Absorption spectrum of a two-level system interacting with two monochromatic fields of equal amplitudes

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The quasienergy method is used to analyze the problem of the interaction of a two-level system with two classical monochromatic fields. A general expression is obtained for the absorption coefficient at the frequency of the scanned field for the case when the amplitudes of the two fields are equal. To solve the quasienergy problem for a two-level system interacting with a strong monochromatic field, a perturbation theory in the free (without field) Hamiltonian is developed. The results of a numerical calculation in accordance with the obtained expressions agree well with the experimental data.

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The problem of the interaction of a two-level system with two given classical monochromatic fields has been posed and studied in a number of papers.[1-6] It has been shown that formally the problem with two fields is analogous (when the resonance approximation for each of the fields is valid) to a problem with periodic coefficients. This analogy can be used, in particular, for the experimental study of a two-level systems under conditions when the resonance approximation is not applicable. This last circumstance is particularly interesting in the optical part of the spectrum, since here the resonance approximation is violated in fields with intensities near the intra-atomic fields.[7]

Considerable interest attaches to an effect obtained experimentally and considered theoretically in Ref. 6, namely, the occurrence of a structure in the absorption spectrum at the frequency of one of the fields when both fields become fairly intense and capable of saturating the homogeneously broadened transition. This effect generalizes the well-known nonlinear interference effect[8] to the case of an arbitrary amplitude of the scanned field. The structure consists of a sequence of peaks in the absorption (amplification) spectrum whose intensity and half-width depend on the peak number. Beginning at a certain number, the peak half-width may be appreciably less than the homogeneous half-width of the line. It was pointed out in Ref. 6 that this effect could be used for frequency stabilization.

The theory developed in Ref. 6 for solving the kinetic equation for the density matrix of the two-level system in the two fields is based on adiabatic perturbation theory. In the case of fields with different amplitudes, the theory gives good agreement with experiment. However, as is noted in Ref. 6 and is demonstrated below, in the case of fields with (nearly) equal amplitudes there exists a broad range of variation of the frequency of the scanned field in which adiabatic perturbation theory is invalid. Nevertheless, this case is of physical interest, since it is precisely in this situation, as has been shown experimentally,[6] that the structure in the absorption is most clearly expressed. Thus, in the case of fields with (nearly) equal amplitudes a special treatment that goes beyond the framework of the method proposed in Ref. 6 is required. This is presented in the present paper.

1. POSSIBLE WAYS OF TREATING THE PROBLEM

The equation for the density matrix $\rho(t)$ of a two-level system interacting with two rotating fields, whose intensities and frequencies we denote by E, E_1, ω, ω_1, respectively, has the form

$$i\hbar\frac{\partial}{\partial t}\rho(t)=[H(t),\rho(t)]-i\gamma(\rho(t)-\rho_0). \tag{1}$$

Here, $\rho_0=(1+\eta\hat\sigma_z)/2$ is the equilibrium density matrix $(-1\leqslant\eta\leqslant 1)$, and $\hbar/\gamma$ is the time of relaxation of the matrix to the equilibrium value.

The Hamiltonian $H(t)$ of the problem can be written in the form

$$H(t)=\hbar\omega\hat\sigma_z/2-F\hat V_\omega(t)-F_1\hat V_{\omega_1}(t), \tag{2}$$

$$\hat V_\omega(t)=^1/_2(\hat\sigma_- e^{i\omega t}+\text{h.c.}). \tag{3}$$

In Eqs. (1)–(3), $F/\hbar=dE/\hbar$, $F_1/\hbar=dE_1/\hbar$ are the Rabi frequencies of the fields E and E_1, and $\hat\sigma_x,\hat\sigma_y,\hat\sigma_z,\hat\sigma_-=(\hat\sigma_x-i\hat\sigma_y)/2$ are the Pauli matrices. In accordance with the experimental situation, the frequency of one of the fields is taken equal to the transition frequency ω.[6]

