Cyclotron resonance maser with background plasma

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An analysis is made of a new type of plasma source of high-frequency electromagnetic oscillations in which the energy is stored by fast electrons and it is converted "explosively" into electromagnetic radiation. The oscillator consists of a magnetic trap filled with a cold plasma and placed in a quasioptical resonator. This background plasma in the trap acts as a nonlinear element which maintains a high excitation threshold in the energy accumulation process and is then rapidly deactivated, giving rise to "giant" electromagnetic radiation pulses. A theory is developed and used to interpret the experimental results on explosive generation of electromagntic radiation in laboratory magnetic traps.

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1. INTRODUCTION

The capabilities of modern pulsed electronic devices in respect of the accumulation of energy in their "active substance" and feasibility of control and duration of the resultant radiation are very limited. This is due to the fact that generation of electromagnetic oscillations in such devices is mostly based on transit effects so that the power and duration of emission of electromagnetic radiation are governed entirely by the electron beam parameters.

The theoretical capabilities of electronic devices can be greatly extended by accumulating the energy of the active substance (electrons) inside a resonator and by releasing this energy in the form of electromagnetic radiation. However, we can easily see that simple accumulation of energetic electrons (even when their space charge is compensated by a background of ions) is practically impossible because realistic accumulation times are many orders of magnitude greater than the characteristic times for the development of instabilities even in the case of fairly low densities of a nonequilibrium plasma.

On the other hand, it is known that energy can be accumulated in an active substance in a laser emitting giant pulses. This accumulation is possible because of the presence of a nonlinear element which maintains a high excitation threshold during the energy accumulation process and is then rapidly deactivated ("saturation") establishing favorable conditions for an abrupt release of a large amount of energy in the form of radiation. A cold background plasma can act as a similar nonlinear element in electronic devices.

We shall consider an electronic device in the form of a magnetic trap with energetic electrons and we shall assume that the trap is inside a quasioptical resonator. Generation of waves in systems of this kind is based on the cyclotron instability resulting from the transverse anisotropy of energetic electrons. A prototype of such an electronic device with a very low plasma density is a cyclotron resonance maser, which is a well known source of centimeter and millimeter wavelengths.[1] The accumulation and release of energy in devices of this kind accumulation and release of energy in devices of this kind can be controlled with a background plasma by two methods.

The first method is in many respects analogous to that used in lasers and it consists in the following. In the presence of a sufficiently dense cold plasma the excitation threshold is governed by the bulk attenuation of waves because of the Coulomb collisions. An important feature is the fact that the frequency of collisions in a totally ionized plasma decreases rapidly on increase in the electron temperature. The radiation, which appears in the system when the instability threshold is exceeded, heats the background plasma and this reduces greatly the excitation threshold causing explosive growth of the instability. A large proportion of the electron energy is then converted into electromagnetic radiation. The pumping is simply provided by the accumulation of energetic electrons inside the trap, which is already filled with a sufficiently dense cold plasma. The energy can be provided by direct injection of fast electrons into the trap, and also by high-frequency and cyclotron heating of the plasma.

The second method is based on the characteristics of propagation of electromagnetic radiation in a magnetoactive plasma. In the case of quasioptical systems of size $l \gg \lambda$ (λ is the radiation wavelength in the medium) the direct emission of radiation into vacuum from a region with a high refractive index is possible only in the case of a sufficiently rarefied plasma, when the following inequality is obeyed[2]

$$\omega_{pL} \leqslant \omega \leqslant \Omega_L,$$

where $\omega_{pL} = (4\pi e^2 n_L/m)^{1/2}$ is the plasma (Langmuir) frequency at the center of the trap; n_L is the plasma density; ω is the radiation frequency; Ω_L is the electron gyrofrequency in the central section of the system. In the case of a denser plasma with $\omega_{pL} > \Omega_L$ the radiation cannot escape into vacuum from the region where it is generated.[1)] Therefore, if energetic electrons accumulate in a magnetic trap filled with a cold plasma placed inside a high-Q resonator, and if the inequality $\omega_{pL} > \Omega_L$ is obeyed, significant generation of radiation begins only from the moment when the amplification length becomes comparable with the length of the system. On the other hand, if the density of the particles in the trap is reduced sufficiently rapidly and if the inequality $\omega_{pL} < \Omega_L$ is obeyed, the appearance of feedback via the

