Electron mobility in two-dimensional layer over the surface of solid hydrogen

A. M. Troyanovskiĭ and M. S. Khaĭkin

Institute of Physics Problems, USSR Academy of Sciences

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The mobility μ of electrons localized in a two dimensional layer near the surface of solid hydrogen is investigated in the temperature interval 3.8–12 K. At $T < 10$ K and in holding fields up to $E_\perp \sim 600$ V/cm, it is found that $\mu \propto T^{-1}$. At $T > 10$ K the mobility is determined by electron scattering by microscopic surface defects with dimension on the order of the crystal cell, $\sim 5\times10^{-8}$ cm. Large defects capture some of the electrons but do not influence the mobility of the free ones. At $T > 10$ K the mobility is determined by scattering by the gas molecules. The interaction of the electrons with Rayleigh waves is quite small and is not observed in the experiments.

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Bound states of electrons exist near the surface of a dielectric in a potential well made up by the image forces and by the potential barrier on the dielectric boundary.[1] Analogous states of electrons can be observed near particles of cosmic dust,[2] which are mainly hydrogen. The mobility of the electrons in a two-dimensional layer above the surface of solid and liquid hydrogen was measured earlier[3] in the temperature interval 10–16 K. In the present study we investigate the mobility of such electrons in detail in a wider temperature interval, 4–16 K, in order to determine the principal mechanisms of electron scattering. We have analyzed the role of scattering by gas molecules, by Rayleigh waves, and by surface defects, and have compared the calculation results with the experimental data.

1. EXPERIMENTAL SETUP

The conductivity of the electron layer was determined by measuring the damping introduced by the electrons in an oscillating circuit. The instrument used in the experiments is shown schematically in Fig. 1. Hydrogen gas is condensed in a glass ampoule *l* of 44 mm diameter, which can be moved in a helium cryostat vertically relative to a bath with liquid helium. This makes it possible to vary the temperature in the ampoule over a wide range. The use of an ampoule has improved the crystal growth condition, compared with the previously used device,[3] by decreasing of the temperature gradients.

To reduce the stresses in the crystal, the lower plate 5 of the capacitor was a grating with a period 0.5–1 mm, made of welded nickel wires of 0.07 mm diameter. The upper plate 2 was made up of three 7 × 25 mm strips, the outer ones connected to the grounded terminal of the tank circuit ($Q \sim 400$, resonant frequency 10.2 MHz), and the central strip to the other terminal. The distance between the capacitor electrodes was ~5 mm. The coil of the circuit was connected through a ~1 pF capacitor to a high-frequency oscillator tuned to the resonant frequency. The signal from a tap at one-third of the coil was fed to a frequency converter (input impedance 100 $k\Omega$ and 3.5 pF). After conversion, the signal was amplified with a selective amplifier, and its amplitude registered with an x-y recorder. The solid hydrogen level (3–4 mm) in the ampoule was measured with a cathetometer. The electrons were emitted by turning on for a short time (~0.1 sec) an incandescent filament 3, a tungsten wire of 0.01 mm diameter and ~3 mm length.

We used in the experiments commercial hydrogen purified by passing through a liquid-nitrogen-cooled trap with activated carbon. In Ref. 4 they considered the effect of ortho–para transitions of the hydrogen molecules on the level population of the surface electrons. The energy of the ortho–para transition is $E_{op} = 14.8$ meV, and the energy of the transition of the electrons from the first level to the second is $E_{1,2} = 12.5$ meV (in the absence of an external field).[5] The probability of a transition of an electron to the second level via interaction with the nuclear spins is $\sim 5\times10^{-11}$ (Ref. 4). This quantity is proportional to $(E_{op} - E_{1,2})^{5/2}$ and decreases with increasing electric holding field (since $E_{1,2}$ increases). The probability of the transition to the second level as a result of temperature fluctuations amounts to $\sim 10^8 \exp(-E_{1,2}/kT) \approx 10^{-8}$ at 3.8 K.[4] It follows from the foregoing that the influence of the ortho–para transitions of hydrogen on the system of the surface electrons can be neglected, and the ortho–para