We denote by $\rho_{st}(t)$ the steady solution of Eq. (1), i.e., the solution of (1) to which the general solution tends at times longer than the field switching-on time and the relaxation time:

$$\rho_{st}(t)=\int_0^\infty e^{-\gamma\vartheta/\hbar}\hat U(t,t-\vartheta)\rho_0\hat U^+(t,t-\vartheta)\frac{d\vartheta\gamma}{\hbar}. \tag{4}$$

Here $\hat U(t,t_0)=\hat U(t)\hat U^+(t_0)$, where $\hat U(t)$ is the evolution operator:

$$H(t)\hat U(t)=i\hbar\frac{\partial}{\partial t}\hat U(t),\quad \hat U(0)=1. \tag{5}$$

The experimentally measured quantity is the power dissipated by one particle from the field of frequency ω_1; it is related to $\rho_{st}(t)$ (4) by[9]

$$W_{\omega_1}=-F_1\{\mathrm{Sp}\,(\rho_{st}(t)\partial\hat V_{\omega_1}(t)/\partial t)\}. \tag{6}$$

The braces in (6) denote averaging over a long time interval (the observation time). The absorption coefficient at the frequency ω_1 is related to this quantity by the simple equation[9]

$$K_{\omega_1}=8\pi(cE_1^2)^{-1}NW_{\omega_1}. \tag{7}$$

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Here, c is the velocity of light, and N is the number of particles in unit volume. By analogy with (6) and (7), we can define W_ω and K_ω.

We rewrite the operator of the interaction with the two fields in (2) in the form

$$-F\hat{V}_\omega(t)-F_1\hat{V}_{\omega_1}(t)=-\hat{\sigma}_-\mathcal{F}(t)e^{i\Omega t}+\text{h.c.} \tag{8}$$

Here, we have introduced the complex (in the general case) amplitude

$$\mathcal{F}(t)=(Fe^{i\varepsilon t}+F_1e^{-i\varepsilon t})/2$$

and the carrier frequency

$$\Omega=(\omega+\omega_1)/2.$$

If

$$|\varepsilon|=|\omega-\omega_1|/2\ll\omega$$

(near-resonance fields), the amplitude $\mathcal{F}(t)$ will be a slow function of the time. This makes it possible to introduce a quasienergy operator of the problem (5), its eigenvalues (the quasienergies) being slow functions of the time. Adiabatic perturbation theory for the case of a slow time dependence of the quasienergies[10] is analogous to ordinary adiabatic perturbation theory for the case of slow time dependence of an energy level.[11]

We fix the "slow" time in (8) and construct the quasienergy operator of the problem (5), performing a unitary transformation by means of the periodic operator (transition to a coordinate system rotating at the carrier frequency)

$$\hat{C}_p(t)=\exp\{-i\Omega\hat{\sigma}_z t/2\}. \tag{9}$$

We obtain the quasienergy operator in the form

$$\hat{Q}(t)=\hbar\varepsilon\hat{\sigma}_z/2-(\hat{\sigma}_-\mathcal{F}(t)+\text{h.c.}). \tag{10}$$

On the basis of (10), the eigenvalues of $\hat{Q}(t)$ are

$$Q_{+,-}(t)=\pm[(\hbar\varepsilon)^2+F^2+F_1^2+2FF_1\cos 2\varepsilon t]^{1/2}/2. \tag{11}$$

At the points (11)

$$t_n=\pi(1+2n)/2\varepsilon,\quad n=0,\pm1,\pm2\ldots \tag{12}$$

the quasiterms $Q_+(t)$ and $Q_-(t)$ have their closest approach in accordance with (11).

In the regions around t_n, nonadiabatic transitions are possible. Let us estimate the relationship between the parameters at which the quasiterms (11) can be adiabatically followed. For this, using the standard criterion,[12] we find that adiabatic perturbation theory is valid if

$$\frac{|\hbar\varepsilon|(F+F_1)}{(\hbar\varepsilon)^2+4(F-F_1)^2}\ll 1. \tag{13}$$

