Interaction between regular and random waves in a nonlinear viscous medium

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The interaction between regular and random waves described by the Burgers equation at large Reynolds numbers in the region of existence of sawtooth waves is investigated. The one-point velocity probability density and recurrence relations for the moments are derived. A closed equation for the average velocity with turbulent viscosity is obtained and the viscosity is calculated. The various stages of evolution of the statistical characteristics of the velocity and their dependence on the parameters of the initial distribution are studied.

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1. INTRODUCTION

In the propagation of finite-amplitude acoustic waves in a liquid or a gas, an essential role is played by the interaction of a regular periodic signal with the noise. The non-coherence of sound and ultrasound ones sources used in laboratory practice and in geophysical experiments leads to the result that their radiations have a noise component. The same problem arises in the recording of sound waves from natural hydrodynamic sources. The spectra of the radiation that arises in such phenomena as explosions, vibrations, cavitation, electric discharges, and jet streams consist as a rule of regular and noise components. Finally, the same questions are raised in connection with the analysis of the parameteric interaction of acoustic waves. The effects observed in similar situations are quite varied and depend on the amplitudes and scales of the signal and the noise, the acoustical Reynolds number, and so on.[1]

One-dimensional finite-amplitude waves satisfy the Burgers equation[1]

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}=\nu\frac{\partial^2 u}{\partial x^2},\tag{1}$$

where u is the vibrational velocity, ν is the viscosity coefficient, t and x in acoustical applications are connected with the real coordinates by the relations $t=c_0^{-1}x'$ and $x=x'-c_0t'$ (c_0 is the sound velocity in the medium).

Equation (1) can also be regarded as a model for nonlinear media without dispersion and is used in radiophysics and hydrodynamics. By considering the evolution of a random or mixed initial disturbance, we reach the state of acoustic turbulence, which is of interest by itself, and which can also be used for the study of vortical turbulence.[2,3]

Although the Burgers equation is solved exactly by the substitution method of Hopf-Cole, there are difficulties, connected with the averaging, in the calculation of the statistical characteristics of the velocity waves. These difficulties are especially significant at large Reynolds numbers, when the spectral methods turn out to be ineffective, since the spectrum is enriched during propagation by a large number of intense and very strongly mutually correlated harmonics.[2]

The evolution of an initial random disturbance with characteristic amplitude a and scale l at large Reynolds numbers $\mathrm{Re}=al/\nu$ passes through three stages. At $t<l/a$ the solution (1) takes the form of a simple wave satisfying (1) at $\nu=0$, and consequently its statistics can be obtained from the statistics of the parameters of a simple wave.[4,5]

At $t>l/a$, i.e., after the toppling of the simple wave, an ensemble of sawtooth waves interacting with one another is formed (acoustic turbulence), the slopes of the fronts of which are limited by the viscosity.[3] In this region, the simple wave is formally multivalued, and to construct the statistics of the velocity in the sawtooth wave, it is necessary to draw on additional considerations. Where the shock waves have just been formed, it is still possible under certain restrictions, to investigate the statistics of the random and mixed signals, using the statistics of the positions of the wavefronts.[6] More universal is the application of the rule of selection of that value of the velocity in the simple wave to which corresponds the minimal value of the action $s=\int udx$. Such a form of the selection rule follows from the Burgers equation, and it is used in Ref. 7 to construct the statistics of the ensemble of sawtooth waves that arise upon evolution of a noise perturbation with a zero dc component in the spectrum.

At t such that the running Reynolds number, which falls off (together with the energy of the wave) as a consequence of the dissipation at the fronts, becomes of the order of unity, degeneracy of the acoustic turbulence takes place.[8]

The problem of the interaction of the determinate and random signals has been considered for various special cases.[1,5,8,9] We consider the evolution of the sum of a regular signal and noise with zero dc component in the spectrum in the region of the intermediate asymptote that corresponds to times that are much longer than the time of toppling of the noise wave, but shorter than the time of degeneracy of the turbulence. For this region we shall obtain a one-point velocity distribution function. It follows from its form that we can obtain a closed equation of type (1) for the average velocity with turbulent viscosity. Questions connected with the closure of the equations for the moments of the random fields by the introduction of turbulent viscosity

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have been discussed for some time in connection with the Burgers equation, and also with the more complicated problems of the interaction of acoustic waves with vortices.[10,11] In the present work we have succeeded in connecting the turbulent viscosity with the characteristic scale of the stirring of the field in the regular wave.

