Kinetic theory of the scattering of atoms by a resonant standing light wave

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A theory is developed of the scattering of two-level atoms by a standing light wave of frequency close to that of an atomic transition. It is shown that for a natural transition width greater than the recoil energy of the atom and for interaction times exceeding the spontaneous decay time the microscopic equations for the atomic density matrix reduce to a single kinetic equation for the atomic distribution function. This is a Fokker-Planck type equation and it includes the optical pressure force and the momentum diffusion tensor. It is shown that in a strong field the maximum force is restricted, whereas the diffusion tensor increases proportionally to the wave intensity. It is concluded that at high standing wave intensities the main cause of the scattering is the diffusion of atomic momenta.

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1. INTRODUCTION. FORMULATION OF THE PROBLEM

It is well known that a self-consistent approach to the description of the motion of atoms in a resonant optical field requires the use of the equation for the atomic density matrix which includes internal and translational states of the atom (see, for example, the review in Ref. 1). This rigorous approach has made it possible to complete recently in the main theory of the motion of atoms in the simplest case of a monochromatic traveling light wave.[1–10] It has been found that the only parameter of the theory governing the qualitative nature of the evolution of the atomic ensemble is the ratio of the recoil energy $r=\hbar k^2/2M$ ($k=\omega/c$ is the wave vector of a light wave those frequency ω is close to an atomic transition frequency ω_0) to the natural width 2γ of an atomic transition line.[5,7] The appearance of this ratio $\varepsilon=r/\gamma=v_r/(2\gamma/k)$ ($v_r=\hbar k/M$ is the recoil velocity) is associated with the circumstance that in the difference integrodifferential equations for the elements of the Wigner density matrix $\rho_{ij}(\mathbf{r},\mathbf{p},t)$ the change in these elements as a result of absorption (emission) of a photon is governed by the shift in the atomic velocity by an amount equal to the recoil velocity v_r, and the characteristic velocity scale of the change in the density matrix Δv is governed by the natural width of the atomic transition line: $\Delta v\sim 2\gamma/k$.

Depending on the value of the parameter ε, we can distinguish two main situations differing in respect of the qualitative nature of the evolution of the density matrix. If $\varepsilon\gtrsim 1$, the considerable magnitude of the recoil momentum causes the density matrix to change abruptly due to discrete changes in the atomic momentum and this is true for any times of the atom-field interaction. In the opposite case of $\varepsilon\ll 1$ (typical of optical transitions in atoms) the nature of the evolution of the density matrix undergoes a qualitative change at times $t\sim\gamma^{-1}$. If $t\lesssim\gamma^{-1}$, the density matrix oscillates strongly with time, whereas for $t\gg\gamma^{-1}$, spontaneous decay smooths out the density matrix variations. Consequently, in describing the atomic motion in the $t\gg\gamma^{-1}$ case it is sufficient to use not all the density matrix, but only the sum of its diagonal components, i.e., the distribution function $w(\mathbf{r},\mathbf{p},t)$. This distribution function satisfies a kinetic equation of the Fokker-Planck type.[4,5,7,8] The exact coefficients in this kinetic equation have been found recently.[7,8] The solutions of the equation have been analyzed already.[7,9,10]

In the theory of motion of atoms in resonant light fields we need to consider not only the case of a traveling wave but also the motion of atoms in a resonant standing wave, which is of fundamental importance. The motion of atoms in such a standing field configuration has been analyzed earlier for the $\varepsilon=\infty$ ($\gamma=0$) case.[11–13] In the case $\varepsilon\ll 1$, which is of practical importance, the motion of atoms has been considered only in the case of low field intensities (assuming that the saturation parameter is $G\lesssim 1$), since attention has been concentrated on the possibility of radiative cooling of atoms[14–18] first suggested by Hänsch and Schawlow[14] for an isotropic light field.

A recent experiment[19] has drawn attention to another problem in the theory of atomic motion, which is the scattering of atoms by a strong standing wave. This problem has been considered before,[20–24] but without allowance for the finite width of an atomic transition line the results of Refs. 20–24 cannot be compared with real experiments carried out under the conditions when $\varepsilon\ll 1$. Moreover, the duration of the interaction of atoms with a standing wave field in the experiments of Ref. 19 was never less than γ^{-1}, whereas the approach of Refs. 20–24 is limited[1)] to the times $t\ll\gamma^{-1}$.

Our aim will be to develop a theory of the scattering of atoms by a resonant standing light wave under conditions typical of experiments on atoms. We mean specifically that the probability of spontaneous decay γ should be greater than the recoil energy r ($r/\gamma\ll 1$) and the scattering time should be longer than the spontaneous decay time: $t>\gamma^{-1}$. Since these are the conditions for the kinetic stage of the evolution of the density matrix, we shall be formulating a kinetic theory of the atomic scattering. This includes the derivation of the kinetic equation for the atomic distribution function and an analysis of the scattering on the basis of the kinetic equation.