On the basis of (13), it is natural to distinguish the three following intervals of values of the parameter ε:

$$0\leqslant|\hbar\varepsilon|\ll(F-F_1)^2/(F+F_1), \tag{14a}$$

$$(F-F_1)^2/(F+F_1)\leqslant|\hbar\varepsilon|\leqslant F+F_1, \tag{14b}$$

$$F+F_1\ll|\hbar\varepsilon|. \tag{14c}$$

In the intervals (14a) and (14c), the probability of nonadiabatic transition between the quasiterms (11) is small, and the solution of (5) can be constructed using adiabatic perturbation theory. In the interval (14b), the applicability of adiabatic perturbation theory breaks down in the neighborhoods of the points (12). Under the condition $|F-F_1|\ll F+F_1$ (fields with nearly equal amplitudes) there is in the second interval the region

$$(F-F_1)^2/(F+F_1)\ll|\hbar\varepsilon|\ll F+F_1, \tag{14d}$$

in which it is possible to use perturbation theory in the diabatic basis $|+\rangle$, $|-\rangle$ (the basis in which $\hat{\sigma}_x$ is diagonal):

$$\hat{\sigma}_x|\pm\rangle=\pm|\pm\rangle. \tag{15}$$

This possibility is due to the circumstance that the probability of a nonadiabatic transition in the neighborhoods of the points (12) when (14d) is satisfied becomes comparable with unity, and the system varies in accord with the diabatic term. In the intermediate cases, perturbation theory cannot be used.

The probabilities of the nonadiabatic transitions in the neighborhoods of the points (12) can be calculated approximately on the basis of the Landau-Zener model. We expand $\mathcal{F}(t)$ in a Taylor series in the neighborhood of the point t_0, and write down the operator $\hat{Q}(t)$ (10) in the basis (15), retaining in the series the constant term and the term linear in the time. We obtain

$$\hat{Q}(t)\approx-\frac{F+F_1}{2}\varepsilon t\hat{\sigma}_z-\frac{\hbar\varepsilon}{2}\hat{\sigma}_x+\frac{F-F_1}{2}\hat{\sigma}_y.$$

This matrix is characteristic of the exactly solvable Landau-Zener model problem[13]: There are linear crossing diabatic terms and a constant interaction between them. To construct the solution in a time interval equal to half the field period, it is necessary to fit the solution in the region of nonadiabaticity to the solutions that describe the adiabatic development to the left and to the right of the point t_0 (12). In the earlier Ref. 14, this method was used to solve the quasienergy problem of a two-level system in a very strong linearly polarized field. In the limiting situations (14a), (14c), and (14d), the solution constructed in this manner goes over into the solutions obtained by means of the corresponding perturbation theories.

In the present paper, we study the relatively simple case $F=F_1$. In accordance with what we have seen above, the interval (14a) disappears for $F=F_1$, and the interval (14d), in which perturbation theory in the diabatic basis is valid, covers almost all of the range of variation of the parameter $0\leqslant|\hbar\varepsilon|<F+F_1$ that is of experimental interest.[6]

2. GENERAL EXPRESSION FOR THE DISSIPATED POWER

We now consider the important special case of fields with equal amplitudes: $E=E_1$. In the coordinate system rotating at the carrier frequency Ω [the transformation (9)], the problem (5) goes over into the problem of the interaction of an effective two-level system with a linearly polarized field:

$$\left(\frac{\hbar\varepsilon\hat{\sigma}_z}{2}-F\hat{\sigma}_x\cos\varepsilon t\right)\hat{u}(t)=i\hbar\frac{\partial}{\partial t}\hat{u}(t),\qquad \hat{U}(t)=\hat{C}_p(t)\hat{u}(t),\quad \hat{u}(0)=1. \tag{16}$$