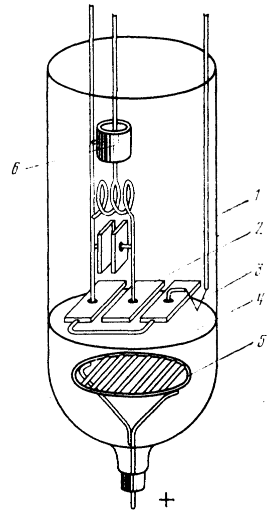

FIG. 1. Diagram of apparatus.

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ratio of the hydrogen does not play any role in the experiments.

2. THE EXPERIMENT

The hydrogen was crystallized by cooling the liquid to the freezing point for ~1 h. The surface of the crystal was next covered with a layer of electrons. The electrons were emitted at a holding potential difference on the capacitor $V_\perp \sim 150-1000$ V (at which the electron density reached 2×10^9 cm^{-2}). The appearance of a conducting electron layer has led to a decrease of the Q of the circuit by 2–7%. The crystal was then cooled for one hour to the specified temperature. After cooling, we plotted the function $Q(V_\perp)$ while decreasing the holding potential (Fig. 2). To charge the surface again, the crystal had to be first heated to 12–14 K, to prevent formation of a static charge on its surface. This charge nevertheless accumulated and became appreciable after 4–6 charge-discharge cycles. It screened almost completely the holding field, and the crystal had to be melted to remove it.

To estimate the possible influence of the helium impurity atoms, control experiments were performed in which helium gas up to 0.1 Torr at 4.2 K was added to the hydrogen vapor. No change in the electron mobility was observed.

Electrons located in an alternating electric field of frequency ω absorb a field energy equal to

$$W=\frac{e^2\tau n f^2}{m(1+\omega^2\tau^2)}\sim\mu e n f^2\quad(\omega\tau\ll 1),$$

where τ is the electron relaxation time, f is the amplitude of the alternating electric field, n is the electron density, and μ is the mobility. If the energy loss introduced in the tank circuit is low, they are equal to

$$W=2U_0\Delta U/R,$$

where U_0 is the voltage on the undamped tank circuit, ΔU is the change of the voltage as a result of damping, and R is the equivalent resistance of the tank circuit. The number of electrons was determined from the condition of complete screening of the field over the surface of the hydrogen: $n=\varepsilon V_\perp/4\pi ed$, where d is the thickness of the hydrogen layer and ε is the dielectric constant of solid hydrogen, equal to 1.29. Thus, the electron mobility is proportional to the ratio $\Delta U/V_\perp$, which was determined from the experimental results.

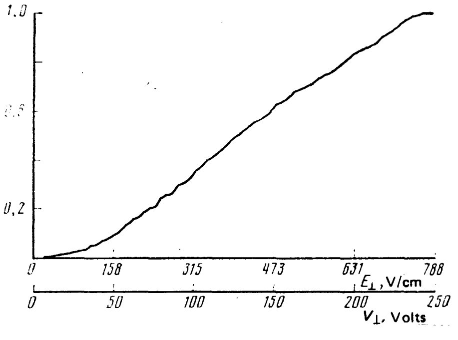

FIG. 2. Experimental plot of the variation the high-frequency voltage on the tank circuit (in relative units) with change in the holding potential in the presence of electrons. At a field 100 V/cm, the surface density of the electrons is 5.5×10^7 cm^{-2}.

