Skin effect and Doppler-shifted cyclotron resonance in metals with open orbits

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A theory is developed of the skin effect in a compensated metal in a perpendicular magnetic field in the presence of open orbits. The case of purely specular reflection of the carriers from the surface is considered. The surface-impedance tensor and the distributions of the radiofrequency-field components are calculated. It is shown that one of the diagonal elements of the impedance tensor is independent of the magnetic field H and has singularities typical of the anomalous skin effect. The second diagonal element is proportional to the square of the magnetic field and has a logarithmic dependence on the carrier mean free path. These singularities are due to the presence of open orbits and to the mixing of the linear polarizations by carriers with closed orbits. Excitation of dopplerons and of Gantmakher-Kaner "waves" is also investigated. It is shown that carriers with open orbits cause collisionless damping of the doppleron; this damping decreases like H^{-8} with increasing field. The amplitudes of plate-impedance oscillations due to either dopplerons or Gantmakher-Kaner waves are the same for both linear polarizations.

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A theory was recently developed for the Doppler-shifted cyclotron resonance (DSCR) in metals in a perpendicular magnetic field. The rigorous treatment applied solely to metals having only closed carrier orbits. Most metals have open Fermi surfaces. The application of the theory to such metals is therefore limited, for at certain magnetic field-directions open carrier orbits constitute a sizeable fraction. It is therefore of interest to construct a DSCR theory and to study the properties of dopplerons and Gantmakher-Kaner oscillations (GKO) for metals with open orbits. This investigation is made necessary, in particular, by the fact that there are many experiments on metals with open orbits[1-4] that have no clear interpretation. In addition to the investigation of the singularities of DSCR, it is also of interest to study the influence of open orbits on the character of the skin effect and the spatial distribution of the long-wave component of the field.

The present paper is devoted to the singularities of the anomalous skin effect and to DSCR in a situation when the orbits of some of the carriers are open. Open orbits produce a strong conductivity anisotropy, as a result of which the field distribution in the metal is determined by a system of coupled integro-differential equations. The problem is therefore substantially more complicated than in the case of closed orbits, when the system of equations breaks up into two independent equations for the plus and minus polarizations. To avoid further complications that appear when the carriers are diffusely scattered from the surface, we confine ourselves to purely specular reflection.

1. THEORY OF NONLOCAL CONDUCTIVITY AND THE DISPERSION EQUATION

We use for the metal a model in which the electron part of the Fermi surface is a corrugated cylinder parallel to the p_z axis, and the hole part consists of two right cylinders parallel to the axes p_x and p_z. The electron dispersion law is described by the expression[5]

$$\varepsilon_1(p)=\frac{p_x^2+p_y^2}{2m_1}+\frac{2}{\pi}p_1v_1\sin^2\left(\frac{\pi p_z}{2p_1}\right)\quad(|p_z|\leqslant p_1), \tag{1}$$

and the hole dispersion law by the expressions

$$\varepsilon_2(p)=\frac{p_y^2}{2m_2}+\frac{2}{\pi}p_2v_2\sin^2\left(\frac{\pi p_z}{2p_2}\right)\quad(|p_z|\leqslant p_2), \tag{2}$$

$$\varepsilon_3(p)=\frac{p_x^2+p_y^2}{2m_3},\quad \varepsilon_{F_2}=\frac{2}{\pi}p_2v_2, \tag{3}$$

where $m_{1,2,3}$, $p_{1,2}$ and $v_{1,2}$ are constants with dimensions of mass, momentum, and velocity, respectively, and ε_{F_2} is the Fermi energy of the second group of carriers. The electromagnetic-wave propagation vector k and the constant magnetic field H are assumed to be directed along the z axis.

Obviously, in this model the orbits of the first and third carrier groups are closed in p-space, while the orbits of the second group are open (straight line parallel to the p_x axis).

