Nuclear widths and shifts of the $dt\mu$ mesic-molecule levels

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The nuclear widths Γ^{Jv} and shifts ΔE^{Jv} of the levels (Jv) of the mesic molecule $dt\mu$, which are due to the resonant interaction of d and t in the s-band, are expressed in terms of the cross section for the reaction $d+t\to n+{}^4\mathrm{He}$. It is established that the influence of the nuclear resonance ${}^5\mathrm{He}^*$ $(3/2^+)$ on the spectrum of the mesic-molecule states is weak because of the low probability of finding d and t in the range of the nuclear forces in the mesic molecule $dt\mu$ and to the large inelastic width of the resonance. The obtained nuclear widths and shifts agree with the results of a calculation that makes use of the generalized optical potential corresponding to the problem of the coupled dt–$n{}^4\mathrm{He}$ channels.

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§1. INTRODUCTION

In the preceding paper[1] we calculated the rates λ_f of the nuclear fusion reaction

$$dt\mu \xrightarrow{\lambda_f} n+{}^4\mathrm{He}+\mu^- \tag{1}$$

from various states (Jv) of the rotational and vibrational motions of the mesic molecule $dt\mu$. The nuclear interaction of d and t was described by a generalized optical potential whose structure was established from an analysis of the problem of the coupled dt and $n{}^4\mathrm{He}$ channels: the anti-Hermitian part had a separable form, while the Hermitian part was approximated by a local potential independent of the energy. An interaction of this type provided a good description of the experimental data on the fusion reaction[2]

$$d+t\to n+{}^4\mathrm{He} \tag{2a}$$

and on elastic scattering[3]

$$d+t\to d+t \tag{2b}$$

near the dt threshold ($E_{\mathrm{cm}}(dt)<200$ keV).

The nuclear level widths Γ^{Jv} of the mesic molecule $dt\mu$, obtained by solving the eigenvalue problem for the total Hamiltonian of the $dt\mu$ system, agreed within ~10% with the values obtained from the known equation[4]

$$\Gamma^{Jv}=A_0|\Psi^{Jv}(0)|^2. \tag{3}$$

Here

$$A_0=\lim_{v\to 0} v\sigma_{in}C_0^{-2} \tag{4}$$

is the constant of the nuclear reaction (2a), v is the relative velocity of d and t, σ_{in} is the cross section of the reaction (2a), C_0^2 is the Gamow factor:

$$C_0^2=2\pi\eta/(e^{2\pi\eta}-1),\quad \eta=\alpha/v\quad (\hbar=c=1), \tag{5}$$

and $\Psi^{Jv}(\mathrm{R})$ is the wave function of the relative motion of d and t in the $dt\mu$ mesic molecule without allowance for the nuclear dt interaction.[1)]

That the traditional equation (3) was valid in the case of the $dt\mu$ system was not obvious beforehand, since the cross section of the reaction (2a) at an energy $E<200$ keV was determined by the resonance ${}^5\mathrm{He}^*(\frac{3}{2}^+)$, the position and width of which are substantially influenced by the Coulomb interaction of d and t,[7] whereas relation (3) is based on the possibility of separating the long-range (Coulomb) and short-range (nuclear) interactions.[4]

We determine in this paper the conditions under which Eq. (3) is satisfied in the case of near-threshold resonance. On the basis of the known fact that the reaction (2a) proceeds via formation of an intermediate nucleus ${}^5\mathrm{He}^*$, we propose a method for calculating the nuclear widths and shifts of the levels of the mesic molecule $dt\mu$, without using the concrete form of the nuclear potential. To describe the resonance mechanism of reactions (2), we introduce the bare state "^{5}He," which acquires physical mass and width as a result of the coupling with the dt and $n{}^4\mathrm{He}$ channels. The distinguishing feature of the Hamiltonian of the corresponding problem of the coupled channels, namely dt (channel 1), $n{}^4\mathrm{He}$ (2), and "^{5}He" (3) is that the nuclear interaction enters only in the form of a dt–"^{5}He" and $n{}^4$He–"^{5}He" channel couplings. This Hamiltonian (which contains also the Coulomb repulsion of d and t) will be shown to yield for the energy dependence of the cross section of reaction (2a) an isolated-level formula[8] that accounts well for the experimental data.[2,3] This Hamiltonian which need not be further detailed, makes it possible to obtain an equation that describes the influence of the nuclear resonance ${}^5\mathrm{He}^*(\frac{3}{2}^+)$ on the $dt\mu$ mesic-molecule levels, and by the same token solve the problem of the nuclear widths and shifts of the levels of the mesic molecule.

On the basis of the available experimental data and the reactions (2), we conclude that the presence of a nuclear level near the $dt\mu$ mesic molecule, and the rate of the nuclear reaction (1) can be calculated with a certain accuracy by using Eq. (3). We have investigated the possible variation of the spectrum of the mesic-molecule states when the nuclear-resonance parameters are varied, and considered, in particular, the previously investigated[1] restructuring of a nuclear level of zero width.

The plan of the article is the following. In Sec. 1 we consider the resonance mechanism of the reaction (2). The determination of the connection between the nuclear widths and shifts of the mesic molecule levels of the $dt\mu$ system, on the one hand, and the parameters of the resonance ${}^5\mathrm{He}^*(\frac{3}{2}^+)$, on the other, is the subject of Sec.

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3. Section 4 contains the results of the numerical calculation of the nuclear widths and shifts of the mesic-molecule levels. In Sec. 5 we consider the restructuring of the mesic-molecule spectrum for various parameters of the nuclear states. The results and concluding remarks are given in Sec. 6.