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2. MICROSCOPIC EQUATIONS

We shall consider the simplest model of a two-level atom and assume that an atomic transition occurs between two nondegenerate levels $|g\rangle$ and $|e\rangle$, of which the lower level $|g\rangle$ is the ground state and the upper level $|e\rangle$ decays to the ground state with the spontaneous decay probability 2γ. We shall assume that a standing light wave is plane

$$\mathbf{E}(\mathbf{r}, t)=2\mathbf{e}E_0 \cos\omega t \cos kz \tag{1}$$

and, to be specific, we shall postulate that it is polarized linearly along the x axis: $\mathbf{e}=\mathbf{e}_x$.

We shall begin with the equations for the atomic density matrix and we shall define the latter as the density matrix averaged over the vacuum states.[1] Consequently, in the Hamiltonian of the "atom +field (1)" system

$$\hat{H}=\hat{H}_0-\frac{\hbar}{2M}\nabla^2+\hat{V} \tag{2}$$

we shall include the energy of the internal state of the atom, the kinetic energy of its translational motion, and the energy of the dipole interaction of the atom with the field

$$\hat{V}=-\hbar^{-1}\hat{\mathbf{d}}\mathbf{E}. \tag{3}$$

Next, we shall introduce the density matrix in the Wigner representation

$$\rho(\mathbf{r}, \mathbf{p}, t)=(2\pi\hbar)^{-3}\int \rho(\mathbf{r}-\mathbf{x}/2, \mathbf{r}+\mathbf{x}/2, t)\, e^{i\mathbf{p}\mathbf{x}/\hbar}\, d\mathbf{x} \tag{4}$$

and we shall write down the equations for its elements

$$\left.\begin{aligned}
&i\frac{d}{dt}\rho_{ee}(\mathbf{r}, \mathbf{p}, t)=\int V_{eg}(\mathbf{k}', t)\rho_{ge}(\mathbf{r}, \mathbf{p}-{}^1/_2\hbar\mathbf{k}', t)e^{i\mathbf{k}'\mathbf{r}}\,d\mathbf{k}'\\
&-\int V_{ge}(\mathbf{k}', t)\rho_{eg}(\mathbf{r}, \mathbf{p}+{}^1/_2\hbar\mathbf{k}', t)e^{i\mathbf{k}'\mathbf{r}}\,d\mathbf{k}'-2i\gamma\rho_{ee}(\mathbf{r}, \mathbf{p}, t),\\
&i\frac{d}{dt}\rho_{eg}(\mathbf{r}, \mathbf{p}, t)=\int V_{eg}(\mathbf{k}', t)[\rho(\mathbf{r}, \mathbf{p}-{}^1/_2\hbar\mathbf{k}', t)\\
&-\rho_{ee}(\mathbf{r}, \mathbf{p}+{}^1/_2\hbar\mathbf{k}', t)]e^{i\mathbf{k}'\mathbf{r}}\,d\mathbf{k}'-i\gamma\rho_{eg}(\mathbf{r}, \mathbf{p}, t),\\
&i\frac{d}{dt}\rho_{gg}(\mathbf{r}, \mathbf{p}, t)=-\int V_{eg}(\mathbf{k}', t)\rho_{ge}(\mathbf{r}, \mathbf{p}+{}^1/_2\hbar\mathbf{k}', t)e^{i\mathbf{k}'\mathbf{r}}\,d\mathbf{k}'\\
&+\int V_{ge}(\mathbf{k}', t)\rho_{eg}(\mathbf{r}, \mathbf{p}-{}^1/_2\hbar\mathbf{k}', t)e^{i\mathbf{k}'\mathbf{r}}\,d\mathbf{k}'+2i\gamma\int d\mathbf{n}\,\Phi(\mathbf{n})\rho_{ee}(\mathbf{r}, \mathbf{p}+\mathbf{n}\hbar k, t).
\end{aligned}\right\} \tag{5}$$

In the system (5) the symbol d/dt denotes the hydrodynamic derivative

$$\frac{d}{dt}=\frac{\partial}{\partial t}+\mathbf{v}\frac{\partial}{\partial \mathbf{r}}, \tag{6}$$

where $\mathbf{v}=\mathbf{p}/M$. The matrix elements $V_{ij}(\mathbf{k}', t)$ are the Fourier transforms of the nondiagonal matrix elements of the operator $\hat{V}$. The function $\Phi(\mathbf{n})$ determines the relative probability of spontaneous emission of a photon in the direction of a unit vector $\mathbf{n}$ (Refs. 3–5 and 7):

$$\Phi(\mathbf{n})=(3/8\pi)[1-(\mathbf{n}\mathbf{e}_x)^2]. \tag{7}$$

The matrix elements of the interaction operator $V_{ij}(\mathbf{k}', t)$ for the field (1) have the following form in the standard approximation of a rotating wave:

$$V_{eg}(\mathbf{k}', t)=-V_0 e^{-i\Omega t}[\delta(\mathbf{k}-\mathbf{k}')+\delta(\mathbf{k}+\mathbf{k}')], \tag{8}$$

