On the theory of spin-lattice relaxation in solid He^3

I. N. Piradashvili

Institute of Physics, Academy of Sciences of the Georgian SSR, Tbilisi
(Submitted 3 January 1981)
Zh. Eksp. Teor. Fiz. **80**, 2508–2510 (June 1981)

The spin-lattice relaxation time in solid He^3 at $T \sim 0.6$ K is calculated under the assumption that a ferromagnetically polarized nuclear-spin region exists around a vacancy.

PACS numbers: 67.80.Jd

Nuclear magnetic resonance in crystalline He^3 is usually described under the assumption that several types of quasiparticles exist in this quantum crystal.[1] In particular, in the spin–lattice relaxation process the energy of the radio-frequency magnetic field is transferred from the Zeeman nuclear-spin system (Z) to the lattice [the phonon thermostat (P)] through the delocalization of the vacancies [vacancions (V)] and tunneling He^3 atoms [system of excitations (T)].

At a He^3-crystal temperature $T \sim 0.6$ K, the spin–lattice relaxation proceeds according to the scheme ZT–VP (the dash denotes a "bottleneck"). There is no correct theory, i.e., one that is compatible with the condition $\omega_T \tau_V \ll 1$, which is realized for the quantum crystal (ω_T is the He^3 atoms' tunneling frequency and τ_V is the mean lifetime of a vacancion at a site), for this temperature region. Thus, for example, in Ref. 1 Zane adopts Richards' approach,[2] which is, however, suitable only in the opposite limit, $\omega_T \tau_V \gg 1$.

Below we calculate the characteristic time of energy transfer in a bottleneck under the assumption that a vacancion produces ferromagnetically polarized region (FPR) of nuclear spins around itself.[3] The Brownian motion of the FPR induces transitions between the energy levels of the vacancion in the potential well, which, to the vacancion, is what the FPR represents. The diffusive motion of the FPR occurs as a result of the passage through it of the fluctuation currents of the T excitations.[4] Thus, the mechanism under consideration ensures the transfer of energy from the T system to the V system, from which the energy is rapidly removed by thermal phonons.[1]

The motion of a vacancion with quasimomentum p and energy $\varepsilon(\mathbf{p})$ is made up of its coherent motion in the FPR potential well and the Brownian motion (with step of the order of the interatomic distance a) of the center of gravity of the FPR in the crystal. In the system of coordinates moving together with the FPR the vacancion energy is $\varepsilon_0(\mathbf{p} - m\mathbf{v})$, where $m = -m_{He^3}$ is the "bare mass" of the vacancion and v is the instantaneous FPR velocity. In the laboratory system of coordinates the vacancion Hamiltonian in the linear—in v—approximation is

$$\varepsilon(\hat{\mathbf{p}}) = \varepsilon_0(\hat{\mathbf{p}}) + (1 - m/M)\hat{\mathbf{p}}\mathbf{v}(t), \tag{1}$$

where $\varepsilon_0(\hat{\mathbf{p}})$ is the vacancion Hamiltonian in the stationary FPR, $M \sim \hbar^2/a^2\Delta$ is the effective vacancion mass, and Δ is the vacancion band width.

The velocity of the FPR is low compared to the vacancion velocity $v_V \sim R\Delta/\hbar$, where R is the radius of the FPR. The perturbing term $\hat{\mathbf{p}}\mathbf{v}(t)$ induces transitions in the spectrum $\varepsilon_0(\hat{\mathbf{p}})$ of the states of the adiabatic approximation.

The probability per unit time for transition between two steady states $|i\rangle$ and $|f\rangle$ depends on the spectral density of the mean square of the fluctuating—about the equilibrium value $\mathbf{v} = 0$—components of the instantaneous FPR velocity vector:

$$w_{fi} = \frac{2\pi}{\hbar^2}\left(1 - \frac{m}{M}\right)^2 \sum_{k=x,y,z} |\langle f|\hat{p}_k|i\rangle|^2 \langle v_k^2\rangle_\omega. \tag{2}$$

The spectral density $\langle v_k^2\rangle_\omega$ is the Fourier transform of the correlation function $\langle v_k(t)v_k(0)\rangle$:

$$\langle v_k^2\rangle_\omega = \frac{D}{\pi}\left[1 + \omega^2\left(\frac{\mathcal{M}D}{T}\right)^2\right]^{-1}, \tag{3}$$

where D is the diffusion coefficient[4,5]; $\mathcal{M}$ is the effective mass of the FPR, i.e., the coefficient attached to $v^2/2$ in the expansion of the FPR free energy about the minimum.[6]

