Plasma oscillations of multicomponent two dimensional systems

R. Z. Vitlina and A. V. Chaplik

L. V. Kirenskiĭ Institute of Physics, Siberian Branch of the Academy of Sciences of the USSR, Krasnoyask
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An investigation is made of plasma waves in films, layer structures or inversion channels in those cases when an electron plasma has several components. These components are groups of electrons differing either in respect of the quantum number of transverse motion or in respect of the layer number. The spectrum of plasma oscillations of such systems includes not only the usual wave with the square-root dispersion law, but also additional branches some of which resemble ion-acoustic waves in a gas plasma whereas others are analogous to excitons. The characteristics of the Landau damping and of the optical absorption spectra of such systems are explained.

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1. INTRODUCTION

Plasma waves in two-dimensional electron systems have been investigated quite thoroughly both theoretically and experimentally. The main objects in these investigations have been electrons in inversion channels of metal–insulator–semiconductor (MIS) structures and, to a lesser degree, deliberately constructed periodic structures (multilayer superlattices in which thin conducting layers are separated by practically insulating intervals). In both cases it is assumed that an electron gas is in an ultraquantum state with respect to the motion which is transverse to the layers, i.e., that all the electrons occupy the lowest transverse quantization level. However, modern technology of preparation of thin films and layer structures makes it possible to vary easily the parameters of such systems. The number of layers, their thickness, degree of doping, and—consequently—the carrier density can all be varied within very wide limits. Therefore, it is interesting to consider systems which are essentially two-dimensional but exhibit to some extent a transverse degree of freedom. In the case of a quantum film this means that several transverse layers are populated and that transitions between them have to be allowed for. In the case of a layer structure one may have to allow for the tunneling between the layers.

We shall consider plasma waves in films and layer structures in those cases when an electron plasma has several components. The components are groups of electrons differing either in respect of the quantum number of transverse motion (film or inversion channel) or in respect of the number of layers (layer structure). The spectrum of plasma waves in such systems has a number of interesting features. New oscillation branches appear and some of them resemble ion-acoustic waves in a gas plasma, whereas others are analogous to excitons. The Landau damping in such systems and the optical absorption spectra of such systems also have special properties.

In Sec. 2 we shall derive general formulas which give the dispersion law of plasma waves obtained in the self-consistent field approximation for systems with the spectrum of the kind expected for a quantum film. The derivation will be made using a matrix dielectric function. Next, we shall investigate two-level and two-layer systems (the rank of the characteristic determinant is generally n^2, where n is the number of plasma components). In Sec. 3 we shall consider a system of spatially separate plasma layers, whereas in Sec. 4 we shall deal with plasma modes in a quantum film. In Sec. 5 we shall explain the characteristics of the optical absorption spectra of the systems under consideration, and in Sec. 6 we shall give the main results in the specific case of inversion channels in MIS structures.

2. PLASMA WAVES IN MULTICOMPONENT SYSTEMS CONSIDERED IN THE SELF-CONSISTENT-FIELD APPROXIMATION

We shall consider a system whose one-electron spectrum is

$$W_n(p)=E_n+p^2/2m, \tag{1}$$

where E_n are the transverse energy levels corresponding to wave functions $\varphi_n(z)$; $\mathbf{p}$ is the two-dimensional momentum. We shall seek the linear response to a perturbation

$$U(z,k)\exp\{i(\mathbf{k}\boldsymbol{\rho}-\omega t)\}.$$

Here, z and $\boldsymbol{\rho}$ are the coordinates in the transverse and longitudinal directions, respectively. Standard perturbation theory calculations yield corrections to the wave functions of the system, and then the correction to the electric charge density caused by the perturbation. If we neglect the retardation effects, which are unimportant in this case with the exception of a narrow range of anomalously low values of k, we find that the self-consistent field equation is simply the Poisson equation for a Fourier component of the potential $U(z,k)$:

$$\frac{d^2U}{dz^2}-k^2U=-\frac{4\pi e^2}{\varepsilon(z)}\sum_{n,m}U_{mn}(\mathbf{k})\varphi_n(z)\varphi_m(z)\times\sum_{\mathbf{q}}\frac{f_n(\mathbf{q})-f_m(\mathbf{k}+\mathbf{q})}{W_n(\mathbf{q})-W_m(\mathbf{k}+\mathbf{q})+\omega+i\delta}. \tag{2}$$

