Effect of electron-phonon interaction on the electron spectrum in a normal metal

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(Submitted 17 December 1980)
Zh. Eksp. Teor. Fiz. **80**, 2445–2451 (June 1981)

The effect of electron-phonon interaction on the electron excitation spectrum in a normal metal is considered for an arbitrary shape of the Fermi surface and without any simplifying assumptions on the phonon dispersion law. The formulas obtained are a generalization of the results obtained by Migdal [Sov. Phys. JETP 7, 996 (1958)]. Singularities, due to interactions with the phonons are predicted in the electron spectrum.

PACS numbers: 71.25.Pi, 71.38. + i

1. As is well known, the energy spectrum of metals consists of two branches, fermion (electrons) and boson (phonons). The existence of the two weakly coupled branches of the energy spectrum is due to the difference in masses of the electrons and ions ($m \ll M$). The basis of the calculation of the properties of metals is the adiabatic approximation, according to which the interaction between electrons and phonons can be considered by perturbation theory in most cases. The electron-phonon interaction leads to many observed effects, of which the most studied are: a) the metal resistance connected with the collision of electrons with phonons, b) the renormalization of the phonon dispersion law of the phonons, by the interaction with electrons (sound absorption, see, for example, Ref. 1, and the Migdal-Kohn singularities[2]). Less studied is the effect of electron-phonon interaction on the electron spectrum of the metal, other than the formation of Cooper pairs and the transition of the metal into the superconducting state (these problems will not be studied by us).

According to Migdal,[3] the electron-phonon interaction renormalizes the electron spectrum, and the renormalization touches mainly on electrons with energies $|\varepsilon-\varepsilon_F| \leq \hbar\omega_D$ (ε_F is the Fermi energy, $\hbar\omega_D$ is the Debye energy). We note that the large ("from the viewpoint of the phonons") energy interval $\alpha\hbar\omega_D$ is small in electron scales ($\hbar\omega_D \ll \varepsilon_F$). Further, since the speed of the electrons v_F is much greater than the speed of sound s, the region of quasimomentum space touched on by the renormalization is very small:

$$\Delta p = |\varepsilon-\varepsilon_F|/v_F \sim ms. \tag{1}$$

This estimate shows that all the geometric singularities of the Fermi surface of the metal should manifest themselves in the renormalization, although on the Fermi surface itself (at $\varepsilon-\varepsilon_F$) the renormalization of the energy is absent (see Ref. 3 and also Ref. 4).

The task of this paper is the generalization of the results of Migdal[3] to the case of an arbitrary dispersion law of the electrons and phonons (in Ref. 3, all the results were obtained for a quadratic electron spectrum and a linear phonon spectrum).

2. At first glance, the words "renormalization" and "arbitrary dispersion" do not go together. How can we renormalize the function $\varepsilon = \varepsilon(\mathbf{p})$ which is not defined precisely? The point is that: a) without account of the interaction with phonons, the range of $\varepsilon(\mathbf{p})$ over which this function changes by an amount of the order of itself is determined by the band structure $(\Delta\varepsilon)_{\text{band}} \approx v_F \Delta p$, while $\Delta p \sim \hbar/a$ (a are the dimensions of the cell of the crystal), and the electron-phonon interaction, according to (1), affects the region of p-space immediately adjoining the Fermi surface; b) account of the electron-phonon interaction leads to a finite lifetime of electrons outside the Fermi surface.