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resonator due to the free escape of radiation into vacuum gives rise to explosive generation of waves and ensures conversion into radiation of a considerable proportion of the energy of the particles stored in the trap. Then, the duration of an electromagnetic pulse governed by the velocity of passing through the "cutoff point" $\Omega_L=\omega_{pL}$ may be very short because the necessary relative change in the plasma density is just a few percent. Escape from the cutoff point can be accelerated greatly by nonlinear effects, particularly by the deposition of a hot plasma at the trap ends during growth of the instability. The velocity of passing through the cutoff point is then governed by the stability growth increment. The cutoff point may be crossed in a natural way in the process of plasma decay. A convenient method of pumping energy into a trap and of crossing the cutoff point is provided by magnetic compression of a plasma. In the case of adiabatic magnetic compression the electron gyrofrequency rises more rapidly then the plasma frequency $[\Omega_L/\omega_{pL}\propto B_L^{1/2}(t)$, where B_L is the magnetic field at the center of the trap] so that when we initially have $(\Omega_L/\omega_{pL})_0<1$, the cutoff point is crossed automatically during compression.

We shall analyze theoretically the above effects and give a quantitative interpretation of some of the laboratory observations of "explosive" generation of electromagnetic radiation in magnetic traps. It is interesting to note that similar but giant oscillators occur in space, particularly in the radiation belts of the Earth.[3]

2. QUALITATIVE THEORY

We shall assume that a magnetic trap placed inside a resonator contains a two-component plasma consisting of a fairly dense, cold, equlibrium plasma (density n_x and temperature T_x) and an anisotropic admixture of hot electrons (density n and average temperature T). We shall be interested only in the case when

$$n_x\gg n,\quad n_xT_x\ll nT. \tag{1}$$

When the inequalities (1) are satisfied, a small proportion of the energy of the active substance (energetic electrons) is used to control the nonlinear element (background plasma). Moreover, if $n_x\gg n$, then the reactive component of the impedance of the system changes only slightly during relaxation of a hot plasma, which ensures frequency stability in the generation of electromagnetic radiation. The inequality $T\gg T_x$ allows us also to ignore the collisionless gyroresonant absorption of waves in the background plasma.

In a relatively dense plasma the branch of oscillations most suitable for the generation of microwave radiation is the electromagnetic branch with frequencies $\omega<\Omega_e$, i.e., the branch for which the frequencies are less than the electron gyrofrequency, and this branch reduces to helicons (whistlers) for high refractive indices. The propagation of this branch along a magnetic field corresponds to a circularly polarized wave and the direction of rotation of the polarization vector is the same as that of the rotation of an electron. As already mentioned, the advantages of this branch include the feasibility of direct escape of radiation into vacuum from a region characterized by $\nu=\omega_p^2/\omega^2\leqslant 1$ (ω_p is the plasma frequency). The escape of radiation from a plasma with $v>1$ is more problematic and usually the intensity of the emitted radiation is very low.

It is known from Ref. 4 that in the case of transverse anisotropy of a hot plasma, the oscillation branch under consideration should exhibit an instability due to the normal Doppler effect[2]

$$\omega-\Omega_e=k_zv_z, \tag{2}$$

where k_z and v_z are the components of the wave vector and velocity of a particle projected onto the direction of the magnetic field **B**. Certain important conclusions can be drawn from a linear theory of the cyclotron instability. We shall obtain specific estimates by assuming that the distribution function of hot electrons is

$$f(\mu,\ v)=C_\alpha\mu^\alpha\exp(-v^2/v_0^2), \tag{3}$$

where $\mu=(B_L/2B)v_\perp^2$ is the adiabatic invariant; $v_\perp$ is the transverse (perpendicular to **B**) component of the particle velocity v; $v=|\mathbf{v}|$; $mv_0^2/2=T$; the index L identifies the values in the central section of the trap. The index α in Eq. (3) represents the degree of transverse anisotropy. When the anisotropy is due to the presence of a loss cone, the index α can be expressed in the following way in terms of the mirror ratio σ:

$$\frac{1}{2}\frac{\bar{v}_\perp^2}{\bar{v}_z^2}=\alpha+1=\frac{2\sigma+1}{2(\sigma-1)}. \tag{4}$$

