Influence of the inhomogeniety on the turbulence spectra of a magnetoactive plasma

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Derivation is given of the spectra of high-frequency turbulence of an inhomogeneous magnetoactive plasma when these spectra are due to the stimulated scattering by ions. It is shown that even a very smooth inhomogeniety results in a considerable turbulence anisotropy: the number of waves traveling along the direction of a gradient is considerably less the number traveling in the opposite direction. In the case of oscillations traveling in the direction of decreasing concentration an inhomogeniety increases considerably the Landau damping. Consequently, a considerable part of the absorbed energy is transferred to fast electrons and a current appears along the magnetic field. A study is made of the influence of a stochastic inhomogeneity, which also gives rise to fast electrons. The role of decay processes is discussed.

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Nonlinear processes in an isotropic plasma result in the transfer of Langmuir oscillations to the long-wavelength part of the spectrum and, therefore, a small change in the wave vector due to a weak inhomogeneity cannot alter the damping waves or modify significantly their spectrum.

The situation changes drastically in the case of a magnetoactive plasma. Rubenchik, Rybak, and Sturman[1] showed that in the case of excitation of skew Langmuir plasmons with $\omega_k=\omega_p|\cos\theta|=\omega_p k_z/k$ (the z axis is directed along the magnetic field), nonlinear processes result in "condensation" of the spectrum in the region of large wave vectors, where the plasma damping is strong. A considerable change in the damping and, consequently, a significant modification in the spectrum results from just a slight broadening caused by an inhomogeneity $\Delta k/k \gtrsim (kr_d)^2$.

We shall show that such broadening appears even as a result of very small gradients:

$$L \leqslant \frac{2\Lambda r_d}{(kr_d)^5}\frac{\omega_p}{\nu_{ei}}\left(\frac{\omega_k}{\omega_p}\right)^2.$$

Here, $L=\omega_p(d\omega_p/dz)^{-1}$ and Λ is the Coulomb logarithm.

When this condition is obeyed, turbulence becomes anisotropic: the number of waves traveling in the direction of increasing concentration differs considerably from the number of waves in the opposite direction. The fraction of the energy absorbed as a result of the Landau damping, i.e., the fraction transferred to fast electrons, also changes. It is shown that this fraction can increase severalfold compared with the case of a homogeneous plasma and can rise to 10–15%. Fast electrons are generated only by those oscillations which are traveling in the direction of lower concentration and, therefore, a plasma inhomogeneity gives rise to a static current.

Broadening of the spectrum, increase in the Landau damping, and consequent generation of fast electrons result also from a weak irregular inhomogeneity of a plasma associated with the excitation of drift or magnetoacoustic oscillations. Such low-frequency turbulence is found to give rise to electron tails at a very moderate level of fluctuations.

§1. PRINCIPAL EQUATIONS AND QUALITATIVE DESCRIPTION OF THE TURBULENCE SPECTRA OF AN INHOMOGENEOUS PLASMA

We shall consider the steady-state problem of the turbulence spectrum of magnetized Langmuir plasmons ($\omega_k=\omega_p k_z/k$, $\omega_H^2 \gg \omega_p^2$) in an isothermal plasma when the spectrum is due to the stimulated scattering on ions. We shall show below that this nonlinear process predominates in, for example, the majority of experiments on high-frequency heating of a plasma.

The evolution of the spectrum of a homogeneous plasma is described by the kinetic equation for the number of plasmons n_k (Ref. 1):

$$\frac{\partial n_{\mathbf{k}}}{\partial t}+\Gamma_k n_{\mathbf{k}}=n_{\mathbf{k}}\int T_{\mathbf{kk}'}n_{\mathbf{k}'}d\mathbf{k}'\equiv n_{\mathbf{k}}\gamma_{nl}. \tag{1}$$

Here, the damping $\Gamma_k=\nu_{ei}+\gamma_L$ includes the collisional ν_{ei} and Landau contributions:

$$\gamma_L=\left(\frac{\pi}{2}\right)^{1/2}\frac{\omega_k}{(kr_d)^3}\exp\left(-\frac{1}{2(kr_d)^2}\right),$$

and the matrix element $T_{\mathbf{kk}'}$ is given by the expression[1-3]

$$T_{\mathbf{kk}'}=\frac{1}{2nT}\frac{\omega_p^4}{\omega_H^2k^2k'^2}\left|\frac{\omega_H k_z k_z'}{(\omega_k\omega_{k'})^{1/2}}-i[\mathbf{kk}']_z\right|^2\operatorname{Im}G. \tag{2}$$