We denote by

$$\bar{\rho}_{st}(t)=\hat{U}_p^+(t)\rho_{st}(t)\hat{U}_p(t)=\int_0^\infty e^{-\gamma\vartheta/\hbar}\hat{u}(t,t-\vartheta)\rho_0\hat{u}^+(t,t-\vartheta)d\vartheta\frac{\gamma}{\hbar} \quad (17)$$

the steady-state solution in the rotating coordinate system [the matrix ρ_0 does not change under the transformation (9)]. Using (17) and the simple relations

$$\hat{U}_p^+(t)\frac{\partial\hat{V}_{\omega_1}(t)}{\partial t}\hat{U}_p(t)=-\frac{\omega_1}{\varepsilon}\frac{\partial\hat{V}_{-\varepsilon}(t)}{\partial t},$$

$$\hat{U}_p^+(t)\frac{\partial\hat{V}_{\omega}(t)}{\partial t}\hat{U}_p(t)=\frac{\omega}{\varepsilon}\frac{\partial V_{\varepsilon}(t)}{\partial t},$$

we obtain expressions for $W_{\omega 1}$ and W_ω in terms of the solutions to the problem (16):

$$W_{\omega_1}=-\omega_1 W_{-\varepsilon}/\varepsilon, \quad W_\omega=\omega W_\varepsilon/\varepsilon. \quad (18)$$

Here

$$W_{\pm\varepsilon}=-F\left\{\mathrm{Sp}\left(\bar{\rho}_{st}(t)\frac{\partial\hat{V}_{\pm\varepsilon}(t)}{\partial t}\right)\right\} \quad (19)$$

are the powers dissipated by one particle from the right- and left-circularly polarized components of the linearly polarized field (16).

Thus, to calculate $\bar{\rho}_{st}(t)$, it is necessary to find the evolution operator $\hat{u}(t)$ of the problem (16), which, by virtue of the periodicity of Eq. (16), we represent in the form[7]

$$\hat{u}(t)=\hat{u}_p(t)\exp\{-i\hat{Q}(F)t/\hbar\}.$$

Here, $\hat{u}_p(t)$ is the $2\pi/\varepsilon$-periodic evolution operator, and $\hat{Q}(F)$ is the quasienergy operator.

In (19), it is convenient to write out Sp in the representation of the quasienergy operator $\hat{Q}(F)$. In the basic zone,[15] the operator $\hat{Q}(F)$ has two eigenvectors $|\alpha\rangle$, $\alpha=\pm1$, and two eigenvalues Q_α, $Q_\alpha=-Q_{-\alpha}$. We introduce the periodic operators

$$\hat{\sigma}_z(t)\equiv\hat{u}_p^+(t)\hat{\sigma}_z\hat{u}_p(t), \quad \hat{D}_{\pm\varepsilon}(t)\equiv\hat{u}_p^+(t)\frac{\partial\hat{V}_{\pm\varepsilon}(t)}{\partial t}\hat{u}_p(t). \quad (20)$$

We denote by $\sigma_m^{\alpha\beta}$ and $D_m^{\alpha\beta}(\pm\varepsilon)$ the Fourier components of the matrix elements of the operators (20) with respect to the eigenvectors of $\hat{Q}(F)$:

$$\langle\alpha|\hat{\sigma}_z(t)|\beta\rangle\equiv\sum_{m=-\infty}^{\infty}\sigma_m^{\alpha\beta}e^{-im\varepsilon t},$$

$$\langle\alpha|\hat{D}_{\pm\varepsilon}(t)|\beta\rangle\equiv\sum_{m=-\infty}^{\infty}D_m^{\alpha\beta}(\pm\varepsilon)e^{-im\varepsilon t}. \quad (21)$$

After integration over ϑ and averaging over t, we obtain from (19), (20), and (21) the general expression for the dissipated powers $W_{\pm\varepsilon}$:

$$W_{\pm\varepsilon}=\frac{-F\eta}{2}\sum_{m=-\infty}^{\infty}\sum_{\alpha,\beta}^{\pm1}\sigma_m^{\alpha\beta}D_{-m}^{\beta\alpha}(\pm\varepsilon)\frac{\gamma}{\gamma+i(Q_\alpha-Q_\beta-m\hbar\varepsilon)}. \quad (22)$$

3. PERTURBATION THEORY FOR $F\gg|\hbar\varepsilon|$

To find $\sigma_m^{\alpha\beta}$ and $D_m^{\alpha\beta}(\pm\varepsilon)$, we must solve the quasienergy problem

$$\left(-i\hbar\frac{\partial}{\partial t}-F\hat{\sigma}_x\cos\varepsilon t+\hbar\varepsilon\hat{\sigma}_z/2\right)\varphi_{Q_\alpha-m\hbar\varepsilon}(t)=(Q_\alpha-m\hbar\varepsilon)\varphi_{Q_\alpha-m\hbar\varepsilon}(t). \quad (23)$$

