Nonlinear effects in the generation by ion lasers

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The theory is considered of the nonlinear effects produced in the generation by ion lasers by the action of the electric field of the gas-discharge plasma. Stability criteria are obtained for various lasing regimes. It is observed that one-mode and two-mode lasing regimes alternate in succession because of the acceleration effects. The action of plasma fields on the profile of the Lamb dip in the lasing spectrum is investigated. It is indicated that the energy repulsion of the lasing frequency can be replaced under the influence of the ion acceleration by energy attraction. It is established that the discharge field can split the frequencies of two oppositely traveling waves in an ion ring laser.

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1. It is known that the investigation of the shapes of the ion spectral lines is one of the principal means of plasma diagnostics and permits the study of many processes that occur in the plasma.[1-6] Fock[2] and Kagan and Perel'[5,6] have developed a linear theory of ion spectra, in which the deformation of the spectral relief in the electric discharge field was connected with the corresponding change of the ion velocity distribution function.

A prominent place among the sources of continuous coherent radiation is occupied at present by lasers using noble gases (see, e.g., Refs. 7–9 and the references therein). The most widely used lasers of this type use singly and doubly ionized argon, which radiate a much higher power than the other cw lasers in the visible and ultraviolet regions of the spectrum.[10,1] Their advent has stimulated an intensive study of nonlinear phenomena in the emission of atoms.

Compared with the Lamb theory of the ordinary gas laser,[12] the ion-laser theory presented in this paper has a number of essential features due to the influence of the electric field of the exciting discharge on the translational motion of the ions. We consider below the action of the discharge field on the spectrum and frequency characteristics of the emission of ion lasers. An analysis of the action of the acceleration in a stationary homogeneous electric field on the dipole-moment relaxation of a single ion was given earlier,[13-15] and the influence of this acceleration on the shapes of both the linear and nonlinear spectral resonances (weak saturation) was revealed. Particular attention is paid in the present paper, to the study of the role of strong saturation and intermode interaction in the evolution of the lasing.

2. We consider an axially symmetrical ion laser emitting at a frequency ω. The semiclassical theory of the laser is constructed in the usual manner on the basis of a description of the electromagnetic field $\mathbf{E}$ by Maxwell's equation and of a quantum-mechanical allowance for the polarization $\mathbf{P}$ of the medium with the aid of the kinetic equation for the density matrix.[12] After introducing the effective conductivity σ, which takes into account the losses in the mirrors, the abbreviated equation for the linearly polarized radiation in the qth longitudinal mode reduces to the form

$$\frac{dG_q}{dt}+\frac{\sigma}{2}G_q=\frac{\pi i\omega d_{mn}}{\hbar}P_q, \tag{2.1}$$

where d_{mn} is the matrix element of the dipole moment of the working $m-n$ transition, and $G_q=E_q d_{mn}/2\hbar$ are the slowly varying amplitudes of the light field.

The active medium of the laser is a gas-discharge plasma containing excited ions. It is described by the system of equations for the density matrix ρ_{ij}, which takes in cylindrical coordinates the following form for two-level ions $(l,j=m,n)$[13-15]

$$\begin{gathered}\left(\frac{\partial}{\partial t}+v_z\frac{\partial}{\partial z}+\hat{\mathscr{L}}+\Gamma_j\right)\rho_j=q_j\mp 2\operatorname{Re}(iV_{mn}{}^*\rho),\\ \left(\frac{\partial}{\partial t}+v_z\frac{\partial}{\partial z}+\hat{\mathscr{L}}+\Gamma\right)\rho=-iV_{mn}(\rho_m-\rho_n);\\ \rho_j=\rho_{jj},\quad \rho=\rho_{mn},\end{gathered} \tag{2.2}$$

where the operator $V_{mn}(\mathbf{r},t)$ takes into account the interaction of the ions with the light field; $\mathbf{a}=e\mathscr{E}/M$ is the acceleration of the ion in the electrostatic field $\mathscr{E}$; q_j are the level-excitation functions; Γ and Γ_j are constants that describe the radiative and collisional broadenings,

$$\begin{aligned}\hat{\mathscr{L}}=v_r\left(\cos\varphi\frac{\partial}{\partial r}-\frac{\sin\varphi}{r}\frac{\partial}{\partial\varphi}\right)+\frac{e\mathscr{E}_r}{M}\Big(\cos\varphi\frac{\partial}{\partial v_r}\\ -\frac{\sin\varphi}{v_r}\frac{\partial}{\partial\varphi}\Big)+a_z\frac{\partial}{\partial v_z}.\end{aligned} \tag{2.3}$$

