Inverse Faraday effect in a relativistic electron plasma

A. Sh. Abdullaev and A. A. Frolov

Institute of Radio Engineering, Electronics, and Automation, Moscow
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An analytic demonstration is given of the excitation of strong quasistatic magnetic fields in a collisionless relativistic electron plasma when strong circularly polarized electromagnetic radiation is propagating through it.

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1. Considerable attention is currently being given to possible mechanisms of generation of strong spontaneous magnetic fields in a laboratory plasma subjected to linearly polarized electromagnetic radiation[1-5] and in a cosmic plasma.[6,7] The interest in the appearance and growth of quasistatic magnetic fields is largely due to their dominant influence on the nature of penetration of electromagnetic radiation into a plasma[8,9] and on the rate of transport processes in a plasma.[9-11]

There is natural interest in the generation of spontaneous magnetic fields in a relativistic plasma when high-power electromagnetic radiation propagates in it. Under laboratory conditions this electromagnetic radiation can be provided by lasers: the power density can be $q \gtrsim 10^{16}$ W/cm^2 for CO_2 lasers emitting at $\lambda_0 = 10.6\ \mu$ and $q \gtrsim 10^{15}$–10^{17} W/cm^2 for Nd lasers emitting at $\lambda_0 = 1.06\ \mu$ (see, for example, Refs. 12 and 13). Under astrophysical conditions similar fields may be provided by electromagnetic radiation of strong radio sources (cores of radiogalaxies, pulsars,[14] etc.).

Our results relating to the generation of spontaneous magnetic fields in a collisionless inhomogeneous relativistic plasma are in qualitative and quantitative agreement (in the nonrelativistic approximation) with the available experimental results,[15,16] particularly those demonstrating the absence of magnetic fields in a plasma irradiated with high-frequency linearly polarized radiation.

2. We shall consider an inhomogeneous relativistic electron plasma of density $n_e(x, y)$ subjected to strong circularly polarized electromagnetic radiation. The intensity of the electric field of this radiation (pump wave) is

$$\tilde{\mathbf{E}}(z, t) = \mathbf{E}_0\{\mathbf{e}_x \cos(\omega_0 t - k_0 z) + \lambda \mathbf{e}_y \sin(\omega_0 t - k_0 z)\}. \tag{1}$$

Here, E_0, ω_0, and k_0 are, respectively, the electric-field amplitude, frequency, and wave number of the external electromagnetic radiation; $\mathbf{e}_x$ and $\mathbf{e}_y$ are unit vectors along the x and y axes; $\lambda = 1$ applies to the right-handed polarization and $\lambda = -1$—to the left-handed polarization. In the case of linear propagation of the pump wave of Eq. (1) in a relativistic plasma, we have

$$\omega_0^2 = \omega_{pe\,\mathrm{eff}}^2 + c^2 k_0^2,$$

where $\omega_{pe\,\mathrm{eff}} = (4\pi n_e e^2/m_e)^{1/2}$ is the effective Langmuir frequency; e and m_e are the electron charge and mass.

In describing the nonlinear interaction of an external electromagnetic wave with a relativistic electron plasma we shall use a system of relativistic hydrodynamic equations for electrons and a system of Maxwell equations for the fields

$$\begin{gathered}\partial \mathbf{p}_e/\partial t + (\mathbf{v}_e \nabla)\mathbf{p}_e = e(\mathbf{E} + [\mathbf{v}_e \times \mathbf{B}]/c), \\ \partial n_e/\partial t + \nabla(n_e \mathbf{v}_e) = 0, \\ \mathbf{p}_e = m_{e0}\mathbf{v}_e/(1 - v_e^2/c^2)^{1/2}, \quad \mathrm{rot}\,\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}, \\ \mathrm{rot}\,\mathbf{B} = \frac{4\pi e}{c} n_e \mathbf{v}_e + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}, \quad \mathrm{div}\,\mathbf{B} = 0,\end{gathered} \tag{2}$$

where $\mathbf{p}_e$, $\mathbf{v}_e$, and n_e are, respectively, the momentum, velocity, and density of electrons; m_{e0} is the rest mass of electrons; $\mathbf{E}$ and $\mathbf{B}$ are the electric-field and the magnetic-induction vectors, respectively.