Because of the strong coupling between the vacancion system V and the lattice P, the vacancions in the FPR are at the ground energy level, which corresponds to the 1s state of a particle in an infinitely deep spherical potential well. The first excited level corresponds to the 1p state, and is described by the wave function

$$\Psi_{1lm}(r, \theta, \varphi) = (2/R^3)^{1/2}[j_0(4.493)]^{-1} \cdot j_1(4.493r/R)\, Y_{lm}(\theta, \varphi), \tag{4}$$

where $m = 0, \pm 1$ is the magnetic quantum number, $j(x)$ is a spherical Bessel function, and the $Y_{1m}(\theta, \varphi)$ are the first-order spherical functions, normalized to unity,

The slow transfer of energy from the T to the V system in the "bottleneck" is realized during the $1s \to 1p$ vacancion transitions. We neglect the transitions to the higher levels. Notice that the well depth which is of the order of Δ, is adequate for the existence of only the levels under consideration.[7]

The system V is strongly coupled to the phonon thermostat P, whose temperature T_p is prescribed by the experimental conditions and is constant: $T_p \ll \varphi_0 \sim 13.5$ K (Ref. 8), where φ_0 is the activation energy of a vacancy (the bottom of the vacancion band). Therefore, the temperature of the V system can be assumed to be equal to zero. For our calculation the inverse transitions of the vacancion are unimportant, since the energy emitted in these transitions is rapidly transferred to the P system. Then for the

 0038-5646/81/061315-02$02.40 © 1982 American Institute of Physics

power dissipated in the $V+P$ system we have, after making allowance for (4), the expression

$$Q=\sum_f \hbar\omega_{fi}w_{fi}=77{,}23\frac{\hbar^2D}{MR^4}\left(1-\frac{m}{M}\right)^2\left[1+26{,}59\left(\frac{\hbar D\mathcal{M}}{R^2TM}\right)^2\right]^{-1}. \quad (5)$$

The observable spin–lattice relaxation time is introduced through the relation $\dot{\beta}=\beta/\tau$, where β is the common reciprocal temperature of the strongly coupled Z and T systems. It is related with the characteristic time τ_{T-V} of energy transfer in the bottleneck as follows:

$$\tau=\tau_{T-V}(1+k_Z/k_T),$$

where $k_a=\partial E_a/\partial\beta$, E_a being the energy of one of the systems in question.[1] In particular, $k_T=-3N(\hbar I)^2/4$, where N is the number of spins in the system and I is the exchange integral for the nuclear spins of He^3.

Assuming that the considered process is the only channel through which the energy of the $Z+T$ system changes, we obtain

$$\tau_{T-V}=-\beta k_T/N_VQ, \quad (6)$$

where N_V is the number of FPR, which coincides with the number of vacancies in the crystal.

The substitution into (5) of the values $R\sim10^{-7}$ cm and $D\sim10^{-5}$ cm^2/sec from Refs. 3–5 and the requirement that the result (6) agree with the experimental data given in Ref. 8 yield for the FPR effective mass the estimate $\mathcal{M}\sim10^{-19}$–10^{-20} $g=10^4$–$10^5m_{He^3}$. Then taking (5) into account, we obtain from (6) the expression:

$$\tau_{T-V}=\frac{3k_B^{-3}\beta^3(\hbar I)^2D\mathcal{M}^2}{11{,}6(1-m/M)^2M}\exp(\beta\varphi_0), \quad (7)$$

where k_B is the Boltzmann constant. Let us note that the accuracy of the experiments reported in Ref. 8 is not high enough for them to be used to verify the temperature dependence of the preexponential factor in (7).

In conclusion I express my thanks to A. F. Andreev, G. E. Gurgenishvili, A. A. Nersesyan, and G. A. Kharadze for extremely useful discussions.

[1]R. A. Guyer, R. C. Richardson, and L. I. Zane, Rev. Mod. Phys. 43, 532 (1971); L. I. Zane, Thesis, Duke University, 1970.
[2]P. M. Richards, Phys. Rev. A137, 1327 (1965).
[3]A. F. Andreev, Pis'ma Zh. Eksp. Teor. Fiz. 24, 608 (1976) [JETP Lett. 24, 564 (1976)].
[4]S. V. Iordanskiĭ, Pis'ma Zh. Eksp. Teor. 26, 183 (1977) [JETP Lett. 26, 171 (1977)].
[5]I. N. Piradashvili, Zh. Eksp. Teor. Fiz. 79, 1908 (1980) [Sov. Phys. JETP 52, 963 (1980)].
[6]M. A. Krivoglaz, Fiz. Tverd. Tela (Leningrad) 12, 3496 (1970) [Sov. Phys. Solid State 12, 2840 (1971)].
[7]L. D. Landau and E. M. Lifshitz, Kvantovaya mekhanika (Quantum Mechanics), Nauka, Moscow, 1974, §33 (Eng. Transl., Pergamon Press, Oxford, 1977).
[8]R. P. Gifford and J. Hatton, Phys. Rev. Lett. 18, 1106 (1967); R. P. Giffard, Thesis, Oxford University, 1968.

Translated by A. K. Agyei