The behavior of the radiation in an ion laser is described by the self-consistent solution of Eqs. (2.1) and (2.2) with polarization

$$P(z,t)={}^1/_2 d_{mn}{}^*\langle\rho\rangle+\text{c.c.} \tag{2.4}$$

The change of variables

$$\varepsilon={}^1/_2 Mv_r{}^2+eV(r),\quad p=rv_r\sin\varphi,$$

where

$$\mathscr{E}_r=-dV/dr,\ V(0)=0,\ V(r)\leqslant 0$$

and $V(r)$ decreases monotonically from the axis to the walls, yields a simpler expression than (2.3) for $\hat{\mathscr{L}}$:

$$\hat{\mathscr{L}}=v_r\cos\varphi\frac{\partial}{\partial r}+a_z\frac{\partial}{\partial v_z}. \tag{2.5}$$

The first term in (2.5) corresponds to transit effects upon acceleration of the ion in the radial field of the

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discharge. Typical parameters of a high-current discharge in an Ar^+ laser $R\sim 1$ cm and $v_r \sim 10^6$ cm/sec. The indicated term can then be of the order of 10^6 sec^{-1}, which is much lower than the relaxation constants ($\Gamma, \Gamma_j \sim 10^8$–$10^9$ sec^{-1}) (i.e., the time of flight of the ion from the axis to the wall is much longer than its emission time). The influence of these effects can therefore be accounted for here by perturbation theory. The second term of (2.5) is always of the order of $a_z/\bar{v} \ll \Gamma, \Gamma_j$ ($\bar{v}$ is the mean thermal velocity of the ion). Only in narrow regions that correspond to Bennett dips in the ion velocity distribution is this term of the order of $\mathbf{k}\cdot\mathbf{a}/\Gamma$, which may turn out to be comparable with the relaxation constants Γ and Γ_j.

We assume hereafter that the excitation rates of the levels q_j are Maxwellian:

$$q_j = Q_j W(\mathbf{v}), \quad W(\mathbf{v}) = (\pi^{1/2}\bar{v})^{-3} \exp(-\mathbf{v}^2/\bar{v}^2). \tag{2.6}$$

3. The Coulomb interaction of the excited ions can be taken into account in the kinetic equations by using the method indicated in a preceding paper.[16] To produce inversion in the active medium of ion lasers or of ion quantum amplifiers, one usually chooses a pair of levels with greatly differing relaxation constants, namely:

$$\Gamma_m \ll \Gamma, \Gamma_n. \tag{3.1}$$

We obtain below an expression for the population difference in the field of a monochromatic standing wave, neglecting the first term of (2.5) and under the condition $\varkappa \gtrsim 1$ (strong saturation). It is convenient to use for this purpose the approximate procedure proposed by Germogenova and Rautian for solving the quantum kinetic equation.[17]

We integrate the second equation of the system (2.2), assuming its right-hand side known and substituting the expression for ρ in the right-hand side of the first equation of the system (2.2) at $j = m$. On the characteristics

$$v = v' + at, \quad z = z' + v't + at^2/2$$

we have

$$(d/dt + \Gamma_m)\rho_m = q_m - 2\mathrm{Re}\left\{V_{mn}^*(t)\int_{-\infty}^{t} dt' \exp[\Gamma(t'-t)]V_{mn}(t')(\rho_m - \rho_n)\right\}. \tag{3.2}$$

