), we obtain at  $\omega = 0$  the expressions

$$\rho(E) \sim \begin{cases} \exp\{-2.9E(1+0.16\omega^2/E^2)/\gamma\}, & d=2 \\ \exp\{-13.2E^{1/2}(1+0.04\omega^2/E^2)/\gamma\}, & d=3 \end{cases} \quad (14)$$

Although the relative change in the index of the exponential function is small, the absolute change can be large. Consequently, even in this case the density of states strongly depends on the magnetic field. The magnetic field dependence of the pre-exponential factor can be neglected.

### 2. Strong field. The two-dimensional case

In the two-dimensional case the impurities smear out all the Landau levels in a strong magnetic field. Each level has an exponential tail both on the side of negative, and on the side of positive, energies. To the positive energies correspond a potential hump, and not a potential well, and we should change the sign of the nonlinear term in Eq. (7). If the energies are close to the Landau-level energies  $E_n = \omega(n + \frac{1}{2})$ , so that  $|E - E_n| \ll \omega$ , then we can solve Eq. (7), treating the nonlinear term as a perturbation. In this case  $\varphi = C_n \psi_n$ , where  $\psi_n$  is the normalized wave function corresponding to the  $n$ -th Landau level with zero spin. Let us find the coefficients  $C_n$  by multiplying Eq. (7) from the left by  $\psi_n$  and integrating:

$$C_n^2 \int |\psi|^4 d^2 r = |E - E_n|. \quad (15)$$

Thus, the integral  $J_0$  entering into the exponent is equal to

$$J_0 = C_n^4 \int |\psi|^4 d^2 r = 4\pi a_0 (E - E_n)^2 / \omega, \quad (16)$$

where the numerical coefficients  $a_n$  are equal to:  $a_0 = 1$ ,  $a_1 = 2$ .

Let us compute the pre-exponential factor only for the lowest level with  $n=0$ . We should, in computing the eigenvalues of the operator (12), take into consideration only the states corresponding to the lowest  $n=0$  level and the various angular-momentum values (i.e., the  $\psi_{0m} \pm \psi_{0m}^*$  states). As a result, we obtain

$$\lambda_m = 1 - (2^m - 1)^{-1}, \quad (17)$$

$$\rho(E) = 2^{1/2} L^2 \varepsilon^2 \omega_H^{-1/2} \gamma^{-1/2} \exp[-(\varepsilon/\Delta)^2].$$

Here and below  $\varepsilon = E_0 - E = \omega/2 - E$ ,  $\Delta^2 = \omega_H \hbar^2 / 2\pi$ . The sample dimension  $L$  and the parameter  $\gamma$  are connected with their values in the original units by the relation (3).

### 3. Strong field. The three-dimensional case

The presence of impurities leads to the smearing out of the density of states and to the appearance of states below the zeroth Landau level. For  $|E - \omega_H/2| \ll \omega_H$ , Eq. (7) can be solved. To do this, let us expand  $\varphi$  in the equation into the product sum

$$\sum \Phi_n(\rho) f_n(z),$$

where the  $z$  axis is oriented along, and the  $\rho$  axis perpendicular to, the field. The solution corresponding to the minimum action is axially symmetric; therefore, if the  $\Phi_n$  are the eigenfunctions of the operator  $\mathcal{D}^2$  in the axial gauge, then the dominant contribution to the sum is made by the term corresponding to the zeroth Landau level. The remaining equation for  $f(z)$  will be one-dimensional:

$$-f'' + \varepsilon f - \frac{1}{2} f^3 = 0.$$

Its solution yields the expressions

$$\varphi = \Phi_{00} f, \quad \Phi_{00} = \exp(-\omega_H \rho^2 / 4), \quad f = 2\varepsilon^{1/2} \text{ch}^{-1} \varepsilon^{1/2} z. \quad (18)$$

Hence we obtain for the exponential density-of-states function the expression:

$$\rho(E) \sim \exp[-(\varepsilon/\Delta)^2], \quad \Delta = (3\gamma\omega_H/16 \cdot 2^{1/2}\pi)^{1/2}. \quad (19)$$