Here, U_{nm} are the matrix elements $U(z)$ of the transverse motion functions (selected to be real); $f_n(q)$ are the Fermi occupation numbers; $\varepsilon(z)$ is the permittivity which, in the case of a layer system, has different values for different ranges of z. If the solution of Eq. (2) is expressed in terms of a Green function and the ma-

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trix elements are obtained for the functions $\varphi_n(z)$, the result is a closed system of equations for the quantities $U_{nm}(\mathbf{k})$. Its specific form depends on the function $\varepsilon(z)$. We shall give here the results for the simplest case when $\varepsilon=\text{const}$, which corresponds to a quantum film immersed in a homogeneous insulating medium with the same value of ε as the permittivity of the film material. This describes approximately multilayer GaAs–$Ga_xAl_{1-x}As$ structures.[1] The strongly inhomogeneous case of an MIS structure will be considered in Sec. 6. If $\varepsilon=\text{const}$, the Green function of Eq. (2) is

$$G(z,z_0)=\frac{1}{2k}\exp(-k|z-z_0|),$$

and we obtain the following system of equations

$$U_{ij}+\frac{2\pi e^2}{\varepsilon k}\sum_{nm} I_{ij,nm}(k)\Pi_{nm}(k)U_{nm}=0, \tag{3}$$

where

$$\left.\begin{aligned} I_{ij,nm}(k)&=\int_{-\infty}^{+\infty}\varphi_i(z)\varphi_j(z)\exp(-k|z-z_0|)\varphi_n(z_0)\varphi_m(z_0)\,dz\,dz_0,\\ \Pi_{nm}(k,\omega)&=-\sum_{\mathbf{q}}\frac{f_n(\mathbf{q})-f_m(\mathbf{k}+\mathbf{q})}{W_n(\mathbf{q})-W_m(\mathbf{k}+\mathbf{q})+\omega+i\delta}.\end{aligned}\right\} \tag{4}$$

The spectrum of plasma waves is obtained by equating to zero the determinant of the system (3), whose rank is equal to the square of the number of transverse energy levels. This condition generalizes the familiar equation for plasma waves $\varepsilon(\omega,k)=0$ to the case of the matrix dielectric function.[2,3] We note that allowance for the dependence of the quantities $I_{ij,nm}$ on the momentum k corresponds to allowance for the finite thickness of a plasma layer. In the limiting case of a film of zero thickness we have $\varphi_n(z)=[\delta(z)]^{1/2}$ and then $I_{1111}=1$. The spectrum of plasma waves in a multilayer superlattice can be obtained in the same approximation (see Ref. 4): we shall do this assuming that $\varphi_n^2(z)=\delta(z-z_n)$; then, only the quantities $I_{nn,mm}=\exp(-k|z_n-z_m|)$ differ from zero. The matrix $I_{ij,nm}$ has the obvious transposition symmetry

$$I_{ij,nm}=I_{ji,mn}=I_{ij,mn}=I_{nm,ij}.$$

Moreover, in the case of a quantum film which is symmetric to its central plane the wave functions $\varphi_n(z)$ are characterized by a definite parity. Therefore, the matrix elements relating one even state to three odd states, and vice versa, all vanish. For example, we find that

$$I_{i,i\pm1,nn}=I_{ii,n,n\pm1}=0.$$

The quantities $\Pi_{nm}(k,\omega)$ form the matrix of a polarization operator which describes renormalization of the Coulomb interaction because of the dynamic screening. In terms of the diagram technique, Eq. (3) is equivalent to summation of electron loops and the zeros of the characteristic determinant coincide with poles of a two-particle Green function.

We shall conclude this section by quoting the solution of the problem of two-dimensional plasma waves in a one-component plasma of finite thickness. A physical model of such a system is a quantum film in which only the lowest transverse energy level is populated. We shall consider an arbitrary dependence of the electron density on the transverse coordinate. The problem is solved in Ref. 5 in the special case of an exponential distribution. Let us assume that the electron density is distributed in accordance with the law $\varphi_1^2(z)\equiv\rho(z)$. Then,

$$\left.\begin{aligned} I_{1111}&=I(k)=\int\rho(z)\rho(z_0)\exp(-k|z-z_0|)\,dz\,dz_0,\\ \Pi_{11}&=\frac{m}{\pi}\left[1-\left(1-\frac{k^2v_0^2}{\omega^2}\right)^{-1/2}\right],\end{aligned}\right\} \tag{5}$$

where v_0 is the Fermi velocity (we shall consider only the case of a degenerate electron gas). The dispersion equation for the $k\ll mv_0$ case has the form

$$1+\frac{2\pi e^2 I(k)}{\varepsilon k}(1-(1-k^2v_0^2/\omega^2)^{-1/2})=0 \tag{6}$$

and it differs from the equation for the case of zero-thickness layer (see Ref. 6) by the renormalization of the charge by the form factor $I(k)$. In the limit of long waves the correction to the plasmon frequency is negative [because $I(k)$ is maximal at $k=0$] and its relative order is kL, where L is the characteristic film thickness,

$$\omega^2=\frac{2\pi e^2 Nk}{\varepsilon m}(1-\text{const}\cdot kL),\quad \text{const}\sim1, \tag{7}$$

