Theory of the deceleration of atomic beams by resonant laser radiation

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The radiative deceleration of atoms by an oppositely propagating resonant light wave is considered. The kinetic and hydrodynamic stages of the deceleration are analyzed on the basis of a kinetic equation of the Fokker-Planck type for the atomic distribution function. Hydrodynamic equations for the motion of the atoms in the field of a plane light wave are derived and are investigated for the case of an atomic beam in a steady state.

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1. INTRODUCTION. FORMULATION OF THE PROBLEM

Among the important applications of resonant light pressure are, as recent studies have shown,[1-16] its use to cool atomic gases and to decelerate atomic beams. A unique feature of the use of light pressure is the possibility it affords of achieving extremely low atomic temperatures. Under optimal conditions, the ultimate cooling is limited only by the natural line width 2γ of the resonant atomic transition, and for typical allowed optical transitions temperatures of 10^{-3}–10^{-4} K can be achieved.[3]

With the realization of deep cooling of atoms in mind, a number of schemes for atomic traps have been proposed[4,9,15,16] which envisage a preliminary radiative cooling of the atoms and the subsequent accumulation of the cold atoms in optical[4,9,15] or magnetic[16] fields. An adequate analysis of the operation of such traps presupposes the solution of two basic problems: that of the evolution of the velocity distribution of atoms in a standing light wave, and that of the evolution of an atomic ensemble irradiated by an oppositely propagating traveling light wave. The first problem is of great importance in clarifying the kinetics of the cooling of atoms within a trap and for determining the conditions necessary for the deepest possible cooling. The solution of this problem has been discussed in Refs. 3, 6, 7, 12, and 13. The second problem is of interest in determining the best methods of injecting atomic beams into traps. In addition, the problem of the radiative deceleration of atoms in a traveling light wave is of more general physical significance, since the ability to produce beams of cold atoms would be of great assistance in many problems of atomic physics and spectroscopy.

In principle, the most general approach to the solution of the problem of radiative deceleration of atoms would be to analyze the evolution of the atomic distribution function $w(\mathbf{r},\mathbf{p},t)$. For the case of ideal two-level atoms interacting resonantly with a traveling light wave, the distribution function satisfies a kinetic equation of the Fokker-Planck type,[17-19] whose exact form was found in Refs. 19 and 20. Actually, that approach is difficult, both because the equation cannot be solved analytically and because the use of the distribution function to describe the motions of the atoms provides more detailed information than is needed for any practical purposes. It is accordingly natural to approach the solution of the problem of radiative deceleration of atoms by describing the motions of the atoms on the macroscopic level.

Using such an approach in this paper, we shall find the hydrodynamic equations for the motion of atoms in the field of a resonant light wave. The equations to be derived, like the initial kinetic equation, are valid for atoms for which the natural line width γ of the resonant transition exceeds the recoil energy r, and are capable of describing the macroscopic motion of the atoms for times $t \gg r^{-1}$. In Section 7 we use these basic equations to analyze the changes in the principal macroscopic variables (local density, average velocity, and temperature) for the case of an atomic beam being decelerated in the field of an oppositely propagating resonant light wave.

2. INITIAL EQUATIONS

In what follows we shall always assume that the ensemble of atoms is in the field of a plane traveling light wave with frequency ω ($\omega = kc$) and the unit polarization vector e, propagating in the negative z direction:

$$\mathbf{E}(z,t) = \mathbf{e}E_0 \cos(\omega t + kz). \tag{1}$$

We shall assume the atoms to have two levels: the ground state, and an excited state that decays to the ground state with the full spontaneous-decay probability 2γ. In accordance with the selected two-level scheme for the atoms, the wave (1) should be either linearly or circularly polarized. The frequency ω_0 of the resonant atomic transition is close to the frequency ω of the wave. We assume that the width γ of the atomic transition and the recoil energy $\hbar r = \hbar^2 k^2/2M$ satisfy the relation

$$\varepsilon = r/\gamma \ll 1, \tag{2}$$

which, keeping in view the most important allowed atomic dipole transitions, is a necessary condition for the motion of the atoms to be describable by a Fokker-

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Planck kinetic equation.[19]