where $\Omega=\omega-\omega_0$, $\mathbf{k}=k\mathbf{e}_x$, $V_0=dE_0/2\hbar$, and d is the matrix element of the projection of the dipole moment onto the x axis. We shall substitute Eq. (8) into Eq. (5) and alter the definition of the nondiagonal elements ρ_{eg}:

$$\rho_{eg}\exp(i\Omega t)\to\rho_{eg}. \tag{9}$$

Then, the final system of microscopic equations for the atomic motion is

$$\left.\begin{aligned}
&i\frac{d\rho_{ee}^{\,0}}{dt}=V_0(\rho_{eg}^{+1}-\rho_{ge}^{-1})e^{ikz}+V_0(\rho_{eg}^{-1}-\rho_{ge}^{+1})e^{-ikz}-2i\gamma\rho_{ee}^{\,0},\\
&i\frac{d\rho_{eg}^{\,0}}{dt}=-V_0(\rho_{gg}^{-1}-\rho_{ee}^{+1})e^{ikz}-V_0(\rho_{gg}^{+1}-\rho_{ee}^{-1})e^{-ikz}-(\Omega+i\gamma)\rho_{eg}^{\,0},\\
&i\frac{d\rho_{gg}^{\,0}}{dt}=V_0(\rho_{ge}^{+1}-\rho_{eg}^{-1})e^{ikz}+V_0(\rho_{ge}^{-1}-\rho_{eg}^{+1})e^{-ikz}+2i\gamma\int d\mathbf{n}\,\Phi(\mathbf{n})\rho_{ee}(\mathbf{r}, \mathbf{p}+\mathbf{n}\hbar k, t),
\end{aligned}\right\} \tag{10}$$

where, for simplicity, we have introduced the notation

$$\rho_{ij}^{\,l}=\rho_{ij}(\mathbf{r}, \mathbf{p}+{}^1/_2l\hbar\mathbf{k}, t);\quad l=0, \pm1. \tag{11}$$

The microscopic difference integrodifferential equations in the system (10) describe the motion of an atom at any time t and for any ratio of the parameters r and γ. In going over to the analysis of the principal case

$$\varepsilon=r/\gamma\ll1, \tag{12}$$

we shall allow for the fact that if the condition of Eq. (12) is satisfied, the photon momentum $\hbar k$ is small compared with the characteristic scale $\Delta p\sim M\gamma/k$ of the change in the density matrix, and we shall expand the elements of the density matrix in powers of $\mu=\hbar k/\Delta p\sim2\varepsilon$. As pointed out earlier,[7] this procedure corresponds to consideration of the kinetic state of the evolution of the system (10), i.e., to the case when the time is $t\gg\gamma^{-1}$. Moreover, we shall introduce the following Bloch variables in these equations:

$$w=\rho_{gg}^{\,0}+\rho_{ee}^{\,0},\quad u=\rho_{gg}^{\,0}-\rho_{ee}^{\,0},\quad c=\rho_{eg}^{\,0}+\rho_{ge}^{\,0},\quad is=\rho_{eg}^{\,0}-\rho_{ge}^{\,0}.$$

We then obtain the following microscopic equations for the new variables and these equations correspond directly to the kinetic stage of the evolution:

$$\frac{dw}{dt}=2\hbar kV_0\frac{\partial c}{\partial p_z}\sin kz+\frac{1}{2}\hbar^2k^2\gamma\sum_{i=1,2,3}\alpha_{ii}\frac{\partial^2(w-u)}{\partial p_i^{\,2}}+\dots, \tag{13a}$$

$$\frac{du}{dt}=2\gamma(w-u)-4V_0 s\cos kz+\dots, \tag{13b}$$

$$\frac{dc}{dt}=-\gamma c-\Omega s+2\hbar kV_0\frac{\partial w}{\partial p_z}\sin kz+\dots, \tag{13c}$$

$$\frac{ds}{dt}=-\gamma s+\Omega c+4V_0 u\cos kz+\dots, \tag{13d}$$

where α_{ii} are given by[3–5,7]

$$\alpha_{11}=\alpha_{xx}={}^2/_{10},\quad \alpha_{22}=\alpha_{yy}={}^2/_5,\quad \alpha_{33}=\alpha_{zz}={}^2/_5.$$

The system (13) is written down using only the first terms of the expansions required in the subsequent discussion.