§2. RESONANCE MECHANISM OF THE REACTION $d+t\to n+{}^4\mathrm{He}$

The reaction (2a) has been well investigated experimentally in the collision-energy range 8 keV $< E <$ 12 MeV (in the c.m. system of $d+t$).[2] Its characteristic feature is the near-threshold resonance ${}^5\mathrm{He}^*(\frac{3}{2}^+)$ in the cross section $\sigma_{tn}(E)$ at an energy $E_R = 64$ keV, with a half-width $\Gamma/2 \approx 70$ keV and with a cross section at the maximum $\sigma_{tn}(E_R) \approx 5$ b, which is close to the unitary limit. The entire aggregate of the experimental data agrees with the assumption that at energies $E < 200$ keV the reaction goes from the s wave ($L=0$) in the dt input channel through intermediate excitation of the state of the ${}^5\mathrm{He}^*$ nucleus ($J^\pi = \frac{3}{2}^+$). The contributions of the remaining states ($L=0$, $J^\pi = \frac{1}{2}^+$, and $L \geq 1$) add up to $\leq 1\%$ in this energy region.[9] It is known that the cross section $\sigma_{tn}(E)$ for the reaction (2a) is well approximated by the Breit-Wigner-Eisenbud isolated-level formula.[7,8,10]

We consider the problem of three coupled channels with Hamiltonian

$$\mathcal{H} = \begin{pmatrix} H_1 & 0 & V_1 \\ 0 & H_2 & V_2 \\ V_1^+ & V_2^+ & E_0|0\rangle\langle 0| \end{pmatrix}. \tag{6}$$

Here H_1 is the Coulomb Hamiltonian of the dt channel (1), H_2 is the free Hamiltonian of the $n^4\mathrm{He}$ channel (2), E_0 and $|0\rangle$ are the energy and wave function of the state "${}^5\mathrm{He}$" (3) without allowance for the coupling with channels 1 and 2. The nuclear interaction in channels 1 and 2 is the result of the coupling of these channels with the resonance "${}^5\mathrm{He}$."

To find the dt-scattering amplitude, it suffices to solve the effective single-channel problem with the Hamiltonian

$$\tilde{H}_1 = H_1 + \frac{V_1|0\rangle\langle 0|V_1^+}{E-E_0-\langle 0|V_2^+G_2(E)V_2|0\rangle} \equiv H_1 + V, \tag{7}$$

in which the nuclear dt interaction is described by a nonlocal and energy-dependent generalized optical potential V. [$G_2(E) = (E-H_2)^{-1}$ is the free Green's function of channel 2.] The amplitude $\mathbf{f}^{11}$ of the s-wave dt scattering is expressed with the aid of the two-potential formula[11] in terms of a generalized optical potential and of the solution of the scattering problem with the Hamiltonian H_1:

$$\mathbf{f}^{11}(E) = \mathbf{f}(E) - 2m_1 f^{-2}(E)\langle\varphi_E|V(1-G_1V)^{-1}|\varphi_E\rangle. \tag{8}$$

Here $\mathbf{f} = (e^{2i\delta}-1)/2ik$ is the scattering amplitude, δ is the scattering phase shift, $f(E)$ is the Jost function, φ_E is the regular solution for the Hamiltonian H_1 in the s wave, $m_1 = m_d m_t/(m_d+m_t)$ and $k = (2m_1E)^{1/2}$. The regular solution $|\varphi_E\rangle$ satisfies the boundary condition

$$\langle r|\varphi_E\rangle|_{r=0} = 1.$$

The matrix element in the right-hand side of (8) can be easily calculated because of the separable form of the operator V. For the S-matrix element corresponding to dt scattering we obtain

$$S^{11} = 1+2ik\mathbf{f}^{11} = e^{2i\delta}\left(1 - \frac{4im_1k|f(E)|^{-2}|\langle\varphi_E|V_1|0\rangle|^2}{E-E_0-\langle 0|V_1^+G_1V_1|0\rangle - \langle 0|V_2^+G_2V_2|0\rangle}\right). \tag{9}$$

We consider now S^{11} in the physical region of the dt channel:

$$S^{11} = S^{11}(E+i0), \quad E>0$$

(the threshold of the dt channel corresponds to $E=0$). Using the spectral representation of the Green's function G_1, we write

$$\langle 0|V_1^+G_1(E+i0)V_1|0\rangle = \frac{(2m_1)^{3/2}}{\pi}\int_0^\infty \frac{\varepsilon^{1/2}|f(\varepsilon)|^{-2}|\langle\varphi_\varepsilon|V_1|0\rangle|^2\,d\varepsilon}{E-\varepsilon} - 2im_1k|f(E)|^{-2}|\langle\varphi_E|V_1|0\rangle|^2. \tag{10}$$

In the resonance region of interest to us ($E < 200$ keV), the quantity $|\langle\varphi_E|V_1|0\rangle|^2$ depends little on the energy E, since the radius R_N of the nuclear interaction is small compared with the characteristic dimension of the Coulomb problem with the value of the reciprocal momentum of the relative motion of d and t:

$$R_N \ll 1/m_1\alpha, \quad R_N \ll 1/k,$$

and can therefore be replaced by a constant:

$$|\langle\varphi_E|V_1|0\rangle|^2 \approx |\langle\varphi_{E=0}|V_1|0\rangle|^2 = g/2m_1. \tag{11}$$