Substituting Eq. (3) into the general expression for the instability growth increment γ and assuming that $\mathbf{k}\parallel\mathbf{B}$, we find that[3,4]

$$\left.\begin{array}{l}\gamma=2\pi^{1/2}\dfrac{n_L(\Omega_e-\omega)^{1/2}}{n_x\beta_0\omega^{1/2}\omega_p}[\alpha\Omega_e-(\alpha+1)\omega]\exp\left\{-\dfrac{(\Omega_e-\omega)^3}{\omega\omega_p^2\beta_0^2}\right\},\\ \beta_0=v_0/c,\quad k=\omega_p\omega^{1/2}/c(\Omega_e-\omega)^{1/2}.\end{array}\right\} \tag{5}$$

It follows from the system (5) that the frequency ω_m corresponding to the maximum increment γ_m and its value for a given ratio n_L/n_x are governed by two parameters: the anisotropy α and $\delta=\Omega_e^2/\beta_0^2\omega_p^2$. We shall be interested in the case when both $\alpha\gg 1$ and $\delta\gg 1$, and the radiation frequency is close to the electron gyrofrequency. As shown below, this corresponds to the maximum efficiency of the system.

Applying the system (5), we find that the average increment is

$$\langle\gamma\rangle=\tau_g^{-1}\int_{-l/2}^{l/2}\frac{\gamma dz}{v_g},\quad \tau_g=\int_{-l/2}^{l/2}\frac{dz}{v_g},$$

where v_g is the group velocity and l is the trap length. We then have

$$\langle\gamma\rangle=1,4\gamma_my^{1/2}\exp(-y^3), \tag{6}$$

$$\gamma_m=\frac{\pi^{1/2}\alpha n_L}{4\delta^{2/3}n_x}\Omega_L,\quad 1-\omega_m/\Omega_L=(6\delta)^{-1/3}, \tag{7}$$

$$y=\delta^{1/3}(1-\omega/\Omega_L),\quad 6\delta/\alpha^3<1,\quad \delta=\Omega_L^2/\beta_0^2\omega_{PL}^2.$$

As shown in the Appendix, the relationships (6) and (7) represent correctly the radiation characteristics also during the nonlinear stage of the instability. We shall now estimate the efficiency η of the system under discussion. We shall do this using the laws of conservation of energy and momentum for the wave–particle interaction in a magnetic field:

$$\Delta p_z = -\hbar k_z, \quad \Delta(p_\perp^2/2) = -s\hbar\Omega_e, \quad s=0, \pm 1, \pm 2, \ldots, \tag{8}$$

where p_z and $p_\perp$ are the components of the momentum of a particle, and $\hbar$ is the Planck constant. The following expressions are obtained from Eq. (8) subject to the cyclotron resonance condition (2):

$$\Delta(v^2/2) = -\hbar\omega/m, \quad \Delta p_z/\Delta p_\perp = (\omega-\Omega_e)/\Omega_e. \tag{9}$$

We can thus see that if $\omega \ll \Omega_e$, the energy of a particle is conserved to within ω/Ω_e and a high efficiency can be expected only at $\omega \approx \Omega_e$. In this case it follows from Eq. (9) that the change in the transverse momentum predominates. Then, the maximum possible efficiency can be estimated as the relative change in the energy of the particles along the $v_z = \mathrm{const}$ lines when these particles reach the boundary with a loss cone in the process of emission. If the initial distribution function is approximated by a triangle with legs $\Delta v_\perp = \nu_0$ and $\Delta \nu_z = (\sigma-1)^{1/2} v_0$, we find that

$$\eta = \frac{2-2\sigma/3+{}^4/_3(\sigma-1)}{2+{}^4/_3(\sigma-1)} = \frac{2}{3}, \quad \sigma\to 1. \tag{10}$$

One-third of the energy is carried away by the energetic electrons reaching the loss cone. However, this has a positive aspect because the "spent" active substance is driven automatically out of the oscillator. This circumstance makes it possible to utilize fully the whole store of the energetic electrons in a magnetic trap. Moreover, a constant degree of anisotropy is maintained in the system during the instability growth and, consequently, the emission frequency remains stable (see the Appendix).