In the final section of the paper, we carry out a concrete study of the statistical characteristics of sawtooth waves at different stages of the evolution of the mean velocity.

2. PROBABILITY DENSITY AND MOMENTS OF THE VELOCITY FIELD

We consider the evolution of a perturbation which satisfies the Burgers equation and which has the form $u_c(x)+u_n(x)$, where $u_c(x)$ is a regular cyclic signal with amplitude a_c and period l_c, and u_n is stationary Gaussian noise with the following properties:

$$\langle u_n\rangle=0,\quad \langle u_n(x_1)u_n(x_2)\rangle=a_n^2B\left(\frac{x_1-x_2}{l_n}\right),\quad \int_0^\infty B(y)\,dy=0,$$

$$\int_0^\infty yB(y)\,dy=1,\quad B(0)=1,\quad -B''(0)=\gamma.$$

We shall assume the acoustic Reynolds number, which is calculated from the parameters of the signal and the noise, to be large: $a_cl_c/\nu\gg1$, $a_nl_n/\nu\gg1$. The Hopf-Cole substitution leads to an exact solution of the problem:

$$u=2\nu\frac{Q'}{Q},\quad Q=\frac{1}{2}(\pi\nu t)^{-1/2}\int_{-\infty}^{+\infty}\exp\left\{-\frac{s_c(x_0)+s_n(x_0)}{2\nu}-\frac{(x-x_0)^2}{4\nu t}\right\}dx_0, \tag{2}$$

where

$$s_c(x_0)=\int_{-\infty}^{x_0}u_c(x_0)\,dx_0,\quad s_n(x_0)=\int_{-\infty}^{x_0}u_n(x_0)\,dx_0.$$

Under the assumptions made, we can calculate the integral of (2) by the saddle-point method, and at t longer than the time of toppling of the simple wave, Q takes the form

$$Q=\sum_m[1+tu_c'(x_{0m})]^{-1/2}\exp\left\{-\frac{s(x,x_{0m})}{2\nu}\right\}, \tag{3}$$

where

$$s=s_c(x_{0m})+s_n(x_{0m})+(x-x_{0m})^2/2t,$$

while x_{0m} satisfies the conditions

$$u_n(x_{0m})+u_c(x_{0m})=(x-x_{0m})/t,\quad \partial x/\partial x_0>0.$$

The action s satisfies the Hamilton-Jacobi equation

$$\frac{\partial s}{\partial t}+\frac{1}{2}\left(\frac{\partial s}{\partial x}\right)^2=0,$$

while $v=\partial s/\partial x$ is the equation of the simple wave.

Each term of the sum (3) corresponds to one of the stable modes of the simple wave that are produced after its toppling and dominate in that region where the action connected with them is minimal. In this region is formed the sloping part of the sawtooth, on which $u=v_m=(x-x_{0m})/t$ (i.e., all the sawteeth have the same slope). Near the shock fronts, the location of which is determined by the equality of two partial actions $s_m=s_{m+1}$, we must take two terms of the sum into account and the expression for the velocity takes the form

$$u=\frac{v_{m+1}+v_m}{2}+\frac{v_{m+1}-v_m}{2}\,\mathrm{th}\,\frac{s_{m+1}-s_m}{2\nu}.$$

The ensemble of sawtooth waves exists as long as the running Reynolds number is large, i.e., $u_{max}^2t/\nu\gg1$, where u_{max} is the characteristic amplitude of the sawtooth wave. In the opposite case there are many commensurate terms in the sum (3) and degeneracy of the acoustic turbulence takes place.