We can also neglect the energy dependence of the matrix element $\langle 0|V_2^+G_2(E+i0)V_2|0\rangle$, since the distance $E_{12} = 17.6$ MeV between the dt and $n^4\mathrm{He}$ thresholds is large compared with the considered energy region:

$$\langle 0|V_2^+G_2V_2|0\rangle = \Delta_2 - i\Gamma_{in}/2 = \text{const}. \tag{12}$$

Denoting

$$E_S = E_0 + \Delta_2, \tag{13a}$$

$$\Delta_1(E) = \mathrm{Re}\langle 0|V_1^+G_1(E+i0)V_1|0\rangle = \frac{(2m_1)^{1/2}}{\pi}g\int_0^\infty \frac{|f(\varepsilon)|^{-2}F(\varepsilon)\varepsilon^{1/2}\,d\varepsilon}{E-\varepsilon}, \tag{13b}$$

$$F(\varepsilon) = |\langle\varphi_\varepsilon|V_1|0\rangle|^2/|\langle\varphi_{\varepsilon=0}|V_1|0\rangle|^2, \tag{13c}$$

we rewrite the matrix element S^{11} in the following form:

$$S^{11} = e^{2i\delta}\frac{E-E_S-\Delta_1(E)+i\Gamma_{in}/2-ikg|f(E)|^{-2}}{E-E_S-\Delta_1(E)+i\Gamma_{in}/2+ikg|f(E)|^{-2}}. \tag{14}$$

The phase shift δ and the Jost function $f(E)$ correspond here to Coulomb s-wave scattering:

$$\delta = \arg\Gamma(1+i\eta), \quad |f(E)|^{-2} = C_0^2(E). \tag{15}$$

The cross section of the reaction (2a) with unpolarized particles is given by

$$\sigma_{in} = \frac{2J+1}{(2S_d+1)(2S_t+1)}\frac{\pi}{k^2}(1-|S^{11}|^2) = \frac{4\pi g\Gamma_{in}|f(E)|^{-2}}{3k[(E-E_S-\Delta_1(E))^2+(\Gamma_{in}/2+kg|f(E)|^{-2})^2]}. \tag{16}$$

Here $J = \frac{3}{2}$ is the angular momentum of the ${}^5\mathrm{He}^*$ resonance, $S_d = 1$ and $S_t = \frac{1}{2}$ are the spins of d and t. We have thus obtained an isolated-level equation similar to the Breit-Wigner-Eisenbud formula known from R-matrix theory (see, e.g., Ref. 7). The position of the resonance is determined by the energy of the bare state E_0 and by the shift Δ_1 and Δ_2 due to the channel couplings

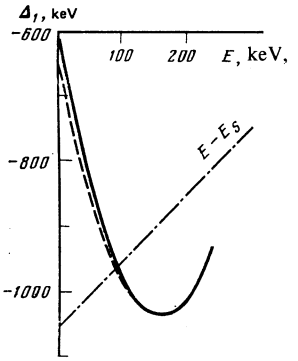

FIG. 1. Energy dependence of the shift $\Delta_1(E)$ due to the coupling of the bare state "^{5}He" with the dt^m channel (solid curve). The dashed curve shows the shift $\Delta_1(E)$.

1–3 and 2–3. In the energy dependence of the elastic width

$$\Gamma_{el}=2kg|f(E)|^{-2}$$

we took into account the proximity of the threshold and the presence of Coulomb repulsion of d and t.

In calculation of the function $\Delta_1(E)$ (Fig. 1) we use the circumstance that the energy region ε that makes the main contribution to the integral (13b) is determined by the action radius of the nuclear forces: $0<\varepsilon<\varepsilon_0\sim 1/m_1R_N^2$. In other words, the form factor $F(\varepsilon)$ is of the order of unity at $\varepsilon\leq\varepsilon_0$ and decreases rapidly with increasing energy at $\varepsilon<\varepsilon_0$. In the region $E\ll\varepsilon_0$ the results are independent of the details of the nuclear interaction that couples channel 1 with 3 and 2 with 3, and of the structure of the bare state "^{5}He," i.e., of the actual form factor. We have used the following form factor

$$F(\varepsilon)=\begin{cases}1, & \varepsilon\leqslant\varepsilon_0\\ 0, & \varepsilon>\varepsilon_0\end{cases}.$$

The parameters E_S, Γ_{in}, and g were determined at a fixed value of ε_0 by obtaining a best fit of the theoretically calculated reaction and scattering cross sections to experiment. We have analyzed the data on reaction (2a) in the energy interval $E=12$–200 keV (Ref. 2) jointly with the data on the dt elastic scattering through an angle $\theta=\pi/2$ in the c.m.s. in the energy interval $E=30$ –200 keV,[3] assuming that the nuclear interaction of d and t is significant only in the state with $L=0$ and $J=\frac{3}{2}$. At $\varepsilon_0=0.5$ MeV, the best agreement with experiment is reached at the following values of the parameters:

$$E_S=1060\text{ keV},\ \Gamma_{in}=336\text{ keV},\quad g=0.16 \qquad (17)$$

($\chi^2=10$ for 24 experimental points and three parameters).

