Dynamics of a domain wall in the spin-wave approximation

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The interaction of a moving domain wall with spin excitations is considered. No assumptions are made in advance regarding the final micromagnetic structure of the wall, the nature of its motion, or the processes responsible for dissipation of energy in the system. A system of equations is derived that describes the motion of the domain wall, the dynamic distortions of its structure, and the precessional and translational spin waves. In the case of nonstationary motion in a sufficiently large constant external field, the processes that emerge from these equations are studied in the lowest orders with respect to the amplitudes of the spin excitations, and the mean velocity of the wall motion caused by them is also calculated.

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Within the last decade, because of the practical development of memory devices based on mobile cylindrical magnetic domains (bubbles), and also because of intensive investigations of various nonlinear excitations in magnetically ordered media, there has been a sudden growth of interest in the dynamical behavior of domain walls (DW) in uniaxial high-anisotropy ferro- and ferrimagnets. A number of interesting results in this direction have been obtained by consideration of free DW motion,[1] on the assumption that both external excitation and damping are absent in the system. But in actual cases, the DW motion occurs under the action of an external magnetic field, and the character of this motion is determined by dissipative processes in the system.

In the investigation of dynamic SW behavior, two methods are usually used. The first is based on the Landau-Lifshitz equation, in which the dissipation of energy is taken into account phenomenologically.[2,3] The equation contains a parameter, describing the damping, which is estimated for each specific material either from the linewidth in ferromagnetic resonance or from the experimentally found value of the initial mobility of the DW. The values of the damping parameter obtained by these two methods in many cases differ sharply.[4] Furthermore, even the solutions of the equation connected by this method to experiment nevertheless describe poorly the motion of a DW in sufficiently large fields. Such an approach to the problem is not connected with real dissipative processes in a ferromagnet, although many observed features of the dynamic behavior of DW may be the result of the action of specific mechanisms of scattering of the Zeeman energy.

The other approach to the problem is free of this inadequacy. It is based on calculation of definite microscopic processes that lead to dissipation of energy: for example, interaction of a moving DW with magnons[5] or with phonons.[6] The velocity of the wall is determined by the condition that the change of Zeeman energy must be equal to the energy scattered by the quasiparticles. But whereas in the first method the primary attention is directed at the micromagnetic structure of the DW, and the damping is taken into account phenomenologically, in this case the picture is the reverse: concentrating their attention on the microscopic dissipative processes, the authors prescribe in advance the structure of the DW and the character of its motion (usually the motion of the wall is assumed to be uniform).

The present paper gives a self-consistent description[1)] of the motion, under the action of a constant magnetic field, of a 180-degree DW in a uniaxial high-anisotropy ferromagnetic dielectric. No assumptions are made in advance regarding its structure and the character of its motion, or regarding the specific mechanisms responsible for the dissipation of energy. Both are derived from the Landau-Lifshitz equation without a dissipative term. This equation describes the DW structure, the spin waves (SW), and also their interaction; this enables us to treat in a unified manner all the processes that are occurring in the magnetic subsystem of the material.

The paper derives, in the most general form, an equation that connects the velocity of motion of the wall with the amplitude of the spin excitations. Various mechanisms of energy dissipation are considered, and their contribution to the DW velocity is calculated.

1. BASIC EQUATIONS

We consider an infinite high-anisotropy ferromagnet with orthorhombic magnetic anisotropy. We choose the coordinate system so that the z axis is directed along the axis of easy magnetization of the ferromagnet, and the yz plane is parallel to a solitary DW. We shall describe the behavior of the magnetization $\mathbf{M}$ in such a system by the equation

$$\partial\mathbf{M}/\partial t=-g[\mathbf{M}\times\mathbf{H}], \tag{1}$$

where $g>0$ is the gyromagnetic ratio. In the effective field $\mathbf{H}$ we shall, for simplicity, take into account the minimum number of magnetic interactions necessary for existence of the DW and for its motion:

$$\mathbf{H}=\alpha\Delta\mathbf{M}+\beta_1\mathbf{e}_z(\mathbf{e}_z\mathbf{M})+\beta_2\mathbf{e}_x(\mathbf{e}_x\mathbf{M})-4\pi\mathbf{e}_x(\mathbf{e}_x\mathbf{M})+h\mathbf{e}_z. \tag{2}$$

Here $\mathbf{e}_j$ are the unit vectors of the chosen coordinate system. The first term is the effective exchange field (α is the exchange parameter), the second and third are the orthorhombic magnetic anisotropy field (the z axis will be an axis of easy magnetization if $\beta_1>|\beta_2|$),

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the fourth is the demagnetizing field in the Winter approximation,[7] and the last is the constant external field, oriented along the z axis. The third and fourth terms have the same form; it is therefore convenient to combine them into one by introducing the quasidipole interaction parameter $\eta = 4\pi - \beta_2$. It must be remembered, however, that the fourth term in (2) accurately describes the demagnetizing field only in the case of a one-dimensional distribution of the magnetization along the x axis, whereas the expression for the plane components (in the xy plane) of the anisotropy field in (2) are applicable in the general case. In this sense, the theory developed below will be exact for $|\eta| \gg 4\pi$, when the contribution of the dipole interaction can be neglected.

We choose a second system of coordinates, which moves with the DW along the x axis with some velocity $v(t)$. We shall seek a solution of equation (1) in this comoving system in the form

$$\mathbf{M}(\xi, y, z, t) = \mathbf{M}_0(\xi, t) + \mathbf{m}(\xi, y, z, t), \quad \xi = x - \int_{t_0}^{t} v(t)\, dt.$$

$\mathbf{M}_0(\xi, t)$ roughly describes the structure of the SW and is determined by the polar angle θ and azimuthal angle φ as follows:

$$\cos\vartheta = \operatorname{th}(\xi/\delta), \quad \varphi = \varphi(t), \quad \delta = [\alpha/(\beta_1 + \nu\eta)]^{1/2}. \tag{3}$$

Here the angle θ is measured from the z axis, φ from the y axis. The distortion of the original DW structure (3) and also the SW are described by the function $\mathbf{m}(\xi, y, z, t)$, which, together with $v(t)$ and $\varphi(t)$, must be found from equation (1). The DW thickness δ also contains the still undetermined parameter ν. It allows for the effect on δ of the quasidipole interaction, and it depends on the micromagnetic structure of the DW. In statics, for a pure Bloch DW ($\varphi = 0, \pi$), $\nu = 0$; for a Néel wall ($\varphi = \pm\pi/2$), $\nu = 1$.[8]