The contributions of the carriers of the different groups to the transverse conductivity tensor are given by

$$\sigma_\pm^{(1)}(k)=\pm i\frac{n_1e^2}{m_1}[(\omega_{c1}\pm i\nu_1)^2-(kv_1)^2]^{-1/2}, \tag{4}$$

$$\sigma_{yy}^{(2)}(k)=\frac{n_2e^2}{m_2}[\nu_2^2+(kv_2)^2]^{-1/2}, \tag{5}$$

$$\sigma_\pm^{(3)}=\mp i\frac{n_3e^2}{m_3(\omega_{c3}\mp i\nu_3)}, \tag{6}$$

$$\sigma_\pm^{(i)}=\sigma_{xx}^{(i)}\pm i\sigma_{yx}^{(i)}\quad(i=1,2,3), \tag{7}$$

where n_i is the density of the carriers of the i-th group, $\omega_{ci}=eH/m_ic$ is their cyclotron frequency, and ν_i is the frequency of the collisions with the lattice. The dependence of σ on the frequency ω is not taken into account, since we are considering radio waves for which $\omega\ll\nu_i$.

Expression (4), which describes the electron part of the conductivity, has at $k=(\omega_{c1}\pm i\nu_1)/v_1$ a square-root singularity that corresponds to the DSCR of the electrons. The carriers of the second group make no con-

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tributions to the conductivity-tensor elements σ_{xx} and σ_{xy}, since the velocity component along the x axis is zero on a right cylinder parallel to the p_x axis. Expression (5) is independent of H, i.e., carriers with open orbits make the same contribution to the conductivity as in the absence of a magnetic field. Finally, the contribution of the carriers of the third group, defined by (6), is local because they have no velocity component v_z. The model considered corresponds thus to a situation with electron DSCR and nonlocal conduction due to holes with open orbits, whereas the DSCR of the holes with closed orbits is not taken into account. The latter is permissible if the displacements of the holes during the cyclotron period are much smaller than the maximum electron displacement. In addition, we assume hereafter that the electron and hole densities are equal, so that

$$n_2=an_1, \quad n_3=(1-a)n_1, \tag{8}$$

where a is the fraction of holes with open orbits.

To simplify the formulas we use the dimensionless coordinate

$$\zeta=z\omega_{c1}/v_1 \tag{9}$$

and the dimensionless wave vector

$$q=kv_1/\omega_{c1}. \tag{10}$$

Assume that a monochromatic radiowave of frequency ω is incident on a metal occupying the half-space $\zeta>0$. The reflection of all carriers from the surface $\zeta=0$ is assumed specular. The electric field in the metal can then be represented by a Fourier integral

$$\mathcal{E}_\alpha(\zeta)=\frac{1}{2\pi}\int\limits_{-\infty}^{\infty}E_\alpha(q)e^{iq\zeta}\,dq, \quad \alpha=x,y, \tag{11}$$

in which the Fourier transforms $E_\alpha(q)$ of the field components are defined by the system of equations

$$q^2E_\alpha(q)-i\xi s_{\alpha\beta}(q)E_\beta(q)=-2\mathcal{E}_\alpha'(0), \quad \alpha=x,\ y, \tag{12}$$

obtained from Maxwell's equations by taking into account the connection between the current density and the field. Here $\mathcal{E}_\alpha(0)$ is the electric field on the surface, the prime denotes differentiation with respect to ζ, summation over repeated indices β is implied,

$$s_{\alpha\beta}(q)=\frac{H}{n_1ec}\sigma_{\alpha\beta}(q), \tag{13}$$

$$\sigma_{\alpha\beta}(q)=\sum_{i=1}^{3}\sigma_{\alpha\beta}^{(i)}(q), \tag{14}$$

$$\xi=\frac{4\pi\omega}{c^2}\frac{n_1ec}{H}\left(\frac{v_1}{\omega_{c1}}\right)^2=\frac{4\pi\omega n_1m_1^2v_1^2c}{eH^3}. \tag{15}$$

Solving the system (12) and substituting E_α in (11), we write the expression for $\mathcal{E}_\alpha(\zeta)$ in the form

$$\mathcal{E}_\alpha(\zeta)=-iT_{\alpha\beta}(\zeta)\mathcal{E}_\beta'(0), \tag{16}$$

$$T_{\alpha\beta}(\zeta)=\frac{1}{\pi i}\int\limits_{-\infty}^{\infty}\frac{dq}{D(q)}e^{iq\zeta}R_{\alpha\beta}(q), \tag{17}$$

$$R_{\alpha\beta}(q)=\begin{pmatrix} q^2-i\xi s_{yy}(q), & i\xi s_{xy}(q) \\ i\xi s_{yx}(q), & q^2-i\xi s_{xx}(q)\end{pmatrix}, \tag{18}$$

$$D(q)=\det R_{\alpha\beta}(q)=(q^2-i\xi s_{xx})(q^2-i\xi s_{yy})+\xi^2s_{xy}s_{yx}. \tag{19}$$