The structure function G can be expressed quite simply in terms of the permittivity:

$$G=\varepsilon_e/\varepsilon-1,$$

where ε is the longitudinal part of the permittivity and ε_e is the contribution of electrons to the permittivity. The properties of this function are described in detail in Refs. 1 and 2. In the case of smooth symmetric spectra we can simplify $T_{\mathbf{kk}'}$ by going over to a differential approximation (the limits of its validity are discussed in detail in Ref. 2), so that γ_{nl} becomes

$$\gamma_{nl}=\frac{2\pi^2}{Mn}x\frac{d}{dx}x\int k'^2(k'^2+k^2)n_{k'}dk',$$

where x is the cosine of the angle between the wave

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vector and the magnetic field.

Under steady-state conditions Eq. (1) is a Fredholm integral equation of the first kind,

$$\gamma_{nl}=\Gamma_k, \tag{3}$$

and, therefore, the solution is largely arbitrary. Clearly, we can add an arbitrary odd function of the wave vector to the steady-state solution (3).

In order to find the steady-state solution we must supplement Eq. (3) by the stability condition[1,2]

$$\gamma_{nl}<\Gamma_k \text{ when } n_k=0. \tag{4}$$

The conditions (3) and (4) can be given a simple geometric meaning.[1] They denote that the γ_{nl} curve lies below the Γ_k curve and is in contact with the latter only at the points where the solution is concentrated. In our case, the γ_{nl} curve is a parabola and Γ_k, equal to v_{ei} when $kr_d \ll 1$, begins to rise strongly at high values of k because of the Landau damping (Fig. 1). Therefore, it is obvious that the two curves can only be in contact at one point and the distribution of oscillations is a singular function of the wave vector $n_k \propto \delta(k-k_0)$ or, as is usually said, is of streaming nature. The wave vector k_0 describing the position of a stream is clearly governed by Eq. (3) and the contact condition

$$\left(\frac{d\Gamma_k}{dk}\right)_{k=k_0}=\left(\frac{d\gamma_{nl}}{dk}\right)_{k=k_0}.$$

The Landau damping is then weak compared with the collisional contribution: $\gamma_L \approx v_{ei}(k_0 r_d)^2$. We then have $1/2(k_0 r_d)^2 \sim \ln \omega_p/v_{ei}$ and for the usual plasma parameters we obtain $k_0 r_d \approx 1/7-1/5$.

In general, Eq. (1) should include the source of oscillations $\gamma_p(x)$. We shall assume that turbulence is excited parametrically by a field $E\cos\omega_0 t$. The parametric instability increment is concentrated in a narrow region near $x_0=\omega_0/\omega_p$ and the length of the resultant stream is much greater than the characteristic width of this increment. Therefore, in finding the spectrum we need consider only the inertial interval and regard pumping as the boundary condition.

When we consider turbulence in an inhomogeneous plasma, we note first of all that the transverse inhomogeneity does not alter the phase velocity of oscillations along the magnetic field ω_k/k_z and, consequently, does not alter their damping. Therefore, it is natural to assume initially a plasma with a longitudinal inhomogeneity.

The kinetic equation for waves in an inhomogeneous plasma is

$$\frac{\partial n_{\mathbf{k}}}{\partial t}+\frac{\partial\omega_k}{\partial\mathbf{k}}\frac{\partial n_{\mathbf{k}}}{\partial\mathbf{r}}-\frac{\partial\omega_k}{\partial\mathbf{r}}\frac{\partial n_{\mathbf{k}}}{\partial\mathbf{k}}=(\gamma_{nl}-\Gamma_k)n_{\mathbf{k}}.$$

It is convenient to separate oscillations traveling in the direction of increasing concentration $n_k^+(y)$ and in the opposite direction $n_k^-(y)$, where $y\equiv|x|$. Generally speaking, they correspond to different nonlinear terms $\gamma_{nl}^\pm$. We shall first consider a plasma in a very strong field $\omega_0\omega_H \gg \omega_p^2$, when we can limit Eq. (2) to just the first term. Going over to the differential approximation, we obtain

$$\left.\begin{aligned}\gamma_{nl}^\pm&=k^2y\frac{d}{dy}yC_0+y\frac{d}{dy}yC_2\pm 2ky^2\frac{d}{dy}y^2C_1,\\ C_i&=\alpha\int k^{2+i}(n_k^-(y)+(-1)^i n_k^+(y))\,dk,\\ \alpha&=\pi^2/Mn,\quad i=0,1,2.\end{aligned}\right\} \tag{5}$$