We choose at each point of coordinate space a local reference system for the magnetic moments, in general time-dependent, in which $\mathbf{M}_0 = (0, 0, M_s)$. On writing (1) in components in these local systems and on introducing, instead of the projections m_j, the cyclic variables $m = m_1 + im_2$, $m^* = m_1 - im_2$, we get, after simple but cumbersome calculations, carried out through terms cubic in m (with exclusion of the cubic quasidpole terms),

$$\begin{aligned}
&i\dot{m} - \beta(\Delta m - \cos 2\vartheta\, m) - iv(t)\,\partial m/\partial\xi \\
&+{}^1/_2\{(1-2\nu)\eta - 2(\dot{\varphi} - h)\cos\vartheta + 3\eta(\nu - \sin^2\varphi)\sin^2\vartheta\}m \\
&+{}^1/_2\eta\{\cos 2\varphi - (\nu - \sin^2\varphi)\sin^2\vartheta - i\sin 2\varphi\cos\vartheta\}m^* \\
&-i\beta\sin\vartheta(m + m^*)\,\partial m/\partial\xi + {}^1/_2\{[\eta\sin 2\varphi + u(t)]\sin\vartheta \\
&-i[\beta - {}^3/_2\eta(\nu - \sin^2\varphi)]\sin 2\vartheta\}mm^* \\
&+{}^1/_4\{\eta\sin 2\varphi\sin\vartheta + i[2\beta - \eta(\nu - \sin^2\varphi)]\sin 2\vartheta\}m^2 \\
&-{}^1/_2\beta\{m^2\Delta m^* + 2m(\nabla m\nabla m^*) + \cos 2\vartheta\, m^2 m^*\} \\
&= u(t)\sin\vartheta + {}^1/_2 i\eta(\nu - \sin^2\varphi)\sin 2\vartheta,
\end{aligned} \tag{4}$$

and also the equation complex-conjugate to this. In these equations,

$$\beta = \beta_1 + \nu\eta, \quad u(t) = v(t) + {}^1/_2\eta\sin 2\varphi + i(\dot{\varphi} - h).$$

Here and everywhere hereafter, the units of measurement for the magnetization and field are the saturation magnetization M_s; for distance, δ; for time, $(gM_s)^{-1}$; and for energy, $M_s^2\delta^3$.

We transform from a coordinate to a momentum representation. For this purpose, we expand $m(\mathbf{r}, t)$ in normalized Winter functions,[7,9] which are eigenfunctions of the operator $\Delta - \cos 2\theta$. The amplitudes obtained by this procedure for the translational ($m_\varkappa(t)$) and precessional ($m_\mathbf{k}(t)$) spin excitations ($\varkappa$ is a two-dimensional wave vector in the DW plane, $\mathbf{k}$ is a three-dimensional wave vector) satisfy equations that can be easily obtained from (4); the free terms of these equations have a δ-function singularity at the point $\varkappa = 0$. One can expect the same singularity to appear also in the solutions; therefore we shall seek them in the form

$$m_\varkappa(t) = a(t)\delta(\varkappa) + c_\varkappa(t), \quad m_\mathbf{k}(t) = a_q(t)\delta(\varkappa) + c_\mathbf{k}(t), \tag{5}$$

where $c_\varkappa$ and $c_\mathbf{k}$ are functions regular at zero, and where $q \equiv k_\xi$.

We substitute (5) in the equations of motion for the amplitudes and separate out from them the terms containing the indicated singularity, by operating with the operator

$$K_0 = \lim_{\varepsilon\to 0}\int_{-\varepsilon}^{\varepsilon}\int_{-\varepsilon}^{\varepsilon} \ldots d\varkappa$$

and taking into account that $K_0\zeta(\varkappa)\delta(\varkappa) = \zeta(0)$ and $K_0\zeta(\varkappa) = 0$, where $\zeta(\varkappa)$ is any function regular at zero. This transformation enables us to obtain two inhomogeneous equations for $a(t)$ and $a_q(t)$. The remaining regular terms give two homogeneous equations for $c_\varkappa(t)$ and $c_\mathbf{k}(t)$. These and the others can be written schematically as follows:

$$\dot{a} + f(a, a_q, c_\varkappa, c_\mathbf{k}, \varphi, v) = -2^{3/2} i\pi u(t), \tag{6}$$

$$\dot{a}_q - i\omega_q a_q + f_q(a, a_q, c_\varkappa, c_\mathbf{k}, \varphi, v) = -2(\pi/2)^{3/2}\eta(\nu - \sin^2\varphi)(1 + iq)\operatorname{ch}^{-1}(\pi q/2), \tag{7}$$

$$\dot{c}_\mathbf{k} - i\omega_k c_\mathbf{k} + f_\mathbf{k}(a, a_q, c_\varkappa, c_\mathbf{k}, \varphi, v) = 0, \tag{8}$$

$$\dot{c}_\varkappa - i\omega_\varkappa c_\varkappa + f_\varkappa(a, a_q, c_\varkappa, c_\mathbf{k}, \varphi, v) = 0, \tag{9}$$

where $\omega_q = \beta(1 + q^2)$, $\omega_k = \beta(1 + k^2)$, and $\omega_\varkappa = \beta\varkappa^2$ are the eigenfrequencies of the spin excitations. The functionals f_j contain terms linear in a, a_q, $c_\varkappa$, and $c_\mathbf{k}$, of quasidipole origin, and terms of higher order; in the linear approximation, f and f_q are independent of $c_\varkappa$ and $c_\mathbf{k}$, and $f_\mathbf{k}$ and $f_\varkappa$ are independent of a and a_q. The functionals f_j depend also on the complex-conjugate amplitudes, which have not been written out for simplicity of notation. Furthermore, for completeness of the system (6)–(9) we must add to them the equations complex-conjugate to those given.

Equations (6) and (7) determine the amplitudes $a(t)$ and $a_q(t)$ of the one-dimensional dynamic distortions of the original DW structure (3), which occur under the action of a driving force of quasidipole origin. We note that $a(t)$ is the amplitude of the translational distortions, which constitute small displacements of the DW as a whole and rotations of the fan of its magnetic moments about the z axis. The homogeneous equations (8) and (9) determine, respectively, the amplitude $c_\mathbf{k}(t)$ of precessional and $c_\varkappa(t)$ of translational SW. Equations (8) and (9), in contrast to (6) and (7), permit, in particular, zero solutions. Such a separation of spin excitations into distortions and SW is convenient, though to a considerable degree conventional.