Thus, we shall start out from the fact that we know the function $\varepsilon = \varepsilon(p)$ that determines the dispersion law of the electrons without account of their interaction with phonons.[1)] The spectrum of the phonons is determined by the dependence of their energy $\hbar\omega_j$ on the quasiwave vector $\mathbf{k}(\omega_j = \omega_j(\mathbf{k}))$. Among the phonons there are bound to be acoustic phonons ($j = 1, 2, 3$) for which $\omega_j = s_j(\mathbf{n})k$ as $k \to 0$ as $k \to 0$ ($\mathbf{n} = \mathbf{k}/k$). In polyatomic metals, in addition, there are optical phonons ($j = 4, 5, 6, \ldots, \omega_j(k) \to 0$ as $k \to 0$). Use of perturbation theory permits us to write down the change electron energy change $\delta\varepsilon = \tilde{\varepsilon}(\mathbf{p}) - \varepsilon(\mathbf{p})$ due to interaction with the phonons [$\varepsilon(\mathbf{p})$ is the renormalized energy] in the form of an integral over p-space (see, for example, Ref. 5, p. 158 of Russ. transl.):

$$\operatorname{Im}\delta\varepsilon(\mathbf{p}) = -\pi \sum_j \int d^3p' |M_{\mathbf{pp}'j}|^2 \delta(\varepsilon-\varepsilon_{\mathbf{p}'}-\hbar\omega_{j,\mathbf{p}-\mathbf{p}'})(1-f_{\mathbf{p}'}), \tag{2}$$

$$\operatorname{Re}\delta\varepsilon(\mathbf{p}) = \sum_j \int d^3p' |M_{\mathbf{pp}'j}|^2 \left\{ (1-f_{\mathbf{p}'}) \times \left[\frac{1}{\varepsilon-\varepsilon_{\mathbf{p}'}-\hbar\omega_{j,\mathbf{p}-\mathbf{p}'}} - \frac{1}{\varepsilon_F-\varepsilon_{\mathbf{p}'}-\hbar\omega_{j,\mathbf{p}-\mathbf{p}'}} \right] + f_{\mathbf{p}'} \left[\frac{1}{\varepsilon-\varepsilon_{\mathbf{p}'}+\hbar\omega_{j,\mathbf{p}-\mathbf{p}'}} - \frac{1}{\varepsilon_F-\varepsilon_{\mathbf{p}'}+\hbar\omega_{j,\mathbf{p}-\mathbf{p}'}} \right] \right\}$$

where

$$M_{\mathbf{pp}'j} = 2^{1/2}\Lambda^j_{\mathbf{pp}'}|\mathbf{p}-\mathbf{p}'|/(2\pi\hbar)^{3/2}(\rho\hbar\omega_{j,\mathbf{p}-\mathbf{p}'})^{1/2} \sim (s/m)^{1/2}$$

with error up to a factor $2^{1/2}/(2\pi\hbar)^{3/2}$ is the matrix element of the electron-phonon interaction and is responsible for the transition of the electron from the

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state $|\mathbf{p}\rangle$ to the state $|\mathbf{p}'\rangle$ by emission of a phonon of the j-th type with quasimomentum $\mathbf{p}-\mathbf{p}'$. It is written in this form to single out the dependence on $\mathbf{p}-\mathbf{p}'$ at $|\mathbf{p}-\mathbf{p}'| \ll p_F$; $\Lambda_{\mathbf{p}}$ then tends toward the deformation potential and is of the order of ε_F. The regions of integration in (2) and (3) are given by the equilibrium Fermi function $f_{\mathbf{p}}$—the Fermi step at $T=0$. In what follows, we shall consider only absolute zero temperature. In the solution of problems of this type (see Refs. 1 and 6) it is convenient to begin with the investigation of $\operatorname{Im}\delta\varepsilon(\mathbf{p})$ because there is a δ function in the integral in (3), the argument of which goes to zero on the surface in $\mathbf{p}'$ space (we call it the F_j surface):

$$\varepsilon(\mathbf{p}')=\varepsilon(\mathbf{p})-\hbar\omega_j(\mathbf{p}-\mathbf{p}'). \tag{4}$$