The theoretical dependence of the cross section σ_{in} and the energy E is shown in Fig. 2, which gives also the experimental data.[2] Figure 3 shows the results of the calculation of the energy dependence of the ratio ζ of the differential cross section for dt scattering through an angle $\theta=\pi/2$ to the differential cross section for Coulomb scattering through the same angle (experimental data from the paper by Balashko[3]):

$$\zeta=\frac{d\sigma/d\Omega}{d\sigma^c/d\Omega}=\frac{1}{3}+\frac{2}{3}\left|\exp\left(-2i\eta\ln\sin\frac{\theta}{2}\right)-\frac{ie^{-2i\delta}}{2\eta}(1-S^{11})\right|^2. \qquad (18)$$

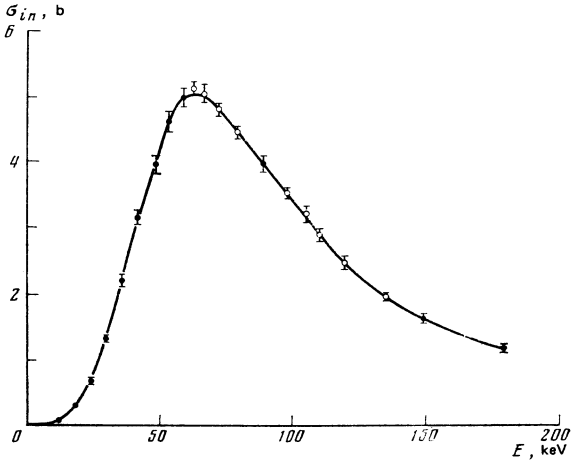

FIG. 2. Cross section $\sigma_{in}(E)$ for the reaction $d+t\to n+{}^4$He. The theoretical curve was calculated from Eq. (16) with the parameters (17). The experimental points are from Ref. 2 (dark from Ref. 2a and light from Ref. 2b).

Figure 4a shows the Argand diagram for the dt scattering amplitude. We have also calculated the n^4He scattering amplitude (the corresponding Argand diagram is shown in Fig. 4b) and verified that our calculation agrees well with the result of a phase-shift analysis of the n^4He scattering in the region of the ^{5}He$^*(\frac{3}{2}^+)$ resonance given in the paper of Hoop and Barschall.[12] As expected, the reduction of the experimental data in the resonance region turned out to be insensitive to the value of ε_0 and to the form factor $F(\varepsilon)$. We have therefore confined ourselves to the use of one set of parameters (17).

§3. EIGENVALUES OF THE HAMILTONIAN OF THE $dt\mu$ SYSTEM WITH ALLOWANCE FOR THE NUCLEAR INTERACTION

We calculate now the eigenvalues of the total Hamiltonian of the $dt\mu$ system

$$H=H^M+V.$$

Here H^M is the Coulomb Hamiltonian of the $dt\mu$ system (its spectrum was obtained in Ref. 13), and V is the nu-

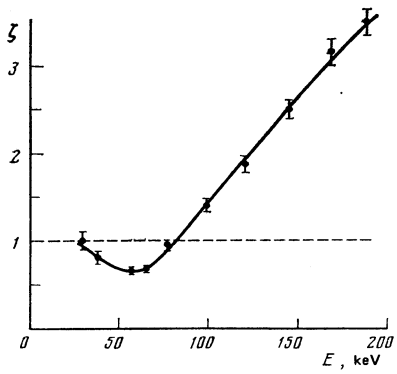

FIG. 3. Energy dependence of at $\theta=\pi/2$ for dt scattering. The experimental points are from Ref. 3. The theoretical curve was calculated from Eq. (18) with the parameters (17).

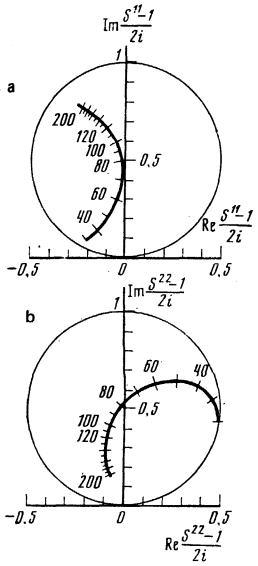

FIG. 4. Argand diagrams [a) for dt scattering $kf^{11} = (S^{11}-1)/2i$; b) for n^4He scattering, $k_2 f^{22} = (S^{22}-1)/2i$] in the region of the resonance ^{5}He$^*(3/2^+)$. The numbers on the curves denote the c.m.s. energy (keV) of collisions, reckoned from the dt threshold.

clear dt interaction whose form we have established in §2:

$$V=\lambda(E)|\xi\rangle\langle\xi|, \quad |\xi\rangle=V_1|0\rangle, \qquad \lambda(E)=(E-E_0-\langle 0|V_2^+ G_2(E) V_2|0\rangle)^{-1}. \tag{19}$$

Using the separable form of the interaction V, we eliminate from the eigenvalue problem of the Hamiltonian H

$$(E-H^M)^{-1}|\xi\rangle\lambda\langle\xi|\tilde{\Psi}^{Jv}\rangle=|\tilde{\Psi}^{Jv}\rangle \tag{20}$$

the unknown eigenfunction $|\Psi^{Jv}\rangle$. To this end, we project Eq. (20) on the vector $\langle\xi|$:

$$\langle\xi|(E-H^M)^{-1}|\xi\rangle\lambda\langle\xi|\tilde{\Psi}^{Jv}\rangle=\langle\xi|\tilde{\Psi}^{Jv}\rangle. \tag{21}$$

For the Green's function

$$(E-H^M)^{-1}=G^M(E)$$

we use the spectral representation

$$G^M(E)=\sum_v \frac{|\Psi^{Jv}\rangle\langle\Psi^{Jv}|}{E-E^{Jv}}+G_c^M. \tag{22}$$

Here $|\Psi^{Jv}\rangle$ are the eigenfunctions of the discrete spectrum of the Hamiltonian H^M, corresponding to the eigenvalues E^{Jv}, and G_c^M is the contribution of the continuous spectrum.