The narrowness of the distribution in the k space allows us to ignore the spatial migration of oscillations and to assume that $\partial/\partial k_z = x\partial/\partial k$. Consequently, the steady-state kinetic equation simplifies to

$$\pm y\frac{\partial\omega_k}{\partial z}\frac{\partial n_k^\pm}{\partial k}=(\gamma_{nl}^\pm-\Gamma_k)n_k^\pm. \tag{6}$$

The structure of the solution (6) is shown qualitatively in Fig. 2.

Since oscillations migrate away, the steady-state conditions are obtained if $\gamma_{nl}^\pm > \Gamma_k$, i.e., if the curves $\gamma_{nl}^\pm$ intersect the curve Γ_k at two points and the distribution of oscillations is no longer singular (Fig. 2). In the case of oscillations traveling along the concentration gradient we find that n_k^+ for $k > k_+$ is of the order of the thermal noise n_k^0. The wave vector of n_k^+ decreases along an inhomogeneity, but it begins to rise in the region $\gamma_{nl}^+ > \Gamma_k$. The maximum of the distribution of n_k^+ obviously coincides with the point $\varkappa_+$ and then in the range $\gamma_{nl}^+ < \Gamma_k$ there is a rapid fall to the thermal noise level. A distribution of n_k^- is of similar form, except that the fall in the range $k > k_-$ is steeper because of the strong Landau damping. Thus, the spectrum has the form of two streams representing waves traveling in the opposite directions. A reduction in the gradient reduces the thickness of the stream and causes them to coalesce into one.

§2. STRUCTURE OF THE STEADY-STATE SOLUTION

We shall assume that the concentration profile is linear: $\partial\omega_k/\partial z = \omega_k/L$. For simplicity, we shall consider fairly low oscillations obeying $y^2 \ll 1$. We can

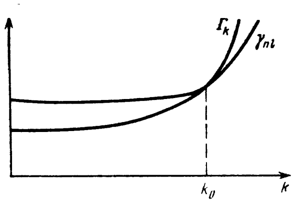

FIG. 1. Dependences of Γ_k and γ_{nl} on the wave vector k for a fixed value of θ in a homogeneous plasma.

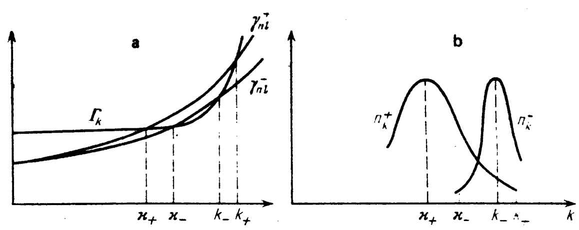

FIG. 2. a) Dependences of γ_{nl}^+, γ_{nl}^-, and Γ_k on the wave vector k. b) Dependences, on k, of the number of waves n_k^+ and n_k^- traveling in the direction of increasing and decreasing concentration, respectively.

then assume that $\gamma^+_{nl}=\gamma^-_{nl}\equiv\gamma_{nl}$, $k_+=k_-\equiv k_2$, and $\varkappa_+=\varkappa_-\equiv k_1$.

Since the dependence of γ_{nl} on k is known, Eq. (6) can be integrated, which gives

$$\left.\begin{aligned} n_k^+&=n_k^0\exp\int_k^{k_2}(\gamma_{nl}-\Gamma_k)\frac{L}{\omega_k y}dk,\\ n_k^-&=n_k^0\exp\int_{k_1}^{k}(\gamma_{nl}-\Gamma_k)\frac{L}{\omega_k y}dk,\end{aligned}\right\}\tag{7}$$

where the limits of integration are selected in accordance with the qualitative nature of the solution described above and n_k^0 is the thermal noise level. Going over to dimensionless variables and integrating, we obtain