Since $\hbar\omega_j \ll \varepsilon(\mathbf{p})$, the F_j surface passes very close to the equal-energy surface (the ε surface):

$$\varepsilon(\mathbf{p}')=\varepsilon(\mathbf{p}). \tag{5}$$

We denote the value of the quasimomentum $\mathbf{p}'$ on the ε surface (5) by $\mathbf{p}$. The distances along the normal from the ε surface to the F_j surface are equal to

$$\Delta p_n' \approx -\hbar\omega_j(\mathbf{p}-\mathbf{p}_\varepsilon)/v(\mathbf{p}_\varepsilon), \tag{6}$$

and if $\hbar\omega_j$ is the energy of the acoustic phonon, then at one point[2)] ($\mathbf{p}'=\mathbf{p}$) the F_j and ε surfaces touch each other (Fig. 1a). In the case of an optical phonon, the F_j surface lies entirely inside the ε surface (Fig. 1b). For definiteness, we have assumed that the ε surface is the electron surface. For comparison we have drawn the hole ε and F_j surfaces on Fig. 2 in the case of an acoustic phonon.

3. We first consider $\operatorname{Im}\delta\varepsilon$, which is due to the interaction of the electron with acoustical phonons. It is seen from (3) that the contribution to the damping is made by the electron states $|\mathbf{p}'\rangle$ which are located outside the Fermi surface. At $|\varepsilon(\mathbf{p})-\varepsilon_F| \ll \hbar\omega_D$, i.e., at $|\mathbf{p}-\mathbf{p}_F| \ll ms$ ($\mathbf{p}_F$ is the point on the Fermi surface closest to $\mathbf{p}$), only a small section of the F_j surface is located outside the Fermi surface. Since, $\omega \propto k$ for small wave vectors, near the contact point of the ε and F_j surfaces the latter surface is a cone (Fig. 3)

$$\Delta p_n = -\frac{s}{v(\mathbf{p})}|\mathbf{p}-\mathbf{p}_s|, \tag{7}$$

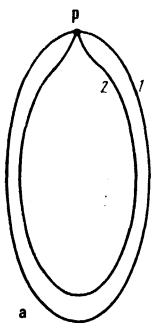
where $\Delta p_n = p_n' - p_n$, and p_n' and p_n are the projections of $\mathbf{p}'$ and $\mathbf{p}$ on the normal to the surface at the point $\mathbf{p}$.

Using formula (7) we can easily carry out the integration in (3). As a result, we get

$$\operatorname{Im}\delta\varepsilon(\mathbf{p}) \approx \lambda_p^{ac}(\varepsilon_p-\varepsilon_F)^3, \quad \varepsilon-\varepsilon_F \ll \hbar\omega_D, \qquad \lambda_p^{ac}=-\pi|\Lambda_{\mathbf{p}_F}^{ac}|^2/3(2\pi\hbar)^3\rho s^4 v_F. \tag{8}$$

Here $\Lambda_{\mathbf{p}_F}^{ac}$ is the value of $\Lambda_{\mathbf{p}'\mathbf{p}}^{ac}$ at $\mathbf{p}'=\mathbf{p}=\mathbf{p}_F$, $v_F=v(\mathbf{p}_F)$ (see above).

It is then seen that the local properties of the Fermi surface appear only because of the factor $|\Lambda_{\mathbf{p}_F}^{ac}|^2/v(\mathbf{p}_F)$, which is different for different points on the Fermi surface. However, there is no basis for expecting any sort of singularities in $\operatorname{Im}\delta\varepsilon(\mathbf{p})$ when the point $\mathbf{p}$ is displaced around the Fermi surface (if $\varepsilon_{\mathbf{p}}-\varepsilon_F \ll \hbar\omega_D$). An exception is a Fermi surface which contains a conical point (at which the velocity is equal to zero). The reason for the weak sensitivity of $\operatorname{Im}\delta\varepsilon$ at $\varepsilon_{\mathbf{p}}-\varepsilon_F \ll \hbar\omega_D$ to the geometry of the Fermi surface is that the contact of the ε and F_j surfaces takes place at the conical point of the latter.