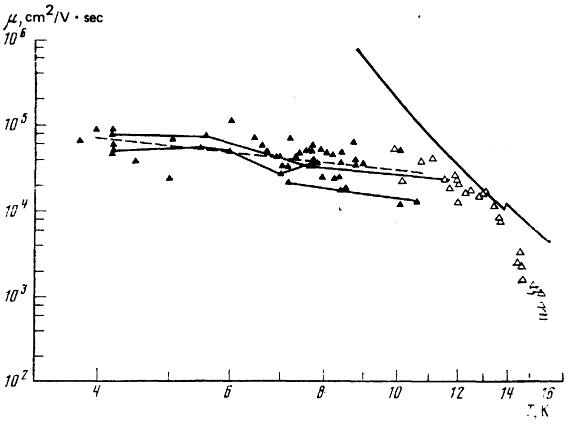

FIG. 3. Mobility of electrons over the surface of solid hydrogen: Δ—data of Ref. 2, ▲—data obtained in the described experiments, right-hand solid curve—calculation of mobility due to scattering by gas molecule, dashed— average value of mobility. The melting temperature of hydrogen is 13.9 K. The kink on the calculated curve corresponds to the jump of the dielectric constant at the melting point.

In the experiment, the amplitude of the alternating voltage U_0 on the coil ranged from 10 to 30 mV, and its increase to 900 mV did not cause a noticeable heating of the electron system (the dependence of ΔU on U_0 remained linear within this region). To measure the absolute values of the electron mobility over the hydrogen, the instrument was calibrated in experiments with liquid ^{4}He condensed in the ampoule in place of hydrogen against the results[6] of measurements of the electron mobility over helium.

3. RESULTS OF EXPERIMENTS AND THEIR DISCUSSION

Figure 3 shows the measured mobility of electrons localized over the surface of solid hydrogen. The straight-line segments join points pertaining to the same sample. The mobility was determined at $V_\perp/d$ = 200–300 V/cm. When electrons move along the surface of a solid dielectric, the following scattering mechanisms are possible: scattering by gas molecules, scattering by Rayleigh waves, and scattering by surface defects. To identify the character of the processes that influence the electron dynamics, we now estimate the values of the mobility due to the interaction of the electrons with the gas molecules and with the Rayleigh waves.

Scattering of electrons by gas molecules is determined by the formula[7]

$$\mu=8e/3\pi\hbar\sigma N\gamma.$$

The calculated values of the mobility as functions of the gas density N (scattering cross section $\sigma = 5.7\times 10^{-16}$ cm^2 and Bohr radius $\gamma^{-1} = 1.7\times 10^{-7}$ cm, Ref. 5) as shown by the right-hand side curve of Fig. 3. The sharp decrease of the measured mobility observed at $T > 14$ K is probably due to formation of electron bubbles in the dense gas.[8] At $T<10$ K, however, the mobility calculated from the collisions with the gas exceeds substantially the measured values; consequently scattering by gas molecules does not play any role in this temperature region.

Calculation of electron scattering by Rayleigh waves is similar to the calculation of the scattering by ripplons in the case of helium.[5] The collision integral of the two-dimensional electron gas is

$$\nu(p)=\frac{m}{2\pi\hbar^3}\int\limits_0^{2\pi}\bar{V}^2(2N_q+1)(1-\cos\varphi)\,d\varphi, \tag{1}$$

where $2N_q+1$ are the occupation numbers, $\bar{V}=\int_0^\infty \psi^2 A\,dz$, A is the perturbation potential, φ is the electron-scattering angle, p is the electron momentum, and $\psi=2\gamma^{3/2}ze^{-\gamma z}$ is the wave function of the electron (the z-part). In Refs. 5 and 9 are considered in detail the influence of the deformation of an oscillating surface on a system of surface electrons, and it is shown in Ref. 10 that when electrons interact with thermal vibrations of a surface the perturbation potential is equal to (in a weak holding field)

$$A=-\frac{1}{2}\left(\frac{\hbar q}{2\omega_q g}\right)^{1/2}\Lambda q^2\left[1+\ln\frac{q}{4\gamma}\right],$$