If the condition (3.1) and the inequality

$$|V_{mn}| \ll \Gamma, \Gamma_m \tag{3.3}$$

are satisfied, then the population of the lower level can be replaced by the stationary value $\rho_n = q_n/\Gamma_n$, and the population of the upper level by the value at $t' = t$. Using (3.2), we obtain an expression for the population difference:

$$N(t) = \rho_m - \rho_n = q_m \int_{-\infty}^{t} e^{f(t,t')} dt' - \frac{q_n}{\Gamma_n}\left[1 - \int_{-\infty}^{t} e^{f(t,t')} g(t') dt'\right], \tag{3.4}$$

where

$$g(t) = 2\mathrm{Re}\left[V_{mn}^*(t)\int_{-\infty}^{t} dt' e^{\Gamma(t'-t)} V_{mn}(t')\right], \tag{3.5}$$

$$f(t,t') = -\Gamma_m(t-t') + \int_{t'}^{t} g(t'')dt''. \tag{3.6}$$

In the case of a standing wave, the matrix element of the interaction of the radiation with the active medium is

$$V_{mn} = Ge^{-i\Omega t}\sin kz, \tag{3.7}$$

where $\Omega = \omega - \omega_{mn}$ is the detuning of the field from the frequency of the working transition. After discarding the terms with the fast spatial oscillations, we obtain the following expressions for the auxiliary functions f and g:

$$g(t) = |G|^2 \mathrm{Re}\sum_{\lambda=\pm 1} K_\lambda(t), \tag{3.8}$$

$$-f(t,t') = \Gamma(t'-t) + \frac{|G|^2}{2}\mathrm{Re}\sum_{\lambda=\pm 1}\left[I\left(-\frac{\lambda ka}{2}, 0, \Gamma_\lambda\right) - I\left(-\frac{\lambda ka}{2}, 0, \Gamma_\lambda - i\lambda ka(t-t')\right)\right], \tag{3.9}$$

$$K_\lambda(t) = \int_0^\infty d\tau \exp(-\Gamma_\lambda \tau + i\lambda ka\tau^2/2), \quad \Gamma_\lambda = \Gamma - i\Omega + i\lambda kv, \tag{3.10}$$

$$I(\alpha, \zeta, z) = \int_0^\infty \frac{d\tau \exp(-z\tau - i\alpha\tau^2)}{\zeta + i\alpha\tau}, \quad a = a_z. \tag{3.11}$$

The integrals $K_\lambda(t)$ are expressed in terms of the probability integral of complex argument. Some properties of integrals such as $I(\alpha, \zeta, z)$ are considered in Refs. 14 and 15.

Using expressions (3.6), (3.8), and (3.9), we obtain an equation for the population difference:

$$N(t) = \frac{q_m}{\Gamma_m} I_1(t) - \frac{q_n}{\Gamma_n}[1 - I_2(t)], \tag{3.12}$$

$$I_1(t) = \int_0^\infty e^{f(t,t-\tau)} d\tau, \quad I_2(t) = \int_0^\infty e^{f(t,t-\tau)} g(t-\tau) d\tau. \tag{3.13}$$

In the case of approximately excitation rates $q_m \sim q_n$, which is of practical interest, we can neglect the second term of (3.12) by virtue of the inequality (3.1). Then the population difference N is determined by the population of the upper level m, owing to the rapid decay of the lower level n. The velocity dependence of N is then given by the integral I_1 and by the velocity dependence of the function q_m. If the condition

$$|\Omega \pm kv| \gg \Gamma \tag{3.14}$$

is satisfied, the expression under the integral sign in (3.9) oscillates rapidly and $I_1 \approx 1$. The function $N(v)$ therefore duplicates the form of $q_m \propto W(v)$.

The integral I_1 differs substantially from unity only in the vicinity of the resonance $|\Omega \pm kv| \lesssim \Gamma$, where I_1 describes the profiles of the acceleration-distorted Bennett dips. In the case of weak saturation, when $\varkappa \ll 1$, the Bennett-dip profile is altered by the acceleration.[13,14] In this case expression (3.13) yields the contour described in these references. In addition, expression (3.13) makes it possible to obtain an asymptotic expansion of the population difference in the case of strong saturation ($\varkappa \gg 1$):

$$I_1(\varkappa) = \sum_{k=0}^{\infty} c_k \varkappa^{-k-1}. \tag{3.15}$$

The coefficients of the series (3.15) can be obtained by the Laplace method (see Ref. 18). Since the obtained expressions are unwieldy, we present only the equation for the coefficient c_0:

$$c_0=\left[2\Gamma\int_0^\infty dte^{-\Gamma t}\cos\Omega t\cos\left(kvt+ka\frac{t^2}{2}\right)\right]^{-1}. \tag{3.16}$$

If there is no acceleration, (3.16) reduces to the coefficient of the first term of the asymptotic expansion of the ordinary Bennett dip.