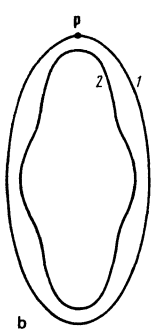


FIG. 1. a) ε and F_j^{ac} surfaces: 1—electron equal-energy surface, 2—F_j^{ac} surface; b) ε and F_j^{opt} surfaces: 1—electron equal-energy surface, 2—F_j^{opt} surface.

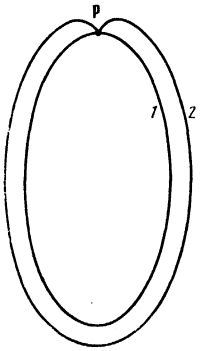


FIG. 2. ε and F_j^{ac} surfaces: 1—hole equal-energy surface, 2—F_j^{ac} surface.

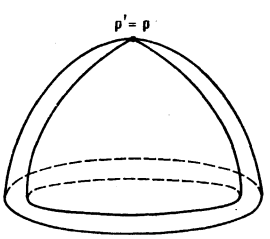


FIG. 3. Fragments of ε and F_j^{ac} surfaces near their point of contact $\mathbf{p}'=\mathbf{p}$.

We note that as $\varepsilon_p - \varepsilon_F \to 0$, the electron-phonon damping mechanism is not the basic one: at $\varepsilon_p - \varepsilon_F \lesssim ms^2$ $(\Delta p \lesssim ms^2/v_F)$ we have $\operatorname{Im}\delta\varepsilon \propto (\varepsilon_p - \varepsilon_F)^2$ due to electron-electron interaction[3] (in particular, via the phonons).

The cubic dependence of $\operatorname{Im}\delta\varepsilon$ on $\varepsilon - \varepsilon_F (\varepsilon - \varepsilon_F \ll \hbar\omega_D)$ was obtained by Migdal[3] in a consideration of an isotropic spectrum. As is seen from the present discussion, this result has a general character—it does not depend on the shape of the Fermi surface. Analysis of formula (3) shows that the conclusion drawn in Ref. 3 as to the weak dependence of $\operatorname{Im}\delta\varepsilon$ on $\varepsilon - \varepsilon_F$ at $\varepsilon - \varepsilon_F \gg \hbar\omega_D$ is valid in the general case. If the F_j surface lies entirely outside the Fermi surface, then

$$\operatorname{Im}\delta\varepsilon(\mathbf{p}) = -\pi \oint \frac{dS_{F'}}{v(\mathbf{p}_{F'})} |M_{\mathbf{p}_{F'}, \mathbf{p}_F}|^2, \quad \varepsilon - \varepsilon_F \gg \hbar\omega_D. \tag{9}$$

Since we are interested in a value of $\mathbf{p}$ close to the Fermi surface, it follows that after integration over the energy we can obtain the limiting transition $\mathbf{p} \to \mathbf{p}_F, \mathbf{p}_\epsilon \to \mathbf{p}'_F$. Assuming $|M_{\mathbf{p}'\mathbf{p}_F}|^2 \approx s/m$, $mv_F^2 \approx Ms^2$, we can establish the fact that $\operatorname{Im}\delta\varepsilon \approx -\hbar\omega_D$ in agreement with Ref. 3.

4. The transition in $\operatorname{Im}\delta\varepsilon$ from a cubic dependence to saturation does not take place smoothly, but is accompanied by singularities. These are bound to include a singularity at that value $\mathbf{p} = \mathbf{p}_c$ at which the F_j surface goes completely outside the Fermi surface (Fig. 4). If $\varepsilon(\mathbf{p}) \lesssim \varepsilon_c (\varepsilon_c = \varepsilon(\mathbf{p}_c))$, then the line of intersection of the F_j surface and the Fermi surface is an ellipse. This allows us to compute the singular part (SP) of $\operatorname{Im}\delta\varepsilon$, which is different from zero at $\varepsilon(\mathbf{p}) \lesssim \varepsilon$:

$$\text{SP } \operatorname{Im}\delta\varepsilon = \frac{\pi |M_c|^2}{v_c} m_c^* (\varepsilon_c - \varepsilon_p). \tag{10}$$