The distribution function $w(\mathbf{r},\mathbf{p},t)$ that describes the motion of the atoms under condition (2) for times $t \gg \gamma^{-1}$ satisfies a kinetic equation, which forms the basis for the subsequent analysis, and which we take directly from Ref. 19:

$$\frac{\partial w}{\partial t}+\mathbf{v}\frac{\partial w}{\partial \mathbf{r}}=-\frac{\partial}{\partial p_z}(Fw)+\sum_{i=x,y,z}\frac{\partial^2}{\partial p_i^2}(D_{ii}w), \tag{3}$$

where F is the light-pressure force along the z axis:

$$F=-\hbar k\gamma G[1+G+(\Omega+kv_z)^2/\gamma^2]^{-1}, \tag{4}$$

and the D_{ii} are components of the momentum diffusion tensor:

$$D_{ii}=0{,}5\hbar^2k^2\gamma\chi_{ii}G[1+G+(\Omega+kv_z)^2/\gamma^2]^{-1}, \tag{5}$$

$$\chi_{ii}=\alpha_{ii}+\delta_{iz}\left(1+\frac{G[(\Omega+kv_z)^2/\gamma^2-3]}{1+G+(\Omega+kv_z)^2/\gamma^2}\right). \tag{6}$$

Here we have used the following notation: $G=(dE_0/\hbar\gamma)^2/2$ is the saturation parameter for the atomic transition, whose dipole-moment matrix element is d, and $\Omega=\omega-\omega_0$ is the difference between the frequency ω of the light wave and the frequency ω_0 of the atomic transition. We assume Ω to be negative ($\omega<\omega_0$), for otherwise the ensemble of atoms and the light wave, propagating in opposite directions, could not interact resonantly. The parameters α_{ii}, which determine the components of the diffusion tensor, depend on the polarization of the wave (1).[17-19]

The problem of the radiative deceleration of an ensemble of atoms, in which we are interested, reduces formally to the solution of Eq. (3). On examining this equation it can be seen that the ensemble of atoms evolves in qualitatively different ways in directions parallel to the z axis, and perpendicular to it. The transverse motion of the atoms is determined by the diffusion of the momenta of the atoms, and as a result, the transverse velocity distribution broadens as time goes on. The transverse velocity distribution accordingly remains permanently in the kinetic stage of its evolution. As will be shown below, the evolution of the longitudinal velocities of the atoms results in the rapid establishment of a local velocity distribution. For the motion of the atoms along the z axis, therefore, the hydrodynamic stage is the most important.

Bearing in mind the simultaneous existence of two qualitatively different stages in the evolution of the distribution function $w(\mathbf{r},\mathbf{p},t)$ and being primarily interested in the longitudinal motions of the atoms, in the beginning we shall neglect the transverse diffusion in Eq. (3) and consider the one-dimensional motion of the atoms along the z axis (Sections 3–5). The analysis will be generalized to the case of a real three-dimensional space in Section 6.

Now let us write the initial kinetic equation for the first part of the problem in dimensionless variables. Taking $(kv_r)^{-1}$, γ/k^2v_r, and γ/k as the units of time, length, and velocity, respectively (here $v_r=\hbar k/M$ is the recoil velocity), we find that (3) reduces to

$$\frac{\partial w}{\partial t}+v\frac{\partial w}{\partial z}=-\frac{\partial}{\partial v}(Fw)+\frac{\partial^2}{\partial v^2}(Dw), \tag{7}$$

where the dimensionless light-pressure force is

$$F=-L=-G[1+G+(\delta+v)^2]^{-1}, \tag{8}$$

and D determines the z component of the dimensionless velocity-diffusion tensor:

$$D=\varepsilon\chi L, \tag{9}$$

$$\chi=1+\alpha+L^2[(\delta+v)^2-3]/G. \tag{10}$$

To simplify Eqs. (7)–(10) we have omitted the subscript z, writing simply v and z for the velocity and coordinate along the z axis. The dimensionless detuning in Eqs. (8) and (10) is $\delta=\Omega/\gamma$.

3. DECELERATION OF ATOMS IN THE KINETIC STAGE

Let us examine the time evolution of the distribution function along the z axis and determine the characteristic time scales of the problem. To do this we first turn to Eq. (7) and find the relaxation time τ_r that determines the kinetic stage of the evolution of the distribution function. In accordance with the structure of Eq. (7), the kinetic stage is characterized by the establishment of a local velocity distribution (see, e.g., Ref. 21). That this statement is correct is clearly evident from the expression for the force F, which, in the case of a sufficiently narrow velocity distribution, can be expanded in a power series in the deviation $u=v-\langle v\rangle$ of the running velocity v from the average velocity $\langle v\rangle$:

$$F=-L(\langle v\rangle)+2G^{-1}L^2(\langle v\rangle)(\delta+\langle v\rangle)u+\ldots. \tag{11}$$

The first term in (11) represents the constant force that alters the average velocity, while the second term represents the friction force (we note that $\delta+\langle v\rangle<0$ when the atoms are being decelerated), which narrows the velocity distribution. Since the time for relaxation to a local velocity distribution depends only on the friction force, we may define it as the reciprocal of the coefficient of friction; then from (11) we obtain

$$\tau_r\approx[1+G+(\delta+v)^2]^2/2G|\delta+\langle v\rangle|. \tag{12}$$

Expression (12) clearly reflects the nonlinear velocity dependence of the light-pressure force and the fact that the greater the deviation of the average velocity from resonance with the average force $\langle F\rangle=-\langle L\rangle$, the longer the kinetic stage of the evolution of the velocity distribution lasts.