To find the matrix elements $\xi|\tilde{\Psi}^{Jv}\rangle$ and $\langle\xi|G^M|\xi\rangle$ we must know the wave functions $\langle\mathbf{r},\mathbf{R}|\Psi^{Jv}\rangle$ and the Green's function $G_c^M(E,\mathbf{r},\mathbf{R},\mathbf{r}',\mathbf{R}')$ for internuclear distances R and R' of the order of the effective radius R_N of the nuclear forces, which are small compared with the characteristic dimension of the mesic molecule (here r is the coordinate of the μ meson relative to the center of the charges of the nuclei). The asymptotic form of the wave function of the three-particle system with Coulomb interaction as $R\to 0$ was obtained by Vinitskiĭ *et al.*[14] in the adiabatic representation of the three-body problem.[15]

As $R\to 0$, the coordinates of the muon and of the relative motion of the nuclei separate:

$$\Psi^{Jv}(\mathbf{R},\mathbf{r})|_{R\to 0}=\sum_j \Phi_j(\mathbf{r}) \sum_{L=|J-l|}^{J+l} \chi_{jL}^{Jv}(\mathbf{R}). \tag{23}$$

Here $\Phi_j(\mathbf{r})$ is the wave function of the mesic atom in a state with quantum numbers $j=(Nlm)$ for a nucleus with mass m_d+m_t and charge $Z=2$. The functions $\chi_{jL}^{Jv}(\mathbf{R})$ describe the relative motion of d and t.

We represent the matrix element $\langle\mathbf{r},\xi|\Psi^{Jv}\rangle$ in the form

$$\langle\mathbf{r},\xi|\Psi^{Jv}\rangle=\sum_j \sum_{L=|J-l|}^{J+l} \Phi_j(\mathbf{r}) b_{jL}^{Jv}, \quad b_{jL}^{Jv}=\langle\xi|\chi_{jL}^{Jv}\rangle. \tag{24}$$

A nonzero contribution to the sum (24) is made only by the terms with $j=(N\mathcal{J}0)$ and $L=0$, since the operator $|\xi\rangle\langle\xi|$ is a projector on a state with zero orbital angular momentum of the relative motion of d and t. We write down the coefficients $b_{N\mathcal{J}0,0}^{Jv}$, using the results of Vinitskiĭ *et al.*[14]:

$$b_{N\mathcal{J}0,0}^{Jv}=B_N^{Jv}\langle\xi|\varphi_N^{Jv}\rangle.$$

Here $|\varphi_N^{Jv}\rangle$ are the s-wave regular solutions of the system of Eqs.[13] that describes the relative motion of the nuclei in their adiabatic representation of the three-body problem: $\langle R|\varphi_N^{Jv}\rangle|_{R=0}=1$, and B_N^{Jv} is a normalization coefficient calculated in a preceding paper (Ref. 1).[2)]

We write the matrix element $\langle\mathbf{r},\xi|\tilde{\Psi}^{Jv}\rangle$ in the form

$$\langle\mathbf{r},\xi|\tilde{\Psi}^{Jv}\rangle=\sum_N \tilde{B}_N^{Jv}\Phi_{N\mathcal{J}0}(\mathbf{r})\langle\xi|\tilde{\varphi}_N^{Jv}\rangle, \tag{25}$$

where $\tilde{B}_N^{Jv}$ are unknown coefficients.

It is known from the numerical solution[13] of the eigenvalue problem of the Coulomb Hamiltonian of the $dt\mu$ system that $|B_N^{Jv}|^2$ exceeds at $N=J+1$ the sum of the spurs of the moduli of all the remaining coefficients B_N^{Jv} by approximately an order of magnitude. In other words, as $R\to 0$ the muon in the $dt\mu$ mesic molecules has an overwhelming probability of being in one of the following states: $j_0=1s\sigma$ at $J=0$, $j_0=2p\sigma$ at $J=1$, and $j_0=3d\sigma$ at $J=2$. This property is preserved also when allowance is made for the dt nuclear interaction,[1] and we confine ourselves therefore to one term with $N=J+1$ in the sum (25). It suffices then to take into account in G_c^M the contribution of the indicated preferred configuration j_0:

$$\langle\mathbf{r},\mathbf{R}|G_c^M|\mathbf{r}',\mathbf{R}'\rangle = \Phi_{j_0}(\mathbf{r})\Phi_{j_0}^*(\mathbf{r}')\frac{(2m_t^*)^{3/2}}{4\pi^2}\int_0^\infty \frac{|f^M(\varepsilon)|^{-2}\varphi_c^M(R)\varphi_c^M(R')\varepsilon^{1/2}d\varepsilon}{E-\varepsilon}. \tag{26}$$

Here $f^M(\varepsilon)$ and $\varphi_c^M(R)$ are the Jost function and the regular solution for the s-wave scattering $t\mu+d$, m_t^* is the reduced mass of the $t\mu+d$ system, and the energy E is reckoned from the $t\mu+d$ threshold.