So far we have not made specific the form of the function $u(t)$ [or, equivalently, $v(t)$ and $\varphi(t)$]. The function $u(t)$ is not determined uniquely by the system (6)–(9), since the number of unknown functions is larger than the number of equations; but its choice can also not be arbitrary. The arbitrariness is restricted by the requirement of smallness of the translational distortions $a(t)$ (in the local reference systems), which are the elementary motions of which the macroscopic DW motion is made up. In order that the system (6)–(9) may become complete and the DW motion self-consistent, we complete the system by the relation $a(t)\equiv 0$. This identity removes the arbitrariness in the choice of $v(t)$ and $\varphi(t)$, and equation (6) at once gives the connection between $u(t)$ and the amplitudes of the SW and of the DW distortions:

$$v(t)=-{}^1/{}_2\eta\sin 2\varphi-2^{-3/2}\pi^{-1}\operatorname{Im} f(a_q,\ c_\varkappa,\ c_{\mathbf{k}},\ \varphi,\ v), \tag{10}$$

$$\dot\varphi(t)=h+2^{-3/2}\pi^{-1}\operatorname{Re} f(a_q,\ c_\varkappa,\ c_{\mathbf{k}},\ \varphi,\ v). \tag{11}$$

Analytical solutions of the equations presented can be obtained in two limiting cases. The first is the region of small external fields, where the equality $2^{3/2}\pi h=-\operatorname{Re} f$ can be satisfied. Under this condition, $\dot\varphi=0$, and the DW moves with constant velocity. The second case is defined by the conditions $2^{3/2}\pi|h|\gg|\operatorname{Re} f|$ and $2^{1/2}\pi|\eta|\gg|\operatorname{Im} f|$. If they are satisfied, then the motion is nonstationary[2,3]; the function $\varphi(t)$ is nearly linear, and the velocity oscillates with frequency

$$\omega_0=2h+2^{-1/2}\pi^{-1}\operatorname{Re}\bar f\approx 2h,$$

where the bar denotes a time average. The first case leads to results similar to those obtained by Abyzov and Ivanov[5]; therefore we shall consider only the second case. It should be mentioned that the conditions corresponding to it, and relating h, η, and f, are not too rigid, since for high-anisotropy ferromagnets, as will be seen from what follows, the estimate $|f|\sim 1$ is usually correct.

In the system (7)–(11), the parameter ν has still remained undetermined. There are no limitations in principle on its choice. But we shall attempt to choose the original DW structure (3) so that its dynamic distortions shall be minimal. Therefore we choose the value $\nu=\frac{1}{2}$, for which the driving force, and also the amplitudes a_q determined by equation (7), will not contain constant components. With such a choice, the mean-thickness parameter of the dynamic DW coincides with the thickness of the original DW. In the general case, when the function $\varphi(t)$ is arbitrary, the requirement of equality of the thicknesses of the original DW and of the DW "dressed" by dynamic distortions leads to the relation $\nu=\sin^2\varphi$.

The functional f, which determines the DW dynamics, takes the form, when all the restrictions presented above have been taken into account,

$$f(a_q,c_\varkappa,c_{\mathbf{k}},\varphi,v)=-\frac{1}{4}i\pi^{1/2}\eta\sin 2\varphi\operatorname{Im} I_1+\frac{1}{24}\pi^{1/2}\eta\cos 2\varphi(3I_2+I_2^*)$$

$$-\frac{i\beta}{2^{9/2}\pi}K_0\left\{\int\frac{(q''^2-q'^2)(q'^2+q''^2+2)}{(1+iq')(1+iq'')}\operatorname{sh}^{-1}\frac{\pi}{2}(q'+q'')c_{\mathbf{k}'}c_{\mathbf{k}''}\right.$$

$$\times\delta(\varkappa-\varkappa'-\varkappa'')d\mathbf{k}'d\mathbf{k}''+2\int\frac{(1-iq')(q''^2-q'^2)}{1-iq''}\operatorname{sh}^{-1}\frac{\pi}{2}(q''-q')c_{\mathbf{k}'}c^*_{\mathbf{k}''}$$

$$\left.\times\delta(\varkappa-\varkappa'+\varkappa'')d\mathbf{k}'d\mathbf{k}''\right\}-\frac{2^{1/2}i}{24\pi}\eta\sin 2\varphi K_0\left\{\int c_{\varkappa'}c_{\varkappa''}\delta(\varkappa-\varkappa'-\varkappa'')d\varkappa'd\varkappa''\right.$$

$$\left.+2\int c_{\varkappa'}c^*_{\varkappa''}\delta(\varkappa-\varkappa'+\varkappa'')d\varkappa'd\varkappa''\right\}; \tag{12}$$

$$I_1=\int_{-\infty}^{\infty}(1-iq)\operatorname{ch}^{-1}\left(\frac{\pi q}{2}\right)a_q dq,\quad I_2=\int_{-\infty}^{\infty}q(1-iq)\operatorname{ch}^{-1}\left(\frac{\pi q}{2}\right)a_q dq,$$

where the integration extends over the whole of momentum space. In (12) we have retained only terms of the lowest order in a_q, $c_{\mathbf{k}}$, and $c_\varkappa$ that are not identically equal to zero.

It is evident that all excitations of the spin system exert an influence on the nature of the DW motion. The maximum contribution can be expected from the amplitudes a_q, which occur in (12) already in the first order; the minimum, from the translational SW, whose amplitudes are contained only in the quadratic quasidipole terms. But actually the relation of the contributions can change, since so far nothing is known about a_q, $c_{\mathbf{k}}$, and $c_\varkappa$, which must be determined from equations (7)–(11); therefore we shall pass on to a systematic calculation of the amplitudes of the dynamic distortions of the DW, and also of the precessional and translational SW.

2. DYNAMIC DISTORTIONS OF DOMAIN-WALL STRUCTURE

The presence in the right side of equation (7) of a driving force of quasidpole origin causes the occurrence of distortions of the DW structure (3) postulated earlier. In order to estimate these distortions, we shall omit f_q in (7). This is justified because in the linear approximation, f_q contains only quasidipole terms much smaller than the first two terms of the equation. Furthermore, with the same degree of accuracy we shall replace 2φ by $\omega_0 t$. After these simplifications, the solution of equation (7) is obtained by elementary methods and has the form

$$a_q(t)=-i\left(\frac{\pi^3}{32}\right)^{1/2}\eta\frac{1+iq}{\operatorname{ch}(\pi q/2)}\left\{\frac{e^{i\omega_0 t}}{\omega_q-\omega_0+i\gamma_q}+\frac{e^{-i\omega_0 t}}{\omega_q+\omega_0+i\gamma_q}\right\}, \tag{13}$$

where we have allowed for possible damping of the distortions by replacing ω_q by $\omega_q+i\gamma_q$. On transforming from the momentum representation (13) again to the coordinate representation, we obtain an expression that describes the distortions of the DW structure in the local reference systems:

$$m(\mathbf{r},t)=\frac{i\pi\eta}{16\beta}\operatorname{sign}\xi\,[e^{i\omega_0 t}\chi(\tilde\omega_0,\xi)+e^{-i\omega_0 t}\chi(-\tilde\omega_0,\xi)]; \tag{14}$$

$$\chi(\tilde\omega_0,\xi)=\frac{q(\tilde\omega_0)+i\operatorname{th}|\xi|}{q(\tilde\omega_0)\operatorname{ch}(\pi q(\tilde\omega_0)/2)}e^{i|\xi|q(\tilde\omega_0)}$$

$$-\frac{4}{\pi}\sum_{n=0}^{\infty}(-1)^n\frac{(2n+1)+\operatorname{th}|\xi|}{(2n+1)^2+q^2(\tilde\omega_0)}e^{-(2n+1)|\xi|}, \tag{15}$$

where $\tilde\omega_0=\omega_0/\beta$, $\tilde\gamma=\gamma/\beta$, $q(\tilde\omega_0)=(\tilde\omega_0-1-i\tilde\gamma)^{1/2}$ [that value of the root is meant for which $\operatorname{Im} q(\tilde\omega_0)>0$]. Here it is assumed that the damping coefficient γ is independent of q.

In the special case when $\gamma=+0$, for $\tilde\omega_0<1$ the value of $q(\pm\tilde\omega_0)$ is pure imaginary, and the DW distortions decrease exponentially at large distances from the wall. The situation changes fundamentally when $\tilde\omega_0\geqslant 1$. In this range of fields, $q(\tilde{\omega}_0)$ is real, and the first term in (15) is nonzero at infinity. Periodic excitations, whose phase velocities are antiparallel, are propagated in the regions $\xi \gg 1$ and $\xi \ll -1$ in a ferromagnet. The DW radiates two SW[2)] that diverge from it. This case was treated by Khodenkov[10] in the approximation of SW short in comparison with the wall thickness.

In the general case, when $\gamma \neq 0$, the value of $q(\pm\tilde{\omega}_0)$ is always complex; therefore the first term in (15), and with it also the whole expression (14), are nonzero only near the DW. The distortions, like the wall itself, are always localized.

3. SCATTERING OF THERMAL MAGNONS BY A MOVING WALL

We consider equation (8), which determines the amplitudes of precessional SW. In the linear approximation, $f_{\mathbf{k}}$ depends only on $c_{\mathbf{k}}$, $c_{\mathbf{k}}^*$, $c_\varkappa$, and $c_\varkappa^*$. The term containing $c_\varkappa$ and $c_\varkappa^*$ may be omitted, since the processes corresponding to them, conserving the wave vector $\varkappa$, are forbidden, at least for $\tilde{\omega}_0 < 1$, by the law of conservation of energy in the first Born approximation. One can proceed similarly in the range $\tilde{\omega}_0 < 2$ with terms containing $c_{\mathbf{k}}^*$. The remaining linear terms describe two-particle scattering of precessional SW by a moving DW whose structure is determined by the relations (3). Scattering of SW by the distortions (14) of this structure is a second-order process; but in contrast to the processes described by the linear terms and having a quasidipolar origin, it is caused by exchange and anisotropy. It may therefore make a contribution comparable with the scattering by the original DW structure, and it must be taken into account.

After these simplifications, Eq. (8), with use of (14), takes the form

$$\dot{c}_{\mathbf{k}} - i\omega_k c_{\mathbf{k}} + \frac{1}{2} i\eta q \sin\omega_0 t c_{\mathbf{k}} - \frac{i}{\hbar} e^{i\omega_0 t} \int \Phi(\mathbf{k},\mathbf{k}') c_{\mathbf{k}'} d\mathbf{k}' - \frac{i}{\hbar} e^{-i\omega_0 t} \int \Phi^*(\mathbf{k}',\mathbf{k}) c_{\mathbf{k}'} d\mathbf{k}' = 0; \tag{16}$$

$$\Phi(\mathbf{k},\mathbf{k}') = \frac{\hbar}{16} \eta \delta(\varkappa - \varkappa') \left\{ \frac{(q'-q)[1+q^2+q'^2+qq'+i(q'+q)]}{(1+iq')(1-iq)\,\mathrm{sh}(\pi(q'-q)/2)} - \pi \int_{-\infty}^{\infty} \left[\psi_{q'}(\xi) \frac{\partial \psi_q^*(\xi)}{\partial \xi} \chi(\tilde{\omega}_0, \xi) + \frac{\partial \psi_{q'}(\xi)}{\partial \xi} \psi_q^*(\xi) \chi(-\tilde{\omega}_0, \xi) \right] \mathrm{ch}^{-1}\xi \,\mathrm{sign}\,\xi \, d\xi - \pi \int_{-\infty}^{\infty} [\chi(\tilde{\omega}_0,\xi) + \chi^*(-\tilde{\omega}_0,\xi)] \psi_{q'}(\xi) \psi_q^*(\xi) \mathrm{ch}^{-1}\xi \,\mathrm{th}\,\xi \,\mathrm{sign}\,\xi \, d\xi \right\}, \tag{17}$$

$$\psi_q(\xi) = (2\pi)^{-1/2} \frac{iq - \mathrm{th}\,\xi}{1+iq} e^{iq\xi}.$$

The first term in (17) determines the interaction of the SW with the DW proper, the two others with its dynamic distortions.

Hereafter, for study of the scattering of thermal SW, it is necessary to go over from the classical equations (16) to a quantum description of the process, according to well known rules. The classical amplitudes $c_{\mathbf{k}}(2\hbar)^{-1/2}$ and $c_{\mathbf{k}}^*(2\hbar)^{-1/2}$ must be replaced, respectively, by operators $\hat{c}_{\mathbf{k}}^+$ and $\hat{c}_{\mathbf{k}}$, which in the linear approximation satisfy the Bose commutation relations. The Hamiltonian of such a quantum system has the form

$$\mathcal{H} = \int \varepsilon_k \hat{c}_{\mathbf{k}}^+ \hat{c}_{\mathbf{k}} d\mathbf{k} - \frac{\hbar}{2} \eta \sin\omega_0 t \int q \hat{c}_{\mathbf{k}}^+ \hat{c}_{\mathbf{k}} d\mathbf{k} + e^{i\omega_0 t} \int \Phi(2,1) \hat{c}_1^+ \hat{c}_2 dk' dk'' + \text{H.c.}, \tag{18}$$

where $1 \equiv \mathbf{k}'$, $2 \equiv \mathbf{k}''$, $\varepsilon_{\mathbf{k}} = \hbar\omega_{\mathbf{k}}$. The second term appears because of the motion of the comoving coordinate system and describes a phenomenon analogous to the Doppler effect. The third and fourth terms describe inelastic transitions in the magnon system under the action of the periodic perturbation due to the oscillating motion of the DW and to the precession of the magnetization in it.