3. KINETIC EQUATION IN THE FIRST ORDER WITH RESPECT TO THE RECOIL MOMENTUM

One closed kinetic equation for the atomic distribution function $w(\mathbf{r}, \mathbf{p}, t)$ will be obtained from the microscopic equation system (13) by applying familiar Bogolyubov analysis[26] to the system (13) (see also Refs. 27 and 28). This approach simplifies greatly the description of the motion of an atomic ensemble during the kinetic stage of the evolution. This simplification of the system (13) is physically obvious in the $\varepsilon\ll1$ case. Since the recoil momentum $\hbar k$ is small, the change in the translational state of an atom is a slower process than the change in

the internal state and the latter state becomes adjusted to the still varying translational state. Formally, the fast relaxation of the internal state means that in the kinetic stage of the evolution the functions $h = u, c, s$, become functionals of the distribution function:

$$h(\mathbf{r}, \mathbf{p}, t) = h(\mathbf{r}, \mathbf{p}; w(\mathbf{r}, \mathbf{p}, t)). \tag{14}$$

The explicit form of the functional dependence (14) can be deduced directly from Eqs. (13b)–(13d). Since these equations contain only the function w and its derivatives with respect to the momentum p_t, the most general form of the function h represents series in powers of the gradients $\partial w/\partial p_t$,

$$h(\mathbf{r}, \mathbf{p}, t) = H^{(0)} w - \frac{\gamma}{4V_0} \hbar k H^{(1)} \frac{\partial w}{\partial p_z} + \dots, \tag{15}$$

where $H^{(i)}$ are the functions only of z and p_t since the whole time dependence is contained in the function w. The factor $-\gamma/4V_0$ is introduced in Eq. (15) with the aim of obtaining a more compact form of further expressions.

We can now use Eq. (15) to derive successively the equation for w in different orders with respect to the parameter $\mu = 2\varepsilon \ll 1$. We shall first consider the zeroth and first orders. In the zeroth order with respect to μ the equation for the atomic distribution function follows from Eq. (13a):

$$\frac{dw}{dt} = \frac{\partial w}{\partial t} + \mathbf{v} \frac{\partial w}{\partial \mathbf{r}} = 0. \tag{16}$$

This equation describes the free motion of atoms because the recoil effect is ignored in the zeroth approximation.

The equation in the first order with respect to the recoil momentum is obtained by deriving from Eqs. (13b)–(13d) the function c in the zeroth order with respect to μ and then substituting it on the right-hand side of Eq. (13a), which already contains the first power of $\hbar k$. Before doing this, we shall calculate the derivatives on the left-hand side of Eqs. (13b)–(13d) in the zeroth order in μ:

$$\left(\frac{dh}{dt}\right)^{(0)} = H^{(0)} \left(\frac{dw}{dt}\right)^{(0)} + v_z w \frac{\partial H^{(0)}}{\partial z} = v_z w \frac{\partial H^{(0)}}{\partial z}, \tag{17}$$

where $(dw/dt)^{(0)}$ is calculated using Eq. (16). Applying Eq. (17), we find that the system (13) for $h = u, c$, and s yields directly the system of equations for the functions $H^{(0)} = U^{(0)}$, $C^{(0)}$, and $S^{(0)}$:

$$\left.\begin{aligned} v_z \frac{\partial U^{(0)}}{\partial z} &= 2\gamma(1 - U^{(0)}) - 4V_0 S^{(0)} \cos kz, \\ v_z \frac{\partial C^{(0)}}{\partial z} &= -\gamma C^{(0)} - \Omega S^{(0)}, \\ v_z \frac{\partial S^{(0)}}{\partial z} &= -\gamma S^{(0)} + \Omega C^{(0)} + 4V_0 U^{(0)} \cos kz. \end{aligned}\right\} \tag{18}$$

The solution of the system (18) can be obtained by expanding the functions $H^{(0)} = U^{(0)}$, $C^{(0)}$, and $S^{(0)}$ as the Fourier series

$$H^{(0)} = \sum_{n=-\infty}^{+\infty} H_n^{(0)} e^{inkz}. \tag{19}$$

After expansion of Eq. (19), the system (18) reduces to the following infinite system of equations

$$\left.\begin{aligned} (2\gamma + inkv_z) U_n^{(0)} &= 2\gamma\delta_{n,0} - 2V_0 (S_{n-1}^{(0)} + S_{n+1}^{(0)}), \\ (\gamma + inkv_z) C_n^{(0)} &= -\Omega S_n^{(0)}, \\ (\gamma + inkv_z) S_n^{(0)} &= \Omega C_n^{(0)} + 2V_0 (U_{n-1}^{(0)} + U_{n+1}^{(0)}), \end{aligned}\right\} \tag{20}$$

whose solution has already been found.[29]

Bearing in mind the possibility of using the results of Ref. 29, i.e., assuming that the solution of the system (20) is known, we shall derive from Eq. (13a) the final equation for w in the first order in μ:

$$\frac{\partial w}{\partial t} + \mathbf{v} \frac{\partial w}{\partial \mathbf{r}} + \frac{\partial}{\partial p_z} (Fw) = 0, \tag{21}$$

where

$$F = -2\hbar k V_0 C^{(0)} \sin kz = F^0 + \sum_{n=1}^{\infty} F_n^c \cos 2nkz + F_n^s \sin 2nkz, \tag{22}$$

$$F^0 = 2\hbar k V_0 \operatorname{Im} C_1^{(0)}, \tag{23}$$

$$F_n^c = 2\hbar k V_0 \operatorname{Im} (C_{2n+1}^{(0)} - C_{2n-1}^{(0)}), \tag{24}$$

$$F_n^s = 2\hbar k V_0 \operatorname{Re} (C_{2n+1}^{(0)} - C_{2n-1}^{(0)}). \tag{25}$$

Thus, in the first order with respect of the recoil momentum the kinetic equation (21) for the distribution function $w(\mathbf{r}, \mathbf{p}, t)$ describes the motion of an atomic ensemble acted upon by the optical pressure force F. It should be pointed out that since the exact form of the force (22) has been found earlier,[29] the Liouville equation (21) can be written down directly omitting the above derivative. However, since we shall continue the derivation of the equations to the next order in μ, we shall definitely require Eqs. (17)–(20).