The Hamiltonian-Jacobi equation is equivalent to the characteristic system of ordinary differential equations for s, v, x, $I=\partial x/\partial x_0$, $q=\partial v/\partial x_0$:

$$\frac{ds}{dt}=\frac{v^2}{2},\quad \frac{dv}{dt}=0,\quad \frac{dx}{dt}=v,\quad \frac{dI}{dt}=q,\quad \frac{dq}{dt}=0. \tag{4}$$

A consequence of (4) is the Liouville equation for the probability density in Eulerian variables (for a fixed point of observation)[12]

$$\frac{\partial W}{\partial t}+\frac{v^2}{2}\frac{\partial W}{\partial s}+v\frac{\partial W}{\partial x}+q\frac{\partial W}{\partial I}-\frac{q}{I}W=0. \tag{5}$$

Solving (5) with the initial condition

$$W_0(s,v,x,I,q)=\frac{\delta(I-1)}{(2\pi)^{3/2}a_n^3(\gamma-1)^{1/2}}\exp\left\{-\frac{[s-s_c(x)]^2}{2a_n^2l_n^2}-\frac{[v-u_c(x)]^2}{2a_n^2}-\frac{[l_n^2q-l_n^2u_c'(x)+s-s_c(x)]^2}{2a_n^2l_n^2(\gamma-1)}\right\}$$

and integrating with respect to q and I with account of the fact that the saddle points in (2) correspond only to $I>0$, we obtain an expression for the probability density of the parameters s and v in the simple wave:

$$W(x,s,v)=\int_0^\infty\int_{-\infty}^{+\infty}W_0\left(s-\frac{v^2}{2}t,v,x-vt,I-qt,q\right)I\,dq\,dI. \tag{6}$$

To obtain the probability density of the parameters of the sawtooth wave, it is necessary to carry out a cutoff of (6), taking into account only that one of the rays that is described by the system (4) and reaching the given point and corresponds the the minimal action. In other words, we must find the probability that if a ray with the parameters $\tilde{s}$, $\tilde{v}$ arrives at the point, then not a single ray $s<\tilde{s}$ arrives at the same point. A rigorous solution of this problem, which is very similar to problems dealing with the crossings of a specified level by a random process, can be obtained in the form of a continual integral in a form suitable for calculations. We can represent it in those cases when the number of random rays arriving at a point can be assumed to be either small or large.

We consider now the second case, assuming that $t\gg\tau_n=l_n/a_n$, i.e., the simple wave of the noise is multivalued. The dominant rays in this case are those connected with large negative overshoots of s. The ray tubes of such rays, as is seen from (4) and (5), diverge rapidly and gradually fill all space. Making use of the lack of correlation between the overshoots of s, as in

Ref. 8, we find the expression for the probability density of s and u in the sawtooth wave:

$$\tilde{W}(s,u)=W(s,u)\exp\left\{-\int_{-\infty}^{s}\int_{-\infty}^{+\infty}W(s,u)\,du\,ds\right\}. \tag{7}$$

Here $W(s,u)$ is given by the expression (6) at $v\equiv u$. The derivation of (7) turns out to be possible if $s_n(x)$ is a stationary process, from which follows the need for requiring absence of a dc component in the velocity spectrum. The expression (7) is valid for s satisfying the inequality

$$-s+s_c^{\min}\gg a_n l_n,$$

where $s_c^{\min}=s_c(ml_c)$ and ml_c are the minimum points of s_c.