Here, $\varphi_{Q_\alpha-m\hbar\varepsilon}(t)$ is the periodic quasienergy state for the quasienergy $Q_\alpha-m\hbar\varepsilon$.[15] Introducing the scalar product $\langle\langle\ldots\rangle\rangle$,[15] and using the property of a periodic quasienergy state

$$\varphi_{Q_\alpha-m\hbar\varepsilon}(t)=e^{-im\varepsilon t}\varphi_{Q_\alpha}(t)=e^{-im\varepsilon t}\hat{u}_p(t)|\alpha\rangle,$$

where $|\alpha\rangle$ is an eigenvector of the operator $\hat{Q}(F)$ in the basic zone, we express $\sigma_m^{\alpha\beta}$ and $D_m^{\alpha\beta}(\pm\varepsilon)$ in the form

$$\sigma_m^{\alpha\beta}=\langle\langle\varphi_{Q_\alpha-m\hbar\varepsilon}(t)|\hat{\sigma}_z|\varphi_{Q_\beta}(t)\rangle\rangle,$$

$$D_m^{\alpha\beta}(\pm\varepsilon)=\left\langle\left\langle\varphi_{Q_\alpha-m\hbar\varepsilon}(t)\left|\frac{\partial\hat{V}_{\pm\varepsilon}(t)}{\partial t}\right|\varphi_{Q_\beta}(t)\right\rangle\right\rangle. \quad (24)$$

We can obtain a solution of (23) and calculate (24) numerically. However, under the condition $|\hbar\varepsilon|\ll F$, which is satisfied experimentally,[6] we can use perturbation theory in the operator $\hbar\varepsilon\hat{\sigma}_z/2$ to solve (23). In the zeroth order in $\hbar\varepsilon\hat{\sigma}_z/2$, the quasienergies are doubly degenerate and equal to zero in the basic zone. This degeneracy is a consequence of the symmetry of theproblem (23). Indeed, the Hamiltonian (23) is invariant under the operator[14,16,17]

$$\hat{C}=\hat{T}\hat{\sigma}_z, \quad (25)$$

where $\hat{T}$ is the operator of a displacement in time through the field half-period π/ε, and $\hat{\sigma}_z$ is the operator of spatial parity (in the two-level case). Since $\hat{C}^2=1$, we can introduce the concept of $\hat{C}$ symmetry have the following form in the basic zone [$|+\rangle$, $|-\rangle$ are introduced in (15)]:

$$\varphi_{Q_\alpha}^{(0)}(t)=\frac{1}{2}\left(\exp\left\{i\frac{1}{2}z\sin\varepsilon t\right\}|+\rangle-\alpha\exp\left\{-i\frac{1}{2}z\sin\varepsilon t\right\}|-\rangle\right), \quad z=2F/\hbar\varepsilon. \quad (26)$$

It is readily verified that

$$\hat{C}\varphi_{Q_\alpha}^{(0)}(t)=\alpha\varphi_{Q_\alpha}^{(0)}(t), \quad \alpha=\pm1.$$

The solution of the secular problem in the basis (26) gives the first order for the quasienergy[7]:

$$Q_\alpha={}^1/_2\hbar\varepsilon\alpha J_0(z). \quad (27)$$

Here and in what follows, $J_m(z)$ are Bessel functions. Using (24) and (26), we obtain in the zeroth order

$$D_{-m}^{(0)\beta\alpha}(\pm\varepsilon)=\frac{-i\varepsilon}{4}\left[\delta_{\alpha,-\beta}(\delta_{m,-1}-\delta_{m,1})\pm\frac{2m}{z}J_m(z)\frac{\beta+(-1)^m\alpha}{2}\right],$$