Here $m_c^* = (\tilde{M}_1 \tilde{M}_2)^{1/2}$ is a parameter with the dimension of mass, which arises in the expansion of the argument of the δ function near the point of contact

$$\varepsilon_p - \varepsilon_{p'} - \hbar\omega_j(\mathbf{p} - \mathbf{p}') = \varepsilon_p - \varepsilon_{p'} - \hbar\omega_c - \frac{x_1^2}{2\tilde{M}_1} - \frac{x_2^2}{2\tilde{M}_2}, \tag{11}$$

$x_{1,2}$ are orthogonal coordinates on the F_j surface, $\tilde{M}_{1,2}$, as can be shown, have the order of magnitude $(mM)^{1/2}$. In (10), all the quantities that depend on $\mathbf{p}'$ are taken at the point of contact[3)] $\mathbf{p}' = \mathbf{p}_{\text{cont}}$. Thus, at $\mathbf{p} = \mathbf{p}_c$, the quantity $\operatorname{Im}\delta\varepsilon$ has a kink as a function of energy, and

$$\frac{d}{d\varepsilon} \operatorname{Im}\delta\varepsilon|_{\mathbf{p}=\mathbf{p}_c} = -\frac{\pi |M_c|^2}{v_c} m^* = K_c \sim 1. \tag{12}$$

It is necessary to note that the signs of $\tilde{M}_1$ and $\tilde{M}_3$ for the obligatory singularity are fully defined independently of the shape of the Fermi surface—the departure of the F_j surface to the outside of the Fermi occurs only through the contact at the elliptical point.

In the case of a complicated shape of the Fermi surface, intermediate singularities are possible, including some due to a contact of the F_j surface with the Fermi surface, accompanied by a change in the topology of the line of intersection of these surfaces (Fig. 5). In these cases, $\tilde{M}_1$ and $\tilde{M}_2$ have different signs, which leads to a strengthening of the singularity

$$\text{SP } \operatorname{Im}\delta\varepsilon = K_c' \Delta \ln|\Delta|, \quad \Delta = \varepsilon_p - \varepsilon_{p_c'}, \tag{13}$$

while K_c' differs from K_c in that $(\tilde{M}_1\tilde{M}_2)^{1/2}$ is replaced by $(|\tilde{M}_1\tilde{M}_2|)^{1/2}$ and there is no factor π.

Since the singularity arises at $|\mathbf{p} - \mathbf{p}_{\text{cont}}| \sim p_F$, it is weakly sensitive to the form of the phonon dispersion law. In particular, the acoustical branches are not singled out in any way in this sense.

Strictly speaking, the singulartities arise because of the use of perturbation theory. Account of the finite lifetime of the electron in the right hand side of formulas (2) and (3) washes out the singularities. However, it can be supposed that in the case of not too strong a coupling between the electrons and the phonons the washing-out will not entirely eliminate the singularities. This can be contributed to by anisotropy of the Fermi surafce. A similar situation occurs with the Migdal-Kohn singularity,[2] which is observed experimentally, although the finite free path length washes it out.

5. The contribution of the optical branches of the phonon spectrum to the absorption of the electrons ($\operatorname{Im}\delta\varepsilon$) begins with that value $\mathbf{p} = \mathbf{p}_c^{\text{opt}}$ at which the F^{opt} surface is touched by the Fermi surface. As is clear from the foregoing section,

$$\operatorname{Im}\delta\varepsilon = \begin{cases} -K_c(\varepsilon_p - \varepsilon_{p_c'}), & \hbar\omega_0 \gg \varepsilon_p - \varepsilon_{p_c'} > 0 \\ 0, & \varepsilon_p - \varepsilon_{p_c'} < 0 \end{cases}. \tag{14}$$