3. GIANT PULSE EMISSION

The condition for the self-excitation of an oscillator, derived allowing for the bulk attenuation and for the finite Q factor of the resonator, can be written in the following way in the case of interest to us:

$$\gamma_m = \nu + \nu_R, \quad \nu_R = \tau_g^{-1}|\ln R|, \tag{11}$$

where ν is the attenuation due to collisions in the background plasma; τ_g is the time of group propagation between the ends of the trap; R is the reflection coefficient of the ends. If $v_L = \omega_{pL}^2/\omega^2 < 1$, the value of R is governed by the quality of the resonator mirrors and in the case of a denser plasma ($v_L > 1$) the value of R is much smaller ($R \ll 1$) and it is governed by the fine effects in the interaction wave.

The self-excitation threshold (11) can be reached in the process of accumulation of fast electrons in a trap (i.e., during the growth of γ_m) or by relaxation of the background plasma (i.e., by reduction in ν or ν_R).

The structure of the equations describing the nonlinear stage of the instability growth is governed by the degree of monochromaticity of the radiation and homogeneity of the magnetic field. In the case of a sufficiently wide (on the frequency scale) wave packet or in the case of a sufficiently strong inhomogeneity of the magnetic field, when there are no particles trapped by the wave potential, we can use the well-known quasilinear theory of relaxation of an anisotropic plasma, which includes the kinetic equation for the velocity distribution function of the particles and the equation of energy transfer for the waves.[6]

Under real conditions we frequently encounter the case when the characteristic relaxation time τ_D of a hot plasma is considerably greater than the time of the bounce oscillations of electrons between magnetic mirrors τ_B and the time of group propagation of an electromagnetic signal between the trap ends τ_g. In this case the quasilinear equations can be averaged over the short times τ_g and τ_B. The necessary calculations are made in the Appendix. The final form of the system of equations describing the cyclotron instability in the "two-level" approximation is

$$dx/d\tau = -\eta^{-1}xw + i, \tag{12}$$

$$dw/d\tau = (x - \varepsilon_\nu - \varepsilon_R)w, \tag{13}$$

where the following dimensionless variables are introduced:

$$x = N/N_0, \quad \tau = \gamma_0 t, \quad i = J/N_0\bar{\mu}, \quad w = \sigma Wl/N_0\bar{\mu},$$

$$\varepsilon_\nu = \nu/\gamma_0, \quad \varepsilon_R = \nu_R/\gamma_0, \quad \eta = \omega(\bar{\mu}-\mu_k)/\Omega_L\bar{\mu},$$

η is the efficiency of the system, N is the number of hot electrons in a magnetic force tube with a unit cross section at the end; N_0 and γ_0 are the initial values of N and of the instability increment of Eq. (19); w is the relative density of the wave energy; i is the source; the rest of the notation is explained in the Appendix.

The system (12), (13) is outwardly identical with the equations describing the dynamics of a two-level laser considered in the rate approximation.[7] The presence of a plasma is manifested by the nonlinear relationships of ε_ν and ε_R with x and w, which determine the feasibility of generating giant pulses in a cyclotron resonance maser. Depending on the ratio of the parameters ε_ν and ε_R, we obtain one of the regimes mentioned in the Introduction: when $\varepsilon_\nu \gg \varepsilon_R$ and the instability threshold is suppressed nonlinearly, or when $\varepsilon_\nu \ll \varepsilon_R$ and the escape from the cutoff point takes place. We shall use the system (12), (13) in developing a quantitative theory of these two oscillation regimes.

a. Regime with nonlinear supression of instability threshold ($\varepsilon_\nu \gg \varepsilon_R$, $v_L < 1$)

Suppresion of the instability threshold is due to reduction in ν caused by heating of the background plasma by electromagnetic radiation. In a totally ionized plasma the collision frequency is

$$\nu = 50 n_x T_x^{-3/2} \ \mathrm{cm}^{-3} \cdot \mathrm{deg}^{-3/2}. \tag{14}$$

The system (12)–(14) can be closed by adding an equation describing the process of heating of the background plasma:

$$d\theta/d\tau = \varepsilon_\nu w - \theta/\tau_0, \quad \theta = T_x n_x \sigma l/N_0\bar{\mu}, \tag{15}$$

where τ_0 is the characteristic temperature relaxation time in units of γ_0^{-1}.