$$\sigma_m^{(0)\alpha\beta}={}^1/_2(\beta+(-1)^m\alpha)J_m(z).$$

Perturbation theory in $\hbar\varepsilon\hat{\sigma}_z/2$ for (23) was carried out to the first order inclusively. However, because they are cumbersome, we do not give the first-order results. The final expressions for $W_{\pm\varepsilon}$ (22) in the leading orders of perturbation theory are

$$W_{\pm\varepsilon}=\frac{-\eta\varepsilon F\gamma}{4}\left\{J_1(z)(\Gamma_1^--\Gamma_1^+)+\frac{1-J_0^2(z)}{z}(\Gamma_1^++\Gamma_1^-)\right.$$
$$+\frac{1}{2}\sum_{m=-\infty}^{\infty}J_m(z)\left[(1-\delta_{m,-1})\frac{J_{1+m}(z)}{1+m}-(1-\delta_{m,1})\frac{J_{m-1}(z)}{m-1}\right](\Gamma_m^++\Gamma_m^-)$$
$$\left.\pm\sum_{m=-\infty}^{\infty}\frac{m}{z}J_m^2(z)(\Gamma_m^++\Gamma_m^-)\right\}. \quad (28)$$

Here,

$$\Gamma_m^\pm=\frac{1+(-1)^m}{2}\frac{m\hbar\varepsilon}{\gamma^2+(m\hbar\varepsilon)^2}+\frac{1-(-1)^m}{2}\frac{\hbar\varepsilon(m\pm J_0(z))}{\gamma^2+(\hbar\varepsilon)^2(m\pm J_0(z))^2}. \quad (29)$$

We note that the nonstationary problem of the interaction of a two-level system with a very strong electromagnetic field was solved by perturbation theory in Ref. 7. It was pointed out in Ref. 18 that the small parameter of the perturbation theory is not z^{-1}, but the values of series of the type

$$\left|\sum_{\substack{m=-\infty\\ n\neq 0}}^{\infty}\frac{J_m(z)}{m}\right|\ll 1, \quad \left|\sum_{\substack{m,n=-\infty\\ n\neq 0,\, n\neq m}}^{\infty}\frac{J_n(z)J_m(z)}{n(n-m)}\right|\ll 1.$$

In Ref. 19, a similar perturbation theory was used to solve the quantum problem of the interaction of a two-level system with a quantized radiation mode. In the present paper the quasienergy problem has been solved by stationary perturbation theory in the extended Hilbert space $T\otimes R$ (where T is the space of functions that are periodic with the field period, and R is the space of the Hamiltonian) in the spirit of the idea of Ref. 15. Such an approach has methodological interest.

4. DISCUSSION OF THE RESULTS

We consider some properties of the relative absorption coefficient $K_{\omega 1}/K$ [where $K=K_0\gamma^1/(\gamma^2+F^2)$, and K_0 is the absorption coefficient for a weak field at the line center] that follow from Eqs. (22), (28), and (29). The denominators in Eqs. (22) and (29), which contain the homogeneous width γ, do not have a resonance nature. More precisely, $Q_\alpha - Q_\beta - m\hbar\varepsilon$ cannot vanish when $m\neq 0$ for any values of ε (except $\varepsilon=0$). This follows directly from the analytic expression (27) for the quasienergy. The case $m=0$, $\alpha\neq\beta$ is eliminated by the selection rules that follow from the $\hat{C}$ symmetry (25): $\sigma_m^{\alpha\beta}$ and $D_m^{\alpha\beta}(\pm\varepsilon)$ are nonvanishing only for $\alpha=\beta$ (m even) and for $\alpha\neq\beta$ (m odd). Thus, the structure in the absorption spectrum is not due to the denominators of the expressions (22) and (29) but to the oscillating behavior of $\sigma_m^{\alpha\beta}$, $D_m^{\alpha\beta}(\pm\varepsilon)$, and the quasienergy as ε varies. The important parameters of the structure (the half-width of the peaks and their positions on the frequency scale) are entirely determined by the field intensities and their frequency detuning ε and are virtually independent of γ, which can affect only the height of the structure. These conclusions differ qualitatively from those that can be drawn if the expressions of Ref. 6 are applied to the case of equal fields.