The motion of the DW is not strictly periodic: the mean velocity of the wall is in general nonzero. This forward motion causes additional scattering of magnons, which was treated by Abyzov and Ivanov.[5] We shall not take account of this process, since in our case the constant component of the velocity is small in comparison with the variable component taken into account in (18).

Inelastic transitions in the magnon system lead to a change of its energy; in other words, there arises a possibility of dissipation of the Zeeman energy, and consequently also of forward motion of the DW. In order to calculate the velocity of this motion, we use the nonequilibrium part of the correlation function $\langle \hat{c}_1^+ \hat{c}_2 \rangle$. It can be calculated by means of formula of Kubo,[11] describing the linear response of a system to a periodic perturbation. Omitting the calculation, we give only the result:

$$\langle \hat{c}_1^+ \hat{c}_2 \rangle = n_1^0 \delta(\mathbf{k}' - \mathbf{k}'') + (n_1^0 - n_2^0) \left\{ \frac{\Phi(1,2) e^{i\omega_0 t}}{\varepsilon_1 - \varepsilon_2 - \hbar\omega_0 + i0} + \frac{\Phi^*(2,1) e^{-i\omega_0 t}}{\varepsilon_1 - \varepsilon_2 + \hbar\omega_0 + i0} \right\}, \tag{19}$$

where n_k^0 is the Bose equilibrium distribution function. The first term of the nonequilibrium part of the correlation function appears because of transitions that involve increase of the magnon energy; the second, decrease. The mean values of other operators can be calculated similarly; for example, it can be shown that $\langle \hat{c}_1^+ \hat{c}_2^+ \rangle = 0$. Hereafter we shall need only these two correlation functions; therefore we pass on to consideration of translational SW.

4. PARAMETRIC EXCITATION OF TRANSLATIONAL SPIN WAVES

Finally, we consider equation (9). In the linear approximation, $f_\varkappa$ in (9), like $f_{\mathbf{k}}$, depends only on the amplitudes $c_\varkappa$, $c_\varkappa^*$, $c_{\mathbf{k}}$, and $c_{\mathbf{k}}^*$. Terms containing $c_{\mathbf{k}}$ and $c_{\mathbf{k}}^*$ may be omitted when $\tilde{\omega}_0 < 1$, for the same reason as were terms containing $c_\varkappa$ and $c_\varkappa^*$ in (8). The remaining part gives the equation

$$\dot{c}_\varkappa - i\omega_\varkappa c_\varkappa + \gamma_\varkappa c_\varkappa - {}^1/{}_2 i\eta \cos\omega_0 t c_\varkappa - {}^1/{}_3 i\eta \cos\omega_0 t c_{-\varkappa}^* = 0,$$

where the damping of translational SW has been taken into account phenomenologically, as in (13). The fourth term gives a slight modulation of the SW energy and may be omitted, like the terms containing $c_{\mathbf{k}}$. The last term leads, for a certain relation between the parameters η and $\gamma_\varkappa$, to the development of parametric instability of the translational SW[12]: $c_\varkappa$ increases with time, without limit. Limitation of the growth of the amplitude occurs because of nonlinear processes,[13] which have not been completely investigated even in a uniformly magnetized ferromagnet.

Without claiming completeness and rigor of our treatment of the problem, we shall attempt to estimate the amplitude of parametric SW, taking into account in f only the terms cubic in the amplitudes of the translational SW, which appear because of exchange and anisotropy. In this case the equation of motion will have the form

$$\dot{c}_\varkappa - i\omega_\varkappa c_\varkappa + \gamma_\varkappa c_\varkappa - \frac{1}{6} i\eta e^{i|\omega_0|t} c_{-\varkappa}^* + \frac{i\beta}{120\pi^2}\int [10(\varkappa_2\varkappa_3) - 5\varkappa_3^2 - 4] c_1 c_2 c_3^* \delta(\varkappa - \varkappa_1 - \varkappa_2 + \varkappa_3)\, d\varkappa_1 d\varkappa_2 d\varkappa_3 = 0. \tag{20}$$

Following S theory,[13] we multiply the integrand in (20) by the sum where

$$\Delta(\varkappa) = \begin{cases} 1, & \text{if } \varkappa = 0; \\ 0, & \text{if } \varkappa \neq 0. \end{cases} \tag{21}$$

As a result of this procedure, only those parametric SW will contribute to the integral for which the product $c_1 c_2 c_3^*$ of the amplitudes has the same phase as $c_\varkappa$. The δ function in (20), in combination with the factor (21), enables us to remove one integral and to reduce equation (20) to quasilinear form, with renormalized frequency and pump. Introducing in this equation new "slow" amplitudes $b_\varkappa(t) = c_\varkappa(t)\exp(-i|\omega_0|t/2)$ and replacing their products by the correlation functions

$$\langle b_\varkappa b_{\varkappa'}^* \rangle = n_\varkappa \delta(\varkappa - \varkappa'), \quad \langle b_\varkappa b_{\varkappa'} \rangle = \sigma_\varkappa \delta(\varkappa + \varkappa'), \tag{22}$$

we get

$$\dot{b}_\varkappa - i\left(\omega_\varkappa + \frac{\beta}{15\pi^2}N - \frac{|\omega_0|}{2} + i\gamma_\varkappa\right) b_\varkappa - \frac{1}{6} i\left[\eta + \frac{\beta}{20\pi^2}(5\varkappa^2 + 4)S\right] b_{-\varkappa}^* = 0, \tag{23}$$

$$N = \int n_\varkappa d\varkappa, \quad S = \int \sigma_\varkappa d\varkappa.$$

It can be shown that this equation leads to the equality $|\sigma_\varkappa| = n_\varkappa$; that is, $\sigma_\varkappa = n_\varkappa \exp i\psi_\varkappa$,[13] where $\psi_\varkappa$ is the total phase of the pair of waves with vectors $\varkappa$ and $-\varkappa$. In the stationary state of the parametric SW, when $\dot{b}_\varkappa = 0$, equation (23) reduces to the form

$$\omega_\varkappa + \frac{\beta}{15\pi^2}N - \frac{|\omega_0|}{2} + i\gamma_\varkappa + \frac{1}{6}e^{-i\psi_\varkappa}\left[\eta + \frac{\beta}{20\pi^2}(5\varkappa^2 + 4)e^{i\psi_\varkappa}N\right] = 0, \tag{24}$$

where it is assumed that $\gamma_\varkappa$, like $\omega_\varkappa$, depends only on $|\varkappa|$.