Equation (21) describes the motion of an arbitrary atomic ensemble. In the important case of a spatially wide atomic distribution of width $\Delta z \gg \lambda = 2\pi/k$ the function w should be averaged over the wavelength of light λ:

$$\bar{w} = \frac{1}{\lambda} \int_0^{\lambda} w \, dz. \tag{26}$$

Averaging of Eq. (21) produces a much simpler equation

$$\frac{\partial \bar{w}}{\partial t} + \mathbf{v} \frac{\partial \bar{w}}{\partial \mathbf{r}} + \frac{\partial}{\partial p_z} (F^0 \bar{w}) = 0, \tag{27}$$

from which, solving the system (2) and using Eq. (23), we obtain[29]

$$F^0 = -2\hbar k \gamma \frac{\operatorname{Im}[(\Omega/(\gamma + ikv_z))Q]}{1 + 2\operatorname{Re} Q}, \tag{28}$$

where Q is an infinite converging fraction

$$Q = \cfrac{p_0}{1 + \cfrac{p_1}{1 + \dots}}, \tag{29}$$

in which

$$p_n = G \frac{\gamma + in_1 kv_z}{2\gamma + in_2 kv_z} \frac{2\gamma^2}{\Omega^2 + (\gamma + in_1 kv_z)^2}, \tag{30}$$

and the integers n_1 and n_2 are defined by

$$\begin{aligned} n_1 &= n+1, \quad n_2 = n, \text{ if } n \text{ is even;} \\ n_1 &= n, \quad n_2 = n+1, \text{ if } n \text{ is odd.} \end{aligned}$$

The parameter $G = 2V_0^2/\gamma^2$ in Eq. (30) represents saturation.

In averaging Eq. (21) we have assumed that the maximum values of the harmonics of the force F_n are of the same order of magnitude as the average force F^0. A rigorous justification of this assumption will be given in Sec. 5.

4. KINETIC EQUATION IN THE SECOND ORDER WITH RESPECT OF THE RECOIL MOMENTUM

It follows from Eq. (13a) that the equation for w can be obtained in the second order with respect to μ by finding first the function u in the zeroth order and the function c in the first order. The first problem is solved by Eqs. (18)–(20). The second problem requires the use of Eqs. (13b)–(13d) in the first order in $\hbar k$. Since in the first order, we have

$$h=H^0w-\frac{\gamma}{4V_0}\hbar kH^{(1)}\frac{\partial w}{\partial p_z}, \tag{31}$$

the derivatives on the left-hand sides of Eqs. (13b)–(13d) considered in the first order are

$$\begin{aligned}\left(\frac{dh}{dt}\right)^{(1)}&=H^{(0)}\left(\frac{dw}{dt}\right)^{(1)}+v_zw\frac{\partial H^{(0)}}{\partial z}-\frac{\hbar k\gamma}{4V_0}v_z\frac{\partial H^{(1)}}{\partial z}\frac{\partial w}{\partial p_z}\\&=-H^{(0)}\frac{\partial}{\partial p_z}(Fw)+v_zw\frac{\partial H^{(0)}}{\partial z}-\frac{\hbar k\gamma}{4V_0}v_z\frac{\partial H^{(1)}}{\partial z}\frac{\partial w}{\partial p_z}.\end{aligned} \tag{32}$$

Using Eqs. (31) and (32), we can reduce the system (13b)–(13d) in the first order in μ to the following equations for $H^{(1)}=U^{(1)}$, $C^{(1)}$, and $S^{(1)}$:

$$\left.\begin{aligned}&\frac{\gamma v_z}{4V_0}\frac{\partial U^{(1)}}{\partial z}+\frac{\gamma^2}{2V_0}U^{(1)}+\gamma S^{(1)}\cos kz=-\frac{U^{(0)}F}{\hbar k},\\&\frac{\gamma v_z}{4V_0}\frac{\partial C^{(1)}}{\partial z}+\frac{\gamma^2}{4V_0}C^{(1)}+\frac{\gamma\Omega}{4V_0}S^{(1)}+2V_0\sin kz=-\frac{C^{(0)}F}{\hbar k},\\&\frac{\gamma v_z}{4V_0}\frac{\partial S^{(1)}}{\partial z}+\frac{\gamma^2}{4V_0}S^{(1)}-\frac{\gamma\Omega}{4V_0}C^{(1)}-\gamma U^{(1)}\cos kz=-\frac{S^{(0)}F}{\hbar k}.\end{aligned}\right\} \tag{33}$$