Under the action of the high-frequency field of Eq. (1) the variables in the system (2) are characterized not only by a slow time dependence but also by a fast dependence with the characteristic time $\tau \propto 1/\omega_0$. Therefore, the momentum, velocity, and density of electrons, as well as the electric and magnetic fields can be represented in the form (a similar procedure was used by Gorbunov[17] in the case of a nonrelativistic electron-ion plasma)

$$\begin{gathered}\mathbf{p}_e = \langle \mathbf{p}_e \rangle + \tilde{\mathbf{p}}_e, \quad \mathbf{v}_e = \langle \mathbf{v}_e \rangle + \tilde{\mathbf{v}}_e, \quad n_e = \langle n_e \rangle + \tilde{n}_e, \\ \mathbf{E} = \langle \mathbf{E} \rangle + \tilde{\mathbf{E}}, \quad \mathbf{B} = \langle \mathbf{B} \rangle + \tilde{\mathbf{B}}.\end{gathered} \tag{3}$$

Here, $\langle \ldots \rangle$ represents averaging over a time interval which is short compared with the characteristic time of changes in the slow quantities but long compared with τ.

Then, the equation describing the excitation and evolution of a quasistatic magnetic field can be derived from equations for the slow motion of electrons (here, $|\langle \mathbf{v}_e \rangle| \ll c, v_{\mathrm{ph}}$, where v_{ph} is the phase velocity) and Maxwell equations for quasistatic fields:

$$\frac{\partial \langle \mathbf{p}_e \rangle}{\partial t} + (\langle \mathbf{v}_e \rangle \nabla)\langle \mathbf{p}_e \rangle = e\langle \mathbf{E} \rangle + \frac{e}{c}[\langle \mathbf{v}_e \rangle \times \langle \mathbf{B} \rangle] - \langle (\tilde{\mathbf{v}}_e \nabla)\tilde{\mathbf{p}}_e \rangle + \frac{e}{c}\langle [\tilde{\mathbf{v}}_e \times \tilde{\mathbf{B}}] \rangle, \tag{4}$$

$$\mathrm{rot}\langle \mathbf{E} \rangle = -\frac{1}{c}\frac{\partial \langle \mathbf{B} \rangle}{\partial t}, \tag{5}$$

$$\mathrm{rot}\,\langle \mathbf{B} \rangle = 4\pi c^{-1}(e\langle n_e \rangle \langle \mathbf{v}_e \rangle + \mathbf{j}_E), \tag{6}$$

$$m_{e0}\langle \mathbf{v}_e \rangle = \frac{\langle \mathbf{p}_e \rangle}{(1 + p_{e0}^2/m_{e0}^2 c^2)^{1/2}}, \tag{7}$$

$$p_{e0}^2 = e^2 E_0^2/\omega_0^2. \tag{8}$$

The influence of the high-frequency fields on the slowly varying fields can be allowed in our case simply by a nonlinear current

$$\mathbf{j}_E=\frac{e\langle\tilde{n}_e\tilde{\mathbf{p}}_e\rangle}{m_{e0}(1+p_{e0}^2/m_{e0}^2c^2)^{1/2}},\qquad(9)$$

found earlier for a nonrelativistic cold plasma.[18]

We shall obtain a closed system of equations by considering the rapidly varying quantities $\tilde{n}_e$ and $\tilde{\mathbf{p}}_e$. We shall initially assume that the following inequalities are obeyed:

$$(L,\lambda_m)\gg|\langle\mathbf{v}_e\rangle|/\omega_0,\quad \omega_0\gg\omega_{ce\ \mathrm{eff}}=e\langle B\rangle/cm_{e0}\gamma,\qquad(10)$$

where $\omega_{ce\ \mathrm{eff}}$ is the relativistic gyroscopic frequency of electrons; L and λ_m are, respectively, the characteristic distances over which there are significant changes in the slowly varying and rapidly varying quantities; $\gamma=(1+p_{e0}^2/m_{e0}^2c^2)^{1/2}$ is a relativistic factor. We then obtain the following system of equations:

$$\partial\tilde{\mathbf{p}}_e/\partial t=e\tilde{\mathbf{E}},\qquad(11)$$

$$\partial\tilde{n}_e/\partial t+\mathrm{div}\langle n_e\rangle\tilde{\mathbf{v}}_e=0,\qquad(12)$$

$$m_{e0}\tilde{\mathbf{v}}_e=\tilde{\mathbf{p}}_e(1+p_{e0}^2/m_{e0}^2c^2)^{-1/2}.\qquad(13)$$