The listed features are confirmed by numerical calculation in accordance with Eqs. (28) and (29), the results of which are given in Fig. 1. The analytic expression for $K_{\omega 1}/K$ has its simplest form in the case $\gamma\ll|\hbar\varepsilon|\ll F$. From (7), (18), (28), and (29), we obtain

$$\frac{K_{\omega_1}}{K}\sim\frac{-F}{2\hbar\varepsilon}J_1(z)J_0(z)\sim\frac{-1}{2\pi}\cos\frac{4F}{\hbar\varepsilon}.$$

In this limiting case, the absorption regions are replaced by amplification regions. The zeros of the absorption coefficient correspond to zeros of the quasienergy and zeros of the Fourier component $\sigma_1^{\alpha\beta}$ [the function $J_1(z)$]. If we define the height of a peak as the distance between the neighboring maximum and minimum of the absorption coefficient, then the peak half-width will correspond approximately to the distance between the successive zeros of the function $J_0(z)J_1(z)$. These

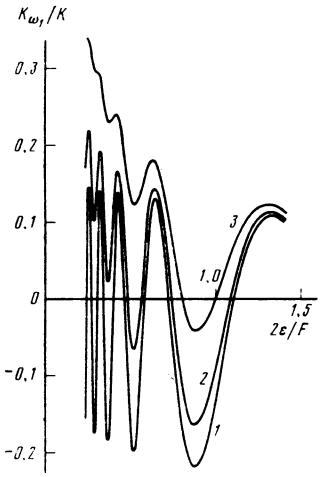

FIG. 1. Calculation of the relative coefficient for the following values of the parameter γ/F: 1) $\gamma/F\ll 1$, 2) $\gamma/F=0.1$, 3) $\gamma/F=0.2$.

zeros correspond to the following values of ε_n (for $z\gg 1$): $\hbar\varepsilon_n=4F/\pi n$; $n=1,2,3,\ldots$. Hence, the half-width of the peak between the n-th and $(n+1)$-th zero will have the form

$$\Delta(\hbar\varepsilon_n)=\frac{4F}{\pi}\frac{1}{n(n+1)}\sim\frac{4F}{\pi n^2}. \tag{30}$$

The boundaries of the region in which all the peaks have approximately equal height are given by the inequality $\gamma\lesssim|\hbar\varepsilon|\lesssim F$. At the lower boundary when $|\hbar\varepsilon|\sim\gamma$ the peak half-width (30) corresponds to

$$\Delta(\hbar\varepsilon_n)_{min}\sim\gamma\pi\gamma/4F. \tag{31}$$

The value $\Delta(\hbar\varepsilon_n)_{\min}$ (31) can be taken as the minimal peak half-width attainable for given F and γ. In accordance with (31), $\Delta(\hbar\varepsilon_n)_{\min}$ is $4F/\pi\gamma$ times less than the homogeneous half-width γ. It can be seen from Fig. 1 that as γ increases only the height of the peaks changes, this being most rapid in the direction of small ε. The structure is smeared out to complete disappearance at about $\gamma/F\sim 0.2$.

In Fig. 2, we compare the results of the experiment in Ref. 20 and the numerical calculation in accordance with the expression (28) proposed here. As follows

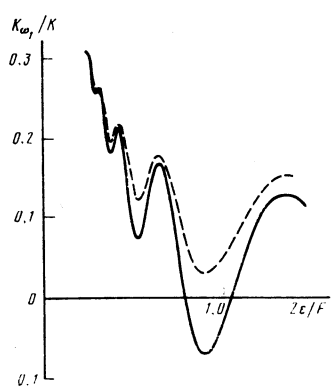

FIG. 2. Comparison of the experimental (dashed curve) and theoretical results. The theoretical curve is plotted for $F/\gamma=5.9$.

from Fig. 2, the positions of the theoretical and experimental absorption peaks on the energy scale agree well. The discrepancy between the measured and calculated absorption coefficients (especially in the region of the minima), which increases with increasing ε, can be attributed to the growth in the error of perturbation theory when the parameter $2\varepsilon/F$ increases.

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