4. To analyze the effect of the longitudinal electric field $\mathscr{E}_z$ on the emission of an ion laser, we consider a resonator made up of flat mirrors and separated by a distance d. We write down the expansion of the light field in terms of traveling waves:

$$E(z,t)=\frac{1}{2}\sum_{\lambda=\pm 1}E_{p\lambda}\exp(-i\omega_p t+i\lambda k_p z)+\text{c.c.} \tag{4.1}$$

The boundary conditions on the mirror surfaces lead to the following relations for the amplitudes $E_{p\lambda}$ and the wave numbers k_p of the modes:

$$E_{p+}=E_{p-}=-iE_p,\quad k_p=\pi p/d,\quad p=1,2,\ldots. \tag{4.2}$$

We confine ourselves next to an analysis of a two-mode regime ($p=1,2$) so that we can present concretely the main regularities of the intermode interaction. The solution of the system of equations for the density matrix (2.2) can be sought in the form

$$\rho_j=R_j+\sum_{\lambda=\pm 1}(r_{j\lambda}e^{i\lambda qz-i\varepsilon t}+\text{c.c.}), \tag{4.3}$$

$$\rho=\sum_{\substack{\lambda=\pm 1\\ p=1,2}}R_{p\lambda}\exp(i\lambda k_p z-i\Omega_p t); \tag{4.4}$$

$$\Omega=\omega_p-\omega_{mn},\quad \varepsilon=\omega_1-\omega_2,\quad q=k_1-k_2.$$

Substituting (4.3) and (4.4) in (2.2), we obtain a system of equations for the functions R_j, r_j, and $R_{p\lambda}$, which assumes on the characteristics (2.2) the form

$$\begin{gathered}(d/dt+\Gamma_j)R_j=q_j\mp 2\operatorname{Re}\sum_{p,\lambda}(iG_{p\lambda}^{*}R_{p\lambda}),\\ (d/dt+\Gamma_j-i\varepsilon+i\lambda qv'+i\lambda qat)r_{j\lambda}=\mp i(G_{2\lambda}^{*}R_{1\lambda}-G_{1\lambda}R_{2\lambda}^{*}),\\ (d/dt+\Gamma_{1\lambda}'+i\lambda k_1 at)R_{1\lambda}=-i[G_{1\lambda}(R_m-R_n)+G_{2\lambda}(r_{m\lambda}-r_{n\lambda})]\\ (d/dt+\Gamma_{2\lambda}'+i\lambda k_2 at)R_{2\lambda}=-i[G_{2\lambda}(R_m-R_n)+G_{1\lambda}(r_{m\lambda}^{*}-r_{n\lambda}^{*})];\\ G_{p\lambda}=E_{p\lambda}d_{mn}/2\hbar,\quad \Gamma_{p\lambda}'=\Gamma-i\Omega_p+i\lambda k_p v'.\end{gathered} \tag{4.5}$$

The difference between the wave numbers of neighboring modes q is small compared with the wave vectors: $q/k=\pi/kd\ll 1$. This allows us to neglect the terms proportional to q and assumed that the wave numbers of the modes are equal ($k=k_1\approx k_2$). Neglecting the higher spatial harmonics and using the Doppler-limit condition

$$\Gamma,\Gamma_j,\ |ka|/\Gamma,\ |ka|/\Gamma_j\ll k\bar{v}, \tag{4.6}$$

we write down, accurate to the first-order corrections for saturation, the expression that follows from the system (4.5) for the population differences $N=\rho_m-\rho_n$:

$$N=N_0\left[1-\sum_{j,p,\lambda}|G_{p\lambda}|^2 I\left(-\frac{\lambda ka}{2},\frac{\Gamma_j}{2},\Gamma_{p\lambda}'+i\lambda kat\right)\right], \tag{4.7}$$

where $N_0=q_m/\Gamma_m-q_n/\Gamma_n$, and the integral is given by (3.11).