Denoting by s_0 the point of the maximum of the distribution (7) at the fixed values $x=ml_c$ and $u=0$, and assuming that

$$-s_0+s_c^{\min}\gg a_n l_n;\quad |s-s_0|\ll|s_0|,$$

at every point where (6) differs appreciably from zero, we get with accuracy to small quantities

$$[s-s_c(x-ut)-{}^1/_2u^2t]^2\approx(s_0-s_c^{\min})^2+2(s_0-s_c^{\min})[s-s_0+s_c^{\min}-s_c(x-ut)-{}^1/_2u^2t],$$
$$[u_c(x-ut)-u]^2\approx u_c^2(x-ut).$$

Making use of these relations and calculating the integral by the saddle-point method, we represent (6) in the form

$$W(\zeta,u,x)=\frac{t(1/\tau_c+\varphi_0/\tau_n)}{2\pi a_n^2l_n}\exp\left\{-{}^1/_2\varphi_0^2-\frac{\varphi_0\zeta+\varphi_0[s_c(x-ut)+{}^1/_2u^2t-s_c^{\min}]}{a_nl_n}-\frac{u_c^2(x-ut)}{2a_n^2}\right\}, \tag{8}$$

with accuracy to small quantities, where

$$\varphi_0=\frac{s_c^{\min}-s_0}{a_nl_n},\quad \zeta=-s+s_0,\quad \frac{1}{\tau_c}=u_c'(ml_c);\quad \tau_n=\frac{l_n}{a_n}.$$

For φ_0 we get the following equation from the extremum condition for (7):

$$\frac{t(1/\tau_c+\varphi_0/\tau_n)}{2\pi a_n\varphi_0}\exp\left\{-{}^1/_2\varphi_0^2\right\}\int\exp\left\{-\frac{\varphi_0[s_c(ut)+{}^1/_2u^2t-s_c^{\min}]}{a_nl_n}-\frac{u_c^2(ut)}{2a_n^2}\right\}du=1. \tag{9}$$

The quantity φ_0 plays the role of a cutoff parameter for the distribution (6), and the condition (9) is equivalent to the normalization condition

$$\int_{-\infty}^{\infty}\int_{-\infty}^{+\infty}W(s,u)\,du\,ds=1.$$

Substituting (8) in (7), we have $\tilde{W}(\zeta,u)=\tilde{W}(\zeta)\tilde{W}(u)$, where

$$\tilde{W}(\zeta)=\frac{\varphi_0}{a_nl_n}\exp\left\{-\frac{\varphi_0\zeta}{a_nl_n}-\exp\left[-\frac{\varphi_0\zeta}{a_nl_n}\right]\right\}$$

has the character of a spike function and shows that in the given region the wave remembers not all the various initial data but only the vicinities of the points where $(-s)$ is sufficiently large. These points are sort of ordering centers and the action s plays the role of an order parameter which forms the sloping part of the sawtooth.

The limits of applicability of the obtained asymptote are determined by the inequality

$$1\ll\varphi_0\ll a_nl_n/\nu.$$

At $\varphi_0\gg l$ the distribution $W(\zeta)$ is concentrated at $\zeta\sim a_nl_n/\varphi_0$; consequently, the condition $|s-s_0|\ll|s_0|$ is satisfied wherever $W(\zeta)$ differs materially from zero. As $t\to\infty$, it follows from (8) that $u_{\max}^2\sim a_nl_n/t\varphi_0$, that is, the requirement $\varphi_0\ll a_nl_n/\nu$ is equivalent to $\mathrm{Re}\sim u_{\max}^2t/\nu\gg1$. The region of values of t where the intermediate asymptote is valid is green by Eq. (9).

Integrating (7) with respect to ζ, we obtain an expression for the one-point velocity probability density:

$$W(u,x)=\Phi(u,x)\left[\int\Phi(u,x)\,du\right]^{-1},$$
$$\Phi(u,x)=\exp\left\{-\frac{\varphi_0}{a_nl_n}\left[s_c(x-ut)+\frac{u^2}{2}t\right]-\frac{u_c^2(x-ut)}{2a_n^2}\right\}. \tag{10}$$

The deviation departures of the velocity from the values in the simple wave, not taken into account in (10), which are noticeable near the shock fronts, introduce into the obtained expression corrections that are small in the Reynolds number.