Being interested in the important case of the evolution of an ensemble that was in resonance with the average force at $t=0$, we must take the minimum value of (12) as τ_r; this minimum value is

$$\tau_r^{min}=(1+G)^{1/2}G^{-1}. \tag{13}$$

We note that in this time the average force alters the velocity $\langle v\rangle$ by a quantity of the order of $(1+G)^{1/2}$, i.e., it brings the ensemble out of resonance with the force $\langle F\rangle$.

The second characteristic time of the problem is the time τ_d in which the atomic ensemble is declerated to zero average velocity. In determining τ_d we need only consider the changes in the average velocity $\langle v\rangle$. We therefore neglect the diffusion term in Eq. (7) and write down the characteristic equations of the linear partial differential equation:

$$dt=\frac{dz}{v}=-\frac{dv}{L}. \tag{14}$$

From (14) we obtain the time dependence of the velocity of interest to us in the form

$$t=G^{-1}\{(1+G)(v_0-v)+[(\delta+v_0)^3-(\delta+v)^3]/3\}, \tag{15}$$

where v_0 is the initial velocity of an atom.

Before calculating τ_d from (15), we introduce the following condition, which is basic for all the subsequent calculations:

$$|\delta|\gg(1+G)^{1/3}, \tag{16}$$

This condition means that the change in the velocity of the atoms during the acceleration process is greater than the characteristic velocity range in which the light-pressure force acts. This condition is satisfied in all cases of practical importance since in decelerating an ensemble of atoms from a typical initial thermal velocity of $\bar v\sim10^2$, the detuning δ should be taken as $-\delta\approx\bar v\sim10^2$, while the saturation parameter G, as a rule, does not exceed 10^3. Now we set $v=0$ in (15) and, taking condition (16) into account, we find the order of magnitude of the deceleration time:

$$\tau_d=|\delta|^3/3G. \tag{17}$$

On comparing Eqs. (17) and (13), we find that when condition (16) holds, we have

$$\tau_r^{min}\ll\tau_d. \tag{18}$$

Thus, when condition (16) holds, as it practically does, the kinetic stage of the evolution of the velocity distribution lasts for only a small fraction of the time during which the atoms are being decelerated, and the hydrodynamic stage of the evolution is the important one. For example, assuming a beam of Ca^{40} atoms irradiated by a light wave in resonance with the $4S-4P$ transition and taking $G=10$ and $\delta=\Omega/\gamma=-70$, we find the characteristic times to be $\tau_r^{min}\approx10^{-5}$ sec and $\tau_d\approx10^{-2}$ sec. These times correspond to the lengths $l_r\approx0.1$ cm and $l_d\approx100$ cm.

A qualitative analysis of the evolution of the distribution function in the kinetic stage encounters no serious difficulties. Despite the fact that, strictly speaking, one should solve the complete equation (7) when $t\gtrsim\tau_r$, the diffusion broadening of the velocity distribution is actually small in the early stages of the evolution,[14]

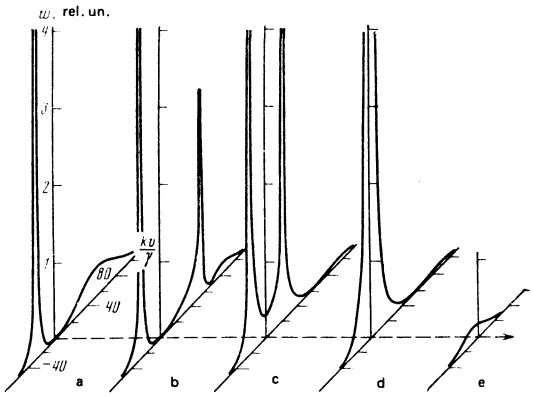

FIG. 1. Evolution of the velocity distribution of a beam of Ca^{40} atoms under the action of the light-pressure force (4). For the calculations, a thermal distribution with the average (dimensionless) velocity $\bar v=48$ was chosen as the initial velocity distribution of the beam, and the values $\delta=-70$ and $G=10$ were chosen for the dimensionless mismatch and the saturation parmeter. The several curves were calculated for the following values of the dimensionless z coordinate: a—0, b—5×10^3, c—10^5, d—2×10^5, e—4×10^5.

and we may set $D=0$ in Eq. (7). After this simplification, the linear equation can always be solved with the aid of the characteristic equations (14).