Substituting (4.7) in the system (4.5), we obtain the functions $R_{p\lambda}$. After averaging over the velocities we have

$$\begin{aligned}\langle R_{1\lambda}\rangle=-iG_{1\lambda}\Delta N\frac{\pi^{1/2}}{k\bar{v}}\exp\left[-\frac{\Omega_1^2}{(k\bar{v})^2}\right]\Bigg\{1-\Phi\left(\frac{\Gamma-i\Omega_1}{k\bar{v}}\right)\\ -\sum_{j=m,n}\Gamma^{-1}\Gamma_j^{-1}\Big[|G_{1\lambda}|^2 Z_{j\lambda}(0)+|G_{1-\lambda}|^2 Z_{j\lambda}(\Omega_1)\\ +|G_{2\lambda}|^2\left(Z_{j\lambda}\left(\frac{\varepsilon}{2}\right)+Z_{1j\lambda}\left(\frac{\varepsilon}{2}\right)\right)+|G_{2-\lambda}|^2 Z_{j\lambda}\left(\frac{\Omega_1+\Omega_2}{2}\right)\Big]\Bigg\},\end{aligned} \tag{4.8}$$

where

$$\Delta N=N_0/W(\mathrm{v}),\quad Z_{1j\lambda}=\Gamma\Gamma_j I(\lambda ka,\Gamma_j,2\Gamma-2i\Omega);$$

$$\Phi(z)=\frac{2}{\sqrt{\pi}}\int_0^z e^{-t^2}dt \tag{4.9}$$

is the probability integral. The formula for $R_{2\lambda}$ is obtained from (4.8) by interchanging indices $1\leftrightarrow 2$ and the signs of the frequency differences $\varepsilon\to-\varepsilon$.

The nonlinear part of (4.8) contains five terms. The first and second describe the effect of the saturation in the mode, and the remaining ones correspond to interaction between the modes. The terms proportional to $|G_{2\lambda}|^2$ correspond to the interaction of traveling waves of the same direction ($\mathbf{k}_1\uparrow\uparrow\mathbf{k}_2$). They reflect the change of population under the influence of the light field and the presence of nonlinear interference effects. The contribution of the nonlinear interference effects is described by the functions

$$Z_{1j\lambda}(\varepsilon/2)=\Gamma\Gamma_j I(\lambda ka,\Gamma_j-i\varepsilon,2\Gamma-i\varepsilon). \tag{4.10}$$

The terms proportional to $|G_{2-\lambda}|^2$ correspond to interaction of waves traveling counter to each other ($\mathbf{k}_1\uparrow\downarrow\mathbf{k}_2$), so that the nonlinear interference effects are suppressed in them by the Doppler broadening.

Calculating the macroscopic polarization from (2.4) and using the relation (4.2) that follows from the boundary conditions on the resonator mirrors, we obtain from (2.1) and (4.8) the abbreviated equations for the complex amplitudes of both modes:

$$\begin{gathered}\frac{dG_p}{dt}+\frac{\sigma}{2}G_p=A_pG_p\Bigg\{1-\Phi\left(\frac{\Gamma-i\Omega_p}{k\bar{v}}\right)-|G_p|^2\sum_{j,\lambda}(2\Gamma\Gamma_j)^{-1}\\ \times[Z_{j\lambda}(0)+Z_{j\lambda}(\Omega_p)]-|G_{3-p}|^2\sum_{j,\lambda}(2\Gamma\Gamma_j)^{-1}\Big[Z_{j\lambda}\left(\frac{\Omega_1+\Omega_2}{2}\right)\\ +Z_{j\lambda}\left(\frac{\varepsilon_p}{2}\right)+Z_{1j\lambda}\left(\frac{\varepsilon_p}{2}\right)\Big];\\ \varepsilon_p=(-1)^{p+1}\varepsilon,\quad A_p=A_0e^{-\Omega_p^2/(k\bar{v})^2},\quad A_0=\frac{\pi^{1/2}\omega|d_{mn}|^2}{\hbar k\bar{v}}\Delta N.\end{gathered} \tag{4.11}$$