Using (10) and making the change of variable $x_0=x-ut$ under the integral sign, we represent the arbitrary moment of the field u in the form

$$\langle u^n\rangle=\int(x-x_0)^n\exp\left\{-\frac{\varphi_0}{a_nl_n}\left[s_c(x_0)+\frac{(x-x_0)^2}{2t}\right]-\frac{u_c^2(x_0)}{2a_n^2}\right\}dx_0/t^n\int\exp\left\{-\frac{\varphi_0}{a_nl_n}\left[s_c(x_0)+\frac{(x-x_0)^2}{2t}\right]-\frac{u_c^2(x_0)}{2a_n^2}\right\}dx_0. \tag{11}$$

Differentiating (11), we obtain a recurrence relation which allows us to express the higher moments in terms of the lower:

$$\frac{\partial\langle u^n\rangle}{\partial x}=\frac{n}{t}\langle u^{n-1}\rangle-\frac{\varphi_0}{a_nl_n}\langle u^{n+1}\rangle+\frac{\varphi_0}{a_nl_n}\langle u^n\rangle\langle u\rangle. \tag{12}$$

Averaging (1) and using (12) at $n=1$, we obtain for the mean velocity, a closed equation which is the Burgers equation with turbulent viscosity:

$$\frac{\partial\langle u\rangle}{\partial t}+\langle u\rangle\frac{\partial\langle u\rangle}{\partial x}=\nu_T\frac{\partial^2\langle u\rangle}{\partial x^2};\quad \nu_T=\frac{1}{2}\frac{a_nl_n}{\varphi_0}. \tag{13}$$

The mean velocity can also be represented in the form

$$\langle u\rangle=2\nu_T(\ln Q)',$$
$$Q=\int\exp\left\{-\frac{1}{2\nu_T}\left[s_c(x_0)+\frac{u_c^2(x_0)\tau_n}{2\varphi_0}+\frac{(x-x_0)^2}{2t}\right]\right\}dx_0. \tag{14}$$

Comparing (14) with (2), it is easy to see that $\langle u\rangle$ actually satisfies the Burgers equation with $\nu=\nu_T$ and with the initial condition

$$u=u_c(x_0)[1+u_c'(x_0)\tau_n/\varphi_0].$$

The weak dependence of ν_T on t need not be taken into account in the solution.

3. DIFFERENT STAGES OF EVOLUTION OF THE CHARACTERISTICS OF THE VELOCITY

In view of the fact that the mean velocity satisfies the Burgers equation it must, in its evolution, go through the same stages as an individual realization. However, by virtue of the difference between the characteristic times, all the stages of the evolution of the mean velocity can be fitted into the region of the intermediate asymptote. We consider each of the stages in more detail.

1. The stage of a simple wave: $\tau_n \ll t \ll \tau_c$. Here the expression (10) has one saddle point and can be represented in the form

$$W(u,x)=\frac{[t+t^2u_c'(x-\tilde{u}t)]^{1/2}}{2(\pi\nu_T)^{1/2}}\exp\left\{-\frac{t+t^2u_c'(x-\tilde{u}t)}{4\nu_T}(u-\tilde{u})^2\right\}, \quad (15)$$

where $\tilde{u}$ satisfies the relation $u_c(x-\tilde{u}t)=\tilde{u}$.

Equation (9) takes the form

$$(t\tau_c)^{1/2}[2\pi\tau_n(\tau_c+t)\varphi_0]^{-1/2}\exp\{-{}^1/_2\varphi_0^2\}=1, \quad (16)$$

whence we get the following for the principal term of φ_0:

$$\varphi_0\approx\left(\ln\frac{t}{\tau_n}\right)^{1/2}.$$

At this stage the average velocity is determined by the regular signal, which almost does not interact with the noise and has the form of the simple wave. The noise component is transformed into fine-scale sawtooth waves, the characteristic scale of which is determined by the distance between the overshoots of the noise action: $l_t \sim (t\nu_T)^{1/2}$. An increase in the scale with increase in t takes place as a result of the motion of the discontinuities, thanks to which the sawteeth, which are connected with ordering centers with smaller values $(-s)$, are absorbed by the neighbors. The energy of the noise waves falls off because of dissipation at the discontinuities, and we have for the dispersion of the velocity fluctuations at the given point

$$\sigma_u^2=2\nu_T[t+t^2u_c'(x-\tilde{u}t)]^{-1},$$

i.e., the noise component turns out to be modulated by the signal.