N is the surface density of particles. If $\rho(z)\sim z^2\exp\{-az\}$ and $\omega\gg kv_0$, the results of Gersten[5] follow from Eqs. (5) and (6).

3. SPATIALLY SEPARATE PLASMA LAYERS

The simplest case of a two-component two-dimensional plasma is represented by a system of two quantum films separated by an insulating gap. We shall assume that in each film only the lowest level is occupied by electrons and the tunneling across the insulator is negligible (allowance for the tunneling makes the system fully equivalent to a film with two populated levels, which is considered in the next section). Assuming also that the layers are infinitesimally thin, we find that $I_{11,11}=I_{22,22}=1$ and $I_{11,22}=\exp(-k\Delta)$, where Δ is the distance between the films. The dispersion equation for coupled waves is then

$$\left[1+\frac{ka_1R_1}{2(R_1-1)}\right]\left[1+\frac{ka_2R_2}{2(R_2-1)}\right]=e^{-2k\Delta},\quad R_{1,2}=(1-k^2v_{1,2}/\omega^2)^{1/2}, \tag{8}$$

where $a_{1,2}$ are the effective Bohr radii and $v_{1,2}$ are the Fermi velocities of the electrons in the films. Equation (8) describes two branches of plasma oscillations. If $k\Delta, ka_{1,2}\ll1$, one of these branches represents cophasal oscillations of particles in both films and it is characterized by the usual (for two-dimensional plasmons) square-root dispersion law

$$\omega_+^2=\frac{2\pi e^2 k}{\varepsilon}\left(\frac{N_1}{m_1}+\frac{N_2}{m_2}\right), \tag{9}$$

where $N_{1,2}$ are the surface densities of the particles in the films. The condition $\omega\gg kv_{1,2}$ is satisfied in this range of wavelengths. On the other hand, if $k\Delta\gg1$, the coupling between the films disappears and Eq. (8) yields two-independent two-dimensional plasma waves. In each of them the phase velocity of plasmons is greater than their "intrinsic" Fermi velocity, i.e., of $\omega_1 > kv_1$ and $\omega_2 > kv_2$ for any value of k no matter how large. If $ka_{1,2} \gg 1$, the dependence $\omega_{1,2}(k)$ becomes of the type characterizing zero sound $\omega_{1,2} \approx kv_{1,2}$. Therefore, if $v_1 \neq v_2$, one of the branches must exhibit the Landau damping even at absolute zero: plasmons in the film with the lower Fermi velocity are damped by the interaction with the electrons in the other film. This is manifested formally by the fact that one of the quantities $R_{1,2}$ becomes imaginary. We can easily see that the branch $\omega_+(k)$ remains undamped for any value of k, irrespective of the parameters of the system. We shall now show that for a definite relationship between Δ, $v_{1,2}$, and $a_{1,2}$ the second solution is also undamped in a finite interval of wave numbers.

In the limit $k \to 0$ the second branch $\omega_-(k)$ describes antiphasal oscillations of electrons in the films and exhibits the acoustic dispersion $\omega_- = sk$. Substituting this value of ω in Eq. (8) and going to the limit $k \to 0$, we obtain the following equation for s:

$$a_1 \frac{(s^2-v_1^2)^{1/2}}{(s^2-v_1^2)^{1/2}-s} + a_2 \frac{(s^2-v_2^2)^{1/2}}{(s^2-v_2^2)^{1/2}-s} + 4\Delta = 0, \quad s>0. \tag{10}$$

To be specific, we shall consider the case when $v_1 > v_2$. Equation (10) gives the real and positive value of s if

$$\Delta > \frac{1}{4} \frac{a_2(v_1^2-v_2^2)^{1/2}}{v_1-(v_1^2-v_2^2)^{1/2}} \equiv \Delta_0. \tag{11}$$

We can thus see that if $\Delta > \Delta_0$, the damping of the branch ω_- vanishes exactly for sufficiently low values of k. In order to determine the limit of the interval of the wave numbers in which $\mathrm{Im}\,\omega_- \equiv 0$, we must clearly find the intersection of the $\omega_-(k)$ curve with the straight line $\omega = kv_1$. In Eq. (8) we shall assume that $R_1 = 0$ and then we find that the point of intersection[1)] k_c is given by

$$2k_c\Delta_0 = 1 - e^{-2k_c\Delta}. \tag{12}$$

If $\Delta \gg \Delta_0$, we find that $k_c \to 1/2\Delta_0$, whereas if $\Delta - \Delta_0 \ll \Delta_0$, we obtain $k_c \approx (\Delta - \Delta_0)/\Delta^2$. The Landau damping of the ω_- wave is "activated" for $k > k_c$.