We shall consider the case when $i=0$ and when the excitation threshold is reached as a result of slow relaxation of the cold plasma density:

$$n_x=n_{x0}(1+t/\tau_x)^{-1}.$$

If time is measured from the moment corresponding to the attainment of the excitation threshold of Eq. (11), then

$$dx/d\tau=-\eta^{-1}xw(1+t/3\tau_x), \quad (16)$$

$$dw/d\tau=(x+t/3\tau_x-\varepsilon_v-\varepsilon_R+\varepsilon_v t/\tau_x)w, \quad (17)$$

$$d\theta/d\tau=\varepsilon_v(1-t/\tau_x)w-\theta/\tau_0, \quad (18)$$

$$\varepsilon_v=50n_{x0}T_x^{-3/2}/\gamma_0\approx(\theta_0/\theta)^{3/2}.$$

In an analysis of fast processes on a time scale of $\gamma_0^{-1}\ll\tau_x, \tau_0$, we can drop the term θ/τ_0 from Eq. (18): the terms $\sim t/\tau_x$ in Eqs. (16)–(18) are important only at the moment when the instability begins to grow. We shall ignore these terms and assume that $\varepsilon_R\ll\varepsilon_v$; then, we obtain the following energy integral from Eqs. (16)–(18):

$$\eta x+w+\theta=\eta+\theta_0+w_0. \quad (19)$$

The second integral can easily be found from Eqs. (16) and (18):

$$x=e^{-a(\xi-1)}, \quad (20)$$

where $a=2\theta_0/5\eta$ and $\xi=(\theta/\theta_0)^{5/2}$.

The intensity of the radiation can be expressed in the following way in terms of θ:

$$w-w_0=\eta[1-e^{-a(\xi-1)}]-\theta_0(\xi^{2/5}-1). \quad (21)$$

The time dependence of ξ is given by the equation

$$d\xi/d\tau=5w_0/2\theta_0+a^{-1}(1-e^{-a(\xi-1)})-5/2(\xi^{2/5}-1). \quad (22)$$

The relationships (19)–(22) describe fully the radiation and plasma characteristics. In accordance with Eq. (1) we assume that $a\ll1$. In this approximation, we find

$$w_{max}=\eta, \quad T_x/T_{x0}\approx[3\ln a^{-1}/2\theta_0]^{2/5}, \quad (23)$$

where w_{max} is the maximum intensity of the radiation inside the resonator, which is reached in a time $\tau_\beta\sim\gamma_0^{-1}$. Outside the resonator the radiation characteristic w is governed by the value of ε_R. If $\varepsilon_R\ll1$, then

$$\tilde{w}_{max}\approx\varepsilon_R w_{max}, \quad t\sim\tau_0/|\ln R|\gg\gamma_0^{-1}.$$

b. Regime with emission of radiation at cutoff point ($\varepsilon_v\to0$, $v_L\sim1$)

As pointed out above, the oscillation conditions change radically when the plasma frequency corresponding to the total density of the cold and hot components decreases and becomes comparable with the gyrofrequency: $\omega_{pL}\leqslant\Omega_L$. Then, unstable waves can escape directly into vacuum and can be amplified repeatedly by reflection from the resonator mirror.

In the linear approximation the characteristic time of activating the instability on passing through the cutoff point is governed by the draft time τ_d of the plasma frequency across the gain profile of Eq. (6) [or Eq. (A. 12)] during plasma decay. The change in the plasma density necessary to cross the half-width of the gain profile from the threshold value of the increment $\gamma=\gamma_n\sim0$ to $\gamma=\gamma_m$ can be obtained from Eq. (A. 2).

$$|\Delta y|\approx y_m\approx\delta^{1/3}\Delta\omega/\Omega_L\approx\delta^{1/3}\Delta n/2n\approx0.55, \quad \Delta n/n=\delta^{-1/3}. \quad (24)$$

The time to cross the gain profile is

$$\tau_d\approx\delta^{-1/3}\tau_x\ll\tau_x, \quad (25)$$

where τ_x is the relaxation time of the plasma density in the absence of the instability.