As an example of the use of this approach, we show in Fig. 1 the results of a calculation of the evolution of the velocity distribution $w(z,v,t)$ for a stationary beam of Ca^{40} atoms. For the chosen parameter values, the kinetic stage of the evolution corresponds to $z\lesssim50$. The curves in Fig. 1 illustrate the general trend of the evolution of $w(z,v,t)$, and in particular, they show the increase in the density of atoms near the turning point (Fig. 1, d) and the turning back (reflection) of the atoms, which separates the beam being decelerated into an incident beam and a reflected one.

4. HYDRODYNAMIC STAGE. MACROSCOPIC EQUATIONS FAR FROM THE TURNING POINT

As was shown in Section 3, the main interest in the theory of the radiative deceleration of atoms lies in the hydrodynamic stage of the evolution of the distribution function. The corresponding macroscopic equations take the simplest form far from the turning point. The equations are complicated in the immediate vicinity of the turning point by the necessity of taking into account the conversion of the incident atomic stream into a reflected stream. We shall therefore first consider the hydrodynamic equations for the motion of the atomic ensemble up to the turning point, and in the next section we shall add the terms responsible for the coupling between the incident and reflected streams.

In the hydrodynamic stage at times $t\gg\tau_r$ the center of the local velocity distribution, according to (15), will have a velocity $\langle v\rangle$ that satisfies the condition (see (16))

$$|\delta+\langle v\rangle|\gg(1+G)^{1/3}. \tag{19}$$

In addition, we assume that when $t\gg\tau_r$ the width of the

local velocity distribution will be smaller than the length of the characteristic velocity interval in which the force varies:

$$\Delta v \ll |\delta + \langle v \rangle|. \tag{20}$$

A rigorous foundation for this assumption will be given below.

Let us introduce the macroscopic variables: the density of atoms

$$n = \int w\, dv, \tag{21}$$

the average velocity

$$\langle v \rangle = n^{-1} \int vw\, dv \tag{22}$$

and the temperature

$$T = n^{-1} \int (v - \langle v \rangle)^2 w\, dv. \tag{23}$$

Since in addition to the principal atomic ensemble there is always, generally speaking, a reflected one, it is actually necessary to define two sets of macroscopic variables: one for the incident atomic stream, and another for the reflected stream. Below we shall always analyze only the incident stream, for it is basically that one that is of experimental interest. Further, in this section we shall assume that the center of the local velocity distribution for the incident atomic ensemble is quite far from the zero-velocity point and shall carry the integrations in (21)–(23) from $-\infty$ to ∞, rather than from 0 to ∞.

Now, using definitions (21)–(23), let us write the following transport equations, which follow from (7) but do not constitute a closed system:

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial z}(n\langle v \rangle) = 0, \tag{24a}$$

$$\frac{\partial \langle v \rangle}{\partial t} + \langle v \rangle \frac{\partial \langle v \rangle}{\partial z} + n^{-1} \frac{\partial}{\partial z}(nT) = -\langle L_0 \rangle, \tag{24b}$$

$$\frac{\partial T}{\partial t} + \langle v \rangle \frac{\partial T}{\partial z} + 2T \frac{\partial \langle v \rangle}{\partial z} + n^{-1} \frac{\partial q}{\partial z} = 2(\varepsilon \chi_0 \langle L_0 \rangle - \langle L_0 u \rangle), \tag{24c}$$

where, because of conditions (19) and (20), the force L and the diffusion constant χ have the following simplified forms in the hydrodynamic stage:

$$L = L_0 = G(v+\delta)^{-2}, \tag{25}$$

$$\chi = \chi_0 = 1 + \alpha. \tag{26}$$

The moment q in Eq. (24c) is

$$q = n^{-1} \int (v - \langle v \rangle)^3 w\, dv. \tag{27}$$

To Eqs. (24) we adjoin the initial equation (7) in a form corresponding to the hydrodynamic stage:

$$\frac{\partial w}{\partial t} + v \frac{\partial w}{\partial z} = \frac{\partial}{\partial v}(L_0 w) + \varepsilon \chi_0 \frac{\partial^2}{\partial v^2}(L_0 w). \tag{28}$$