Equation (10) is easily solved in the $s \gg v_{1,2}$ case. It is then found that

$$\omega_-^2 = \frac{4\pi e^2\Delta}{\varepsilon} \frac{N_1N_2k^2}{N_1m_2+N_2m_1}, \quad k\Delta \gg 1, \tag{13}$$

and this assumption is justified if $\Delta \gg a_{1,2}$ (the particle densities are assumed to be quantities of the same order of magnitude). If $\Delta < \Delta_0$, the wave ω_- is damped for all values of k. This damping is weak in the sense that $\mathrm{Im}\,\omega \ll \mathrm{Re}\,\omega$, if $v_1 \gg v_2$ (for example, when the electron densities in the films differ strongly).

We shall now give the result for the most realistic experimental case $a_{1,2} \ll \Delta$, $ka_{1,2} \ll 1$:

$$\omega_-^2 = \frac{2\pi e^2 N_2 k}{\varepsilon m_2}(1-e^{-2k\Delta}) \left[1 - i\frac{v_2}{v_1} \frac{ka_1 \exp\{-2k\Delta\}}{[ka_2(1-e^{-2k\Delta})]^{1/2}}\right]. \tag{14}$$

As expected, the damping decreases exponentially in the range $k\Delta \gg 1$. The relative smallness of $\mathrm{Im}\,\omega_-$ is ensured by the condition $kv_1 \gg \omega$ (a small proportion of the particles is in phase with the wave).

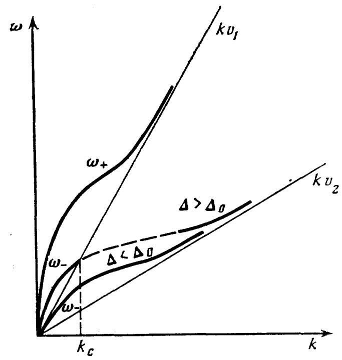

FIG. 1.

It should be stressed that Eqs. (11) and (12) give the exact criterion for the existence of an undamped part of the branch ω_-. Naturally, this criterion is governed only by the parameters of the system and not by the value of ω/kv and the waves described by Eqs. (13) and (14) cannot exist simultaneously (see Ref. 7).

A special situation occurs when $v_1 = v_2$. We then have $\Delta_0 = 0$ [see Eq. (11)] and $k_c \to \infty$, i.e., the Landau damping vanishes in both branches for all values of k. In fact, Eq. (8) with $v_1 = v_2$ can be solved exactly and both solutions satisfy the condition $\omega > kv$. If $k\Delta \ll 1$ and $\Delta/a_{1,2}$ has an arbitrary value, $\omega_+(k)$ is described by Eq. (9) with $N_1 = N_2$ whereas ω_- is given by

$$\omega_-(k) = kv\frac{\Delta+b}{(2\Delta b+b^2)^{1/2}}, \quad b \equiv \frac{a_1+a_2}{4}. \tag{15}$$

Thus, in all the cases discussed above the branch ω_- is analogous, in respect of the dependence on k and also the damping mechanism, to ionic sound in a gaseous plasma. One should simply bear in mind that for the same sign of charge carriers in the films the wave ω_- corresponds to antiphasal oscillations of the particles, whereas for a pair of n- and p-type films it corresponds to cophasal oscillations.

The dispersion curves $\omega_\pm(k)$ are shown schematically in Fig. 1. The strong damping range is identified by a dashed curve.

We can easily see that in the case of a system of n layers (again without allowance for transitions from layer to layer) the spectrum of plasma oscillations consists of n branches and one of them obeys $\omega \propto k^{1/2}$, whereas others obey $\omega \sim k$ in the limit $k \to 0$. For example, in a system of three identical films distributed equidistantly with intervals Δ, we obtain $\omega_1^2 = 3\omega_0^2$, $\omega_2^2 = 2k\Delta\omega_0^2$, $\omega_3^2 = \frac{2}{3}k\Delta\omega_0^2$, where $\omega_0^2 \equiv 2\pi e^2 Nk/\varepsilon m$.