As a result of renormalization of the pump, the system of parametric SW freezes, as it were, on the threshold of excitation; the only nonzero amplitudes are those of waves whose wave vectors terminate on the resonance curve

$$\omega_\varkappa + \beta N/15\pi^2 = |\omega_0|/2, \tag{25}$$

which is a circle. Equations (24) and (25) enable us to calculate N and $\psi_\varkappa$:

$$N \approx 40\pi^2(\eta^2 - 36\gamma_\varkappa^2)^{1/2}/(5|\omega_0| + 8\beta), \quad \sin\psi_\varkappa = 6\gamma_\varkappa/\eta. \tag{26}$$

If the damping depends on $\varkappa$, then to eliminate this dependence it is necessary to solve equations (25) and (26) simultaneously.

In small fields, when $15\pi^2|\omega_0| < 2\beta N$, the relation (25) is not fulfilled. In this case, SW are excited with $\varkappa = 0$ and with frequencies different from the resonance frequency, and N is determined by the expression

$$N = \frac{5\pi^2}{\beta}\left\{|\omega_0| + \frac{1}{3}(\eta^2 - 36\gamma_0^2)^{1/2}\right\}.$$

The role of pump in the system considered is played by the sum of the demagnetizing field of the wall and of the plane component of the anisotropy field; these fields oscillate as a result of the precession of the magnetization about the z axis. The SW amplitude is nonzero only when $\eta^2 > 36\gamma_\varkappa^2$. If this inequality is not satisfied, a subliminal mode of excitation prevails.

5. VELOCITY OF A DOMAIN WALL

In the preceding sections, we calculated, in certain approximations, the spin excitations that exist in a ferromagnet with a DW, placed in a constant magnetic field. This enables us now, by means of equations (10) and (11), to determine the contribution of the processes considered above to the mean velocity of a DW.

We note that the functional (12), in the approximations adopted, contains no mixed terms; therefore the processes considered will make additive contributions to the velocity. In fact, all the described mechanisms of excitation of SW are independent of one another in the high-external-field range, since they are caused by precession of the magnetization about the z axis, which in turn is determined by the external field. Consequently, the effect of the spin excitations on the character of the DW motion, in the case considered, is also individual. Coupling between the different processes can show up only when, in the equations of motion, account is taken of terms of higher order in comparison with those considered in the present paper, and also in the low-external-field range, where the $\varphi(t)$ relation is significantly nonlinear and is dependent on the amplitudes of the spin excitations as well as on h.

The contribution of distortions of the DW structure to $f(a_q, c_\varkappa, c_k, \varphi, v)$, according to (12), will be

$$f(a_q) = -{}^1/_4 i\pi^{1/2}\eta \sin\omega_0 t \,\mathrm{Im}\, I_1 + {}^1/_{24}\pi^{1/2}\eta\cos\omega_0 t\,(3I_2 + I_2^*). \tag{27}$$

On substituting here a_q from (13), one can easily show that $I_2 \equiv 0$ because of the oddness of the integrand. The remaining part of $f(a_q)$, after substitution in (10) and averaging over time, leads to the following expression, which determines the contribution of the amplitudes a_q to the mean velocity of the DW:

$$V_1 = -(2^{3/2}\pi)^{-1}\,\mathrm{Im}\,\overline{f(a_q)} = -\pi^2\eta^2 Q(\tilde{\omega}_0, \tilde{\gamma})/128\beta;$$

$$Q(\tilde{\omega}_0, \tilde{\gamma}) = \mathrm{Re}\left\{\frac{i\tilde{\gamma} - \tilde{\omega}_0}{q(\tilde{\omega}_0)}\mathrm{ch}^{-2}\frac{\pi}{2}q(\tilde{\omega}_0) - \frac{i\tilde{\gamma} + \tilde{\omega}_0}{q(-\tilde{\omega}_0)}\mathrm{ch}^{-2}\frac{\pi}{2}q(-\tilde{\omega}_0) - \frac{16}{\pi^2}(\tilde{\gamma} + i\tilde{\omega}_0)\sum_{n=0}^{\infty}\frac{2n+1}{[(2n+1)^2 + q^2(\tilde{\omega}_0)]^2} + \frac{16}{\pi^2}(\tilde{\gamma} - i\tilde{\omega}_0)\sum_{n=0}^{\infty}\frac{2n+1}{[(2n+1)^2 + q^2(-\tilde{\omega}_0)]^2}\right\}, \tag{28}$$

where $q(\tilde{\omega}_0)$ is the same as in (15). The main contribution to $Q(\tilde{\omega}_0, \tilde{\gamma})$ comes from the first two terms.

When $\gamma = +0$, the value of $q(\pm\tilde{\omega}_0)$ will be pure imaginary in the field range $\tilde{\omega}_0 < 1$; therefore V_1 vanishes. When $\tilde{\omega}_0 \geq 1$, the value of $q(\tilde{\omega}_0)$ is real, and radiation of SW ensures scattering of the Zeeman energy and a nonzero velocity of motion of the DW. On further increase of the external field, the first two terms in $Q(\tilde{\omega}_0, 0)$ decrease exponentially; this occurs because of the ineffectiveness of excitation by the wall of SW with a wavelength much smaller than its thickness. At the point $\tilde{\omega}_0 = 1$, the velocity V_1 becomes infinite.

In the general case, when $\gamma \neq 0$, typical functions $Q(\tilde{\omega}_0, \tilde{\gamma})$ are shown in Fig. 1. The discontinuity at the point $\tilde{\omega}_0 = 1$, which was characteristic of the previous case, is absent here; furthermore, the velocity is nonzero even when $\tilde{\omega}_0 < 1$. As the damping increases, the value of the maximum on the curve decreases. This is caused by a decrease of the amplitude of distortion of the DW.