The representation of the pump wave in the form of circularly polarized radiation of Eq. (1) allows us to apply the procedure of Eq. (3), since p_{e0}^2 does not contain the fast time dependence.

3. The equation describing the evolution of a quasistatic magnetic field $\langle\mathbf{B}\rangle$ in time, deduced from Eqs. (1)–(13), is

$$\frac{\partial}{\partial t}\left(\langle\mathbf{B}\rangle+\frac{c}{e}\,\mathrm{rot}\,\langle\mathbf{p}_e\rangle\right)+\mathrm{rot}\left[\langle\mathbf{B}\rangle+\frac{c}{e}\,\mathrm{rot}\langle\mathbf{p}_e\rangle,\frac{\langle\mathbf{p}_e\rangle}{m_{e0}\gamma}\right]=0,\qquad(14)$$

where

$$\langle\mathbf{p}_e\rangle=\frac{cm_{e0}\gamma}{4\pi e\langle n_e\rangle}\mathrm{rot}\langle\mathbf{B}\rangle-\frac{m_{e0}\gamma\mathbf{j}_E}{e\langle n_e\rangle},\qquad(15)$$

$$\mathbf{j}_E=\lambda\frac{e^3}{2m_{e0}^2\gamma^2}\left[\frac{E_0^2}{\omega_0^3}\left(\mathbf{e}_x\frac{\partial}{\partial y}-\mathbf{e}_y\frac{\partial}{\partial x}\right)\langle n_e\rangle\right].\qquad(16)$$

In a hot plasma the thermal pressure does not influence generation of magnetic fields if the temperature distribution is homogeneous and isotropic, but in the presence of a constant anisotropy of the thermal pressure in a plasma[5] there may be a source of excitation of spontaneous magnetic fields additional to or competing with the nonlinear current of Eq. (16). It follows from Eqs. (14) and (15) that the dependence of the magnetic induction vector on the slow quasistatic current is

$$\langle\mathbf{B}\rangle=\frac{\gamma m_{e0}c^2}{4\pi e^2}\left\{\frac{4\pi}{c}\left(\mathrm{rot}\,\frac{\mathbf{j}_E}{\langle n_e\rangle}\right)-\mathrm{rot}\left(\frac{1}{\langle n_e\rangle}\mathrm{rot}\,\langle\mathbf{B}\rangle\right)\right\}.\qquad(17)$$

We shall estimate the amplitude of spontaneous magnetic fields by considering two limiting cases of plasma inhomogeneity. In the case of a strong inhomogeneity of the plasma density corresponding to $\omega_{pe\ \mathrm{eff}}L/c\ll1$, which is comparable here with the characteristic homogeneity length of the magnetic field $\langle B_z(x,y)\rangle$, we obtain

$$\langle\mathbf{B}\rangle=\lambda\frac{2\pi e^3}{cm_{e0}^2\gamma^2}\frac{E_0^2}{\omega_0^3}n_e(x,y)\mathbf{e}_z.\qquad(18)$$

It is clear from Eq. (18) that in the ultrarelativistic case $\gamma\gg1$ the dependence of the amplitude of a quasistatic magnetic field on the value of E_0 becomes weaker but the dependence on the plasma density inhomogeneity is still retained.