2) The stage of the sawtooth wave: $\tau_c \ll t \ll l_c^2/l_n a_n$. During this stage Eq. (10) has many saddle points, and the argument $x_0=x-ut$ approaches independently of x, the points of the minima of s_c where

$$s_c\approx s_c^{\min}+(x_0-ml_c)^2/2\tau_c.$$

From (9), we get in this stage

$$\left(\frac{1}{\tau_c}+\frac{\varphi_0}{\tau_n}\right)^{1/2}\frac{\tau_c^{1/2}}{(2\pi)^{1/2}\varphi_0}\exp\left\{-\frac{\varphi_0^2}{2}\right\}=1,$$

whence it follows that the inequality $\varphi_0 \gg 1$ can be satisfied only at $\tau_c \gg \tau_n$, and in this case $\varphi_0 \approx [\ln(\tau_c/\tau_n)]^{1/2}$.

With account of what has been pointed out, (10) takes the form

$$W(u,x)=\sum_m\exp\left\{-\frac{(x-ml_c)^2}{4\nu_T(t+\tau_c)}-\frac{t^2}{4\nu_T\tau_c}\left(u-\frac{x-ml_c}{t+\tau_c}\right)^2\right\}$$

$$\Big/\left[2(\pi\nu_T\tau_c)^{1/2}t^{-1}\sum_m\exp\left\{-\frac{(x-ml_c)^2}{4\nu_T(t+\tau_c)}\right\}\right]. \quad (17)$$

The sum (17) shows that at each point a value of the velocity close to one of the values in the regular simple wave can be realized with a definite probability. At most of the points, one of the terms of the sum is dominant. The average velocity at these points duplicates the sawtooth wave formed by the regular signal. In the vicinity of fronts, one of the two dominant values of the velocity is realized with a definite probability. These are the causes of the fluctuations of the location of the boundary between sawteeth, which lead to the statistical broadening of the shock front. The scale of the front width is defined as $\Delta \sim t\sigma_\xi/l_c$, where $\sigma_\xi^2 \sim (a_n l_n/\varphi_0)^2$ is the variance of the fluctuations of the action. On the other hand, the viscosity and the front width are connected by a relation which agrees with the expression obtained above for the turbulent viscosity: $\nu \sim \Delta l_c/t$.

The average velocity in the range $0<x<l_c$ is described by the expression

$$\langle u\rangle=\frac{1}{t+\tau_c}\left[x-\frac{l_c}{2}-\frac{l_c}{2}\operatorname{th}\frac{(x-l_c/2)l_c}{2\nu_T t}\right],$$

whence we have for the variance of the velocity fluctuations

$$\sigma_u^2=\frac{2\nu_T\tau_c}{t^2}+l_c^2/2t^2\operatorname{ch}^2\left[\frac{l_c(x-l_c/2)}{2\nu_T t}\right]. \quad (18)$$

The first term (18) is connected with the fluctuations of the position of the center of the random sawtooth, which has the scale $l_t \sim l_c$ in this stage, about the point $x=ml_c$. The second term describes the noise modulation of the position of the boundary between sawteeth. The fluctuations associated with this term are concentrated near the points $(m+\frac{1}{2})l_c$ and form a sequence of random pulses with characteristic length $\nu_T t/l_c$.