where $\omega_q=cq$ is the spectrum of the Rayleigh waves (c is the velocity of the wave; for hydrogen, $c=0.93c_t$, where $c_t=1.08\times10^5$ cm/sec (Ref. 11) is the velocity of the transverse bulk sound wave), g is the density of solid hydrogen, and $\Lambda=(1/4)e^2(\varepsilon-1)/(\varepsilon+1)$. After the calculations, the collision integral takes the form

$$\nu(p)=\frac{64}{15}\frac{mT\Lambda^2p^3}{\pi\hbar^6gc^2}\left[\left(\ln\frac{p}{2\hbar\gamma}+0.91\right)^2+0.016\right]. \tag{2}$$

The mobility of the electrons of a two-dimensional gas under the condition $\omega\tau\ll1$ (ω is the frequency of the exciting field) is given by[10]

$$\mu=\frac{e}{m}\frac{2}{(2mT)^2}\int\limits_0^\infty\frac{p^3dp}{\nu(p)}\exp\left(-\frac{p^2}{2Tm}\right). \tag{3}$$

Substituting (2) in (3) we obtain (by computer calculation) the values of the electron mobility due to scattering by Rayleigh waves:

T, K	4.0	8,0	14.0
μ, cm^2/V · sec	$4.46\cdot10^7$	$1.27\cdot10^7$	$5.57\cdot10^6$

These quantities exceed by more than two orders of magnitude the experimental data (Fig. 3).

Thus, the principal scattering mechanism at $T<10$ K should be scattering by the surface defects of the dielectric. We consider two possible cases: scattering by defects that are large compared with the average distance between the electron and the surface $\langle\psi|z|\psi\rangle=(3/2)\gamma^{-1}=25$ Å, which are capable of capturing electrons, and scattering by neutral roughnesses of the surface, with dimensions on the order of the lattice constant. In the former case the scattering potential averaged over the z-coordinate (normal to the surface) is[12]

$$A=e\int\limits_0^\infty\frac{k}{k+s}J_0(\rho k)\,dk, \tag{4}$$

where $J_0(x)$ is a Bessel function of zero order and $s=2\pi e^2n/t$ is the reciprocal screening radius of the field. The potential (4) is a Coulomb potential screened by a two-dimensional electron layer: at distances $\rho\gg1/s$ we have $A\approx e/s^2\rho^3$. The matrix element of the scattering by the potential (4) is of the form

$$V_{pp'}=e\int\limits_0^\infty\rho J_0(q\rho)A\,d\rho=\frac{e^2}{q+s}, \tag{5}$$

where q is the change of the electron momentum, $\hbar\mathbf{q}=\mathbf{p}-\mathbf{p}'$, and $q=(2p/\hbar)\sin\varphi/2$. The Bessel function $J_0(x)$ in (5) is the result of integration with respect to the angle in the xy plane.

Since the defects on the surface of the dielectric are randomly located, the scattering is not coherent and the collision integral is proportional to the surface density N of the defects:

$$\nu(p)=N\frac{m}{2\pi\hbar^3}\int\limits_0^\infty V_{pp'}^2(1-\cos\varphi)\,d\varphi. \tag{6}$$

Substituting (5) in (6) and using (3), we can estimate the mobility μ [this calculation only an estimate, since the scattering potential (4) cannot be regarded as weak in the sense of perturbation theory]. If it is assumed that N corresponds to the number of captured electrons, which can be determined from experiment ($N\sim10^7-10^8$ cm^{-2}), then we obtain $\mu=3\times10^4$ cm^2/V · sec ($N=10^8$ cm^{-2}). At $q\gg s$ the screening can be neglected and the potential (5) becomes equal to e^2/q, while the mobility $\mu\propto T^{1/2}$. The condition $q\gg s$ corresponds to the condition $T\,[K]\gg(10^{-9}n\,[\mathrm{cm}^{-2}])^{2/3}$. Under the condition $q\ll s$ [which corresponds to $T\ll(10^{-8}n)^{2/3}$], the mobility $\mu\propto n^2/T^2$. In experiment ($n\sim10^8$ cm^{-2}, $T=4-10$ K), however, neither a relation $\mu\propto T^{1/2}$, nor any connection whatever between μ and the number of captured electrons was observed, and the presence of a static charge on the surface of solid hydrogen leads only to a shift of the $\Delta Q(V_\perp)$ curve without a noticeable change of its slope. Thus, no scattering by captured electrons is observed in the experiments.