Now the problem is to derive a closed set of macroscopic equations from Eqs. (24). To do this we make use of conditions (19) and (20) to express the right-hand sides of Eqs. (24b) and (24c) as power series in the local velocity $u = v - \langle v \rangle$:

$$\langle L_0 \rangle = L_0(\langle v \rangle) - 3L_0(\langle v \rangle)(\langle v \rangle + \delta)^{-2} T + \ldots, \tag{29}$$

$$\varepsilon \chi_0 \langle L_0 \rangle - \langle L_0 u \rangle = \varepsilon \chi_0 L_0(\langle v \rangle) + 2L_0(\langle v \rangle)(\langle v \rangle + \delta)^{-2} T + \ldots. \tag{30}$$

We also expand the right-hand side of Eq. (28) in powers of u:

$$L_0(\langle v \rangle) \frac{\partial w}{\partial v} - \frac{2L_0(\langle v \rangle)}{\langle v \rangle + \delta} \frac{\partial}{\partial v}(uw) + \varepsilon \chi_0 L_0(\langle v \rangle) \frac{\partial^2 w}{\partial v^2} + \ldots. \tag{31}$$

Equations (24) with the expressions (29) and (30) on the right, and Eq. (28) with expression (31) on the right constitute the initial equations for the derivation of accurate macroscopic equations valid far from the turning point.

Let us briefly consider the solution of Eq. (28) in the spatially uniform case,[19] in which $\partial/\partial z = 0$ in the kinetic equation (28) and the transport equations (24). We shall first consider only the first term in expansion (29) and the first two terms in expansions (30) and (31). Then, treating $w(z, v, t)$ as a functional of the average velocity $\langle v \rangle$ and employing the Enskog-Chapman method, we obtain the following series for the solution of Eq. (28):

$$w^{(0)} = \frac{n}{(2\pi T_0)^{1/2}} \exp\left(-\frac{u^2}{2T_0}\right) \times \left[1 + \frac{1}{8}\left(1 - \frac{u^2}{T_0}\right) - \frac{1}{128}\left(1 + 2\frac{u^2}{T_0} - \frac{u^4}{T_0}\right) + \ldots\right], \tag{32}$$

$$T_0 = -{}^1/_2 \varepsilon \chi_0 [\langle v \rangle + \delta], \tag{33}$$

where n is a constant in the spatially uniform case.

The expression of the solution of (28) as a power series in u^2 is connected with the difference between the temperature $T = T_0$ defined by the zeroth-approximation function [the first term in (32)] and the true asymptotic temperature $T_a = (4/5)T_0$, defined by Eq. (24c) with $\partial/\partial z = 0$. Expansion (32) is therefore actually an expansion of w in powers of $(T - T_0)/T_0 = 1/5$. Thus, the zeroth approximation to the distribution function gives the temperature as $T = T_0$; when the first correction [the second term in (32)] is included, it becomes $T = (3/4)T_0$; and the first three terms of (32) determine the temperature as $T = (13/16)T_0$, which differs from the true asymptotic temperature $T_a = 0.8T_0$ by only one percent.

The solution (32) obtained above shows, in particular, that relation (20), in which Δv is equal in order of magnitude to $(2T_0)^{1/2}$, is automatically satisfied when conditions (2) and (19) hold. We therefore retain only the principal terms in (29)–(31) and rewrite Eqs. (24), which are the equations we are interested in, in the following form, which corresponds to the hydrodynamic stage of the process:

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial z}(n\langle v \rangle) = 0, \tag{34a}$$

$$\frac{\partial \langle v \rangle}{\partial t} + \langle v \rangle \frac{\partial \langle v \rangle}{\partial z} + n^{-1} \frac{\partial}{\partial z} nT = -L_0(\langle v \rangle), \tag{34b}$$

$$\frac{\partial T}{\partial t} + \langle v \rangle \frac{\partial T}{\partial z} + 2T \frac{\partial \langle v \rangle}{\partial z} + n^{-1} \frac{\partial q}{\partial z} = 2\varepsilon \chi_0 L_0(\langle v \rangle)\left(1 - \frac{T}{T_0}\right). \tag{34c}$$

Then the kinetic equation (28) takes the form

$$\frac{\partial w}{\partial t}+v\frac{\partial w}{\partial z}-L_0(\langle v\rangle)\frac{\partial w}{\partial v}=\varepsilon\chi_0 L_0(\langle v\rangle)\left(\frac{1}{T_0}\frac{\partial}{\partial v}(uw)+\frac{\partial^2}{\partial v^2}w\right). \quad (35)$$

In this equation the average force, which moderates the entire atomic ensemble, and the friction force, which narrows the local velocity distribution, are explicitly separated.