The system (33) can be solved by analogy with Eq. (19), i.e., expanding the functions $H^{(1)}=U^{(1)}$, $C^{(1)}$, and $S^{(1)}$ in a Fourier series. In this case, the system (33) yields an infinite system of equations for $H_n^{(1)}$ whose solution may be represented in the form of infinite converging fractions. We shall not give these very cumbersome results but write down directly the general form of the equation for w in the second order in μ:

$$\frac{\partial w}{\partial t}+\mathbf{v}\frac{\partial w}{\partial \mathbf{r}}+\frac{\partial}{\partial p_z}(Fw)=\sum_{i=1,2,3}\frac{\partial^2}{\partial p_i^2}(D_{ii}w), \tag{34}$$

where the force $\mathbf{F}$ is defined by the relationships (22)–(25), and the components D_{ii} define the diffusion tensor in the momentum space:

$$D_{ii}=D_{ii}^0+\sum_{n=1}^{\infty}D_{ii}^{cn}\cos 2nkz+D_{ii}^{sn}\sin 2nkz, \tag{35}$$

$$D_{ii}^0=\tfrac{1}{2}\hbar^2k^2\gamma[\alpha_{ii}(1-U_0^{(0)})+\delta_{3,i}\operatorname{Im}C_1^{(1)}], \tag{36}$$

$$D_{ii}^{cn}=\tfrac{1}{2}\hbar^2k^2\gamma[-2\alpha_{ii}\operatorname{Re}U_{2n}^{(0)}+\delta_{3,i}\operatorname{Im}(C_{2n+1}^{(1)}-C_{2n-1}^{(1)})], \tag{37}$$

$$D_{ii}^{sn}=\tfrac{1}{2}\hbar^2k^2\gamma[2\alpha_{ii}\operatorname{Im}U_{2n}^{(0)}+\delta_{3,i}\operatorname{Re}(C_{2n+1}^{(1)}-C_{2n-1}^{(1)})]. \tag{38}$$

We can thus see that in the second order in respect of the recoil momentum the motion of atoms is described by a kinetic equation of the Fokker-Planck type.

In the case of a spatially wide distribution ($\Delta z\gg\lambda$), which is important in practice, Eq. (34) reduces to a Fokker–Planck equation for the function $\bar{w}$:

$$\frac{\partial \bar{w}}{\partial t}+\mathbf{v}\frac{\partial \bar{w}}{\partial \mathbf{r}}+\frac{\partial}{\partial p_z}(F^0\bar{w})=\sum_{i=1,2,3}\frac{\partial^2}{\partial p_i^2}(D_{ii}^0\bar{w}), \tag{39}$$

where the average force F^0 is defined by Eqs. (28)–(30) and the components of the momentum diffusion tensor D_{ii}^0 are given in Eq. (36). It should be noted that Eq. (34) is averaged on the assumption that not only the maximum values of the harmonics F_n do not exceed F^0, but also the maximum values of D_{ii}^n do not exceed D_{ii}^0. We shall justify these assumptions later.

We shall conclude a discussion of the structure of the kinetic equation by pointing out that, although strictly speaking it is valid only for times $t\gg\gamma^{-1}$, we can in fact use it for times $t\geq 3\gamma^{-1}$ because it has been shown by Mandel[6] that three spontaneous decays are sufficient for a considerable smoothing out of the density matrix.

5. DISCUSSION OF EQUATIONS

The Liouville equations (21) and (27) and the Fokker-Planck equations (34) and (39) are determined entirely by the optical pressure forces and diffusion tensors which occur in them. We shall now consider this force and diffusion drawing attention to their dependences on the parameter G (i.e., on the light wave intensity).

We shall first consider the average force F^0. The dependence of this force on the velocity of an atom along the z axis is governed mainly by the sign of the detuning Ω. If $\Omega<0$, the force is directed mainly against the atomic velocity (Fig.1). If $\Omega>0$, the force F^0 is directed along the atomic velocity. For $\Omega=0$ the force vanishes. All the small-scale changes in the force (Fig. 1) due to multiresonance processes specific to a standing wave have been discussed in detail earlier (see the review in Ref. 1 and the experimental investigations reported in Refs. 30 and 31). From the point of view of the scattering of atoms the main interest lies in the contribution of these processes to the optical pressure force at high intensities of the standing wave. A direct

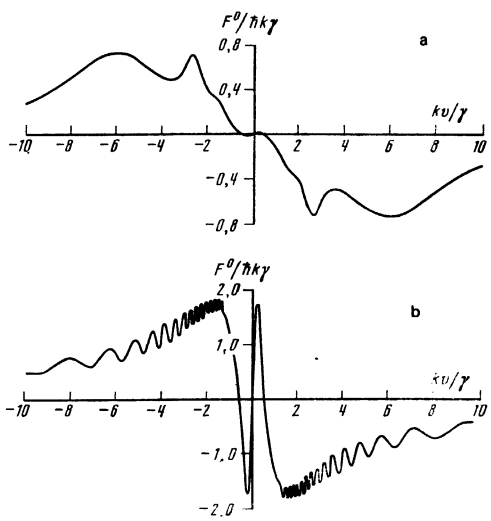

FIG. 1. Dependence of the average optical pressure force on the atomic velocity $v=v_z$ for $\Omega/\gamma=-3$, $G=10$ (a), and $G=1000$ (b).