$$\left.\begin{aligned} \ln n_k^+(y)&=l\left\{\frac{k_2^3-k^3}{3}y\frac{d}{dy}yC_0+(k_2-k)\left(y\frac{d}{dy}yC_2-1\right)\right.\\ &\left.-\left(\frac{\pi}{2}\right)^{1/2}\frac{\omega_k}{\nu_{ei}}\left(\exp\left(-\frac{1}{2k_2^2}\right)-\exp\left(-\frac{1}{2k^2}\right)\right)\right\},\\ \ln n_k^-(y)&=l\left\{\frac{k^3-k_1^3}{3}y\frac{d}{dy}yC_0+(k-k_1)\left(y\frac{d}{dy}yC_2-1\right)\right.\\ &\left.-\left(\frac{\pi}{2}\right)^{1/2}\frac{\omega_k}{\nu_{ei}}\left(\exp\left(-\frac{1}{2k^2}\right)-\exp\left(-\frac{1}{2k_1^2}\right)\right)\right\}.\end{aligned}\right\}\tag{8}$$

Here, $n_k^\pm$ are orthonormalized to n_k^0 and all the frequencies are normalized to v_{ei}; k is understood to mean kr_d, whereas C_0, C_1, and C_2 are still described by the formulas in the system (5) but with other values of the coefficient α: $\alpha=\pi^2n_k^0/Mnv_{ei}r_d^5$.

Knowing the explicit dependence of $n_k^\pm$ on the wave vector, we can integrate in Eq. (5) and obtain a closed system of equations. Obviously, the main contribution to the integral of n_k^+ is the region of the maximum where $k\sim k_1$ and the integral of n_k^- is dominated by the region near $k\sim k_2$. We thus obtain

$$\left.\begin{aligned} &n_k^\pm=\exp l(f^0-b_\pm^2(\Delta k_\pm)^2),\\ &\Delta k_+=k-k_1,\quad \Delta k_-=k-k_2,\\ &f^0\approx k_1h^2y\frac{d}{dy}yC_0,\quad h=k_2-k_1;\end{aligned}\right\}\tag{9}$$

$$b_+^2\approx k_1y\frac{d}{dy}yC_0,\quad b_-^2\approx\left(\frac{\pi}{2}\right)^{1/2}\frac{\omega_k}{2\nu_{ei}k_2^6}\exp\left(-\frac{1}{2k_2^2}\right);\tag{10}$$

$$\left.\begin{aligned} C_0&\approx\alpha\left(\frac{\pi}{2}\right)^{1/2}\left(\frac{k_2^2}{b_-}+\frac{k_1^2}{b_+}\right)\exp(lf^0),\\ C_2&\approx\alpha\left(\frac{\pi}{2}\right)^{1/2}\left(\frac{k_2^4}{b_-}+\frac{k_1^4}{b_+}\right)\exp(lf^0).\end{aligned}\right\}\tag{11}$$

The system (9)–(11) can be closed by adding the conditions for the intersection of $\gamma^\pm_{nl}$ and Γ_k, which determine k_1 and k_2:

$$\left.\begin{aligned} &k_1^2y\frac{d}{dy}yC_0+y\frac{d}{dy}yC_2=1,\\ &(k_2^2-k_1^2)y\frac{d}{dy}yC_0=\left(\frac{\pi}{2}\right)^{1/2}\frac{\omega_k}{\nu_{ei}k_2^3}\exp\left(-\frac{1}{2k_2^2}\right).\end{aligned}\right\}\tag{12}$$

When oscillations are excited in a homogeneous plasma, we have $\gamma_L\approx v_{ei}(k_0r_d)^2$ (Ref. 1). If the broadening of the spectrum is small ($h\ll k^3$), the nature of the solution and the value of γ_L change only slightly. Therefore, we shall be interested in the opposite limiting case $h\gg k^3$. Streams n_k^+ and n_k^- then diverge to the left and right of k_0. The collisionless damping increases in the case of n_k^-, whereas for n_k^+ we can ignore the term representing this damping.

It is clear from Eqs. (5) and (11) that the order of magnitude of lf^0 is equal to the logarithm of the ratio of the oscillation energy W to the thermal noise energy in the region where the oscillations are excited.