The explicit form of the elements of the tensor $s_{\alpha\beta}$ can be obtained from (4)–(7) and (13)–(15):

$$s_{xx}(q)={}^1/_2i\{[(1+i\gamma_1)^2-q^2]^{-1/2}-[(1-i\gamma_1)^2-q^2]^{-1/2}\}+(1-a)\gamma_3/(1+\gamma_3^2), \tag{20}$$

$$-s_{xy}(q)=s_{yx}(q)={}^1/_2\{[(1+i\gamma_1)^2-q^2]^{-1/2}+[(1-i\gamma_1)^2-q^2]^{-1/2}\}-(1-a)/(1+\gamma_3^2), \tag{21}$$

$$s_{yy}(q)=s_{xx}(q)+s_0(q), \quad s_0(q)=ab(q^2+\gamma^2)^{-1/2}, \tag{22}$$

$$\gamma_i=\nu_i/\omega_{ci} \quad (i=1,\ 2,\ 3), \quad \gamma=b\gamma_2, \tag{23}$$

$$b=m_1v_1/m_2v_2; \tag{24}$$

we assume hereafter that b is of the order of unity.

We calculate the electric field in the metal by deforming the integrating contour in (17) in the upper complex q plane. The integral (17) is then the sum of the residues of the poles of the integrand and of the integrals along the edges of the cuts drawn from the branch points. The poles correspond to the roots of the dispersion equation

$$D(q)=0, \tag{25}$$

which determine the eigenmodes of the electromagnetic field in an unbounded metal.

Even though in our model the local conductivity has the simplest possible form, the integral (17) is the general case complicated and cannot be calculated analytically. We restrict ourselves therefore to a field region in which the parameter ξ is small and the carrier cyclotron frequencies are much higher than the collision frequencies:

$$\xi\ll 1, \quad \gamma_i\ll 1 \quad (i=1,\ 2,\ 3). \tag{26}$$

In this field region, the different poles of the integrand are far from one another in the q plane. The same holds also for the different branch points. All the poles and branch points are divided here into two groups. The first includes the singular points located in the region of small q:

$$|q|\ll 1, \tag{27}$$

and the second the points located near ± 1, for which

$$|q^2-1|\ll 1. \tag{28}$$

It is these regions of q which make the main contribution to the integral (17). Since the sizes of the regions are small compared with the distances between them, their contributions can be separated. In other words, in the magnetic-field range defined by (26) the tensor $T_{\alpha\beta}$ can be represented in the form

$$T_{\alpha\beta}(\zeta)=U_{\alpha\beta}(\zeta)+V_{\alpha\beta}(\zeta), \tag{29}$$

where $U_{\alpha\beta}$ is obtained from (17–(19) by putting $q=0$ in (20) and (21), and $V_{\alpha\beta}$ is obtained by putting $q^2=1$ in the second term of (22) and integrating only in the vicinities of the points ± 1.

The tensor $U_{\alpha\beta}(\zeta)$ describes the long-wave part of the field, which is connected with the skin effect and which we shall call the skin component. The short-wave part $V_{\alpha\beta}(\zeta)$ on the other hand, is connected with the DSCR of the first group of carriers and describes the doppleron and the Gantmakher-Kaner "wave."

2. SKIN COMPONENT OF THE FIELD

We consider in this section the long-wave field component and disregard effects connected with DSCR.

Neglecting the spatial dispersion of the conductivity $\sigma^{(1)}$, we write down the tensor $U_{\alpha\beta}(\zeta)$ in the form

$$U_{\alpha\beta}(\zeta)=\frac{1}{\pi i}\int_{-\infty}^{\infty}\frac{dq}{D_0(q)}R_{\alpha\beta}^{(0)}(q)e^{iq\zeta}, \tag{30}$$

$$R_{\alpha\beta}^{(0)}(q)=\begin{pmatrix} q^2-ib\xi_0(q^2+\gamma^2)^{-1/2}-i\xi_0\gamma_0 & -i\xi_0 \\ +i\xi_0 & q^2-i\xi_0\gamma_0\end{pmatrix}, \tag{31}$$

$$D_0(q)=(q^2-ib\xi_0(q^2+\gamma^2)^{-1/2}-i\xi_0\gamma_0)(q^2-i\xi_0\gamma_0)-\xi_0^2, \tag{32}$$

$$\xi_0=a\xi,\quad \gamma_0=\frac{\gamma_1}{a}+\frac{1-a}{a}\gamma_3. \tag{33}$$

In the considered approximation, the spatial dispersion of the conductivity is connected with the carriers of the second group, which have open orbits. In noble metals, zinc, cadmium, and in a number of others the number of carriers with open orbits is relatively small, and we assume therefore that

$$a\ll 1. \tag{34}$$

In addition, we shall be interested hereafter only in the field region

$$b^2\gamma_0\ll\xi_0, \tag{35}$$

where the presence of open orbits alters most substantially the RF properties of the metal.