4. PLASMA WAVES IN A QUANTUM FILM

In the case of plasma oscillations in a quantum film with several populated transverse energy levels there is a basically new feature which is the possibility of transitions between the levels. We shall write down the characteristic determinant for a two-level system on the assumption that the film in question is symmetric and that the parity selection rules given in Sec. 2 apply. We shall introduce the matrix motation in accordance with the following scheme: $(11)=1$, $(12)=2$, $(21)=3$, $(22)=4$. We then obtain

$$\begin{vmatrix} 1+\gamma I_{11}\Pi_1 & 0 & 0 & \gamma I_{14}\Pi_4 \\ 0 & 1+\gamma I_{22}\Pi_2 & \gamma I_{23}\Pi_3 & 0 \\ 0 & \gamma I_{32}\Pi_2 & 1+\gamma I_{33}\Pi_3 & 0 \\ \gamma I_{41}\Pi_1 & 0 & 0 & 1+\gamma I_{44}\Pi_4 \end{vmatrix}=0, \qquad (16)$$

where $\gamma=2\pi e^2/\varepsilon k$. It is clear from Eq. (16) that the dispersion equation has two groups of solutions. One of them describes zero-gap plasma-wave excitations, similar to those investigated in the preceding section:

$$1+\gamma(I_{11}\Pi_1+I_{44}\Pi_4)+\gamma^2\Pi_1\Pi_4(I_{11}I_{44}-I_{14}^2)=0. \qquad (17)$$

We can easily show that the quantity $D(k)=I_{11}I_{44}-I_{14}^2$ vanishes linearly in the limit $k\to 0$ and is positive for $k>0$ (Bunyakovskiĭ–Schwarz inequality).

The derivative dD/dk at $k=0$, denoted below by $D'(0)$, is of the same order of magnitude as the film thickness L and it governs the criterion of existence of an un damped branch of the ω_- type. This criterion is obtained from Eq. (10) by substituting $a_1=a_2=a$ and replacing 4Δ with $2D'(0)$. We shall express the result in terms of the surface densities of particles at the first and second film levels $N_{1,2}$ (we shall assume that $N_2<N_1$):

$$(1-N_2/N_1)[1+a/2D'(0)]^2<1. \qquad (18)$$

If $D'(0)\sim L\gg a$, the dispersion law $\omega_-(k)$ is readily found:

$$\omega_-^2(k)=\frac{2\pi e^2 D'(0)k^2}{\varepsilon m}\frac{N_1N_2}{N_1+N_2}. \qquad (19)$$

In the case of a GaAs film of thickness 200 Å the branch $\omega_-(k)$ exists beginning from electron densities (4–5) $\times 10^{12}$ cm^{-2}. Naturally, for any electron density there is a branch ω_+ with the square-root dispersion law at low values of k. It is interesting to note that ω_+^2 is proportional to the total number of particles only in the limit $kL\to 0$. If the film thickness is much greater than the effective Bohr radius a, there is a range of wavelengths where $ka\ll kL<1$. The inequality $ka\ll 1$ ensures that the condition $\omega\gg kv_{1,2}$ is satisfied and then we have

$$\omega_+^2(k)=\frac{2\pi e^2 k}{\varepsilon m}[I_{11}(k)N_1+I_{44}(k)N_2]. \qquad (20)$$

Thus, for a given value of k, the plasmon frequency is governed by the total electron density $N=N_1+N_2$ and by the separation between the film levels $E_2-E_1=\Omega$, because the occupation numbers N_1 and N_2 are functions of N and Ω.

An additional solution of Eq. (16) is related to the nondiagonal elements of the matrix of the polarization operator Π_2 and Π_3. It follows from the properties of the matrix I that $I_{22}=I_{23}=I_{32}=I_{33}=J(k)$. Bearing this point in mind, we obtain the dispersion equation

$$1+\frac{2\pi e^2 J(k)}{\varepsilon k}[\Pi_2(\omega,k)+\Pi_3(\omega,k)]=0. \qquad (21)$$

In the limit $k\to 0$ the value $J(k)$ vanishes because of the orthogonality of the functions $\varphi_n(z)$, whereas Π_2 and Π_3 remain finite:

$$\Pi_2=\Pi_{12}=\frac{N_2-N_1}{\omega-\Omega+i\delta}, \quad \Pi_3=\Pi_{21}=\frac{N_1-N_2}{\omega+\Omega+i\delta}. \qquad (22)$$