We consider now the role of defects with characteristic dimensions of the order of the atomic lattice-dimension $a\sim5\times10^{-8}$ cm. It is known that the force of interaction between a dielectric sphere of diameter a and an electron at a distance $R\gg a$ is equal to $F=\Lambda a^3/r^5$. When this interaction between the electrons and defects is taken into account, the mobility of the surface electrons turns out to be independent of temperature, in contradiction to the experimental data. It appears that the interaction of the electrons with the surface defects is described by another potential, the form of which is difficult to determine, since we know neither the character of the defects nor their size distribution. We have therefore chosen an electron scattering potential that yields for the mobility a temperature dependence close to the experiment. We chose a potential in the form $A=\Lambda a/r^2$.

According to the Born approximation, the scattering is determined by the quantity

$$V_{pp'}=\langle\psi_p|A|\psi_{p'}^*\rangle=4\gamma^3a\Lambda\int\limits_0^\infty z^2e^{-2\gamma z}dz\int\limits_0^\infty\frac{\rho J_0(q\rho)\,d\rho}{z^2+\rho^2}. \tag{7}$$

Using the relation

$$\int\limits_0^\infty\frac{\rho J_0(q\rho)\,d\rho}{z^2+\rho^2}=K_0(qz)$$

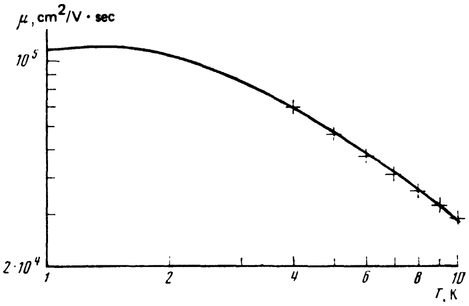

FIG. 4. Calculation of the temperature dependence of the mobility due to scattering by surface defects with dimensions ~5 × 10^{-8} cm, points— average values of the measured mobility.

and expanding the modified Bessel function $K_0(qz)$ in powers of qz (this can be done, since $z \sim \gamma^{-1}$, $qz < 0.2$ at $T < 10$ K), we obtain

$$V_{pp'} = -a\Lambda\left[1{,}5 + \ln\frac{q}{4\gamma} + \frac{13q^2}{16\gamma^2} + \frac{3q^2}{4\gamma^2}\ln\frac{q}{4\gamma}\right]. \tag{8}$$

Substituting (8) in (6), we obtain the collision integral in the form

$$\nu(p) = 0.25Nma^2\Lambda^2\hbar^{-3}R(x), \quad R(x) = \ln^2 x + 5.2\ln x + 7.12$$
$$+90x^2[(\ln x+2)^2+0.0463]+18x[(\ln x+2.36)^2-0.078],$$
$$x = p^2/4\hbar^2\gamma^2.$$

The mobility of the surface electrons is expressed, in accordance with (3), by the formula

$$\mu = \frac{e\hbar^3}{m^2Na^2\Lambda}\int\limits_0^\infty \frac{ydye^{-\nu}}{R(Tmy/2\hbar^2\gamma^2)}. \tag{9}$$