We investigate now the singular points of the integrand in (30). The function D_0 has two branch points: $q=i\gamma$ and $q=-i\gamma$. We draw the cut from the first to the left and from the second to the right, as shown in Fig. 1 On the first sheet of the complex variable q we have then six roots of the dispersion equation

$$D_0(q)=0. \tag{36}$$

In the upper half plane are located the roots

$$q_1\approx(b\xi_0)^{1/3}e^{\pi i/6},\quad q_2\approx(b\xi_0)^{1/3}e^{5\pi i/6}, \tag{37}$$

$$q_3\approx i\xi_0/b, \tag{38}$$

and in the lower the roots $-q_1$, $-q_2$, and $-q_3$.

The roots (37) and (38) differ significantly. The wave vectors k_1 and k_2, which correspond to q_1 and q_2, are independent of the magnetic field and characterize the distribution of the electromagnetic field under conditions of the normal skin effect at $H=0$. This field component is determined entirely by the carriers of the second group.

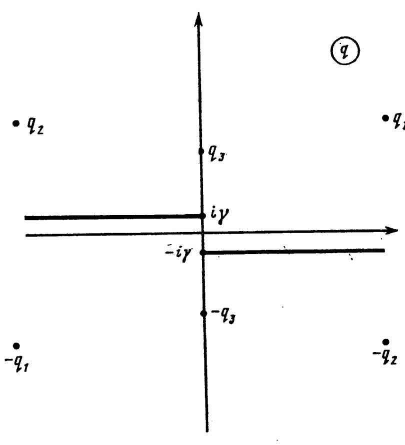

FIG. 1. Arrangement of the branch points and of the zeros of the function $D_0(q)$ and of the cuts in the complex q plane.

The root q_3 is much less than q_1 or q_2, and the component corresponding to it is attenuated over a distance proportional to H^2. An important role in the formation of this component is played not only by the carriers with open orbits, but also with closed ones. We shall show below that the root q_3 is connected mainly with the x component of the electric field, and the roots q_1 and q_2 with the y component.

We calculate now the elements of the tensor $U_{\alpha\beta}(\zeta)$, which determine the distribution of the field in the skin layer. We begin with the element $U_{xx}(\zeta)$. The pre-exponential factor in the corresponding integrand is

$$\frac{1}{D_0(q)}\left[q^2-\frac{ib\xi_0}{(q^2+\gamma^2)^{1/2}}-i\xi_0\gamma_0\right]$$
$$=\left\{q^2-i\xi_0\gamma_0-\xi_0^2\left[q^2-\frac{ib\xi_0}{(q^2+\gamma^2)^{1/2}}-i\xi_0\gamma_0\right]^{-1}\right\}^{-1}. \tag{39}$$

The function (39) increases rapidly with increasing q, and the main contribution to the integral is made by the range of values $q\lesssim q_3$. We need retain in the square brackets of the right-hand side pzrt of (39) only the second term, so that the expression for U_{xx} takes the form

$$U_{xx}(\zeta)\approx\frac{1}{\pi i}\int_{-\infty}^{\infty}dq e^{iq\zeta}\left(q^2-i\xi_0\gamma_0-\frac{i\xi_0}{b}(q^2+\gamma^2)^{1/2}\right)^{-1}$$
$$\approx-\frac{b}{\pi\xi_0}\int_{-\infty}^{\infty}dq e^{iq\zeta}\left(\frac{1}{(q^2+\gamma^2)^{1/2}-i\xi_0/b}-\frac{1}{(q^2+\gamma^2)^{1/2}+b\gamma_0}\right). \tag{40}$$