Since W/W_0 is large, it follows that $\ln(W/W_0)$ is almost independent of the noise level. Moreover, the condition $nT/W_0\gg W/nT$ is practically always satisfied and, consequently, we have $\ln(W/W_0)\approx\ln(nT/W_0)\approx\Lambda$ (Λ is a Coulomb logarithm, $\Lambda\approx10-15$),

$$lf^0\approx\Lambda.\tag{13}$$

Since the n_k^- streams are narrow compared with n_k^+, Eq. (11) yields the expression $C_2\approx k_1^2C_0$, and then from Eq. (12) we obtain

$$k_1^2y\frac{d}{dy}yC_0\approx{}^1/_2,\tag{14}$$

i.e., the integral characteristics of the distribution remain the same as for a homogeneous plasma. The distance between streams follows from Eqs. (13) and (9):

$$h=(2\Lambda k_1/l)^{1/2},\tag{15}$$

and the position of a stream is obtained from Eqs. (12) and (15)

$$\frac{1}{2k_2^2}=\ln\left[\frac{\omega_k}{\nu_{ei}}\left(\frac{\pi}{2}\right)^{1/2}\frac{1}{k_0^2h}\right].\tag{16}$$

We can also see that the distribution of n_k^+ and n_k^- have the same amplitude and their characteristic widths Δk_+ and Δk_- are much smaller than the distance between the streams if $h\gg k^3$ or $(2\Lambda/lk^5)^{1/2}\gg1$:

$$\left.\begin{aligned} &\Delta k_+\sim(l^{1/2}b_+)^{-1}\sim(2k_1/l)^{1/2},\\ &\Delta k_-\sim(l^{1/2}b_-)^{-1}\sim(2k_1/l)^{1/2}k_2(2k_1l/\Lambda)^{1/4},\\ &\Delta k_+/h\sim\Lambda^{-1/2},\quad \Delta k_-/h\ll\Lambda^{-1/2}.\end{aligned}\right\}\tag{17}$$

As assumed above, a stream n_k^+ is wider than n_k^-:

$$\frac{\Delta k_+}{\Delta k_-}=\frac{b_-}{b_+}\sim\left(\frac{\Lambda}{2lk^5}\right)^{1/4}>1.$$

We shall now calculate the Landau damping for a stream n_k^-:

$$\gamma_L\approx(\nu_{ei}k_2^2)\frac{\omega_k}{\nu_{ei}}\left(\frac{\pi}{2}\right)^{1/2}k_2^{-5}\exp\left(-\frac{1}{2k_2^2}\right)\approx\nu_{ei}k_2^2\frac{h}{2k_2^3}\gg\nu_{ei}k_0^2.\tag{18}$$

We can see that there is a considerable increase in the Landau damping compared with a homogeneous plasma.

We shall now consider oscillations of lower frequency characterized by $\omega_0\omega_H\ll\omega_p^2$, when Eq. (2) is dominated by the second term describing the scattering on velocity fluctuations caused by the drift in the wave field. In this case, γ_{nl} is described by

$$\gamma_{nl}^\pm=k^2\frac{dC_0}{dy}+\frac{dC_2}{dy}\pm2ky\frac{d}{dy}yC_1.$$

As in §2 we can simplify $\gamma^\pm_{nl}$ by dropping the term with C_1 and this gives equations analogous to Eqs. (8)–(12), except that the operator $y(d/dy)y$ is replaced by d/dy, and the definitions of C_0 and C_2 contain a different coefficient $\alpha=(\pi^2/2Mn)\omega_p^2/\omega_H^2$.

We can easily show that the positions of streams n_k^+ and n_k^-, their widths, and the distances between the maxima are given by Eqs. (15)–(17). The ratio of the collisional damping to the collisionless Landau damping remains the same as in the case when $\omega_0\omega_H \gg \omega_p^2$.

§3. RANGE OF VALIDITY OF THE RESULTS AND DISCUSSION OF THE SPECTRAL BROADENING MECHANISM

We shall now discuss the range of validity of the results obtained. It follows from the condition $h \gtrsim k^3$ that an inhomogeneity affects significantly the structure of turbulence if $l \lesssim 2\Lambda/k^5$ or, in terms of dimensional variables,

$$L \leqslant r_d \frac{2\Lambda}{(kr_d)^5}\frac{\omega_p}{\nu_{ei}} y^2. \tag{19}$$

Such negligible concentration gradients can occur in any real plasma.