To obtain a closed set of macroscopic equations from Eqs. (34), we apply the Enskog-Chapman method to Eqs. (34) and (35). In doing this we must take the function $w^{(0)}$ as the zeroth approximation in the gradients of the macroscopic variables. Since in the spatially uniform case, however, the difference between the complete series (32) for $w^{(0)}$ and the first term

$$w_0^{(0)}=n(2\pi T_0)^{-1/2}\exp[-(v-\langle v\rangle)^2/2T_0] \quad (36)$$

of that series amounts to only a 20% difference between the true asymptotic temperature $T_a=(4/5)T_0$ and the temperature T_0, we shall neglect this difference and assume the locally equilibrium distribution function to be $w_0^{(0)}$. Then setting $T=T_0$ in Eqs. (34), we see at once that the desired closed hydrodynamic equations are Eqs. (34a) and (34b) with $T=T_0$. One can obtain a rigorous proof of this assertion by calculating the first correction to $w_0^{(0)}$, which is proportional to the gradients of the macroscopic variables n and $\langle v\rangle$. We shall give such a proof, treating w as a functional of the density and the average velocity. We use the distribution function (36) to calculate the left-hand side of Eq. (35) and, using Eq. (34c) with $T=T_0$ and $q=0$, we obtain from Eq. (35) the following equation for the first correction $w_0^{(1)}$ to the zeroth-order distribution function $w_0^{(0)}$:

$$\varepsilon\chi_0 L_0(\langle v\rangle)\left[\frac{1}{T_0}\frac{\partial}{\partial u}(uw_0^{(1)})+\frac{\partial^2}{\partial u^2}w_0^{(1)}\right]=-\frac{3}{2T_0}\frac{\partial T_0}{\partial\langle v\rangle}\frac{\partial\langle v\rangle}{\partial z}\left(1-\frac{u^2}{3T_0}\right)uw_0^{(0)}. \quad (37)$$

Solving the inhomogeneous equation (37), we obtain the following expression for the first correction:

$$w_0^{(1)}=-\frac{w_0^{(0)}}{4L_0(\langle v\rangle)}\frac{\partial\langle v\rangle}{\partial z}\left(1-\frac{u^2}{3T_0}\right)u. \quad (38)$$

Now we use the function $w_0^{(0)}+w_0^{(1)}$ to calculate the macroscopic variables, and easily verify that the correction $w_0^{(1)}$ to the locally equilibrium distribution function $w_0^{(0)}$ does not alter the density n, the average velocity $\langle v\rangle$, or the temperature $T=T_0$. The first moment to which w_0^1 contributes is the heat flux q:

$$q=\frac{T_0^2}{2L_0(\langle v\rangle)}\frac{\partial\langle v\rangle}{\partial z}. \quad (39)$$

Thus, the hydrodynamic equations, valid far from the turning point, are

$$\frac{\partial n}{\partial t}+\frac{\partial}{\partial z}(n\langle v\rangle)=0, \quad (40a)$$

$$\frac{\partial\langle v\rangle}{\partial t}+\langle v\rangle\frac{\partial\langle v\rangle}{\partial z}+T_0n^{-1}\frac{\partial n}{\partial z}=-L_0(\langle v\rangle). \quad (40b)$$

We note that the temperature T_0 has been taken out from under the derivative sign in Eq. (40b) because $\partial T_0/\partial z\sim\varepsilon\ll 1$ is a small correction to the average velocity $\langle v\rangle$ and should be dropped to avoid excess accuracy.

5. MACROSCOPIC EQUATIONS WITH ALLOWANCE FOR REFLECTION

As was noted above, Eqs. (40) are valid when $\langle v\rangle\gg(2T_0)^{1/2}$. This condition is violated near the turning point [$\langle v\rangle\sim(2T_0)^{1/2}$] and then it becomes necessary to take account of the reflection of atoms, which reduces the atomic flux. To obtain accurate macroscopic equations that would be valid not only before the turning point, but also right at it, is a difficult problem. It is sufficient to say that the expansion of the local velocity distribution in powers of the gradients of the macroscopic variables is problematic at the turning point because there the gradients are no longer small. Actually, however, neither the turning point itself, nor the region beyond it, is of any great practical interest because of the sharp decrease of the atomic flux in that region. We shall accordingly seek only to find approximate equations that will make it possible to take account of the evolution of the atomic ensemble as it approaches the turning point.

We note first that in this case the integrations in the definitions (21)–(23) of the density, average velocity, and temperature must be taken from 0 to ∞. Further, using the same considerations as in Section 4, we find that the kinetic equation that determines the behavior of w when $v>0$ will have the form (35) as before, but with $\langle v\rangle$ defined only by the nonnegative velocities.