The activation time of the instability can be reduced considerably by rapid deposition of the hot plasma at the trap ends in the process of instability growth. Clearly, these effects become important when the hot plasma density exceeds a critical value given by Eq. (24):

$$1>n_L/n_x\geqslant\delta^{-1/3} \quad (26)$$

This regime can be described qualitatively employing the following model ($i=0$):

$$dx/d\tau=-\eta^{-1}w\varphi(x), \quad dw/d\tau=[x\varphi(x)-\varepsilon_R]w, \quad (27)$$

$$\varphi(x)=\begin{cases} y^{1/2}\exp(-y^3), & 0\leqslant y\leqslant0.55 \\ \varphi_m\approx0.7, & y\geqslant0.55, \end{cases} \quad (28)$$

where, as before, we have $y=\delta^{1/3}(1-\omega/\Omega_L)$. The function $\varphi(x)$ represents the dependence of the growth increment on the plasma density when the radiation escapes on attainment of the cutoff point. The increment reaches its maximum value and remains constant, and the emission frequency during the subsequent reduction in the plasma density changes slightly in accordance with the equation $y(x,\omega)\approx0.55$.

Bearing in mind that on crossing the cutoff point we have $\omega\approx\omega_{pL}$, we shall represent y in the form ($n\ll n_x$)

$$y=\delta_0^{1/3}(\Delta-n/2n_{x0}), \quad \Delta=1-\omega_{pL}^{(0)}/\Omega_L, \quad \delta_0=(\Omega_L/\beta_0\omega_{pL}^{(0)})^2, \quad (29)$$

where $\omega_{pL}^{(0)}=(4\pi e^2n_{x0}/m)^{1/2}$ and n_{x0} is the density of the background plasma at the center of the trap corresponding to the moment when the instability is activated ($y\approx0$). In the process of instability growth the value of n varies within the limits: $0\leqslant n/2n_{x0}\leqslant\Delta$, which corresponds to $0\leqslant y\leqslant y_m=\Delta\delta^{1/3}=0.55$.

Using Eqs. (28) and (29), we can rewrite the equations in the system (27) in the form

$$\left.\begin{aligned} \frac{d\zeta}{d\tau}&=-\delta^{1/6}\eta^{-1}\zeta(\Delta-\zeta)^{1/2}w, \\ \frac{dw}{d\tau}&=[\zeta_0^{-1}\delta^{1/6}\zeta(\Delta-\zeta)^{1/2}-\varepsilon_R]w, \end{aligned}\right\} \quad (30)$$

where $\zeta=n/2n_{x0}$ and $\zeta_0=n_0/2n_{x0}$. If $\varepsilon_R\ll1$, we can use the approximate integral $w+\eta\zeta/\zeta_0=\text{const}$. We then obtain the equation

$$d\zeta/d\tau=-\delta_0^{1/6}\eta^{-1}\zeta(\Delta-\zeta)^{1/2}[w_0+2\eta(1-\zeta/\zeta_0)]. \quad (31)$$

The characteristic duration of the leading edge of radiation pulse can be estimated by assuming that the quantity ζ, which occurs as a separate factor on the right-hand side of Eq. (31), is constant. Integrating Eq. (31), we find that

$$\left(1-\frac{\zeta}{\zeta_0}+w_0\eta^{-1}\right)\left\{\frac{\varepsilon_R}{\zeta_0}+1-\frac{\zeta}{\zeta_0}+\left[\frac{\varepsilon_R}{\zeta_0}\left(\frac{\varepsilon_R}{\zeta_0}+1-\frac{\zeta}{\zeta_0}\right)\right]^{1/2}\right\}^{-1}$$

$$=\frac{\zeta_0w_0}{2\eta\varepsilon_R}\exp(\zeta_0^{1/2}\delta^{1/6}\tau). \quad (32)$$

If we bear in mind that $(\zeta_0\delta^{1/3})^{1/2}\sim y_m^{1/2}\sim 1$, we find that the characteristic instability growth time is equal to the maximum value of the increment (6) at the cutoff moment.