We now consider the contribution of precessional SW to the velocity of motion of the DW. In (12) we replace $c_{k'} c^*_{k''}$ by $2\hbar\langle \hat{c}^+_{k'} \hat{c}_{k''}\rangle$, $c_{k'} c_{k''}$ by $2\hbar\langle \hat{c}^+_{k'} \hat{c}^+_{k''}\rangle$. It has already been mentioned that the second correlation function is equal to zero. Therefore the part of $f(a_q, c_\varkappa, c_{\mathbf{k}}, \varphi, v)$ that depends on the amplitudes of the precessional SW can be transformed, with use of (19), to the form

$$f(c_{\mathbf{k}}) = \frac{iT}{2^{5/2}\hbar\beta}\{e^{i\omega_0 t}\alpha_1(\tilde{\omega}_0) + e^{-i\omega_0 t}\alpha_2(\tilde{\omega}_0)\};$$
$$\alpha_1(\tilde{\omega}_0) = \int \frac{(1-iq')(q''^2-q'^2)\Phi(q', q'')G(q', q'')}{(1-iq'')(q'^2-q''^2-\tilde{\omega}_0+i0)} \mathrm{sh}^{-1}\frac{\pi}{2}(q''-q')\,dq'\,dq'',$$
$$\alpha_2(\tilde{\omega}_0) = \int \frac{(1-iq')(q''^2-q'^2)\Phi^*(q'', q')G(q', q'')}{(1-iq'')(q'^2-q''^2+\tilde{\omega}_0+i0)} \mathrm{sh}^{-1}\frac{\pi}{2}(q''-q')\,dq'\,dq'',$$
$$G(q', q'') = \ln\{1-\exp[-\hbar\beta(1+q'^2)/T]\} - \ln\{1-\exp[-\hbar\beta(1+q''^2)/T]\}, \tag{29}$$

where T is the temperature in energy units, and where $\Phi(\mathbf{k}', \mathbf{k}'') = \delta(\varkappa' - \varkappa'') \cdot \Phi(q', q'')$.

The functional $f(c_{\mathbf{k}})$, in contrast to $f(a_q)$, has no constant component; therefore a direct averaging of it over time gives zero. But as is evident from (10) and (11), the period of variation of $f(c_{\mathbf{k}})$ and therefore also of $\dot{\varphi}$ coincides with the period of oscillation of the DW velocity. Scattering of precessional magnons may modulate the precession frequency of the magnetization in such a way that the time of motion of the DW in one direction will exceed the time of motion in the opposite direction, and the mean velocity turns out to be nonzero. Integrating (11) with use of (29), substituting the resulting $\varphi(t)$ relation in (10), and averaging the resulting velocity over time, we find

$$V_1 = -\frac{T\eta}{32\pi\omega_0\hbar\beta}\mathrm{Re}[\alpha_1(\tilde{\omega}_0) - \alpha_2(\tilde{\omega}_0)]. \tag{30}$$

The problem has been reduced to calculation of the functions $\alpha_j(\tilde{\omega}_0)$. It can be shown that the last two terms in (17), which are responsible for the scattering of magnons by DW distortions, make no contribution to α_j. The magnons that take part in this process, interacting with the DW distortions, produce damping of them but do not directly affect the wall itself. The remaining term in (17) describes scattering of magnons by the DW and leads to the following contribution to the mean velocity:

$$V_2 = -\frac{T\eta^2\tilde{\omega}_0^2}{32\beta^2}\int_0^\infty \frac{x^3\,\mathrm{sh}^{-2}(\pi x/2)}{(x^4+4x^2+\tilde{\omega}_0^2)^2 - 4\tilde{\omega}_0^2 x^4}$$
$$\times G\left[\frac{1}{2}\left(\frac{\tilde{\omega}_0}{x}+x\right), \frac{1}{2}\left(\frac{\tilde{\omega}_0}{x}-x\right)\right]dx.$$

At high temperatures ($T \gg \hbar\beta$) and sufficiently small external fields ($\tilde{\omega}_0 \ll 2$), one can obtain the simpler expression

$$V_2 \approx -\frac{T\eta^2\tilde{\omega}_0}{8\beta^2}I(\tilde{\omega}_0),$$

$$I(\tilde{\omega}_0) = \tilde{\omega}_0^2 \int_0^\infty \frac{x^5\,\mathrm{sh}^{-2}(\pi x/2)\,dx}{[(x^4+4x^2+\tilde{\omega}_0^2)^2 - 4\tilde{\omega}_0^2 x^4](x^4+4x^2+\tilde{\omega}_0^2)}. \tag{31}$$

The variation of this integral with $\tilde{\omega}_0$ is shown in Fig. 2. In the small-field range, I may be considered constant; therefore the velocity V_2 in this range is a nearly linear function of the external field.

We shall now calculate the contribution of parametric SW to the mean DW velocity. For this purpose, we transform in (12) from the $c_\varkappa$ to the slow amplitudes $b_\varkappa$ and average the terms containing them over an ensemble of individual phases of the parametric SW. After this, the part of (12) that depends on translational SW, with use of (22), takes the form

$$f(c_\varkappa) = -\frac{2^{1/2}i}{24\pi}\eta\sin\omega_0 t$$
$$\times[2+\exp i(\psi_\varkappa + |\omega_0|t)]N.$$

Substituting this expression in (10), averaging over time, and taking account of (26), we get for $15\pi^2|\omega_0| \geq 2\beta N$

$$V_3 = -\frac{5\gamma_\varkappa\,\mathrm{sign}\,\omega_0}{10|\omega_0|+16\beta}(\eta^2 - 36\gamma_\varkappa^2)^{1/2}. \tag{32}$$

This velocity as a function of $\gamma_\varkappa$ has a maximum at $\gamma_\varkappa = \eta/6\sqrt{2}$ and goes through zero at the points 0 and $\eta/6$.

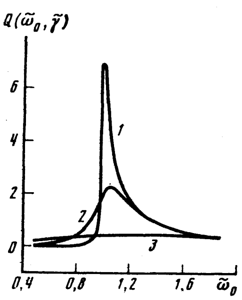

FIG. 1. The function $Q(\tilde{\omega}_0, \tilde{\gamma})$ for various fixed values of $\tilde{\gamma}$: 1, 0.01; 2, 0.1; 3, 1.0.

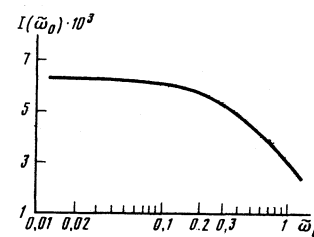

FIG. 2. The function $I(\tilde{\omega}_0)$.

In the first case, the parametric SW do not attenuate and do not produce dissipation of the Zeeman energy; the second case corresponds to the threshold of parametric instability of SW. The field dependence of V_3 is determined not only by the explicit dependence of the velocity on ω_0, but also by the function $\gamma_\varkappa$, since $\varkappa$ is related to ω_0 by the relation (25).