Substituting (24)–(26) in (21) and recognizing that at $\ll\varepsilon_0$

$$|\langle\xi|\varphi_N^{Jv}\rangle|^2\approx|\langle\xi|\varphi_c^M\rangle|^2\approx|\langle\xi|\varphi_c\rangle|^2=g/2m_t,$$

we arrive at the eigenvalue equation[3)]

$$(E-E^{Jv})[E-E_s-\Delta_1^{M}(E)+i\Gamma_{in}/2]=g|B^{Jv}|^2/2m_t, \quad (27)$$

$$\Delta_1^{M}(E)=\frac{g(2m_t^*)^{3/2}}{\pi}\int_0^{\infty}\frac{|f^{M}(\varepsilon)|^{-2}F^{M}(\varepsilon)\varepsilon^{1/2}d\varepsilon}{E-\varepsilon}+\sum_{v'\neq v}\frac{|B^{Jv'}|^2}{E-E^{Jv'}}, \quad (28)$$

$$F^{M}(\varepsilon)=|\langle\xi|\varphi_\varepsilon^{M}\rangle|^2/|\langle\xi|\varphi_{\varepsilon=0}^{M}\rangle|^2\approx F(\varepsilon), \quad (29)$$

$$B^{Jv}=B^{Jv}_{J+1}. \quad (30)$$

As shown in Fig. 1, the function $\Delta_1^{M}(E)$ differs insignificantly from the function $\Delta_1(E)$ in the case $J=0$ (the difference is even smaller at $J\geq 1$).

Equation (27) describes the influence of the nuclear resonance in the dt system on the levels of the $dt\mu$ mesic molecule. In the case of weak coupling of the nuclear and mesic-molecule levels, when the condition

$$\frac{2g|B^{Jv}|^2/m_t}{[E_s-E^{Jv}+\Delta_1^{M}(E^{Jv})]^2+\Gamma_{in}^2/4}\ll 1 \quad (31)$$

is satisfied, the solution of (27) corresponding to the mesic-molecule state is close to the pure Coulomb solution:

$$E=E^{Jv}+\frac{g|B^{Jv}|^2/2m_t}{E^{Jv}-[E_s+\Delta_1^{M}(E^{Jv})]+i\Gamma_{in}/2}. \quad (32)$$

§4. RESULTS OF NUMERICAL CALCULATIONS OF THE WIDTHS AND SHIFTS OF THE MESIC-MOLECULE LEVELS

Using the values of the resonant parameters E_S, Γ_{in}, and g determined in §2, and the coefficients $|B^{Jv}|^2$ obtained by numerical solution of the eigenvalue problem for the Coulomb Hamiltonian of the $dt\mu$ system, we have verified that the condition (31) for weak coupling of the levels is satisfied. In this case the nuclear widths Γ^{Jv} and shifts ΔE^{Jv} of the mesic-molecule levels (Jv) are given by

$$\Gamma^{Jv}=\frac{g\Gamma_{in}|B^{Jv}|^2/2m_t}{[E_s+\Delta_1^{M}(0)]^2+\Gamma_{in}^2/4}, \quad (33)$$

$$\Delta E^{Jv}=-\frac{[E_s+\Delta_1^{M}(0)]}{\Gamma_{in}}\Gamma^{Jv}. \quad (34)$$

The calculated widths and shifts of the levels of the $dt\mu$ mesic molecule are listed in Table I. The accuracy of the results, which is governed by the uncertainty with which the resonance parameters were determined from experiment, is 5%. According to our estimates, the contribution of the non-adiabatic corrections, which we have neglected in the derivation of (27), does not exceed 10% for states with $J=1$ and 1% for states with $J=0$.

TABLE I. Nuclear widths Γ^{Jv} and shifts ΔE^{Jv} of the levels (Jv) of the $dt\mu$ mesic molecule. Rate of nuclear fusion reaction $\lambda_f^{Jv}=\Gamma^{Jv}/\hbar$.

(Jv)	$-E^{Jv}$, eV	$\lvert B^{Jv}\rvert^2$, cm^{-3}	Γ^{Jv}, eV	ΔE^{Jv}, eV	λ_f^{Jv}, sec^{-1}
(00)	319.2	$7.47\cdot10^{26}$	$8.2\cdot10^{-4}$	$-9.6\cdot10^{-4}$	$1.2\cdot10^{12}$ ($1.0\cdot10^{12}$)
(01)	34.9	$6.18\cdot10^{26}$	$6.8\cdot10^{-4}$	$-8.0\cdot10^{-4}$	$1.0\cdot10^{12}$ ($0.80\cdot10^{12}$)
(10)	232.4	$7.07\cdot10^{22}$	$6.6\cdot10^{-8}$	$-7.8\cdot10^{-8}$	$1.0\cdot10^{8}$ ($1.1\cdot10^{8}$)
(11)	0.64	$2.71\cdot10^{22}$	$2.5\cdot10^{-8}$	$-3.0\cdot10^{-8}$	$3.9\cdot10^{7}$ ($4.2\cdot10^{7}$)
(20)	102.5	$7.20\cdot10^{19}$	$6.7\cdot10^{-11}$	$-8.0\cdot10^{-11}$	$1.0\cdot10^{5}$ ($1.1\cdot10^{5}$)

Note. The quantities in the parentheses are the results of Ref. 1.

The calculated nuclear level widths of the $dt\mu$ mesic molecule refer to the case when the total spin of the nuclei d and t is $I=\frac{1}{2}$ to the cross section of the reaction (2a) does not exceed 1% in the region next to the threshold,[9] the rate of the nuclear fusion reaction (1) from molecular states with total nuclear spin $I=\frac{3}{2}$ exceeds by two orders the rate of the reaction (1) from states with $I=\frac{1}{2}$. This circumstance must be kept in mind in calculations of the kinetics of mesic-molecule processes with allowance for the hyperfine structure of the $dt\mu$ mesic-molecule levels.

One can expect for the mesic-molecule states with $J=1$, where an important role is played by the wave-function component corresponding to the p-wave of the relative motion of d and t (Ref. 1), that a noticeable contribution to the level widths will be made by the p-wave nuclear interaction of d and t. To take this effect into account we must determine the p-wave amplitude of the dt scattering from experimental data.