Resolving (4.11) into amplitude and phase relations

$$G_p=G_p^0\exp(i\psi_p),\quad \operatorname{Im}G_p^0=\operatorname{Im}\psi_p=0, \tag{4.12}$$

we obtain the equation for the parameters of the saturation

$$\varkappa_p=2|G_p|^2/\Gamma\gamma,\quad \gamma^{-1}=\sum_j\Gamma_j^{-1}$$

and of the phases ψ_p of the first and second modes:

$$\dot{\varkappa}_p=\varkappa_p(\alpha_p-\beta_p\varkappa_p-\theta_p\varkappa_{3-p}), \tag{4.13}$$

$$\dot{\psi}_p=\sigma_p-\rho_p\varkappa_p-\tau_p\varkappa_{3-p}, \tag{4.14}$$

where

$$\alpha_p=2A_p\left[1-\Phi'\left(\frac{\Gamma-i\Omega_p}{k\bar{v}}\right)\right]-\sigma,\quad \beta_p=\gamma A_p\sum_j\Gamma_j^{-1}\left[Z_{js}'(0)+Z_{js}'(\Omega p)\right.$$

$$\theta_p=\gamma A_p\sum_j\Gamma_j^{-1}\left[Z_{js}'\left(\frac{\Omega_1+\Omega_2}{2}\right)+Z_{js}'\left(\frac{\varepsilon}{2}\right)+Z_{1js}'\left(\frac{\varepsilon}{2}\right)\right],$$

$$\sigma_p=-A_p\Phi''\left(\frac{\Gamma-i\Omega_p}{k\bar{v}}\right),\quad \rho_p=\frac{\gamma A_p}{2}\sum_j\Gamma_j^{-1}Z_{ja}''(\Omega_p),$$

$$\tau_p=\frac{\gamma A_p}{2}\sum_j\Gamma_j^{-1}\left[Z_{ja}''\left(\frac{\Omega_1+\Omega_2}{2}\right)+Z_{ja}''\left(\frac{\varepsilon_p}{2}\right)+Z_{1ja}''\left(\frac{\varepsilon_p}{2}\right)\right];$$

$$Z_j(\Omega)=Z_{j+}(\Omega).$$

The primes and double primes denote the real and imaginary parts of the quantities; the subscripts s and a denote respectively the symmetrical and antisymmetrical parts of the function. In the derivation of (4.13) and (4.14) we used the fact that the real and imaginary parts of the functions $I(\alpha, \zeta, z)$ are even[14,15]:

$$Z_{j-\lambda}'(-\Omega)=Z_{j\lambda}'(\Omega),\quad Z_{j-\lambda}''(-\Omega)=-Z_{j\lambda}''(\Omega). \qquad (4.15)$$

To analyze the one-mode lasing regime we put in (4.13) and (4.14)

$$p=1,\quad \theta_1=\tau_1=0. \qquad (4.16)$$

This condition is satisfied, for example, if $\Omega_1 < \Gamma$ and $\Omega_2 > \Gamma$. If $\alpha < 0$, i.e., the loss exceeds the gain, Eq. (4.13) with condition (4.16) has a stable null solution. At $\alpha > 0$ the following stationary solution becomes stable:

$$\varkappa_1^\infty=\frac{\alpha_1}{\beta_1}=\left(1-\frac{\sigma}{2A_1}\right)\Big/\sum_j\frac{2\gamma}{\Gamma_j}[Z_{js}'(0)+Z_{js}'(\Omega_1)]. \qquad (4.17)$$

The plot of the function $\varkappa_1^\infty(\Omega)=\varkappa_1^\infty(-\Omega)$ has a symmetrical dip against the background of a broad Doppler contour, which differs in shape from the Lamb dip at $\mathscr{E}_z \neq 0$.

When the acceleration is small enough

$$|ka|^{1/2}\ll\Gamma,\Gamma_j, \qquad (4.18)$$

the profile of the dip can be approximated by two dispersion curves

$$2Z_{js}'(\Omega)=\gamma_j\Gamma/(\gamma_j^2+\Omega^2),\quad \gamma_j=\Gamma(1+3k^2a^2/4\Gamma^2\Gamma_j^2). \qquad (4.19)$$

For a laser using noble-gas ions, where the inequality (3.1) is valid, the decisive factor is the broadening of the long-lived upper level, whose relative value is $2(ka/2\Gamma\Gamma_j)^2$. An estimate for an Ar^+ laser at $\lambda = 4880$ Å, $\mathscr{E}_z = 10$ W/cm, $\Gamma_m \sim 10^8$ sec^{-1}, and $\Gamma \sim 10^9$ sec^{-1} yields a relative broadening ~5%. Plots of the function $Z_{js}'(\Omega)$, which determines the shape of the Lamb dip, are shown in Fig. 1 for various values of the parameter $\nu_j = ka/2\Gamma\Gamma_j$. It is seen from the figure that in the limiting case that is the inverse of the inequality (4.18) the dip becomes less pronounced and can split.