At $\varepsilon_p - \varepsilon_{p_c}^0 \gg \hbar\omega_0$, the contribution of the optical branches of the spectrum is of the same order as that of the acoustical [see (9)]. The departure of the F^{opt} surface to the outside of the Fermi surface is naturally accompanied by a jump in the derivative (12) and between $\mathbf{p}_c^{\text{opt}}$ and the obligatory singularity there can be

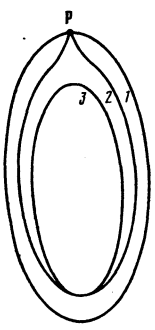


FIG. 4. Arrangement of the Fermi surface and ε and F_j surfaces at $\mathbf{p} = \mathbf{p}_c$: 1—electron equal-energy surface, 2—F_j^{3c} surface, 3—Fermi surface.

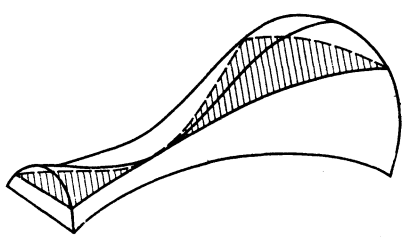


FIG. 5. Intersection of the Fermi surface and the F_j surface (the F_j surface is shaded) at $\mathbf{p} = \mathbf{p}_c'$. The line of intersection contains the point of self-intersection.

intermediate singularities that include singularities of the type (13). Neglect of the dispersion of optical the phonons $\omega_0(\mathbf{k})=\omega_0$, $\mathbf{k}=\mathbf{p}-\mathbf{p}'$ leads to a jumpwise change in $\operatorname{Im}\delta\varepsilon$ (compare with Ref. 7):

$$\operatorname{Im}\delta\varepsilon=\begin{cases}0, & \varepsilon_p-\varepsilon_F<\hbar\omega_{opt}\\ \text{const}\sim-\hbar\omega_{opt}, & \varepsilon_p-\varepsilon_F>\hbar\omega_{opt}\end{cases}. \tag{15}$$

6. We now determine the spectrum renormalization proper, i.e., $\operatorname{Re}\delta\varepsilon$. For elucidation of the dependence on $\varepsilon_p-\varepsilon_F$, we must integrate over ε' in (2). This can be done since, in the first place, the good convergence of the integrand allows us to extend the integration to infinity and, in the second place, the implicit dependence on ε' can be omitted because $v_F \gg s$ and the change in the momentum on going from one equal-energy surface to another in a layer $\approx\hbar\omega_D$ is very small and can be neglected. The latter circumstance allows us, after the integration, to take the limiting transition $\mathbf{p}_\varepsilon''\to\mathbf{p}_F'$. The result is thus obtained in the form of an integral over the Fermi surface:

$$\operatorname{Re}\delta\varepsilon=\oint\frac{dS'}{v(\mathbf{p}_F')}|M_{\mathbf{pp}'_F}|^2\ln\left|\frac{\varepsilon_p-\varepsilon_F-\hbar\omega(\mathbf{p}-\mathbf{p}_F')}{\varepsilon_p-\varepsilon_F+\hbar\omega(\mathbf{p}-\mathbf{p}_F')}\right|. \tag{16}$$

We then immediately obtain a generalization of the formulas derived in Ref. 3:

$$\begin{aligned}\operatorname{Re}\delta\varepsilon&=-2(\varepsilon_p-\varepsilon_F)\oint\frac{dS'}{v(\mathbf{p}_F')}|M_{\mathbf{pp}'}|^2/\hbar\omega(\mathbf{p}-\mathbf{p}'),\\ &\qquad \varepsilon_p-\varepsilon_F\ll\hbar\omega_D;\\ \operatorname{Re}\delta\varepsilon&=-\frac{2}{\varepsilon_p-\varepsilon_F}\oint\frac{dS'}{v(\mathbf{p}_F')}|M_{\mathbf{pp}'}|^2\hbar\omega(\mathbf{p}-\mathbf{p}'),\\ &\qquad \varepsilon_p-\varepsilon_F\gg\hbar\omega_D.\end{aligned} \tag{17}$$