At large distances $\zeta\gg\gamma^{-1}$ the value of U_{xx} is determined by a small vicinity of the branch point of the second term in (40), and is given by the last expression of (42). To calculate U_{xx} at $\zeta\ll\gamma^{-1}$ we deform in the first integral the integration contour towards the upper cut on the figure, and in the second towards a cut drawn from the point $i\gamma$ vertically upwards. This modification of the cut in the second term is necessary to prevent the cut from passing near the pole $q_4=-b\gamma_0$ in the second sheet. Taking the residue of the integrand in the pole q_3 and neglecting small terms of order γ, we reduce the equation for U_{xx} to the form

$$U_{xx}(\zeta)=\frac{2b}{i\xi_0}e^{-\xi_0\zeta/b}-\frac{2b}{\pi\xi_0}\int_0^{\infty}\frac{e^{-ix}x\,dx}{x^2+(\xi_0\zeta/b)^2}+\frac{2b}{\pi\xi_0}\int_0^{\infty}\frac{e^{-x}x\,dx}{x^2+(b\gamma_0\zeta)^2} \tag{41}$$

This function has a complicated dependence on the coordinate. Its behavior in different regions of values of ζ is described approximately by the first three expressions in (42):

$$U_{xx}(\zeta)=\frac{2b}{\pi\xi_0}\ln\frac{\xi_0}{b^2\gamma_0}\quad\left(\zeta\ll\frac{b}{\xi_0}\right),$$
$$U_{xx}(\zeta)=\frac{2b}{\pi\xi_0}\ln\frac{1}{b\gamma_0\zeta}\quad\left(\frac{b}{\xi_0}\ll\zeta\ll\frac{1}{b\gamma_0}\right),$$
$$U_{xx}(\zeta)=\frac{2b}{\pi\xi_0}\frac{1}{(b\gamma_0\zeta)^2}\quad\left(\frac{1}{b\gamma_0}\ll\zeta\ll\frac{1}{\gamma}\right),$$
$$U_{xx}(\zeta)=\frac{2b}{\pi\xi_0}\frac{1}{(b\gamma_0\zeta)^2}\left(\frac{\pi}{2}\gamma\zeta\right)^{1/2}e^{-\gamma\zeta}\quad\left(\frac{1}{\gamma}\ll\zeta\right). \tag{42}$$

It is seen that the function U_{xx} is constant near the surface, and then falls off logarithmically to a constant of the order of

$$l_0=\frac{v_1}{\omega_{c1}b\gamma_0}=\frac{n_2m_2\nu_2}{n_1m_1\nu_1+n_3m_3\nu_3}\frac{v_2}{\nu_2}\sim al_2, \tag{43}$$

where $l_2 = v_2/\nu_2$ is the path length of the carriers on the open orbitals. At distances larger than l_0 but smaller than l_2, U_{xx} decreases like $1/z^2$, and at $z \gg l_2$ it decreases exponentially.

The element $U_{yy}(\zeta)$ is determined by the formula

$$U_{yy}(\zeta)=\frac{1}{\pi i}\int_{-\infty}^{\infty}\frac{(q^2-i\xi_0\gamma_0)\,e^{iq\zeta}dq}{(q^2-i\xi_0\gamma_0)(q^2-ib\xi_0(q^2+\gamma^2)^{-1/2})-\xi_0^2}. \qquad (44)$$

At large and small distances from the surface, the main contribution to this integral is made by different regions of q. At $\zeta \ll (b\xi_0)^{-1/3}$ the significant region is $q \sim (b\xi_0)^{1/3}$. In this case the term ξ_0^2 in the denominator of (44) can be neglected, and the expression for U_{yy} becomes

$$U_{yy}(\zeta)\approx\frac{1}{\pi i}\int_{-\infty}^{\infty}\frac{e^{iq\zeta}dq}{q^2-ib\xi_0(q^2+\gamma^2)^{-1/2}}. \qquad (45)$$

Shifting the integration contour towards the cut in the upper halfplane, taking the residues of the integrand in the poles q_1 and q_2, and neglecting terms of order γ, we obtain

$$U_{yy}(\zeta)\approx\frac{2}{3q_1}e^{iq_1\zeta}+\frac{2}{3q_2}e^{iq_2\zeta}+\frac{2b\xi_0}{\pi}\int_0^{\infty}e^{-iq\zeta}\frac{q\,dq}{q^6+(b\xi_0)^2}. \qquad (46)$$