calculation of the force F^0 shows that these processes increase very slightly the force even when the saturation parameter G increases considerably. For example, for $\Omega/\gamma = -3$ an increase in the saturation parameter by a factor of 100 [Figs. 1(a) and 1(b)] increases the force only twofold. A rigorous analysis shows that an increase in the force F^0 occurs only up to a certain value because the maximum force F^0 is restricted.

We shall prove this conclusion by establishing the order of the maximum value of the force F^0. Since F^0 is governed by the value of $C_1^{(0)}$ and the order of the latter is identical with the order of the function $C^{(0)}$, we shall turn back to the system (18). Neglecting the derivatives $\partial H^{(0)}/\partial z$, which do not alter the order of the equations, and assuming that $\cos kz = 1$, we obtain directly from the system (18) an estimate of $C^{(0)}$, which gives the following estimate for the force:

$$F^0 \sim \hbar k\Omega \frac{8V_0^2}{8V_0^2+\gamma^2+\Omega^2}. \tag{40}$$

An analysis of Eq. (40) shows that the maximum force F^0 at high values of G,

$$G \gg G_0 = 1+\Omega^2/\gamma^2, \tag{41}$$

is limited to

$$F^0_{max} \sim \hbar k\Omega. \tag{42}$$

Repeating the above discussions, we can easily establish that both the maximum values of F_n^c and F_n^s and the maximum value of the total optical pressure force F are restricted by Eq. (42). In connection with the estimate (42) it should be noted that among the components of the force there is none which could be called the induced force in the sense used in Refs. 20–23 and 25, since the maximum values of all the components of the force are limited by the scattering of photons at the rate Ω and not at the rate $V_0 = dE_0/2\hbar$.

We shall now consider the average diffusion tensor. This is represented in Eq. (36) by two terms:

$$D_{ii}^0 = D_{ii}^0(1) + \delta_{3i}D_{33}^0(2). \tag{43}$$

The first of these terms describes the "continuous" diffusion[3-5,7] due to fluctuations of the direction of the spontaneous emission of photons. The second term describes discrete diffusion[5,7] whose existence is due to fluctuations of the number of the scattered photons for a fixed scattering direction (z axis). This type of diffusion is related directly to the finite width of an absorption line. The dependences of both terms on the velocity of an atom along the z axis are symmetric curves whose widths are of the same order as the width of the velocity distribution of the force F^0. At high wave intensities these curves exhibit small-scale oscillations which are due to, as in the case of the force F^0, multiresonance processes.[1]

The order of $D_{ii}^0(1)$ can be determined by repeating an analysis of the system (18) and estimating the order of $1 - U^{(0)}$; this shows that the order of the components of the continuous diffusion tensor is given by the expression

$$D_{ii}^0(1) \sim \frac{1}{2}\hbar^2k^2\gamma\alpha_{ii}\frac{8V_0^2}{8V_0^2+\gamma^2+\Omega^2}. \tag{44}$$

It follows from this expression that in the case of high values of $G \gg G_0$ corresponding to the condition (41) the maximum values of $D_{ii}^0(1)$ are limited to

$$D_{ii}^0(1) \sim {}^1/_2\hbar^2k^2\gamma\alpha_{ii}. \tag{45}$$

The order of the z-th component of the discrete diffusion tensor can be found from Eq. (33). We shall find this order by substituting in Eq. (33) the estimates of $H^{(0)}$ from the system (18) and by substituting $\sin kz = 1$ in Eq. (33); this gives

$$D_{33}^0(2) \sim \hbar^2k^2\gamma\frac{4V_0^2}{\gamma^2+\Omega^2}\left[1+\frac{32\Omega^2V_0^2}{(\gamma^2+\Omega^2+8V_0^2)^2}\right]. \tag{46}$$

The expression (46) shows that in the $G \gg G_0$ case the discrete diffusion tensor increases proportionally to the light wave intensity. It should be noted that the selection of the numerical values of the trigonometric functions in estimates of the solutions of the systems (18) and (33) is not accidental. For example, if in the analysis of the system (33) we follow the procedure in the case of the system (18) and assume that $\cos kz = 1$, we find that the role of the saturation parameter G in the discrete diffusion tensor is distorted.

The above considerations can be applied also to the tensors D_{ii}^{cn} and D_{ii}^{sn}. The first parts of these tensors can be described by the estimates (44) and (45), whereas the second parts are described by the estimate (46).