Now let us consider this equation in the spatially uniform case. Expressing the solution of Eq. (35) in the form of the series (32) and neglecting the 20% difference between the temperature T_0 and the true asymptotic temperature T_a, we can, as before, regard the equilibrium distribution function as locally Maxwellian. In order to be able to treat the last as the zeroth approximation to the local velocity distribution, we bring it into conformity with the new definitions of n, $\langle v\rangle$, and T. To do this we introduce the velocity v_m at the peak of the local distribution and write $w_0^{(0)}$ in the form

$$w_0^{(0)}=2n(2\pi T_0)^{-1/2}[1+\Phi((2T_0)^{-1/2}v_m)]^{-1}\exp[-(v-v_m)^2/2T_0], \quad (41)$$

$$\Phi(x)=\int_0^x e^{-t^2}dt,$$

where $\Phi(x)$ is the error integral and T_0 is defined as before by formula (33). In view of Eq. (41), the average velocity $\langle v\rangle$ will be related to v_m by the formula

$$\langle v\rangle=v_m+T_0n^{-1}w_0^{(0)}(0). \quad (42)$$

Since Eq. (41) is only approximate, the velocity v_m in

(41) and (42) must be considered in the region $v_m \gtrsim -(2T_0)^{1/2}$.

Having the explicit form of the velocity distribution, we use Eq. (35) to write down the first two moment equations, using (41) to calculate the inhomogeneous terms:

$$\frac{\partial n}{\partial t}+\frac{\partial}{\partial z}(n\langle v\rangle)=-L_0(\langle v\rangle)w_0^{(0)}(0), \tag{43a}$$

$$\frac{\partial\langle v\rangle}{\partial t}+\langle v\rangle\frac{\partial\langle v\rangle}{\partial z}+n^{-1}\frac{\partial}{\partial z}(nT)=-L_0(\langle v\rangle)(1-\langle v\rangle n^{-1}w_0^{(0)}(0)). \tag{43b}$$

Setting $T=T_0$ in (43) (see Section 4), we write down the final equations in dimensionless variables:

$$\frac{\partial n}{\partial t}+\frac{\partial}{\partial z}(n\langle v\rangle)=-G\gamma^3 nf(\Omega+k\langle v\rangle)^{-2}, \tag{44a}$$

$$\frac{\partial\langle v\rangle}{\partial t}+\langle v\rangle\frac{\partial\langle v\rangle}{\partial z}+\frac{k_BT_0}{Mn}\frac{\partial n}{\partial z}=-\frac{G\gamma^3v_r(1-\langle v\rangle f/v_r)}{(\Omega+k\langle v\rangle)^2}, \tag{44b}$$

where

$$f=2\pi^{-1/2}(v_r/u_0)[1+\Phi(v_m/u_0)]^{-1}\exp[-(v_m/u_0)^2], \tag{45}$$

$$T_0=\hbar\chi_0|k\langle v\rangle+\Omega|/4k_B, \quad u_0=(2k_BT_0/M)^{1/2}, \tag{46}$$

and the relation between $\langle v\rangle$ and v_m is given by the formula

$$\langle v\rangle=v_m+u_0^2f/2v_r. \tag{47}$$

In these equations k_B is Boltzmann's constant and v_r is the recoil velocity.

The hydrodynamic equations (44) describe the changes in n, $\langle v\rangle$, and $T=T_0$ clear out to $v_m=-u_0$. Equations (44) reduce to Eqs. (40) when $\langle v\rangle\sim v_m\gg u_0$, i.e., far from the turning point.

6. ALLOWANCE FOR TRANSVERSE DIFFUSION

Up to now we have been analyzing the Fokker-Planck equation (3) without taking into account the broadening of the velocity distribution perpendicular to the z axis. Now let us take into account the transverse diffusion of the velocities, neglecting the transverse broadening of the atomic ensemble and assuming as a zeroth approximation that the longitudinal velocity distribution represents local equilibrium. Under these conditions it is sufficient to take the Maxwell distribution (36) as the distribution function for the velocities along the z axis. Then, noting that $\partial w/\partial t=0$ for the zeroth approximation distribution (36), we derive the following equation from (3) for $w(v_x,v_y,t)$ in dimensionless variables:

$$\frac{\partial w}{\partial t}=\varepsilon\alpha_{xx}\frac{\partial^2}{\partial v_x^2}(L_0w)+\varepsilon\alpha_{yy}\frac{\partial^2}{\partial v_y^2}(L_0w). \tag{48}$$