Substituting the values of q_1 and q_2 and retaining the higher-order terms, we obtain

$$U_{yy}\approx\frac{2(1-i\sqrt{3})}{3^{3/2}(b\xi_0)^{1/3}}. \qquad (47)$$

At values $\zeta \gg (b\xi_0)^{-1/3}$ the main contribution to the integral is made by the region $q \ll (b\xi_0)^{1/3}$, and the expression for $U_{yy}(\zeta)$ can be written in the form

$$U_{yy}(\zeta)\approx\frac{1}{\pi i}\int_{-\infty}^{\infty}e^{iq\zeta}(q^2-i\xi_0\gamma_0)\left[(q^2-i\xi_0\gamma_0)\frac{-ib\xi_0}{(q^2+\gamma^2)^{1/2}}-\xi_0^2\right]^{-1}dq. \qquad (48)$$

It is understood here that the integration contour will be deformed in the calculation in the upper half-plane, and the exponential ensures convergence of the integral. The calculation of (48) is similar to the calculation of the integral in (40). Incidentally, to find the function $U_{yy}(\zeta)$ in the region $\zeta \gg b/\xi_0$ it is more convenient to use the relations

$$U_{yx}(\zeta)=\left(\gamma_0-\frac{i}{\xi_0}\frac{d^2}{d\zeta^2}\right)U_{xx}(\zeta),$$
$$U_{yy}(\zeta)=\left(\gamma_0-\frac{i}{\xi_0}\frac{d^2}{d\zeta^2}\right)U_{yx}(\zeta),\quad U_{yx}(\zeta)=-U_{xy}(\zeta),$$

which follow from the equations for the field $\mathscr{E}(\zeta)$. In different regions of ζ, the function U_{yy} is described by the expressions

$$\begin{aligned}
U_{yy}(\zeta)&=-\frac{2}{\pi b\xi_0\zeta^2}\quad\left((b\xi_0)^{-1/3}\ll\zeta\ll\frac{b}{\xi_0}\right),\\
U_{yy}(\zeta)&=\frac{12b}{\pi\xi_0^3\zeta^4}+\frac{4ib\gamma_0}{\pi\xi_0^2\zeta^2}+\frac{2b\gamma_0^2}{\pi\xi_0}\ln(b\gamma_0\zeta)\quad\left(\frac{b}{\xi_0}\ll\zeta\ll\frac{1}{b\gamma}\right)\\
U_{yy}(\zeta)&=-\frac{2}{\pi b\xi_0\zeta^2}\quad\left(\frac{1}{b\gamma_0}\ll\zeta\ll\frac{1}{\gamma}\right),\\
U_{yy}(\zeta)&=-\frac{2}{\pi b\xi_0\zeta^2}\left(\frac{\pi}{2}\gamma\zeta\right)^{1/2}e^{-\gamma\zeta}\quad\left(\frac{1}{\gamma}\ll\zeta\right).
\end{aligned} \qquad (49)$$

The function $U_{xy}(\zeta)$ is calculated similarly. Its behavior in different regions of ζ is described approximately by the formulas

$$\begin{aligned}
U_{xy}(\zeta)&=-\frac{4i}{3\pi b}\ln\frac{b^2}{\xi_0}\quad(\zeta\ll(b\xi_0)^{-1/3}),\\
U_{xy}(\zeta)&=-\frac{2i}{\pi b}\ln\frac{b}{\xi_0\zeta}\quad\left((b\xi_0)^{-1/3}\ll\zeta\ll\frac{b}{\xi_0}\right),\\
U_{xy}(\zeta)&=+\frac{2ib}{\pi\xi_0^2\zeta^2}+\frac{2b\gamma_0}{\pi\xi_0}\ln(b\gamma_0\zeta)\quad\left(\frac{b}{\xi_0}\ll\zeta\ll\frac{1}{b\gamma_0}\right),\\
U_{xy}(\zeta)&=-\frac{2}{\pi b\xi_0\gamma_0\zeta^2}\quad\left(\frac{1}{b\gamma_0}\ll\zeta\ll\frac{1}{\gamma}\right),\\
U_{xy}(\zeta)&=-\frac{2}{\pi b\xi_0\gamma_0\zeta^2}\left(\frac{\pi}{2}\gamma\zeta\right)^{1/2}e^{-\gamma\zeta}\quad\left(\frac{1}{\gamma}\ll\zeta\right).
\end{aligned} \qquad (50)$$