3) the stage of degeneracy: $t \geq l_c^2/l_n a_n$. In this stage it is convenient to start out from the expression (14) and use for $\tilde{Q}(x)$ the spectral representation

$$\tilde{Q}(x)=\frac{2(\pi t\nu_T)^{1/2}}{l_c}\sum_n A_n\exp\left\{-\frac{2\pi^2n^2t\nu_T}{l_c^2}+i\frac{2\pi nx}{l_c}\right\},$$

$$A_n=\int_0^{l_c}\exp\left\{-\frac{s_c(x_0)}{2\nu_T}-\frac{u_c^2(x_0)}{2a_n^2}-i\frac{2\pi nx_0}{l_c}\right\}dx_0. \quad (19)$$

Equation (9) has the following form for this case:

$$\left(\frac{1}{\tau_c}+\frac{\varphi_0}{\tau_n}\right)\frac{(\tau n t)^{1/2}}{(2\pi\varphi_0^3)^{1/2}}\frac{A_0}{l_c}\exp\left\{-\frac{\varphi_0^2}{2}\right\}=1,$$

whence we have for the principal term φ_0:

$$\varphi_0\approx\left(\ln\frac{ta_n^2}{a_c l_c}\right)^{1/2},\quad \frac{\tau_c}{\tau_n}\gg1,\quad \frac{a_c l_c}{a_n l_n}\gg1,$$

$$\varphi_0\approx\left(\ln\frac{t}{\tau_n}\right)^{1/2},\quad \frac{\tau_c}{\tau_n}\gg1,\quad \frac{a_c l_c}{a_n l_n}\ll1,$$

$$\varphi_0\approx\left(\ln\frac{ta_n l_n}{l_c^2}\right)^{1/2},\quad \frac{\tau_c}{\tau_n}\ll1,\quad \frac{a_c}{a_n}\gg1,$$

$$\varphi_0\approx\left(\ln\frac{t\tau_n}{\tau_c^2}\right)^{1/2},\quad \frac{\tau_c}{\tau_n}\ll1,\quad \frac{a_c}{a_n}\ll1.$$

The mean velocity can be calculated with the help of (14) and (19). In the case $t \geq l_c^2/l_n a_n$ we need take into account only a single term each in the numerator and denominator of the resultant expression. It is seen from the formula that the mean velocity decreases exponentially with increase in t, while its spatial distribution is identical with the fundamental of the regular wave. In this stage, the basic energy is connected with random, large-scale sawteeth. For the variance of the velocity fluctuations we have from (12) $\sigma_u^2 = \nu_T/t$ for the scale $l_{st} \sim (\nu_T t)^{1/2}$.

In the degeneracy stage, the turbulent viscosity can be connected with the characteristic scale at which the mixing of the initial pattern (with a diffusion interval of sorts) takes place. It follows from the exact solution of the Burgers equation that this scale is connected with the viscosity by the relation $\Delta \sim (\nu t)^{1/2}$. On the other hand, the spatial scale of the teeth in the given stage is connected with the turbulent velocity by the same relation, $l_t \sim (\nu_T t)^{1/2}$.

4. CONCLUSION

In the considered problem, the interaction of the regular signal with the noise leads to an additional damping of the mean field, which can be taken into account by the introduction of turbulent viscosity in the Burgers equation. Physically, the turbulent viscosity can be connected with the statistical broadening of the shock front in the sawtooth wave stage, and with the statistical scale of mixing of the initial patterns in the stage of degeneracy of the mean velocity. The analysis that has been carried out shows that the noise and the signal interact parametrically, so that, independently of the amplitude of the noise, the fluctuations of the velocity reach high values primarily in the vicinity of the shock fronts of the regular signal. This explains why the attempts at application of the mean field approximation to the Burgers equation do not lead to success.[13]

In the analysis of the interaction of signal and noise in the region of the intermediate asymptote, we can distinguish the following cases:

1) The time of toppling of the noise is smaller than that of the signal: $\tau_n \ll \tau_c$. The mean velocity in this case is determined by the regular wave while the turbulent viscosity determines the time of transfer of its energy to the large-scale noise. The evolution of the mean velocity also depends on the ratio $a_c l_c / a_n l_n$ which determines the value of the turbulent Reynolds number. At large values of this parameter, the mean velocity passes through all three of the stages of evolution described above, while at small values, the middle stage does not arise. The limits of applicability of the intermediate asymptote follow in this case from the inequalities

$$1 \ll \ln\frac{t}{\tau_n};\quad \ln\frac{t a_n^2}{a_c l_c} \ll \left(\frac{a_n l_n}{\nu}\right)^2,\quad \frac{a_c l_c}{a_n l_n} \gg 1,$$
$$1 \ll \ln\frac{t}{\tau_n} \ll \left(\frac{a_n l_n}{\nu}\right)^2,\quad \frac{a_c l_c}{a_n l_n} \ll 1.$$

2) The time of toppling of the noise is greater than that of the signal: $\tau_c \ll \tau_n$. Only the third stage of evolution of the mean velocity lands in the region of the intermediate asymptote, and this region is itself determined by the inequalities

$$1 \ll \ln\frac{t a_n l_n}{l_c^2} \ll \left(\frac{a_n l_n}{\nu}\right)^2,\quad \frac{a_c}{a_n} \gg 1,$$
$$1 \ll \ln\frac{t \tau_n}{\tau_c^2} \ll \left(\frac{a_n l_n}{\nu}\right)^2,\quad \frac{a_c}{a_n} \ll 1.$$

In this case the mean velocity depends on some effective initial distribution $u_c(x_0)u_c'(x_0)\tau_n/\varphi_0$, which indicates that the noise wave plays in some sense the role of a pump and transfers its energy to the signal. However, during the stage considered, the damping of the regular wave because of the turbulent viscosity turns out to be more noticeably, so that no amplification arises. The preceding regions of the intermediate asymptote of the evolution stage have been considered by other methods.[1,5,9]

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[1] O. C. Rudenko and S. I. Soluyan, Teoreticheskie osnovy nelineinoĭ akustiki (Theoretical Foundations of Nonlinear Acoustics), Moscow Nauka 1975.

[2] B. B. Kadomtsev and V. I. Petviashvili, Dokl. Akad. Nauk SSSR **208**, 794 (1973) [Sov. Phys. Doklady **18**, 115 (1973)].

[3] T. Tatsumi and Sh. Kida, J. Fluid Mech. **55**, 659 (1972).

[4] A. N. Malakhov and A. I. Saichev, Zh. Eksp. Teor. Fiz. **67**, 940 (1974) [Sov. Phys. JETP **40**, 467 (1975)].

[5] O. V. Rudenko and A. S. Chirkin, Zh. Eksp. Teor. Fiz. **67**, 1903 (1974) [Sov. Phys. JETP **40**, 945 (1975)].

[6] O. V. Rudenko and A. S. Chirkin, Dokl. Akad. Nauk SSSR **225**, 520 (1975) [Sov. Phys. Doklady **20**, 748 (1975)].

[7] Sh. Kida, J. Fluid Mech. **93**, 337 (1979).

[8] S. N. Gurbatov, A. A. Dubkov and A. N. Malakhov, Zh. Eksp. Teor. Fiz. **72**, 456 (1977) [Sov. Phys. JETP **45**, 239 (1977)].

[9] S. N. Gurbatov, Akust. Zh. **26**, 551 (1980) [Sov. Phys. Acoustics **26**, 302 (1980)].

[10] S. S. Moiseev, R. Z. Sagdeev, A. V. Tur and V. V. Yanovskiĭ, in: Nelineĭnye volny (Nonlinear Waves) Moscow, Nauka, 1979, p. 105.

[11] S. S. Moiseev, A. V. Tur and V. V. Yanovskiĭ, Izv. vuzov Radiofiz. **23**, 68 (1980).

[12] V. I. Klyatskin, Stokhasticheskie uravneniya i volny v sluchaĭno-neodnorodnykh sredakh (Stochastic Equations and Waves in Randomly Inhomogeneous Media) Moscow, Nauka, (1980).

[13] E. N. Pelinovskiĭ, in: Nelineĭnye vony (Nonlinear Waves) Moscow, Nauka, 1979, p. 331.

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