This solution can be completed by employing an oscillator model of a film, i.e., by assuming that

$$\varphi_1=(\pi L^2)^{-1/4}e^{-z^2/2L^2}, \quad \varphi_2=2^{1/2}(\pi L^2)^{-1/4}\frac{z}{L}e^{-z^2/2L^2}. \qquad (23)$$

Then, if $kL\ll 1$, we have $J=kL/2^{3/2}\pi^{1/2}$ and the solution of Eq. (21) becomes

$$\omega^2(k)=\Omega^2+\frac{2^{3/2}\pi^{1/2}}{\varepsilon}\Omega e^2 L(N_1-N_2)+\frac{Ne^2Lk^2}{\varepsilon m}f\left(\frac{a}{L},\frac{m\Omega}{N_1-N_2}\right). \qquad (24)$$

We shall not give the fairly cumbersome expression for the dimensionless function f. We simply note that f is positive if $N_1>N_2$ (by definition, we have $\Omega=E_2-E_1>0$).

Equation (24) describes an excitation of an exciton-type wave. A similar expression was obtained in Refs. 8 and 9 for the case when $N_2=0$ and $k=0$ and it was interpreted as representing a shift of the energy of an intersubband transition due to the depolarization field. It should be noted that this shift [second term in Eq. (24)] increases on increase in the electron density as long as only the lowest level is populated, which was assumed to be the case in Refs. 8 and 9. As soon as the Fermi level rises above E_2, the difference N_1-N_2 becomes constant $N_1-N_2=m(E_2-E_1)/\pi$. At absolute zero there is no damping of an exciton wave right up to values of the momentum $k_0=(\omega-\Omega)/v_1$, whereas at temperatures $T>0$ the damping is proportional to $\exp[-(\omega-\Omega)^2/k^2U_T^2]$, where U_T is the thermal velocity. Since $(N_1-N_2)\propto L^{-2}$, the quantity in the argument of the exponential function can be regarded as equal to $(e^2/\varepsilon U_T kL)^2$.

5. ABSORPTION OF ELECTROMAGNETIC WAVES

We shall now discuss the question of the optical activity of the investigated oscillations. It is clear that the branch ω_+ for a film or for a layer system is fully analogous to a plasmon in a one-component plasma and that it interacts resonantly with a longitudinal inhomogeneous electric field $\mathscr{E}_0\exp[i(kx-\omega t)]$, as found experimentally.[10,11] The branch ω_- is also, in principle, optically active (for the same fields), but the amplitude and width of the absorption resonance differ considerably from the corresponding parameters in the ω_+ case.

We shall first consider spatially separate plasma layers. The scattering of electrons in the two layers will be described by collision frequencies v_1 and v_2. The absorption coefficient will be defined as the ratio of the power dissipated per unit area to the surface energy density of a plasma wave amounting to $\mathscr{E}_0^2/8\pi k$. In the case of constant frequencies v_1 and v_2 the calculations are elementary. The resonance values of Q and of the widths Γ of the resonances are given by the following expressions.

For the ω_+ branch, we have

$$Q_+^{\text{res}}=\frac{(\omega_+)^4}{\omega_1^2\nu_1+\omega_2^2\nu_2}, \quad \Gamma_+=\frac{\nu_1\omega_1^2+\nu_2\omega_2^2}{\omega_1^2+\omega_2^2}. \qquad (25)$$

For the ω_- branch, we obtain

$$Q_-^{\text{res}}=\frac{1}{4}\frac{\omega_-^4}{\omega_1^2\nu_2+\omega_2^2\nu_1}\left(\frac{\omega_1}{\omega_2}-\frac{\omega_2}{\omega_1}\right)^2, \quad \Gamma_-=\frac{\nu_1\omega_2^2+\nu_2\omega_1^2}{\omega_1^2+\omega_2^2}. \qquad (26)$$