The heat conduction equation (48) can be solved with the aid of the relation

$$\partial\langle v\rangle/\partial t=-L_0(\langle v\rangle). \tag{49}$$

We make use of (49) to change from the variable t to the variable $\xi=\langle v\rangle_0-\langle v\rangle$, where $\langle v\rangle_0$ is the initial average velocity of the ensemble. Then the solution of Eq. (48) can be expressed as a Poisson integral, from which we obtain the following expression for the transverse temperatures of the ensemble ($i=x,y$):

$$T_i=(Mv_r/k_B)\alpha_{ii}(\langle v\rangle_0-\langle v\rangle). \tag{50}$$

Thus, the anisotropy of the polarization of the laser radiation causes the velocity distribution to broaden differently along the three axes x, y, and z, and it is convenient to describe this behavior by three temperatures: $T_z=T_0$ [Eq. (46) and $T_x=T_y$ (Eq. (50)]. Of course the introduction of transverse temperatures is justified only when they are higher than the transverse temperatures of the initial beam.

7. STEADY-STATE DECELERATION

All the basic parameters of an ensemble of atoms undergoing deceleration can be obtained from Eqs. (44)–(47) and (50). For definiteness, let us consider the case of practical importance in which an atomic beam issuing from the origin of coordinates is being decelerated. Being interested in a steady-state stream of atoms, we set $\partial/\partial t=0$ in (44). The equations obtained in this case can be easily solved far from the turning point. In this region the atomic flux is constant

$$j=n\langle v\rangle=j_0, \tag{51}$$

where j_0 is the initial flux, while the change in the velocity is determined by the parametric relationship

$$z=\chi(\gamma/kv_r)(\Omega+k\langle v\rangle)^3(\Omega/3-k\langle v\rangle)/4G\gamma^4, \tag{52}$$

where we have chosen the resonant velocity $\langle v\rangle_0=-\Omega/\gamma$ as the initial velocity. We recall that Ω must be negative for deceleration. The increase in the temperature in this region is determined from Eqs. (46) and (50).

When $\langle v\rangle\sim(2k_BT_0/M)^{1/2}$, Eqs. (44) must be solved numerically. The variations along the z axis of the flux j, the average velocity $\langle v\rangle$, the density n, and the temperature T are indicated qualitatively in Fig. 2.

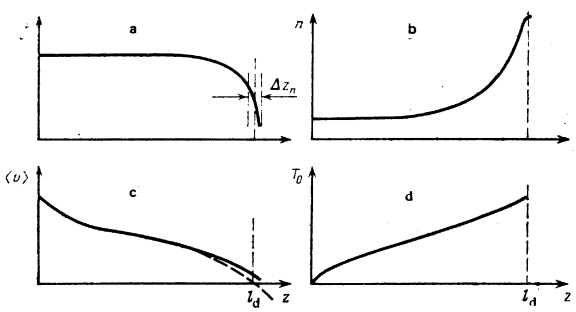

FIG. 2. Various parameters as functions of z for an atomic beam with the initial average velocity $\langle v\rangle_0=-\Omega/k$: a—flux, b—average velocity (full curve) and the velocity v_m at the peak of the distribution function (dashed curve), c—density, d—temperature.

In concluding, let us make a few estimates. According to (44b), the distance to the turning point is

$$l_d=\bar{\lambda}(\gamma/kv_r)\Omega^4/12G\gamma^4. \tag{53}$$

The turning point is characterized by the velocities $v_m=0$ and $\langle v\rangle_t=u_0/2$. The density n reaches its maximum near the turning point, while the flux j and the average velocity $\langle v\rangle$ decrease exponentially:

$$j,\ \langle v\rangle\propto\exp(-z/\Delta z_n), \tag{54}$$

where the characteristic length for the rapid decrease in j and $\langle v\rangle$ is

$$\Delta z_t=\bar{\lambda}\chi_0|\Omega|^3/4G\gamma^3. \tag{55}$$

Near the turning point the longitudinal and transverse temperatures, T_0 and T_t, respectively, are

$$T_0^t=\hbar\chi_0|\Omega|/4k_B,\quad T_t^t=\hbar\alpha_{tt}|\Omega|/k_B.$$

For a beam of Ca^{40} atoms, for example, being decelerated by laser radiation at the $4S$-$4P$ transition ($\lambda=422.7$ nm) under the conditions chosen for Fig. 1, the basic parameters have the values $l_d=110$ cm, $\Delta z_t=2\times10^{-3}$ cm, and $T_0^t\approx T_t^t\approx10^{-2}$ °K.

All the estimates presented above are typical for allowed optical transitions in atoms and show that radiative deceleration of atomic beams is a practical means of obtaining cold atoms.

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