To compute the pre-exponential function in the formula (11), we must compute the product  $D$ . For this purpose, we regularize, as is done in Ref. 3,  $M$  by adding to the Hamiltonian the term  $-\delta\varphi^2$ , which leads to the disappearance of the zero and infinite eigenvalues of  $M$ . The eigenvalue corresponding to shifts parallel to the field will be equal to  $\delta/2$ , the one corresponding to shifts in the direction perpendicular to the field will be equal to  $\delta$ , and the infinite eigenvalue will become equal to  $2/\delta$ . Therefore,

$$D = \lim_{\delta \rightarrow 0} \delta^{-2} \text{Det } M_\delta. \quad (20)$$

The operator  $M_\delta$  is axially symmetric; therefore, let us expand it in terms of the functions having the angular

dependence  $e^{im\theta}$ . For  $m \neq 0$ , one of the terms in the sum  $[(E-H)^{-1} + (E-H^*)^{-1}]$  is much smaller than unity; for  $m=0$  they are equal. Neglecting the small terms, and factoring the resulting determinant into a product of two determinants, we obtain

$$\begin{aligned} \text{Det } M_R &= \text{Det}(1+2\varphi(E-H)^{-1}\varphi)_0 \left[ \prod_m \text{Det}(1+\varphi(E-H)^{-1}\varphi) \right]^2 \\ &= \frac{\text{Det}(E-H+2\varphi^2)_0}{\text{Det}(E-H)_0} \left[ \prod_{m>0} \frac{\text{Det}(E-H+\varphi^2)_m}{\text{Det}(E-H)_m} \right]^2. \end{aligned} \quad (21)$$

For  $\varepsilon \ll \omega_H$ , it is sufficient to limit ourselves in the determinants entering into (20) to the expansion in terms of the functions corresponding to the zeroth Landau level; therefore, in the formula (20)

$$(Y)_m = \int \Phi_{0m} \cdot Y \Phi_{0m} d^2\rho.$$

Using the explicit forms of  $\varphi^2$  and  $\Phi_{0m}$ , we find that  $(\varphi^2)_m = 2^{-m}(\varphi^2)_0$ ; therefore, all the determinants in the product (21), except three, cancel out:

$$\text{Det } M_R = \frac{\text{Det}(E-H-\delta\varphi^2/2)_0}{\text{Det}(E+d^2/dz^2)} \frac{\text{Det}(E-H+2\varphi^2)_0}{\text{Det}(E+d^2/dz^2)} \frac{\text{Det}(E-H-\delta\varphi^2/2)_0}{\text{Det}(E-H)}, \quad (22)$$

where

$$H_0 = -d^2/dz^2 + 2\varepsilon(1+\delta) \text{ch}^{-2} \varepsilon^{1/2} z, \quad (\varphi^2)_0 = 2\varepsilon \text{ch}^{-2} \varepsilon^{1/2} z.$$

All the eigenvalues of the operators entering into the the last quotient in the formula (22), except the discrete, nearly zero ones, are equal. The eigenfunctions and the spectrum of the one-dimensional operators entering into (22) are well known. The first operator in the right member of (22) has one discrete level with energy  $\delta/3$  and a continuous spectrum; the second operator, two discrete levels with energies  $2\delta/5$  and 1 and a continuous spectrum.

Let us compute the contribution from the continuous spectrum. It does not have a singularity at  $\delta=0$ . The eigenfunctions of these operators do not have an echo wave. Let us use their asymptotic forms:

$$\begin{aligned} z \rightarrow -\infty, \quad f_{1,2} \rightarrow e^{i\mu z}, \quad z \rightarrow \infty, \quad f_{1,2} \rightarrow \exp(ikz + \varphi_{1,2}), \\ \varphi_1 = \arg[(ik-1)/(ik+1)], \quad \varphi_2 = \varphi_1 + \arg[(ik-2)/(ik+2)], \end{aligned}$$

where  $f_1$  and  $f_2$  are the eigenfunctions of the first and second operators in the right member of (22).