6. GENERAL ANALYSIS OF THE SCATTERING AND COMPARISON WITH EXPERIMENTAL RESULTS[19]

We shall now use the above kinetic equation to discuss the features of the scattering of atoms by a resonant standing wave characteristic of the times $t > \gamma^{-1}$. Bearing in mind the conditions in real experiments, we shall consider an atomic ensemble to be a beam of atoms whose spatial width Δz is considerably greater than the wavelength of light λ. When this condition is satisfied, the problem of the scattering of atoms reduces to an analysis of Eq. (39). Moreover, since the main interest lies in the variation of the distribution function along the z axis (along the direction of propagation of a light wave), It is permissible—in principle—to consider only the one-dimensional equation and to ignore the trivial diffusive broadening of the distribution along the x and y axes. In this case the equation of interest to us becomes

$$\frac{\partial\bar{w}}{\partial t} + v\frac{\partial\bar{w}}{\partial z} + \frac{\partial}{\partial p}(F^0\bar{w}) = \frac{\partial^2}{\partial p^2}(D^0\bar{w}). \tag{47}$$

In Eq. (47) we have omitted the index z and we have used v and p to denote the velocity and momentum of an atom along the z axis.

According to Eq. (47) and the analysis in Sec. 5, the scattering of atoms at high wave intensities is governed by the average force F^0 and by the discrete diffusion coefficient[2)] $D^0 \approx D^0(2)$. We can distinguish here two qualitatively different scattering patterns. In one of them, the scattering of atoms is governed by the force F^0, and in the other it is governed by the diffusive broadening of the velocity distribution. In the former case we observe a complex deformation of the velocity distribution

associated with the steep dependence of the force F^0 on the velocity (Fig. 1). In the latter case, the diffusion broadens symmetrically the velocity distribution.

We shall estimate the intensity of a wave at which the first type of scattering changes to the second. We shall assume that the scattering time t is fixed. Then, equating the change in the velocity under the action of the force (42) to the diffusion-induced change in the velocity in the case when the diffusion coefficient is given by Eq. (46), we find that the critical saturation parameter G_{cr} is

$$G_{cr}\sim {}^1/_2(\Omega/\gamma)^4\gamma t. \tag{48}$$

We can thus see that the role of the force F^0 is greatest for $G_0 \ll G \ll G_{cr}$. If $G \gtrsim G_{cr}$, the result of the scattering can only be the diffusive broadening of an atomic beam. The change in the transverse velocity of the atomic beam under the action of this force can be estimated from Eq. (42):

$$\Delta v\sim v_r\Omega t. \tag{49}$$

We shall now consider the experiments reported in Ref. 19. In these experiments the frequency of a light wave was identical with the frequency of one of the transitions between the components of the hyperfine structure of the states $3S_{1/2}$ and $3P_{3/2}$ of the sodium atom. The saturation parameter was $G \sim 10^4$. The interaction time of atoms with the field exceeded γ^{-1}.

For $\Omega = 0$ the force is $F^0 = 0$ and the *z-th* component of the diffusion tensor D^0 obtained from Eqs. (33) and (36) are

$$D^0=\hbar^2k^2\gamma\left(\alpha_{33}\frac{\operatorname{Re}Q(0)}{1+2\operatorname{Re}Q(0)}+G\frac{\gamma^2}{\gamma^2+(kv)^2}\right), \tag{50}$$

where $Q(0) = Q(\Omega = 0)$. Since $G \gg G_0 = 1$, it follows from Eqs. (47) and (50) that the experimental results of Ref. 19 can be described by the simple diffusion equation

$$\frac{\partial\bar{w}}{\partial t}=\frac{\partial^2}{\partial v^2}(D\bar{w}), \tag{51}$$

where

$$D=D^0(2)/M^2=\gamma v_r^2G\gamma^2/[\gamma^2+(kv)^2]. \tag{52}$$

[In Eq. (51) we have omitted the term $v\partial\bar{w}/\partial z$ which plays no significant role in the case of short scattering.]

The expressions (51) and (52) are in full agreement with the symmetric equation for the velocity distribution given in Ref. 19 and with the square-root dependence of the broadening on the wave intensity, because Eqs. (51) and (52) give the diffusive broadening:

$$\Delta v\sim(Dt)^{1/2}\propto G^{1/2}. \tag{53}$$

Therefore, our analysis shows that in the case of moderately weak atomic transitions ($\gamma \gg r$) and for times $t > \gamma^{-1}$, the scattering of atoms by a standing light wave is governed by the evolution of the atomic distribution under the action of a velocity-dependent force which is constant in space and by the diffusion of the atomic velocities. In the case of a strong light wave ($G \gg G_{cr}$) the scattering is entirely due to the diffusion of atoms in the velocity space.

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[1)] In this connection we should mention an attempt[25] to explain the results of Ref. 19 using the representations in Refs. 20-23 known to be invalid under the experimental conditions in Ref. 19.

[2)] It should be pointed out that the scattering under the influence of continuous diffusion typical of $G \lesssim 1$ has recently been discussed in Ref. 32.

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