It should also be noted that a magnetic field expected in the nonrelativistic approximation is directed, as found experimentally,[15,16] at right-angles to the plane of rotation of the electric field vector and its direction is reversed on reversal of the rotation of the electric field vector E_0. In the case of linear polarization of the external radiation ($\lambda=0$), Eq. (18) predicts no magnetic fields.[15,16]

In the opposite case of $\omega_{pe\ \mathrm{eff}}L/c\gg1$, when the spatial structure of the field changes little over a distance $\sim c/\omega_{pe\ \mathrm{eff}}$, we can ignore the spreading of a magnetic field in the XY plane. We then obtain the following expression for $\langle\mathbf{B}\rangle$:

$$\langle\mathbf{B}\rangle=\lambda\frac{ce}{2\gamma m_{e0}}\frac{E_0^2}{\omega_0^3}\left(\frac{\partial^2\ln\langle n_e\rangle}{\partial x^2}+\frac{\partial^2\ln\langle n_e\rangle}{\partial y^2}\right)\mathbf{e}_z.\qquad(19)$$

It follows from Eq. (19) that in the ultrarelativistic limit $1\ll\gamma\ll\omega_{pe}L/c$ the magnetic field intensity is largely determined by the amplitude of the external pump wave E_0 and, which is self-evident, in a homogeneous plasma the source of spontaneous magnetic fields disappears. This corresponds to mutual compensation of the circular currents, analogous to the diamagnetic current in a homogeneous plasma.

There are as yet no experimental data on the generation of magnetic fields by circularly polarized electromagnetic radiation traveling in a laser plasma. However, as pointed out above, in the microwave range of pump frequencies[15,16] a well-designed experiment was carried out on a laboratory plasma and it was intended to simulate the interaction of laser radiation with a dense plasma. In this experiment an inhomogeneous plasma ($L_x=L_y=6.5$ cm) with $P=6\times10^2$ Torr was subjected to circularly polarized electromagnetic radiation of frequency $f=3\times10^9$ Hz. The power of this radiation was 1 MW and it was applied in the form of pulses of ~12 μsec duration. This pulse duration was considerably greater than the growth time of a magnetic field. Two magnetic field pulses were observed on an oscilloscope screen: the first (larger) was the result of a change in the density and electric field, whereas the second (smaller) was due to switching of the field. In addition to its qualitative agreement with the experimental results, the numerically determined magnetic induction $B\sim10^{-2}$ G was in good qualitative agreement with Eq. (19) for a nonrelativistic plasma giving an estimate of $B\sim0.5\times10^{-2}$ G.

4. In investigating nonlinear propagation of strong electromagnetic fields in a laser plasma with a particle density $n_e(z)$ it is very interesting to consider the self-interaction of the pump wave. Strong relativistic nonlinearity observed in our case because of the dependence of the electron mass on the amplitude of the incident field may give rise to three-dimensional, localized, transverse, and circularly polarized electromagnetic pulses.[19] These pulses travel in a plasma at a frequency ω_{pe} which can be larger or smaller than the carrier frequency ω. The intensity of the electric field in a pump wave can generally be described by

$$\mathbf{E}(\mathbf{r},t)=E(x,y)\{\mathbf{e}_x\sin(\omega t-kz+\varphi)+\lambda\mathbf{e}_y\cos(\omega t-kz+\varphi)\},\qquad(20)$$

where φ is the phase, which we shall assume to be constant.

We shall use a system of equations for the rapidly varying quantities (11)–(13), in which the relativistic factor is a function of the arguments x and y, and the profile of the plasma electron density varies in the direction of propagation of a three-dimensional soliton along the Z axis. This gives the following expression for a quasistatic nonlinear current

$$\mathbf{j}_E=\lambda\frac{c^2e\langle n_e\rangle}{2\omega\gamma}\psi\left(\mathbf{e}_y\frac{\partial}{\partial x}-\mathbf{e}_x\frac{\partial}{\partial y}\right)\psi_\gamma. \tag{21}$$

Here, $\psi=eE/m_{e0}\omega c$ is a Lorentz-invariant function of the relativistic nature of the electromagnetic field and $\psi_\gamma=\psi/(1+\psi^2)^{1/2}$ is a nonlinear term due to the nonrelativistic nature of particle motion.