There are several conditions which set the lower limit to the size of an inhomogeneity described by the above results. The most stringent of these conditions is related to the fact that in the case of high gradients we cannot regard the profile n_k^+ as Gaussian and the next higher terms in the expansions have to be included in the calculation of C_0 and C_2 by the steepest descent method. We can easily find the conditions that the gradient has to satisfy:

$$1/10\Lambda k^5 \leqslant l \leqslant 2\Lambda/k^5. \tag{20}$$

At high gradients the strong Landau damping makes the structure of n_k^- asymmetric but the turbulence pattern described above remains the same. If, moreover, the condition $\gamma_p \gg \Lambda v_{gr}/L \sim \Lambda\omega_p/kL$ is satisfied, it then follows from Ref. 4 that the inhomogeneity does not affect the absorption and the energy flux reaching the plasma is given by the expressions derived in Refs. 1 and 2, and the boundary condition for the spectrum at $x = x_0$ can be set using the results obtained for a homogeneous plasma.[1,2]

As pointed out above, a homogeneous plasma is characterized by $\gamma_L \approx v_{ei}(k_0 r_d)^2$. This means that almost all the energy is transferred to the bulk of electrons by the collisional damping process and the fast electrons receive the fraction of the total energy amounting to $\eta_0 \sim \gamma_L/v_{ei} \sim 2-4\%$. However, if the plasma concentration is inhomogeneous, the relationship between the collisional and collisionless damping changes. As shown in §2, the Landau damping should be allowed only for the stream n_k^-. Then, the fraction η of the energy transferred to fast electrons is

$$\eta \sim \frac{\gamma_L(k_2)}{\nu_{ei}} \frac{\int n_k^- dk}{\int (n_k^- + n_k^+) dk}. \tag{21}$$

We shall now calculate this fraction. The amplitudes of the n_k^+ and n_k^- peaks are the same and, therefore, we can express the ratio of the integrals in Eq. (30) in terms of the stream widths Δk_+ and Δk_- ($\Delta k_+ \gg \Delta k_-$). Using also Eq. (21), we obtain

$$\eta \sim \frac{\Delta k_-}{\Delta k_+}\frac{\gamma_L(k_2)}{\nu_{ei}} \sim \left(\frac{\Lambda}{2k_2^5 l}\right)^{1/4} k_2^2 > \eta_0. \tag{22}$$

We can see that an increase in the inhomogeneity (on reduction in l), increases η and then at the limit of validity we have $\eta \sim 10-15\%$.

At first sight the reason for the broadening of the spectra of an inhomogeneous plasma is obvious: the wave vector of oscillations traveling in a medium with an inhomogeneous concentration changes in accordance with the equation $\partial k_z/\partial t = -\partial\omega_k/\partial z$. However, if we estimate the change in the wave vector also during an oscillation lifetime v_{ei}^{-1}, we find that this change is considerably less than the broadening obtained above. The answer is as follows: in a homogeneous medium the contraction of a spectrum into a stream is due to nonlinear processes, associated with the Fredholm structure of the equations, and this is why the spectrum is so sensitive to the change in the structure of these equations. It follows that even a weak inhomogeneity can influence significantly the process of stream "condensation."

We shall illustrate this by a simple estimate of the characteristic inhomogeneity length L in which an inhomogeneity begins to affect the structure of a solution. In this case the width of the streams and the distance between them are of the same order of magnitude: δk and $\delta k \sim k_0(k_0 r_d)^2$. Bearing in mind the smallness of the inhomogeneity, we shall expand Eq. (6):

$$y^2 \frac{\omega_p}{L}\frac{n_k}{\delta k} \sim \frac{\partial^2}{\partial k^2}(\gamma_{nl} - \Gamma_k)\delta k^2 n_k.$$

Substituting here

$$\frac{\partial^2}{\partial k^2}(\gamma_{nl} - \Gamma_k) \sim \frac{\nu_{ei} r_d^2}{(kr_d)^4},$$

we obtain a result which agrees with Eq. (19):

$$L \sim \frac{\omega_p}{\nu_{ei}}\frac{r_d y^2}{(kr_d)^5}.$$

If the gradient is high an estimate of this kind is more difficult to obtain because the spectrum is characterized by three different scales: the widths of the two streams and the distance between them.