A comparison of the calculated dependence of the electron mobility on the temperature (9) with experiment takes the following form. By approximating (with a computer) the mobility values measured at $T < 11$ K (53 points) by a polynomial of first degree, we obtain by least squares the relation

$$\lg\mu = 5.4 - 0.96\lg T \tag{10}$$

(shown dashed in Fig. 3 and by the points in Fig. 4). As seen from Fig. 4, this dependence agrees with the calculation results shown by the solid curve, but in view of the large scatter of the points (Fig. 3, the large scatter is due to the different values of N for different samples) it is difficult to determine the accuracy of the relation (10). At the same time, the slope of the segments joining measurements points obtained for one sample (Fig. 3) agrees with the $\mu \propto 1/T$ law. The calculated curve in Fig. 4 is normalized to the average value of the mobility measured at 4 K by choosing the value of N; the result is $N = 2\times10^{12}$ cm^{-2}.

In the derivation of (9), to simplify the calculation, we disregarded the screening effect. This is permissible, since the electron density is lower than the defect density, and the screening of the stray field by the surface electrons becomes meaningless. The dielectric screening near the surface is incomplete, and the error that can arise in the calculation when the dielectric screening is neglected is <15%.[5] We note also that in the derivation of (9) we have assumed that the defects scatter independently of one another, so that for an exact calculation it is necessary to take into account the fact that the electron wavelength is of the order of magnitude of the distance between the defects $q \sim N^{1/2}$.

We have thus investigated here the properties of a two-dimensional gas of electrons localized near the surface of a hydrogen crystal. The results of the investigations were used to ascertain the character of the singularities of the surface defects of the crystal and their density. Measurements of the charge accumulating on the surface have yielded the density (10^7–10^8 cm^{-2}) of the defects that are large in comparison with the distance from the surface to the electron layer. From the measurements of the mobility of the electrons we obtain the density (2×10^{12} cm^{-2}) of defects having dimensions of the order of the crystal cell. These numbers should be compared with the surface density ~4 × 10^{14} cm^{-2} of the hydrogen atoms in the crystal. There is one small defect for 200 cells, and the average distance between defects is 10^{-6} cm. At the same time, the electron mean free path calculated from the measured mobility is of the order of 10^{-4} cm and does not depend on the number of large defects. From these figures we can estimate the probability of electron scattering by the small defects, which equal ~0.01. We note in conclusion that the quantity $N = 2\times10^{12}$ cm^{-2} characterizes the quality of the surface in the atomic scale, and that measurement of the mobility of the electrons localized over the crystal surface is, alongside electron microscopy, a method that permits an estimate of the number of defects with dimensions <10^{-7} cm.

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[1] V. S. Edel'man, Usp. Fiz. Nauk **130**, 675 (1980) [Sov. Phys. Usp. **23**, 227 (1980)].
[2] M. S. Khaĭkin, Pis'ma Zh. Eksp. Teor. Fiz. **27**, 706 (1978) [JETP Lett. **27**, 668 (1978)].
[3] A. M. Troyanovskiĭ, A. P. Volodin, and M. S. Khaĭkin, *ibid.* **29**, 525 (1979) [**29**, 382 (1979)].
[4] J. S. Helman, F. Mejia Lira, and R. Williams, Sol. State Commun. **20**, 917 (1972).
[5] M. W. Cole, Phys. Rev. **B2**, 4239 (1970).
[6] W. T. Sommer and D. J. Tanner, Phys. Rev. Lett. **27**, 1345 (1971).
[7] T. R. Brown and C. C. Grimes, *ibid.* **29**, 1233 (1972).
[8] J. L. Levine and T. M. Sanders, Phys. Rev. **154**, 138 (1967).
[9] V. B. Shikin and Yu. P. Monarkha, Fiz. Nizk. Temp. **1**, 957 (1975) [Sov. J. Low Temp. Phys. **1**, 459 (1975)].
[10] P. M. Platzman and G. Beni, Phys. Rev. Lett. **36**, 626 (1976).
[11] US NBS Technical Note 641, 1973, p. 28.
[12] F. Stern and W. Howard, Phys. Rev. **163**, 816 (1967).

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