Let us impose periodic boundary conditions at the walls of a large box of dimension  $L$ . Then  $kL + \varphi(k) = 2\pi n$ , and each quotient in (22) is equal to

$$\prod_n \frac{\lambda_n}{\lambda_n^0} = \exp\left(\sum_n \ln \frac{\lambda_n}{\lambda_n^0}\right) = \exp\left(\int \frac{dk}{2\pi} \varphi'(k) \ln \lambda(k)\right). \quad (23)$$

Evaluating the integral in (23), and multiplying it by the eigenvalues of the discrete spectrum, we obtain

$$\text{Det } M_R = 2^3 \cdot 3 \cdot 5^{-1} \delta^2. \quad (24)$$

Collecting all the factors entering into the expression (11) for the density of states, we obtain

$$\rho(\varepsilon) = 16 \cdot 6^{1/2} \cdot 3^{-1} \varepsilon^{1/2} \omega_H^{-1} \gamma^{-2} \exp[-(\varepsilon/\Delta)^{1/2}]. \quad (25)$$

## II. CYCLOTRON RESONANCE

1. Let us consider the absorption of a high-frequency electromagnetic field in the presence of a strong magnetic field.

The absorption rate is given by the formula

$$I = 2\pi e^2 \mathcal{E}^2 \omega \int j(\omega, \varepsilon) n_f(\varepsilon) d\varepsilon, \quad (26)$$

where  $n_f$  is the Fermi distribution function, while

$$\begin{aligned} j(\omega) &= \int |\langle \psi_i | d | \psi_o \rangle|^2 \delta(\varepsilon_i - \omega - \varepsilon) \delta(\varepsilon_o - \varepsilon) \\ &\times \exp\left[-\frac{V^2}{2\gamma}\right] DV / \int \exp\left[-\frac{V^2}{2\gamma}\right] DV. \end{aligned} \quad (27)$$

Here  $\psi_o$  and  $\psi_i$  are the wave functions of an electron with energies  $E_o$  and  $E_i$  in the potential  $V(\mathbf{r})$  and  $d$  is the dipole-interaction operator. At high electron densities or high temperatures (i.e., for  $\varepsilon_f, T \gg \omega_H$ ), the semiclassical procedure is suitable for the description of the electrons, and in this case the absorption line has the Lorentz shape with width

$$\tau^{-1} = 2\pi \rho_f \gamma. \quad (28)$$

At low electron concentrations and temperatures (i.e., for  $\varepsilon_f, T \ll \omega_H$ ) all the electrons are at the zeroth Landau level. If, however,  $\varepsilon_f$  or  $T$  is higher than  $\Delta$ , then the absorption-line width is of the order of  $\Delta$ . Below we shall derive the explicit form of the tails of this line. If  $\varepsilon_f$  lies in the forbidden band (i.e., if  $\omega_H > |\varepsilon_f| > \Delta, T < \Delta$ ), then the cyclotron resonance line splits up into several sublevels. The position and widths of these lines depend on the temperature. The integral (26) over  $\varepsilon$  has a maximum at  $\varepsilon = \varepsilon_f$  if  $T \ll \Delta_1$  and at  $\varepsilon = 4\Delta^3/9T^2(3D), \varepsilon = \Delta^2/2T(2D)$  when  $T > \Delta_1$ , where  $\Delta_1 = (\Delta^3/\varepsilon_f)^{1/2}(3D), \Delta_1 = \Delta^2/\varepsilon_f(2D)$ .