6. DISCUSSION OF RESULTS

The expressions (28), (31), and (32) determine the contributions to the mean velocity of motion of a DW from, respectively, dynamic distortions of the DW structure, a_q, scattering of thermal precissional SW, c_k, and parametric excitation of translational SW, $c_\varkappa$. In the case when, for some reason, a_q, c_k, and $c_\varkappa$ simultaneously vanish, the functional (12) also vanishes, and with it the mean velocity of the wall. Only the first term in (10) produces oscillations of the DW about the position that it had at the instant of application of the external field. In fact, an external field parallel to the axis of easy magnetization produces a uniform precession about this axis. Departure of the magnetization from the plane of the wall leads to the appearance of a demagnetizing field, and consequently to a torque, which causes displacement of the DW. But the periodic variation of the demagnetizing field produces only oscillations of the DW about some initial position, as depicted schematically in Fig. 3a.

A progressive motion of the DW is possible in the presence of an additional torque, which either destroys the equilibrium of the precession of the magnetization about the external field, or directly reverses the magnetic moments from direction $-\mathbf{e}_z$ to direction $\mathbf{e}_z$. Both of these possibilities are realized in the interaction of a DW with SW. As was shown above, thermal magnons scattered by the wall produce a torque, which modulates (and in the case of small external field and uniform motion of the DW, stops) the precession of spins about the axis of easy magnetization. The frequency of this modulation coincides with the frequency of oscillation of the DW velocity.

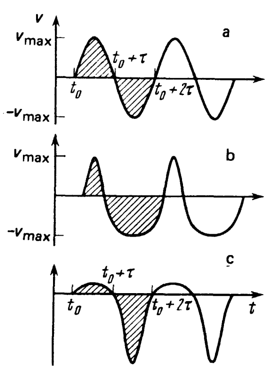

FIG. 3. Illustrative form of the $v(t)$ relation: a, in the absence of interaction of the DW with SW; b, with allowance only for scattering of thermal magnons; c, with allowance only for radiation of precessional and/or parametric excitation of translational SW.

The time intervals corresponding to the positive and negative sections of the $v(t)$ relation, shown qualitatively in Fig. 3b, are different, and the mean velocity is nonzero.

The interactions of dynamic distortions and of parametric translational SW with a DW are examples of a process of the second type. The constant components of the torques produced by these excitations directly reverse spins in the DW, causing an increase of its velocity during motion in one direction and a decrease during the opposite motion. An approximate form of the time dependence of the resulting velocity of the wall is shown in Fig. 3c. We note that if we take account only of processes of this type, uniform motion of the DW is altogether impossible, since the torque produced by the external field and causing precession of the spins about the z axis cannot be compensated by the torque produced by these excitations.

Thus interaction of a DW with quasiparticles that scatter the Zeeman energy of the ferromagnet produces a progressive motion of it, and not damping, as is often assumed. One can speak of damping only in the absence of an external field. In this case the principal type of DW motion is motion with constant velocity, and dissipative processes cause damping of it.

We shall estimate the contributions (28), (31), and (32) to the mean DW velocity, using the following values of the parameters, which are typical for materials with cylindrical magnetic domains, for example epitaxial iron-garnet films: $4\pi M_s \sim 100$ G, $\beta \sim 100$, $\eta \sim 10$, $gM_s\delta \sim 10$ m/s. For temperatures ~300 K (in dimensionless units, $T\sim 0.3$), we easily obtain from (31) $V_2 \sim 10^{-5}\tilde{\omega}_0$ m/s. Even for $\tilde{\omega}_0 \sim 1$, this value is several orders of magnitude smaller than the experimentally observed values (~10 m/s[13]). A considerably larger contribution to the DW velocity is made by parametric excitation of SW. For $\gamma_\varkappa \sim 1$ [the maximum on the $V_3(\gamma_\varkappa)$ curve], V_3 is found to be ~0.3 m/s. Finally, distortions of the DW structure give $V_1 \sim Q(\tilde{\omega}_0, \tilde{\gamma})$ m/s (see Fig. 1). For $\tilde{\omega}_0 \approx 1$ and for small damping of spin excitations, this contribution to the velocity may turn out to be ~10 m/s, and the resonance peak characteristic of it on the $V(h)$ curve can be detected experimentally. But in small fields, $V_1 < V_3$.

The relative value of the interaction of DW with SW is determined by the ratio η/β. When $\eta = 0$, the DW, in the approximations adopted, does not interact with SW; therefore $V_{1,2,3} = 0$. With increase of η/β, the contributions $V_{1,2,3}$ to the mean velocity also increase. It may be expected that the processes considered will determine the dynamic behavior of a DW in materials with a sufficiently low quality factor $\beta/4\pi$.

We again emphasize that the observed velocity of motion of a wall is the result of the action of many dissipative processes. In the present paper, only the one- and two-magnon contributions to the velocity have been calculated. Many-magnon processes may exert a significant influence on the dynamics of a DW. Preliminary estimates show that a large contribution to the velocity is made by interaction of the DW with elastic waves. In the low-temperature range, it is necessary to take into account the interaction of a DW with the rare-earth ions, which are an intermediate like in the transfer of the Zeeman energy to the thermostat.[15,16] Furthermore, we have considered only steady nonstationary motion of a DW. Transitional processes may have great importance. For example, in the initial stage of development of parametric instability of translational SW, the Zeeman energy is used not only to maintain but also to increase their amplitude. Therefore at the first instant after application of the external field, the mean DW velocity should be larger than at later instants.

In conclusion, we shall discuss still another approximation made in this paper: namely, uniformity of the micromagnetic structure (3), of the magnetic properties, and of the external field along the DW. It is clear that in the experiments these conditions are never strictly fulfilled. Even if the DW was initially one-dimensional, the nonuniform precession of the magnetization about the external field, caused for example by the presence of a slight dependence of h on the coordinates y and z, will lead to the result that the angle φ after a certain time will be different at different points of the wall; furthermore, these differences will increase with time, since $\dot{\varphi} \approx h$. There will occur a "twisting" of Bloch lines, whose strong influence on DW dynamics is common knowledge.[17]

Thus the method developed in this paper for self-consistent calculation of DW motion makes it possible to investigate at least qualitatively, on a model of the magnetic subsystem of the material, the dynamic processes in a ferromagnet containing a SW and placed in a constant magnetic field. A more rigorous theory, in contrast to that set forth above, must take into account a possibly larger number of interactions, and consequently also fo dissipative processes in the system, and must allow a three-dimensional structure of the DW itself, and not only of the spin excitations.

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1) The DW motion determines dissipative processes in the system and at the same time is itself determined by these processes.

2) The term SW is used here in a general sense. These SW, according to our definition, are periodic distortions of the DW structure. They must not be confused with the SW described by the amplitudes $c_{\varkappa}$ and $c_{\mathbf{k}}$.

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