We have assumed above that in the case of a dense plasma ($v_L>1$) the radiation does not escape from the generation region to the ends of the trap. However, it is known[2] that if the angle ψ between the wave vector **k** and the magnetic field **B** is sufficiently small, the waves in question can escape from the region where $v_L>1$ into vacuum because of the interaction of normal waves in the case when $v_L\approx 1$. The effect is characterized quantitatively by the transmission coefficient $D=\exp(-2p)$, where the interaction parameter p in the $\Omega_L/\omega>1$ case is given by[2]:

$$2p=\pi\omega l_N\psi_0^2/c(1-\omega/\Omega_e)^{1/2}, \tag{33}$$

where ψ_0 is the value of ψ in the $v_L=1$ interaction region and $l_N^{-1}=N^{-1}dN/dz$. The upper limit of the leakage effect can be found by taking the minimum angle $\psi\sim\lambda_L/d$, where λ_L is the central section of the trap and d is the transverse size of the plasma. Then, allowing for refraction, we find that $\psi_0=\psi kc/\omega=\lambda_0/d$, where $\lambda_0=2\pi c/\omega$ is the wavelength in vacuum. Consequently, we have

$$2p\geqslant 2p_{min}\approx\frac{2\pi^2\lambda_0 l_N}{d^2(1-\omega/\Omega_e)^{1/2}}. \tag{34}$$

We can ignore the leakage effect if $2p>1$. Under the experimental conditions of Ref. 8, when $\lambda_0\sim d$, this criterion is known to be satisfied. However, when the ratio λ_0/d is sufficiently small, the leakage effect must be allowed for.

4. DISCUSSION

An abrupt activation of the instability and generation of intense and short radiation pulses during plasma relaxation in a magnetic trap has been observed in a number of experiments. We shall consider particularly the experiments of Alikaev, Glagolev, and Morozov[8] in which these effects have been manifested most clearly. In their experiments a magnetic trap was placed in a high-Q resonator and a plasma was created in the process of microwave breakdown at a frequency $\omega_0=2\Omega_L$. This created both cold and hot components of a plasma with $n_0/n_{x0}\ll 1$ inside the trap. The energy of fast electrons was $T\sim 10-40$ keV. The cold plasma had initially a density of $n_{x0}\sim 10^{12}-10^{13}$ cm^{-3} and then it decayed slowly (with a characteristic time $\tau_x\approx 10^2$ μsec). During the decay stage at the moment when $\omega_{pL}\approx\Omega_L$ an intense electromagnetic pulse of duration $t_p\sim 0.1$ μsec appeared at a frequency $\omega\approx(0.6-0.9)\Omega_L$. Practically the whole energy of the hot plasma (~80%) was transformed into radiation and directed partly to the trap ends.

These experimental results can be explained satisfactorily by the model of giant pulses in the case of passing through the cutoff point, discussed in Sec. 3b. Under the experimental conditions of Ref. 8 it was found that $\delta\approx\beta_0^{-2}\sim 10-40$ so that in accordance with Eq. (A.14) the radiation frequency should satisfy the ratio $\omega/\Omega_L\approx 1-0.55\delta^{-1/3}\approx 0.7-0.9$. Using the value $\Omega_L\approx 1.4\times 10^{10}$ sec^{-1}, we find from Eqs. (6) and (A.13) that $\gamma_m\approx(1-0.25)\times 10^9 n/n_x$. We now have to satisfy the inequality $\gamma_m\gg t_p^{-1}\approx 10^7$ sec^{-1}, so that $n/n_x\approx 0.1-0.03$, which is in agreement with the experimental results of Ref. 8 [in this case the linear time for passing through the cutoff point τ_d of Eq. (25) is ~10 μsec $\gg t_p$]. We shall now estimate both ν and τ_g^{-1}: at $T_x\approx 10$ eV, we have $\nu\approx 10^6$ sec^{-1} and $\tau_g^{-1}=(l/v_g)^{-1}\approx 5\times 10^8$ sec^{-1}. It follows that in order to excite giant pulses in the case when $\gamma_m>\tau_g^{-1}|\ln R|$ under the experimental conditions of Ref. 8, we have to satisfy the condition $R>0.9$, which has indeed obeyed in these experiments. Our theory allows us to explain also finer effects, such as the excitation of lower frequencies at latter moments, i.e., according to Eq. (7), at lower densities.

It should be pointed out that when energetic electrons escape to the trap ends, the plasma becomes charged and this may have a significant effect on the instability growth. This effect can be ignored if the time for the emission of a pair consisting of an energetic electron and a cold ion from a Debye boundary layer of the background plasma is less than the instability growth time γ_m^{-1}. Under the experimental conditions of Ref. 8 the Debye radius is $r_d<10^{-1}$ cm and the velocity of a pair is $v_{pair}\approx(T/M)^{1/2}\sim 10^8$ cm/sec. It follows that the time of escape of a pair $t_{pair}\sim 10^{-9}$ sec is comparable with the time taken by an energetic electron to cross the trap and is considerably less than the pulse duration $t_p\sim 10^{-7}$ sec.