The values of Γ^{Jv} obtained in the present paper agree within 10–20% with the results of the preceding paper,[1] and within ~10% with the result of the classical formula (3). An equation similar to (3) and relating the width of the $dt\mu$ mesic-molecule levels with the nuclear-reaction constant A_0 (4) is obtained from (33) by using expression (16) for the cross section of the reaction (2a):

$$\Gamma^{Jv}=\frac{3}{2}\frac{|B^{Jv}|^2}{4\pi}A_0\gamma. \quad (35)$$

Here

$$\gamma=\frac{[E_s+\Delta_1(0)]^2+\Gamma_{in}^2/4}{[E_s+\Delta_1^{M}(0)]^2+\Gamma_{in}^2/4}=\begin{cases}1.16 & (J=0)\\ 1 & (J\neq 0)\end{cases} \quad (36)$$

is a coefficient that characterizes the deviation from the classical factorization relation (3). We recall that

$$|\Psi^{Jv}(0)|^2=|B^{Jv}|^2/4\pi.$$

The additional factor $\frac{3}{2}$ was due to that fact that we are considering mesic-molecule states with a definite nuclear spin $I=\frac{3}{2}$. The resonance parameters obtained in §2 correspond to a reaction constant

$$A_0=\frac{4\pi}{3m_t}\frac{g\Gamma_{in}}{[(E_s+\Delta_1(0))^2+\Gamma_{in}^2/4]}=1.2\cdot10^{-14}\ \text{cm}^3\ \text{sec}^{-1}. \quad (37)$$

The proximity of the coefficient γ to unity can be explained in the following fashion. According to our calculations the energy dependence of the cross section of the reaction

$$t\mu+d\to n+{}^4\text{He}+\mu^-$$

at 5 keV $< E <$ 200 keV has a resonant character.[4)] The resonance in the $t\mu+d$ system is shifted from the resonance in the dt system by an amount $\delta E_R=-4$ keV for the states with $J=0$ ($\delta E_R<4$ keV for $J\neq 0$) (see also Fig. 1). Since the shift δE_R is small compared with the resonance energy $E_R=64$ keV and with the half-width $\Gamma/2\approx 70$ keV, the factorization relation (3) is satisfied with good accuracy.

§5. MUTUAL INFLUENCE OF THE NUCLEAR AND MESIC-MOLECULE LEVELS

In §3 we derived Eq. (27), which describes the mutual influence of the molecular and nuclear levels in the $dt\mu$ system. It is of interest to investigate this equation for arbitrary E_S in the range from $-\infty$ to $+\infty$. The solutions obtained will describe the spectrum of the $dt\mu$ system at different values of the energy of the nuclear resonance ^{5}He* and at a given intensity of the coupling with the channels dt and n^4He (g and Γ_{in} are fixed). As will be made clear presently, the nontrivial phenomena take place only in the case when Eq. (27) has closely lying roots. We can then use for the function $\Delta_1^M(E)$ the approximation

$$\Delta_1^M(E)=\Delta_1^M(0)+Ed, \quad d=\frac{d\Delta_1^M(E)}{dE}\Big|_{E=0} \tag{38}$$

and Eq. (27) takes the form

$$(E-E^{Jv})(E-E_R+i\Gamma_{in}/2)=\tilde{g}|B^{Jv}|^2/2m_1, \tag{39}$$

$$E_R=\frac{E_S+\Delta_1^M(0)}{1-d}, \quad \tilde{\Gamma}_{in}=\frac{\Gamma_{in}}{1-d}, \quad \tilde{g}=\frac{g}{1-d}. \tag{40}$$

If the coupling between the levels is weak ($2\tilde{g}|B^{Jv}|^2/m_1(\tilde{E}_R^2+\Gamma_{in}^2/4)\ll 1$) one of the roots of Eq. (39) corresponds to nuclear resonance with energy E_R:

$$E^{(r)}=\left(E_R-\frac{i\tilde{\Gamma}_{in}}{2}\right)+\frac{\tilde{g}|B^{Jv}|^2/2m_1}{E_R-E^{Jv}-i\Gamma_{in}/2}. \tag{41}$$

The other solution corresponds to the mesic-molecule level:

$$E^{(m)}=E^{Jv}+\frac{\tilde{g}|B^{Jv}|^2/2m_1}{E^{Jv}-E_R+i\Gamma_{in}/2}. \tag{42}$$

In the general case the roots of Eq. (39)

$$E^{\pm}=\frac{1}{2}\left\{E^{Jv}+E_R-\frac{i\Gamma_{in}}{2}\pm\left[\left(E^{Jv}-E_R+\frac{i\Gamma_{in}}{2}\right)^2+\frac{2\tilde{g}|B^{Jv}|^2}{m_1}\right]^{1/2}\right\} \tag{43}$$

can behave in three different ways with changing E_R, depending on the value of $\beta=8\tilde{g}|B^{Jv}|^2/m_1\Gamma_{in}^2$.