At  $T < D$  the electrons reside in deep potential-fluctuation wells that differ little from the optimal wells whose potential is given by the formulas (6) and (7). This potential lifts the infinite-multiple degeneracy of the first Landau level, and leads to the splitting of the cyclotron-resonance peak. The location of each peak is determined by the energy level in the potential (6). In order to find the shape of the peaks, we should minimize  $\int V^2 d^4\mathbf{r}$  with the additional conditions  $\text{Det}(E-H) = 0$  and  $\text{Det}(E+\omega-H) = 0$ . As a result, we find that

$$V = \varphi^2 + \varphi_1^2, \quad (29)$$

where  $\varphi$  and  $\varphi_1$  are found from the set of equations

$$-\mathcal{D}^2 - E - (\varphi^2 + \varphi_1^2)\varphi = 0, \quad -\mathcal{D}^2 - E - \omega - (\varphi^2 + \varphi_1^2)\varphi_1 = 0. \quad (30)$$

The positions of the peak centers can be found by solving (30) if  $\varphi_1^2$  is neglected in comparison with  $\varphi^2$ ;  $\varphi_1^2$  can be taken into account in the vicinity of the center of a peak (i.e., in the region  $\omega - \omega_m \ll \varepsilon$ ) with the aid of perturbation theory.

2. The cyclotron resonance in the two-dimensional case. The functions  $\varphi$  and  $\varphi_1$  in the system (30) are proportional to the eigenfunctions of the operator  $\mathcal{D}^2$ :

$$\varphi = a_0 \Phi_{0m}, \quad \varphi_1 = a_1 \Phi_{1m}.$$

For the coefficients  $a_0^2$  and  $a_1^2$ , we obtain a linear system of equations; e.g., for the  $m=-1$  case we have

$$2\varepsilon = a_0^2 + a_1^2, \quad 4\varepsilon_1 = a_0^2 + 2a_1^2, \quad (31)$$

where  $\varepsilon_1 = E + \omega - 3\omega_H/2 = \varepsilon + \omega - \omega_H$ .

Solving this system, and substituting the values obtained into the formula (29), we obtain an expression for

the index of the exponential function:

$$\int V^2 d^2r/2\gamma = 2\pi[\varepsilon^2 + A_m(\omega - \omega_m)^2]/\gamma\omega_H, \quad (32)$$

where

$$\omega_m = \omega_H + \beta_m \varepsilon, \quad \beta_m = 1 - (m+2)2^{-m-2}/m+1, \\ A_m = 4 \left[ \frac{(2m-1)!!}{2m!} \frac{3m+2}{(m+1)^2} - 2^{-2m-2} \left( \frac{m+2}{m+1} \right)^2 \right]^{-1}$$

for  $m = -1$ ,  $\omega_{-1} = \omega_0$ , and  $A_{-1} = A_0$ .

The height of the peak is determined by the matrix element entering into (27); in the zeroth approximation in the potential, this matrix element is nonzero only for the transitions with  $m = -1$ . The transitions with  $m = +1$  occur in the first-order approximation in the optimal potential; the remaining transitions occur only as a result of the unsymmetric fluctuations of the potential, and therefore have the additional smallness

$$|\langle \psi_1 | d | \psi_0 \rangle|^2 = \alpha_m \omega_H^{-1}, \quad \alpha_{-1} = 1/2, \quad (33) \\ \alpha_1 = (\varepsilon/40\omega_H)^2; \quad m \neq \pm 1, \quad \alpha_m \sim (\Delta/\omega_H)^2.$$

The dependence of the pre-exponential function on  $\omega$  can be neglected in the vicinity of the maximum of each peak. For not very large  $m$  the neighboring peaks do not overlap; therefore, the total intensity of the peak is equal to the product of the density of states and the matrix element, which allows us to calculate the pre-exponential function in (27). It now remains to evaluate the  $E$  integral in the expression (26). At low  $T$  ( $T < \Delta_1$ ),  $n_f(\varepsilon)\rho(\varepsilon)$  has at  $\varepsilon = \varepsilon_f$  a sharp peak whose width is smaller than  $\Delta$ ; therefore, the integration amounts to the replacement of  $\varepsilon$  by  $\varepsilon_f$ :

$$I = 2\pi^{3/2} \varepsilon_f^2 N \Delta^{-1} \alpha_m A_m^{1/2} \exp[-A_m \Delta^{-2} (\omega - \omega_m)^2], \quad (34) \\ \omega_m = \omega_H + \beta_m \varepsilon_f,$$

where  $N$  is the total number of electrons.