We can now write down the spatial evolution of a spontaneous magnetic field directed parallel or antiparallel to the Z axis and considered to be a function of the index λ:

$$\langle\mathbf{B}\rangle+\frac{c^2\gamma}{\omega_{pe}^2}\operatorname{rot}\operatorname{rot}\langle\mathbf{B}\rangle=\lambda\frac{m_{e0}c^3}{2e\omega}\left\{\psi\frac{\partial^2\psi_\gamma}{\partial x^2}+\psi\frac{\partial^2\psi_\gamma}{\partial y^2}+\gamma\left(\frac{\partial\psi_\gamma}{\partial x}\right)^2+\gamma\left(\frac{\partial\psi_\gamma}{\partial y}\right)^2\right\}\mathbf{e}_z. \tag{22}$$

It follows from Eq. (22) that in the case of formation and propagation of a stable two- or three-dimensional rotating electromagnetic pulse, we can expect excitation of quasistatic magnetic fields.

5. In order to estimate the magnitudes of spontaneous magnetic fields, we must find the amplitude of a localized pulse and its characteristic inhomogeneity length. The system of the Maxwell and hydrodynamics equations for the rapidly varying quantities together with an allowance for the excited magnetic field yields the following wave equation for ψ:

$$\left(\nabla^2-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\psi\mathbf{e}_1=\left(\frac{\omega_{pe}}{c}\right)^2\left[1+\frac{\omega_{ce}}{\omega}\frac{1}{(1+\psi^2)^{1/2}}\right]\frac{\psi\mathbf{e}_1}{(1+\psi^2)^{1/2}}. \tag{23}$$

Here, $\omega_{ce}=e\langle B\rangle/m_{e0}c$ is the gyroscopic frequency of electron rotation and

$$\mathbf{e}_1=\mathbf{e}_x\sin(\omega t-kz+\varphi)+\mathbf{e}_y\cos(\omega t-kz+\varphi).$$

A parabolic equation describing the propagation of a localized three-dimensional pulse, similar to the steady-state nonlinear Schrödinger equation, can be obtained from Eq. (23)

$$\Delta_\perp\psi+\beta^2\frac{\partial^2\psi}{\partial z^2}-\frac{\omega^2}{c^2}\beta^2\psi=\frac{\omega_{pe}^2}{c^2}\left[1+\frac{\omega_{ce}/\omega}{(1+\psi^2)^{1/2}}\right]\frac{\psi}{(1+\psi^2)^{1/2}}, \tag{24a}$$

$$k\frac{\partial\psi}{\partial z}+\frac{\omega}{c^2}\frac{\partial\psi}{\partial t}=0,\quad \beta=\left(1-\frac{k^2c^2}{\omega^2}\right)^{1/2}. \tag{24b}$$

In Eq. (24a) we have neglected longitudinal perturbations of the particle density because it is assumed that the relativistic nonlinearity[20] predominates over the striction nonlinearity.

Since a detailed investigation of the system (24a)–(24b) is not at this stage necessary, we shall determine the amplitude of spontaneous magnetic fields in a laser plasma by a numerical analysis in the spherically symmetric case assuming that the influence of $\langle B\rangle$ on the evolution of solitons is not the dominant effect.[19] We shall first note that for specific eigenvalues $l=\omega/c\beta$, varying in the interval $0<l<1$, soliton solutions appear as a result of self-channeling, i.e., when the power of circularly polarized electromagnetic radiation has the critical value $p_{cv}=\lambda_0^2c/32\pi^2\varepsilon_2^{1/2}$ [see Eq. (3.2) in the review by Akhmanov *et al.*[21]]; here, $\varepsilon_2=(e\omega_{pe}/m_{e0}c\omega^2)^2$ and λ_0 is the wavelength of the incident radiation.

This critical power of a three-dimensional electromagnetic pulse determines, in fact, the minimum value of the amplitude of the generated magnetic field. In the case of the newly formed nonlinear normal modes (see Fig. 1 in Ref. 19) the intensity associated with the first soliton at the wavelength of the incident radiation $\lambda_0=1\ \mu$ is $q\sim1.4\times10^{18}$ W/cm^2, so that the characteristic inhomogeneity length in the transverse direction is $r\sim5c/\omega_{pe}$. Without allowance for the diffusion term it then follows from Eq. (22) that $\langle B\rangle\gtrsim10$ MG.

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