1. If $\beta>1$, then in the limit as $\tilde{E}_R\to\pm\infty$ the solutions E^* take the form

$$E^+\to\begin{cases}E^{(m)}, & \tilde{E}_R\to+\infty\\ E^{(r)}, & \tilde{E}_R\to-\infty\end{cases}, \quad E^-\to\begin{cases}E^{(r)}, & \tilde{E}_R\to+\infty\\ E^{(m)}, & \tilde{E}_R\to-\infty\end{cases}.$$

The solution E^-, which corresponds as $\tilde{E}_R\to+\infty$ to the nuclear level $E^{(R)}$ approaches with decreasing $\tilde{E}_R$ that region of the complex energy E plane in which the mesic-molecule level $E^{(m)}$ is located, and occupies this place as $\tilde{E}_R\to-\infty$ (see Fig. 5a). In turn, the solution E^+ corresponding as $E_R\to+\infty$ to the mesic-molecule level, is transformed into a nuclear level as $E_R\to-\infty$. This case, in which the character of the levels changes when the level energies corresponding to interactions with different effective radius of the forces come together (the mesic moleculer level turns into a nuclear level and vice versa) is known in the literature as the spectrum restructuring phenomenon.[1,16]

The restructuring of the spectrum of the $dt\mu$ mesic molecule in the absence of absorption ($\Gamma_{in}=0$, $\beta=\infty$) was considered earlier.[1] It was shown that the probability of landing in the restructuring region, where the solutions E^* differs substantially from $E^{(m)}$ and $E^{(r)}$, is extremely small because d and t in the $dt\mu$ mesic molecule have a low probability of being in the effective r range of the nuclear forces. The mesic-molecule state wave function localized at large distances goes over continuously, in the course of the restructuring, into a nuclear-state wave function localized at short distances. In the restructuring region, the system stays at short and at long distances with equal probabilities.

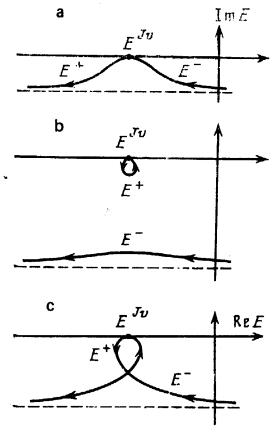

FIG. 5.

2. If $\beta<1$, then the limiting values of the solutions take the form

$$E^+\to E^{(m)}, \ \tilde{E}_R\to\pm\infty, \ E^-\to E^{(r)}, \ \tilde{E}_R\to\pm\infty.$$

With changing $\tilde{E}_R$, the solution E^+ carries out finite motion in the complex plane near $E^{(m)}$ (Fig. 5b), and the solution E^- passes outside the region $E\approx E^{(m)}$. Thus, at arbitrary $\tilde{E}_R$ the corresponding wave function of the relative motion of d and t is localized at the characteristic mesic-molecule distances. The absence of spectrum restructuring in this case is due to the presence of intense absorption (strong coupling with the open channel: $\Gamma_{in}^2>8g|B^{Jv}|^2/m_1$). Using the calculated values of Γ_{in} and g [Eq. (17)], and of B^{Jv} (Table I), we obtain for all the $dt\mu$ mesic-molecule levels a value $\beta\leq 2.6\times10^{-7}\ll 1$, and consequently, no matter what the value of $\tilde{E}_R$, restructuring of the mesic-molecule spectrum is excluded. The reasons for this were already noted above: the low probability of finding d and t in the effective range of action of the nuclear forces for the mesic-molecule states of the $dt\mu$ system, and the large inelastic width Γ_{in} of the nuclear resonance.

The result means that the theoretical predictions of the nuclear widths and level shifts of the $dt\mu$ mesic molecule are "stable," despite the possibility of varying the parameters E_S, g, and Γ_{in} within the limits admitted by the errors in the experimental data on reaction (2a) and on the scattering (2b).

3. The case $\beta=1$ is intermediate. At $\tilde{E}_R=E^{Jv}$ we have degeneracy: $E^+=E^-$ (Fig. 5c).

§6. CONCLUSION

We have calculated the nuclear widths Γ^{Jv} and shifts ΔE^{Jv} of the levels (Jv) of the mesic molecule $dt\mu$, due to resonant interaction of d and t in the s-wave with spin $I=\frac{3}{2}$. It was established that the influence of the nuclear resonance on the mesic-molecule states is weak be-

cause of the low probability of finding d and t in the effective range of the nuclear forces in the $dt\mu$ mesic molecule and to the large inelastic width of the resonance, and that the spectrum of the $dt\mu$ molecule is stable to variations of the nuclear-interaction parameters within the limits allowed by the available experimental data.

It was shown that the position of the nuclear resonance in the $dt\mu$ system coincides with the position of the dt resonance, with accuracy of the order of the average energy of the interaction of the muon in the $dt\mu$ system. Since the muon, which is bound to d and t by electromagnetic forces, has a low probability of being located in the region of the d and t nuclear interaction, the shift of the nuclear resonance is also small, and a restructuring of the spectrum of the mesic-molecule levels is excluded. Under these conditions, the coupling of the nuclear widths with the reaction constant (4) is described with good accuracy by the classical factorization formula (3).

The results agree well with the widths Γ^{Jv} calculated by using a generalized optical potential[1] corresponding to the model of the coupled channels dt and n^4He.

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[1)] Equation (3) was used earlier[5,6] only to calculate the nuclear widths of the ground state ($J = 0$, $v = 0$) of the $dt\mu$ mesic molecules.

[2)] The coefficient $B^{Jv}_{N=J+1}$ was designated in Ref. 1 by B^{Jv}_0.

[3)] An eigenvalue equation can be written also for the case of an arbitrary number of terms in the sums (24) and (25): it is the condition that the resultant system of linear homogeneous equations have a solution with respect to the coefficients $\tilde{B}^{Jv}_N$.

[4)] The caluation was carried out using Eq. (16) in which $f(\varepsilon)$ was replaced by $f^m(\varepsilon)$ and $\Delta_1(E)$ by $\Delta_1(E)$.

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