In the other temperature region,  $\Delta_1 < T < \Delta$ ,  $n_f \rho(\varepsilon)$  has at  $\varepsilon = \Delta^2/2T$  a Gaussian peak of width  $\Delta$ ; computing the integral (26), we obtain

$$I = 2\pi^{3/2} \varepsilon_f^2 \Delta^{-1} N (1 + \beta_m^2 A_m)^{-1/2} \alpha_m A_m^{1/2} \exp(-A_m) \\ \times [1 - (1 + \beta_m^2 A_m^{-1})^{-1}] (\omega - \omega_m)^2 / \Delta^2, \quad \omega_m = \omega_H + \beta_m \Delta^2 / 2T. \quad (35)$$

Thus, the cyclotron resonance line splits up into several lines, each of which has a Gaussian shape. As  $m$  increases, the peak spacing rapidly decreases, and they begin to overlap, but the distance between the first peaks is much greater than their widths.

All the peaks merge into one broad peak of width  $\Delta$  at  $\omega_H > T > \Delta$ . For  $\omega - \omega_H \gg \Delta$ , the tail of this line is determined by the rare potential fluctuations, and the index of the exponential function in this dependence is given by the formula (32) ( $m = -1$ ):

$$I \sim \exp[-4\pi(\omega - \omega_H)^2/\gamma\omega_H]. \quad (36)$$

This formula is also valid at low temperatures for frequencies far from  $\omega_H$  (in comparison with the level spacing).

3. *The cyclotron resonance in the three-dimensional case.* A qualitative difference between the three- and two-dimensional cases is connected with the fact that, besides the discrete spectrum, there is a continuous spectrum in the three-dimensional case. At low tem-

peratures and low electron concentrations the electrons occupy largely the discrete states below the zeroth Landau level. For  $\omega \sim \omega_H$ , the transitions into the continuous spectrum of the zeroth level yield a weak—in terms of the quantity  $\varepsilon/\omega_H$ —smooth background, which will be neglected below. The transitions to the first level yield, besides the peaks characteristic of films, a broad peak lying above these peaks. The discrete levels lying below the first Landau level are not stationary levels, but their width,  $\gamma\omega_H^{1/2}$ , is much smaller than the width of the isolated peak, and shall be neglected below.

Let us, to begin with, compute the parameters of the discrete peaks. The dependence of the functions  $\varphi$  and  $\varphi_1$  of the system (30) on  $\rho$  is given by the eigenfunctions of the operator  $\mathcal{D}^2$ :

$$\varphi = f_0(z) \Phi_{00}(\rho), \quad \varphi_1 = f_1(z) \Phi_{10}(\rho).$$

For the functions  $f_0$  and  $f_1$ , we obtain a system of equations; for example, in the  $m = -1$  we have:

$$[2(\varepsilon - d^2/dz^2) - (f_0^2 + f_1^2)] f_0 = 0, \quad (37) \\ [4(\varepsilon - d^2/dz^2) - (f_0^2 + f_1^2)] f_1 = 0.$$

The peak centers are found by solving the first equation of the system (37) with  $f_1 = 0$ , and determining the eigenvalues of the second equation:

$$\omega_m = \omega_H + \beta_m \varepsilon, \quad \beta_m = 1 - \{[1 + 2^{-m+1}(m+2)/m+1]^{1/2} - 1\}^2/4, \\ \omega_{-1} = \omega_0, \quad \alpha = \{[1 + 2^{-m+1}(m+2)/m+1]^{1/2} - 1\}/2. \quad (38)$$