We discuss now the obtained expressions. According to (16), the tensor $T_{\alpha\beta}(\zeta)$ determines the distribution of the electric field and the impedance at different polarizations of the exciting magnetic field. The short-wave part of this tensor, $V_{\alpha\beta}$, does not influence the value of the impedance (see Sec. 3). Therefore, for example, the component $U_{yy}(\zeta)$ determines the impedance Z_{yy} and describes the distribution of the long-wave part of the field $\mathscr{E}_y(\zeta)$ in the case when the exciting magnetic field is directed along the x axis. (According to Maxwell's equation, $\mathscr{E}_y' = -i\omega H_x/c$.) It follows from (46) that in this case, at short distances from the surface, the field $\mathscr{E}_y$ is determined by the roots q_1 and q_2, i.e., it is formed by holes on open orbits, and the presence of other carriers plays no role. Accordingly, the impedance $Z_{yy} = 4\pi q_0 U_{yy}(0)/c$, where $q_0 = \omega v_1/\omega_{e1}c$, does not depend on the magnetic field and has the singularities that characterize the skin effect at $H=0$.[6]

The behavior of the field $\mathscr{E}_y$ at distances exceeding the thickness of the anomalous skin layer is described by expressions (49). It is seen from them that in the region where $|q_1|^{-1} \ll \zeta \ll |q_3|^{-1}$ the field $\mathscr{E}_y$ is determined as before by the carriers on the open orbits, and its value decreases in inverse proportion to the square of the distance from the surface.[6] On the other hand, at distances ζ larger than $|q_3|^{-1}$, an important role in the formation of the field $\mathscr{E}_y$ is assumed by carriers with closed orbits, so that its value becomes a function of H and has a more complicated dependence on ζ. The carrier motion over closed orbits, which causes Hall conduction, leads to a mixing of the linear polarizations of the RF and to so unusual a character of the variation of $\mathscr{E}_y$ at large distances from the surface. This mixing manifests itself even more strongly when the field $\mathscr{E}_x$ is excited by an alternating magnetic field directed along the y axis, and leads to an even more unexpected $\mathscr{E}_x(\zeta)$ dependence [see (42)]. If the open orbits that do not contribute to the current j_x were not to influence this field, then its value would vary with the coordinate in accord with the simple exponential law that is typical of the normal skin effect in a magnetic field. On the other hand, owing to the presence of open orbits, the field $\mathscr{E}_x$ varies logarithmically up to distances of the order of l_0. The role of the skin-layer thickness is then assumed by the quantity $u/2\pi q_3$, which we shall call the thickness of the "skin layer in the presence of open orbits."

The function $U_{xy}(\zeta)$ characterizes the distribution of the field $\mathscr{E}_x$ when it is excited by an alternating magnetic field polarized along the x axis.

To conclude this section, we discuss the properties of various elements of the impedance tensor. According to (42), (47), and (50)

$$Z_{\alpha\beta}=\frac{4\pi q_0}{c}\begin{pmatrix}\frac{2b}{\pi\xi_0}\ln\frac{\xi_0}{b^2\gamma_0} & -\frac{4i}{3\pi b}\ln\frac{b^2}{\xi_0}\\ +\frac{4i}{3\pi b}\ln\frac{b^2}{\xi_0} & \frac{2(1-i\sqrt{3})}{3^{3/2}(b\xi_0)^{1/2}}\end{pmatrix}, \tag{51}$$

where $q_0=\omega v_1/\omega_{c1}c$. It follows from (51) that the element Z_{yy} does not depend on the magnetic field, the element Z_{xx} varies with the field like $H^2\ln H$, and the elements Z_{xy} and Z_{yx} like $H^{-1}\ln H$. Furthermore, Z_{xx} depends logarithmically on the carrier mean free path, while the remaining elements do not depend on it and consequently also on the temperature.

3. DOPPLERON AND GANTMAKHER KANER WAVE

The expression for the short-wave component $V_{\alpha\beta}(\zeta)$ is determined by the contribution of the region (28). Therefore the variable q in the argument of the function s_0 in (17)–(19) should be replaced by unity. Accurate to terms of order ξ^2, it is then possible to represent the function $D(q)$ by the product

$$D(q)\approx[q^2-i\xi s_+(q)-{}^1/_2 i\xi s_0(1)]\,[q^2-i\xi s_-(q)-{}^1/_2 i\xi s_0(1)]\equiv D_V(q). \tag{52}$$

Using (7), we express the function $V_{xx}(\zeta)$ in the form

$$V_{xx}(\zeta)=\frac{1}{\pi i}\int\frac{e^{iq\zeta}dq}{D_V(q)}\left[q^2-i\xi\left(\frac{s_+(q)+s_-(q)}{2}+s_0(1)\right)\right], \tag{53}$$

where the integration is over the region (28).