The resonance frequencies $\omega_\pm$ occurring in Eqs. (25) and

(26) are given in the $k\Delta \ll 1$ range by Eqs. (9) and (13), respectively; ω_1 and ω_2 are the frequencies of two-dimentional plasmons in the layers. The expression for Q_-^{res} given above is valid on condition that $\nu_1\nu_2(\omega_1^2+\omega_2^2) \ll (\omega_1^2+\omega_2^2)$. We can easily show that in the case of a symmetric structure ($\omega_1=\omega_2$, $\nu_1=\nu_2$) the value of Q_-^{res} is proportional to ν in the frequency range $\omega \sim \omega_-$, whereas outside this interval the absorption decreases in accordance with the law $\nu\omega_-^2/\omega^2$. Thus, in the case of a symmetric structure the optical activity of the ω_- branch is anomalously weak and the absorption differs only slightly from that due to the Drude background of free carriers. This is due to the fact that the branch ω_- corresponds to antiphasal motion of like charges in the layers. The total current in the system at resonance differs from zero only because of the electron scattering. In the calculation of the absorption by the $\omega_\pm$ branches in a quantum film we have to allow for the scattering of electrons accompanied by transitions between the subbands. The related problem of the conductivity of a quantum film in a homogeneous field in the case when electrons are scattered elastically by impurities reduces to solution of a system of coupled transport equations so as to find the distribution functions f_n (n is the number of the film subband).[2] As a result, it is possible to introduce the relaxation time τ_n, which occurs additively in the total conductivity. However, in our case an alternating electric field is strongly inhomogeneous so that the absorption coefficient cannot be expressed solely in terms of the quantities τ_n.

For simplicity, we shall consider the model of δ-like scatters. The probability of a $np \to mp'$ transition averaged over the positions of the impurity centers will be denoted by $v_{nm}\delta[W_n(p)-W_m(p)]$, where $v_{nm}=v_{mn}$ and it is independent of the momenta. The acceleration of a particle, which has to be substituted into the transport equation, is governed by the sum of the external field $\mathscr{E}_0$ and the field $\mathscr{E}(z)$ of a plasma wave. The latter should obviously be averaged over z with the weighting $\varphi_n^2(z)$ in the equation for the function f_n. Bearing this point in mind, we obtain the following system of equations for the nonequilibrium parts of the distribution g_n:

$$i(\mathbf{kv}-\omega)g_m(p)+\sum_{n,p'}(g_m(p)-g_n(p'))v_{nm}\delta(W_m(p)-W_n(p'))$$
$$+(e\mathbf{v}\vec{\mathscr{E}}_m)f_m{}^{0\prime}=0, \quad (27)$$

where $\mathscr{E}_m \equiv \mathscr{E}_0+\int \mathscr{E}(z)\varphi_m^2 dz$. The "outflow" term in Eq. (27) contains only zeroth cylindrical harmonics $g_n(\mathrm{p}')$ so that the problem can be reduced to a system of linear algebraic equations for small quantities $\langle g_m \rangle$, where $\langle \ldots \rangle$ denotes averaging over the angles in a two-dimensional p space. The expressions for $Q_\pm$ are very cumbersome even in the case of a two-level model. We shall give only the formulas for the resonance widths $\Gamma_\pm$. The absorption coefficients themselves correspond qualitatively to Eqs. (25) and (26); the value of Q_-^{res} cannot obviously be anomalously small because in the case of a film we have $N_1 \neq N_2$. We shall introduce $\nu_1=v_{11}+v_{12}$ and $\nu_2=v_{22}+v_{12}$, which determine the conductivity of a film σ_0 in a static field

$$\sigma_0=\frac{e^2}{m}\left(\frac{N_1}{\nu_1}+\frac{N_2}{\nu_2}\right).$$

Then, Γ_+ and Γ_- are described by

$$\Gamma_+=\frac{\nu_1N_1+\nu_2N_2}{N_1+N_2},\quad \Gamma_-=\frac{\nu_1N_2+\nu_2N_1}{N_1+N_2}+2v_{12}. \quad (28)$$

Comparing Eq. (28) with Eqs. (25) and (26), we can see that the formulas for Γ_+ are identical for a film and a layer structure, and they can be expressed in terms of the relaxation times $\tau_{1,2}=1/\nu_{1,2}$ introduced in Ref. 2. In contrast, the formulas for Γ_- contain an additional term which cannot be expreesed via $\tau_{1,2}$ and which appears precisely because of the intersubband scattering.

6. METAL–INSULATOR–SEMICONDUCTOR STRUCTURE

The most thoroughly investigated MIS structures are those made of silicon. The intersubband infrared absorption, plasmons, and magnetoplasma resonance have been observed precisely in $Si-SiO_2$–metal systems. In this section we shall discuss the main results of our study specifically in the case of MIS structures.