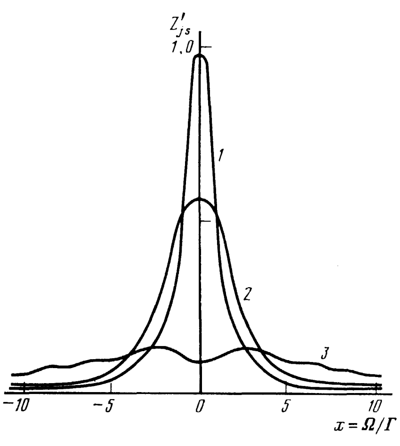

FIG. 1. Plot of the function Z'_{js} at $\Gamma = 5\Gamma_j$. Curves: 1) at $\nu_j = 0.1$, 2) 1.0, 3) 10.

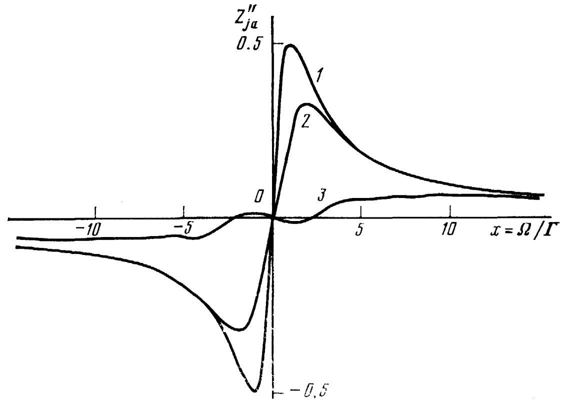

FIG. 2. Plots of the function Z''_{js} at $\Gamma = 5\Gamma_j$. Curves: 1) $\nu_j = 0.1$, 2) 1.0, 3) 10.

In Eq. (4.14) for the phase with condition (4.16), the term $\sigma_1 \approx A_1 \cdot 2\Omega/\pi^{1/2}k\bar{v}$ ($\Omega \ll k\bar{v}$) determines the effect of the linear contraction of the frequency, while the second term corresponds to energy repulsion determined by the functions $Z_{ja}''(\Omega)$. Plots of these functions are shown in Fig. 2. The acceleration decreases the repulsion, and hence also the detuning Ω_0 at which the repulsion is balanced by the locking. In the case of large accelerations, as shown in Fig. 2, oscillations appear in the frequency dependence of the effect. These are due to interference between different spectral components of the radiation, and at small detunings the energy repulsion gives way to energy contraction.

An investigation of the stability of various stationary solutions of the system (4.13) for the two-mode regime yields the following results: The zero solution $\varkappa_p = 0$ is stable only when the loss in the resonator exceeds the gain of the medium $\alpha_p < 0$. A regime in which one mode p is excited is stable if the effective gain of the second mode is negative:

$$\alpha_{3-p}'=\alpha_{3-p}-\theta_{3-p}\alpha_p/\beta_p<0$$

(mode competition). The two-mode regime $\varkappa_1\varkappa_2 \neq 0$ is stable in the case of weak coupling between the modes

$$\theta_1\theta_2<\beta_1\beta_2. \qquad (4.20)$$

If the coupling is strong, however, i.e., $\theta_1\theta_2 > \beta_1\beta_2$, the state that becomes stable is the one with only one mode of oscillations, the choice of mode being dictated by the history of the system.

The difference between an ion laser and an ordinary gas one is that the limits of the different lasing regimes are shifted by the discharge field. The weak-coupling criterion (4.20), i.e. that of a stable two-mode regime, is

$$\left|\sum_j \Gamma_j^{-1}\left[Z_{js}'\left(\frac{\Omega_1+\Omega_2}{2}\right)+Z_{js}'\left(\frac{\varepsilon}{2}\right)+Z_{1js}'\left(\frac{\varepsilon}{2}\right)\right]\right| < \left\{\prod_{p=1,2}\sum_j \Gamma_j^{-1}[Z_{js}'(0)+Z_{js}'(\Omega_p)]\right\}^{1/2}. \quad (4.21)$$