For  $\omega - \omega_m \ll \varepsilon$ , we can solve the system (37), using perturbation theory to take account of the term  $f_1^2$ . Substituting the expressions obtained for  $\varphi^2$  and  $\varphi_1^2$  into the formula (29) for the potential, we obtain the index of the exponential function for the dependence  $j(\omega)$ :

$$\int V^2 d^2r/2\gamma = (\varepsilon/\Delta)^{3/2} + (\varepsilon/\Delta)^{3/2} B_m (\omega - \omega_m) \Delta^{-1}, \quad (39)$$

where

$$B_m = \frac{3\pi^{1/2}}{8} \frac{(m+2)!}{(2m-1)!!(m+1)^2} \frac{\Gamma^2(\alpha)}{\Gamma^2(\alpha+1/2)} \frac{\alpha}{\alpha+1/2} \\ \times \left[ \frac{3m+2}{m+1} \frac{\Gamma(2\alpha)}{\Gamma(2\alpha+1/2)} - \frac{1}{2} \frac{[(m+2)!]^2}{2m!} \frac{\Gamma(\alpha+1)}{\Gamma(\alpha+3/2)} \frac{1}{\alpha+1} \right]^{-1}$$

As in the two-dimensional case, in the low-temperature region, the integral over  $\varepsilon$  is determined by the region in the vicinity of  $\varepsilon_f$ , and the absorption rate is equal to

$$I = \pi \varepsilon_f^2 \varepsilon_f^2 N \Delta^{-3/2} \alpha_m B_m g(\omega - \omega_m) / \Omega, \quad (40)$$

where

$$g(\zeta) = \begin{cases} \zeta > 0, & \frac{\exp(-B_m)\zeta}{1-2B_m/3} - \frac{4B_m/3}{1-4B_m^2/9} \exp(-3\zeta/2) \\ \zeta < 0, & (1+2B_m/3)^{-1} \exp(-B_m\zeta) \end{cases}$$

$$\Omega = \Delta^{3/2} \varepsilon_f^{-3/2}, \quad \omega_m = \omega_H + \beta_m \varepsilon_f.$$

Here  $\alpha_m$  is the dimensionless matrix element's value, which is determined in much the same way as (33):

$$\alpha_{-1} = \frac{\pi^{3/2} \Gamma^2[(5^{3/2}+1)/4] \Gamma(5^{3/2}/2)}{4 \Gamma^2[(5^{3/2}+3)/4] \Gamma[(5^{3/2}-1)/4]}, \quad \alpha_1 \sim \left( \frac{\varepsilon}{\omega} \right)^2 \\ \alpha_m \sim (\Delta/\omega)^2, \quad m \neq \pm 1.$$

At temperatures  $\Delta_1 \ll T \ll \Delta$ , the product  $n_f(\varepsilon)\rho(\varepsilon)$  has at the point  $\varepsilon = 4\Delta^3/9T^2$  a peak of width  $(16\Delta^3/9T^2)^{1/2}$ . This width is much greater than  $\Delta^{3/2} \varepsilon^{-1/2}$ ; therefore, on being integrated over  $\varepsilon$ , the lines broaden and their

shape becomes Gaussian:

$$I = (\nu/2) \pi^{1/2} e^2 \mathcal{E}^2 N (T/\Delta^2)^{1/2} \alpha_m \beta_m^{-2} \exp[-\beta_m^2 9T(\omega - \omega_m')^2 / 16\Delta^2], \quad (41)$$

$$\omega_m' = \omega_H + 4\beta_m \Delta^2 / 9T,$$

where the  $\beta_m$  are the same quantities defined in the formula (38).

The lines have an interesting shape at  $T \sim \Delta_1$ . The right (i.e.,  $\omega > \omega_m$ ) line edge is determined by the formula (40), while the left ( $\omega < \omega_m$ ) edge is determined by the formula (41). At  $T < 2\Delta_1/3$ ,  $\omega_m' < \omega_m$  and the line width decreases sharply.

Let us now compute the absorption due to the transition into the continuous spectrum above the first Landau level. Since the width of this peak is much greater than  $\Delta$ , we can neglect the deviation of the potential from the optimal potential. The transition probability is determined by the matrix element  $|\langle \psi_1 | d | \psi_0 \rangle|^2$  connecting the electron bound state at the zeroth Landau level and the continuous-spectrum bound state of the first Landau level.