§4. INFLUENCE OF A TRANSVERSE INHOMOGENEITY

We have shown above that the effect of an inhomogeneity is to alter the structure of the equations, giving rise to an effective "noise" so that a transverse inhomogeneity should also deform the spectrum. Since in real experiments such an inhomogeneity is considerably greater than a longitudinal inhomogeneity, and also because oscillations travel mainly across the magnetic field, this aspect is very important.

We can easily show that if

$$L_\perp \leqslant \frac{\omega_p}{\nu_{ei}}\frac{r_d}{(kr_d)^5}$$

the oscillation spectrum is greatly modified. However, an inhomogeneity results primarily in an angular change in the spectra, disturbing their axial symmetry, but may not give rise to fast electrons. We shall demonstrate this for the case when $\omega_0\omega_H \gg \omega_p^2$.

We shall select the following coordinate system: the k_z axis is directed, as before, along the magnetic field; the k_x axis is along the concentration gradient. The distribution of oscillations $n_k = n_k(x, \varphi)$ should satisfy the symmetry relationships (here, φ is the azimuthal angle measured from the k_x axis):

$$n_k(x,\varphi)=n_k(-x,\varphi)=n_k(x,-\varphi).$$

Equation (5) becomes

$$\frac{\omega_k}{L_\perp}\frac{\partial n_k}{\partial k_x}=n_k(\gamma_{nl}-\Gamma_k), \tag{23}$$

and then γ_{nl} is readily described by

$$\gamma_{nl}(k,x,\varphi)=\frac{\pi}{Mn}x\frac{\partial}{\partial x}x\int k'^2(k'^2+k^2)n_{k'}(x,\varphi')dk'\,d\varphi'$$
$$-\frac{2\pi}{Mn}x(1-x^2)^{1/2}\frac{\partial}{\partial x}x(1-x^2)^{1/2}k\int k'^3 n_{k'}(x,\varphi')\cos(\varphi'-\varphi)dk'\,d\varphi'. \tag{24}$$

In this case the scattering of oscillations is through an angle π. Therefore, we shall consider the solution of Eq. (23) in the form

$$n_k(x,\varphi)=n_k(x)\left[\delta\left(\varphi+\frac{\pi}{2}\right)+\delta\left(\varphi-\frac{\pi}{2}\right)\right]. \tag{25}$$

Then, γ_{nl} is given by Eq. (5) and is independent of φ, so that for all values of φ, we have $\gamma_{nl}=v_{ei}$. Inhomogeneity gives rise to additional damping for all angles $\varphi\neq\pi/2$ and, consequently, Eq. (25) is a stable solution.

If $\omega_0\omega_H<\omega_p^2$, so that the scattering occurs through an angle $\pi/2$, it is not possible to obtain the solution so that an inhomogeneity clearly generates fast electrons. However, the long-wavelength stochastic plasma inhomogeneity may be more important.

§5. INFLUENCE OF A STOCHASTIC INHOMOGENEITY

We have confined ourselves so far to a regular inhomogeneity. However, from the practical point of view a study of a stochastic long-wavelength inhomogeneity is of greater importance. Such an inhomogeneity can be due to, for example, drift or magnetoacoustic oscillations, which always occur in real experiments. The scattering of oscillations by such an inhomogeneity results in the diffusion in the k space, so that the steady-state equation (1) can be written in the form

$$D_z\frac{\partial^2 n_k}{\partial k_z^2}+D_\perp\Delta_\perp n_k=n_k(\Gamma_k-\gamma_{nl}). \tag{26}$$

Since such a low-frequency turbulence can naturally be assumed to be axially symmetric, the most important effect is the diffusion in the transverse direction. The diffusion coefficient $D_\perp$ can be obtained consistently from the kinetic equation for waves, but in our case it is sufficient to obtain simple estimates. Let the change in the concentration in an inhomogeneity be δn and the characteristic transverse wave vector be q. Then, the order of magnitude of the diffusion coefficient is $D_\perp\sim(\partial k_\perp/\partial t)^2\tau$, where τ is the time for high-frequency oscillations to travel a scale distance q^{-1}, $\tau\sim(qv_{gr})^{-1}\sim k_0/q\omega_k$, and $\partial k_\perp/\partial t$ is governed by the condition $\partial \mathbf{k}_\perp/\partial t=-\nabla_\perp\omega_k$, so that the diffusion coefficient is

$$D_\perp\sim\omega_k q k_0(\delta n/n)^2.$$

This diffusion broadens a stream by an amount δk, which can be determined from Eq. (26) exactly as has been done in the preceding section:

$$\frac{D_\perp}{\delta k^2}\sim\frac{\partial^2}{\partial k^2}(\gamma_{nl}-\Gamma_k)\delta k^2\sim\frac{v_{ei}}{(kr_d)^2}\left(\frac{\delta k}{k}\right)^2.$$