By way of example we consider a noble-gas-ion laser whose active medium satisfies the inequality (3.1). Let the laser be so constructed that the center of the gain lines is equidistant from the frequencies of two neighboring modes of the empty resonator. In this case $\Omega_1 = -\Omega_2 = \varepsilon/2$ and the criterion (4.21) simplifies to

$$Z_{1ms}'(\varepsilon/2)<0. \quad (4.22)$$

Plots of the function $Z_{1ms}'(\varepsilon/2)$, which describes the nonlinear interference effects, are shown in Fig. 3. At zero acceleration the condition (4.22) is satisfied if $\varepsilon > \varepsilon_0 = \Gamma$. The small acceleration (4.18) expands the stability region of the one-mode regime, i.e., increases ε_0 by an amount on the order of $\varepsilon_0 \nu_m^2$. Large acceleration, as seen from Fig. 3, causes oscillations of the function $Z_{1ms}(\varepsilon/2)$. It follows from (4.22) that $\varepsilon = 0$ corresponds to a stable one-mode regime, and the zeros of the function Z_{1ms} correspond to an alternations between one-mode and two-mode lasing.

The effects of locking and repulsion of the frequency in the stability region of the two-mode regime are described by Eqs. (4.14). The condition for the effects to be equal follows from (4.14) at $\psi_p = 0$:

$$\varkappa_p=(\sigma_p\rho_{3-p}-\sigma_{3-p}\rho_p)/(\rho_1\rho_2-\sigma_1\sigma_2)>0, \quad \rho_1\rho_2\neq\sigma_1\sigma_2. \quad (4.23)$$

5. The expressions obtained in the preceding sections are valid also when it comes to describing an ion ring laser. An exception is relation (4.2), which follows from the boundary conditions on the mirrors of the Fabry-Perot resonator. The competing power resonances of a ring laser with a nonlinear absorbing cell, with acceleration taken into account, were considered in Ref. 14, where stability criteria were derived for the standing- and traveling-wave regimes. These criteria can be obtained from the real part of (4.8) by putting $G_{2\lambda} = 0$. The imaginary part of (4.8) makes it possible to investigate the frequency pulling and repulsion in an ion ring laser. In the standing-wave stability region the intensity-dependent increments to the frequency have opposite signs for opposing traveling waves. The acceleration-induced frequency difference between these waves is

$$\delta\omega=A_0 \exp\left[-\frac{\Omega^2}{(k\bar{v})^2}\right]\varkappa\sum_j \frac{\gamma}{\Gamma_j} Z_{ja}''(\Omega). \quad (5.1)$$

The longitudinal electric field transforms thus a standing wave into one slowly traveling with a velocity

$$v_{ph}=\delta\omega/k, \quad (5.2)$$

proportional to the gain (absorption) and to the saturation parameter $\varkappa$. The frequency dependence of the effect is determined by the functions $Z_{ja}''(\Omega)$, plots of which are shown in Fig. 2.

The nonlinear splitting of the lasing frequency of the opposing wave can take place also in an ordinary gas ring laser (e.g., as a result of diffraction or of the different Q of the resonator for the two opposing waves[19]). In contrast to the effect (5.1) of frequency splitting under the influence of the electric field, it takes place for ions only when the intensities of the opposing waves are unequal because of the difference between the nonlinear frequency repulsion. If the detuning Ω is small compared with the homogeneous width Γ, the beat frequency $\delta\omega$ varies in proportion to Ω, and this makes it possible in principle to use the splitting effect to satisfy the emission frequency of an ion laser against the center of the gain line.

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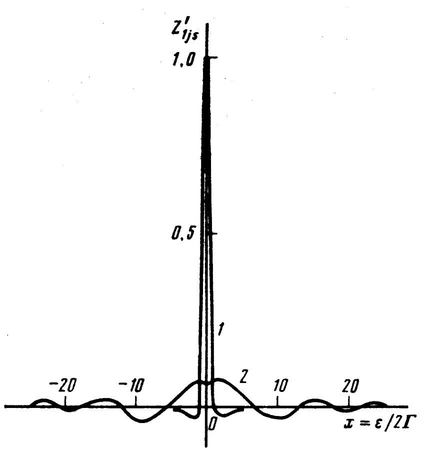

FIG. 3. Plots of the function Z_{1js} at $5 = 5\Gamma_j$: curve 1) $v_j = 0$ 2) $v_j = 100$.

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