We shall assume that the semiconductor layer occupies the region $z>0$, that the insulator is located at $-d<z<0$, and the metal electrode is at $z=-d$. The matrix $\hat{I}$ of Eq. (4) is now described by

$$I_{ijmn}^k=\int_0^\infty \varphi_i(z)\varphi_j(z)\left[e^{-k|z-z_0|}+\frac{\varepsilon-\varepsilon_0 \operatorname{cth} kd}{\varepsilon+\varepsilon_0 \operatorname{cth} kd}e^{-k(z+z_0)}\right]\varphi_n(z_0)\varphi_m(z_0)\,dz\,dz_0, \quad (29)$$

where ε and ε_0 are the permittivities of the semiconductor and insolator, respectively. The functions $\varphi_n(z)$ are known from self-consistent calculations based on the direct variational method.[12] Since the functions φ_n are no longer classified in accordance with the parity, all the elements of the determinant (16) differ from zero. However, in the long-wavelength limit the situation simplifies greatly. In fact, if $kL \to 0$ (L is the thickness of the inversion channel), all the elements of $\hat{I}$ vanish, with the exception of I_{11}, I_{14}, I_{41}, and I_{44} and this is due to the orthogonality of φ_n. The dispersion law for the branches $\omega_\pm$ is then easily found:

$$\omega_+{}^2=\frac{4\pi e^2k(N_1+N_2)}{m(\varepsilon+\varepsilon_0 \operatorname{cth} kd)},\quad \omega_-{}^2=\frac{4\pi e^2D'(0,kd)}{m(\varepsilon+\varepsilon_0 \operatorname{cth} kd)}\frac{N_1N_2}{N_1+N_2}, \quad (30)$$

where $D'(0,kd)$ denotes (in contrast to Sec. 4) the derivative with respect to k of $I_{11}I_{44}-I_{14}^2$ in the limit $k\to 0$, but subject to the condition that $\coth kd$ is regarded as a constant. This difference from Eq. (19) is due to the fact that we usually have $L \ll d$ and when $kL \ll 1$ the value of kd need not be small. The quantity $D(0,kd)$ contains the combination $(\varepsilon-\varepsilon_0 \coth kd)(\varepsilon+\varepsilon_0\coth kd)^{-1}$, which tends to a constant both at high and low values of kd. We can thus see that when two electron subbands are populated in an MIS structure, we obtain $\omega_- \propto k^{3/2}$ when $kd \ll 1$ and $\omega_- \propto k$ in the opposite limiting case.

The exciton mode is characterized by a finite frequency at $k=0$. Hence, it follows that $\Pi_1=\Pi_4=0$ and $\Pi_2, \Pi_3 \neq 0$, and that the dispersion equation is identical with Eq. (21). Then, the contribution to $J(k)$ of the second term in Eq. (29) is proportional to k^2, whereas the dispersion equation contains $J(k)/k$. Therefore, Eq. (24) applies to an inversion channel with the exception of a numerical coefficient in the second term. Finally, we shall use the experimental results discussed in Ref. 13 to estimate the importance of the finite thickness of an inversion layer in the one-level regime. It has been possible to record experimentally the fourth spatial harmonic of plasmons for which $k \sim k_0/7$, but even in the case of such large momentum there is no deviation from the long-wavelength asymptote to the dispersion law [first of the formulas in Eq. (30)]. We wish to draw attention to the fact that the corrections for the finite thickness of the layer (expansion with respect to kL) and for the spatial dispersion (expansion with respect to ka) have opposite signs and in the present case they largely balance out. It follows from Eq. (6) of Ref. 6 (in the case when $ka \ll 1$) that the correction to the frequency is

$$\frac{\delta\omega_1}{\omega}=\frac{3}{16}\frac{\hbar^2 k}{me^2}(\varepsilon+\varepsilon_0 \operatorname{cth} kd).$$

Having calculated the constant in Eq. (7) of the present paper for the function of the ground subband selected in accordance with Ref. 12 in the form $\psi \sim z\exp(-\alpha z/2)$, we obtain

$$\frac{\delta\omega_2}{\omega}=-\frac{k}{\alpha}\left(\frac{63}{32}-\frac{33}{32}\frac{\varepsilon_0 \operatorname{cth} kd}{\varepsilon}\right).$$

Under the experimental conditions discussed here, we have $\delta\omega_1+\delta\omega_2 \approx 1.25\times 10^{-2}\omega$, which is approximately one-third less than the correction for the spatial dispersion.

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[1)]Strictly speaking, at the point k_c there is no intersection, but the $\omega_-(k)$ curve is in contact with the straight line kv_1. If $k > k_c$, the value of R_1 formally changes its sign, which corresponds to a change to a different sheet of a Riemann surface. Near k_c the behavior of the ω_- curve is described by $\omega_- - kv_1 \propto (k_c-k)^2$, $k < k_c$.

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