A considerable increase in the Landau damping occurs when $\delta k/k\gtrsim(k_0r_d)^2$, so that the intensity of long-wavelength oscillations corresponding to the onset of significant heating of electrical tails is given by

$$\left(\frac{\delta n}{n}\right)^2\geqslant\frac{v_{ei}}{\omega_k}\frac{k}{q}(kr_d)^6. \tag{27}$$

We can see that the fluctuation level defined by Eq. (27) is very low. In the case of characteristic parameters of tokamaks at frequencies of the order of the lower hybrid, low-frequency fluctuations have a considerable influence on the spectra of high-frequency turbulence even for $\delta n/n\lesssim 1\%$ and they increase considerably the number of fast electrons.

It should be stressed that even if a transverse inhomogeneity dominates generation of fast electrons, an analysis of a longitudinal inhomogeneity is very important because it makes the heating of electrons and generation of the current anisotropic.

§6. ROLE OF DECAY PROCESSES

We have assumed above that the main nonlinear process governing the level of turbulence is the stimulated scattering by ions. However, there are many other processes that limit the oscillation amplitude, the main of which is the decay of a plasmon into two other plasmons (for a discussion of other nonlinear processes see Ref. 2):

$$\omega_k\to\omega_{k_1}+\omega_{k_2}. \tag{28}$$

This process is due to electron nonlinearities and, therefore, its description contains a small quantity $(kr_d)^2$. The stimulated scattering is characterized, because of the smallness of the step of energy transfer in the case of spectra of Δx, by a small term $(x_{dif}/\Delta x)^2$. The width of the spectrum increases on increase in the instability increment γ_p: $\Delta x\propto x_{dif}(\gamma_p/v_{ei})$. Therefore, for large values of the excess above the threshold we can expect decay processes to play an important role: for example, it is concluded in Ref. 5 that decay processes determine the characteristics of high-frequency plasma heating. They are also important in the case of beam heating of a plasma, when even a small excess above the threshold results in the excitation of oscillations in a wide range of angles. When the excess above the threshold is small, decay is unimportant. It can also be ignored in the most interesting case (from the practical point of view) of the excitation of oscillations with frequencies close to the lower hybrid. This occurs because of a reduction in the characteristic increments of the process (28) when $\omega_k\ll\omega_p$ (see, for example, Ref. 5). Moreover, when $\omega_k\sim\omega_{LH}$, decay processes become forbidden by the nature of the spectrum.

The mechanism of stabilization of a parametric instability can be determined experimentally by recording the turbulence frequency spectra. If stabilization is due to the scattering on ions, the spectra of high-frequency oscillations are broadened in the direction of red wavelengths by an amount of the order of several acoustic frequencies. Decay processes give rise immediately to oscillations of frequencies 2–3 times less than the pump frequency. The spectra recorded in all the experiments on high-frequency heating demonstrate the dominant role of the stimulated scattering, but so far there is no experimental evidence of the importance of decay processes. It follows that the results obtained above have a wide range of validity and are in agreement with the published experimental evidence.

CONCLUSIONS

It is shown that the spectra of high-frequency turbulence are very sensitive to plasma inhomogeneity. This inhomogeneity gives rise to anisotropy of the spectra and, which is more important, increases the proportion of the energy transferred to fast electrons. These fast electrons, which contain 10–20% of the absorbed energy, have been observed in a number of experiments on the lower-hybrid plasma heating.[6,7] Their origin is not yet clear. In our opinion, the above results show that the generation of these electrons may be due to a real inhomogeneity of the plasma which occurs in these experiments.

Another point should be noted. As shown above, fast electrons are generated only by waves traveling in the direction of decreasing density, i.e., a regular longitudinal inhomogeneity in a closed trap gives rise to a current. Such an inhomogeneity can be created in a tokamak in the absorption region by modulation of the longitudinal magnetic field. Then, a considerable proportion of the absorbed energy is expended to generate the current.

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