The first of the formulas (17) can be rewritten in the form of a renormalization of the Fermi velocity:

$$\frac{\delta v_F}{v_F}=-2\oint\frac{dS'}{v(\mathbf{p}_F')}|M_{\mathbf{pp}'}|^2/\hbar\omega(\mathbf{p}-\mathbf{p}'), \tag{18}$$

and it is seen that $\delta v_F/v_F$ does not depend on the ratio of the masses of the electrons and ions. We recall that this formula is strictly applicable only as long as $|\delta v_F/v_F|\ll 1$. At $|\delta v_F/v_F|\sim 1$ use of perturbation theory is incorrect. However, account of the succeeding higher approximations cannot change the character of the result. With increase in energy upon departure from the Fermi surface, the velocity approaches its unrenormalized value [see the second formula of (17)].

The singularities in the $\operatorname{Im}\delta\varepsilon(\mathbf{p})$ dependence, of course, manifest themselves in the $\operatorname{Re}\delta\varepsilon(\mathbf{p})$ dependence. Their character is easiest to determine by using a dispersion relation of the Kramers-Kronig type.[8] It is not difficult to establish the following relation:

$\operatorname{Im}\delta\varepsilon$:	jump in the derivative	$\Delta\ln\lvert\Delta\rvert$
$\operatorname{Re}\delta\varepsilon$:	$\Delta\ln\lvert\Delta\rvert$	jump in the derivative.

The coefficients of the singular parts in $\operatorname{Im}\delta\varepsilon$ and $\operatorname{Re}\delta\varepsilon$ are identical apart from the sign and a numerical factor.

The detailed studies of the electron spectrum that are being carried out at the present time allow us to hope for the possibility of an experimental test of the relations obtained here.

In conclusion, we take this opportunity to thank I.M. Lifshitz and L.P. Pitaevskiĭ for stimulating discussions.

[1)] We shall not take into account the multiband character of the electron spectrum. Account of multiband transitions due to electron-phonon interaction is of most interest for phase transitions of order $2\frac{1}{2}$, which require a special consideration.

[2)] If among the vectors $\mathbf{p}-\mathbf{p}$ there are vectors $2\pi\hbar\mathbf{b}$, where $\mathbf{b}$ is one of the vectors of the reciprocal lattice, then the ε and the F_j surfaces touch each other at more than one point.

[3)] The contact point depends on $\mathbf{p}$. Usually, $|\mathbf{p}_c-\mathbf{p}_{cont}|$ differs from $2\mathbf{p}_F$ by an amount $\sim ms$. For a sphere, $|\mathbf{p}_c-2\mathbf{p}_F|=2ms$.

[1] G. T. Avanesyan, M. I. Kaganov, and T. Yu. Lisovskaya, Zh. Eksp. Teor. Fiz. **75**, 1786 (1978) [Sov. Phys. JETP **48**, 900 (1978)].
[2] W. Kohn, Phys. Rev. Lett. **2**, 393 (1959).
[3] A. B. Migdal, Zh. Eksp. Teor. Fiz. **34**, 1438 (1958) [Sov. Phys. JETP **7**, 996 (1958)].
[4] E. M. Lifshitz and L. P. Pitaevskiĭ, Statisticheskaya fizika (Statistical Physics) vol. I, Moscow, Nauka, 1978.
[5] J. R. Schrieffer, Theory of Superconductivity, Benjamin, New York, 1970.
[6] M. I. Kaganov and A. I. Semenenko, Zh. Eksp. Teor. Fiz. **50**, 630 (1950) [Sov. Phys. JETP **23**, 419 (1966)].
[7] S. Engelsberg and J. R. Schrieffer, Phys. Rev. **131**, 993 (1963).
[8] L. D. Landau and E. M. Lifshitz, Statisticheskaya fizika (Statistical Physics), Moscow, Nauka, 1976.

Translated by R. T. Beyer