Representing the expression in the square brackets in (53) in the form of two terms and using (52), we get

$$V_{xx}(\zeta)=\frac{1}{2\pi i}\int e^{iq\zeta}dq\left[\frac{1}{q^2-i\xi s_+(q)-i\xi s_0(1)/2}+\frac{1}{q^2-i\xi s_-(q)-i\xi s_0(1)/2}\right]. \tag{54}$$

In the upper half-plane, the first term of (54) has a branch point $1+i\gamma_1$, and the second a branch point $-1+i\gamma_1$ and a doppleron pole q_D. The position of q_D is determined approximately by the expression

$$q_D\approx-1+{}^1/_2\xi^2+i\gamma_1+{}^1/_2 iab\xi^3. \tag{55}$$

The last term in (55) is due to collisionless absorption of the field by holes on open orbits. The corresponding part of the doppleron damping is proportional to the density of the second carrier group, its value is independent of temperature and decreases with increasing field like H^{-8}. Thus, in strong-field regions the doppleron propagates also in the presence of open orbits.

An analysis of (54) shows that at $\zeta=0$ the element V_{xx} is the of order of ξ. It therefore does not influence the surface impedance. At $\zeta\gg1$ the integral (54) can be represented in the form of a sum of contributions of the residue at the pole q_D and of the integrals along the edges of the cuts:

$$V_{xx}(\zeta)=\xi^2e^{iq_D\zeta}+e_{GK}(\zeta)+e_{GK}^*(\zeta), \tag{56}$$

where the asterisk denotes the complex conjugate, and $e_{GK}(\zeta)$ is of the form

$$e_{GK}(\zeta)=\frac{\xi^2}{2\pi}e^{i\zeta}\int_0^\infty\frac{x^{1/2}dx}{x+1}\exp\frac{i\xi^2\zeta x}{2}. \tag{57}$$

The first term in (56) describes the doppleron field, while the second and third describe the Gantmakher-Kaner wave.

The elements $V_{xy}(\zeta)$ and $V_{yy}(\zeta)$ are calculated similarly. Their contribution to the impedance of a semi-infinite metal is also negligibly small. Accurate to terms of higher order in ξ, the element V_{yy} is equal to V_{xx}. At $\zeta\ll1$ the element V_{xy} is given by

$$V_{xy}(\zeta)=-i\xi^2e^{iq_D\zeta}+ie_{GK}(\zeta)-ie_{GK}^*(\zeta). \tag{58}$$

Excitation of a doppleron and of a Gantmakher–Kaner component in a plate does not lead to oscillations of the sample impedance as functions of the constant magnetic field. When the transmitted signal is reflected from the opposite side of the plate, the amplitude of the RF field is simply doubled[5] (we are considering the case of specular reflection of the carriers). To calculate the oscillating part of the plate impedance (under antisymmetrical excitation) it is therefore necessary to introduce an appropriate multiplier in (56) and (58) and set in these equations $\zeta=L$, where $L=\omega_{c1}d/v_1$ and d is the plate thickness,

$$\Delta Z_{xx}=\Delta Z_{yy}=-\frac{16\pi q_0}{c}V_{xx}(L),$$
$$\Delta Z_{xy}=-\Delta Z_{yx}=-\frac{16\pi q_0}{c}V_{xy}(L). \tag{59}$$

It is seen that the amplitudes of both the doppleron oscillations and of the GKO are the same in both linear polarizations, although the spin components differ radically.

Doppler-shifted cyclotron resonance in a compensated metal with open orbits was investigated, for example in cadmium.[2-4] The authors have noted there that the oscillations are different for linear polarization of the current along the hexagonal and binary axes of the crystal, in patent contradiction to the constructed theory. The reason, apparently, is the predominantly diffuse character of the reflection of the carriers in the investigated cadmium samples.

The theory developed in the present paper can possibly be used for other compensated metals with open orbits, or for the same cadmium if specular carrier reflection becomes realizable in it.

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