As in the case of the discrete spectrum,  $\psi_1 = f_1(z) \Phi_{1-1}(\rho)$  and  $\psi = f_0(z) \Phi_{00}(\rho)$ ,  $f_1$  being the solution, normalized to unit flux density, of the linearized second equation of the system (37).

The cyclotron absorption is expressible in terms of the matrix element connecting the functions  $f_1$  and  $f_0$  in the form

$$I = \pi e^2 \mathcal{E}^2 N g [(\omega - \omega_H)/\varepsilon],$$

$$g = \left| \int f_0(z) f_1(z) dz \right|^2 / \int f_0^2(z) dz [2(\omega - \omega_H - \varepsilon)]^{1/2}.$$

As for the function  $g$ , its asymptotic form can be found:

$$g \left[ \frac{(\omega - \omega_H)}{\varepsilon} \right] = \frac{\pi^2 \varepsilon^{1/2}}{(\omega - \omega_H)^{1/2}} \exp \left\{ \frac{-\pi(\omega - \omega_H)}{\varepsilon} \right\}^{1/2},$$

$$(\omega - \omega_H)/\varepsilon \gg 1.$$

Using the completeness condition for the solutions of the second equation in (37) and the fact that, besides the continuous spectrum (of the functions  $f_1$ ), this equation has only one discrete level, we obtain the integral of  $g$  corresponding to the total intensity of the peak:

$$\int g d\omega = 1 - 2\alpha_{-1}.$$

## CONCLUSION

A suitable object for the experimental verification of the results obtained is a semiconductor with a comparatively high concentration of isovalent impurities. The number of charged impurities should be small. Because of the long-range character of the Coulomb potential, charged impurities cannot be considered to be white noise, and they give rise to another form of the density of states.<sup>5</sup> Furthermore, the number of electrons in the conduction band and their temperature should be low, so that the electrons will occupy only the deep fluctuation levels. The magnitude of the random potential can be roughly estimated from the mean free time  $\tau$  at not

very low temperatures or densities, when the electrons exist as free carriers, or from the exciton-resonance-line width. Thus, the conditions for the observation of the "tail" of the density of states in a strong magnetic field and the cyclotron resonance at the fluctuation levels can be written in the form

$$ehH/mc \gg h/\tau \gg T.$$

The argument of the exponential function in the expression for the density of states in a strong magnetic field can be computed in the same way, and has the same form, as in the one-dimensional case. But the pre-exponential factor has a different form, since the three-dimensional fluctuations are essential to it.

The cyclotron resonance at the fluctuation levels has several peaks connected with transitions to levels with different values of the magnetic quantum number  $m$ . The uppermost and lowest peaks correspond to transitions into  $m = -1$  states of the continuous and discrete spectra respectively. The ratio of their integrated intensities is  $1 - 2\alpha_{-1}$ , and they are more intense than the remaining peaks lying between them. The width of the top peak is of the order of the peak spacing; that of the lower peaks is smaller than their spacing, but can be greater than the cyclotron resonance width at high temperatures. The distance from the bottom peak to the lower edge of the top peak is equal to the bound-state energy  $\varepsilon_1$  of an electron at the first Landau level. This energy is smaller than the bound-state energy  $\varepsilon_0$  of an electron at the zeroth Landau level in the same potential well:  $\varepsilon_1 = 0.38\varepsilon_0$ . Therefore, even the bottom peak has a frequency somewhat higher than the cyclotron-resonance frequency in the same magnetic field at high temperatures, specifically, by the amount  $\varepsilon_0 - \varepsilon_1 = 1.63\varepsilon_1$ .

In two-dimensional systems, e.g., in MOS systems, continuous-spectrum states do not occur. The density of states has Gaussian tails on both sides of each Landau level, and cyclotron resonance does not have a top broad peak.

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