In the systems with crossing of the cutoff point the maximum gain in the resonator length is limited: $\tau_g\gamma_m<1$; there is also a limit on the minimum ratio $n/n_x\geqslant\delta^{-1/3}$. There are no such restrictions in the case of nonlinear suppression of the instability threshold (Sec. 3.a). However, it is quite difficult to ensure that the collision frequency is sufficiently high, since the cold plasma density is limited by the inequality $\omega_{pL}\leqslant\Omega_L$.

Nevertheless, estimates indicate that in the case of nonlinear suppression of the instability threshold we can, in principle, generate high-power electromagnetic radiation of millimeter and submillimeter wavelengths.

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APPENDIX

We shall now write down a system of quasilinear equations describing the cyclotron instability of an anisotropic plasma in an inhomogeneous magnetic trap. We shall assume that a quasimonochromatic electromagnetic wave of circular polarization with a frequency $\omega\lesssim\Omega_L$ and a wave vector $\mathbf{k}\parallel\mathbf{B}$ (helicon) is excited during the instability growth.

In the approximation of weak diffusion, when the electron velocity in a single transit between magnetic mirrors changes only slightly because of the interaction with the radiation and the amplification experienced by the waves over the length of the system is weak, the initial quasilinear equations[6] can be averaged over the period of the bounce oscillations of the particles and over the period of oscillations of a wave packet between the mirrors of the system (for details see Ref. 3). We thus obtain

$$\frac{df}{dt}=\langle j\rangle+(2\pi)^3\int\frac{k^2dk}{\tau_B}\left[\frac{\Omega_L}{\omega}\frac{\partial}{\partial\mu}+\frac{\partial}{\partial u}\right]G(z^*)W_k(z^*)\times\left[\frac{\Omega_L}{\omega}\frac{\partial f}{\partial\mu}+\frac{\partial f}{\partial u}\right], \tag{A.1}$$

$$\frac{dI_\omega}{dt}=(\langle\gamma\rangle-\nu-\tau_g^{-1}|\ln R|)I_\omega,\quad I_\omega=BW_kk^2/B_{max}, \tag{A.2}$$

$$\langle\gamma\rangle=\frac{16\pi^3}{mv_g\tau_g}\int du\,d\mu G(z^*)\left(\frac{\Omega_L}{\omega}\frac{df}{\partial\mu}+\frac{df}{\partial u}\right), \tag{A.3}$$

where integration is carried out over the region $|v_{zL}|\geqslant(\Omega_L-\omega)/k$; f is the electron velocity distribution function; W_k is the spectral energy density of cyclotron waves ($W=\int W_kk^2dk$ is the energy density of waves per unit volume); $\mu=(B_L/2B)v_\perp^2$; $mu=m(v_\perp^2+v_z^2)/2$ is the energy of a particle; z^* is the coordinate of the cyclotron resonance where $\omega-kv_z-\Omega_e=0$; Ω_L is the electron gyrofrequency in the central cross section of the trap; R is the reflection coefficient of the trap ends

$$\langle j\rangle=\tau_B^{-1}\int_{-l/2}^{l/2}jdz/v_z,\quad \tau_B=\int_{-l/2}^{l/2}dz/v_z;$$

j is the source of particles; the coefficient $G(z^*)$ is given by the expression

$$G(z^*)=\left[\frac{e^2\omega^2(\Omega_e(z)-\omega)\mu}{\pi c^2k^2\Omega_e(z)v_z(z)|\partial(\omega-\Omega_e-kv_z)/\partial z|}\right]_{z=z^*}.$$

In systems with a small mirror ratio $\sigma-1<1$, we can assume approximately that $z^*\approx0$. The values of τ_B and of $|\partial(\omega-kv_z-\Omega_e)/\partial z|$, which depend on the actual geometry of the magnetic field, will be assumed—for simplicity—to be given by

$$\tau_B\sim l/v_{zL},\quad |\partial(\omega-kv_z-\Omega_e)/\partial z|\sim\Omega_L/l,$$

where l is the characteristic longitudinal dimension of the system.

Going over in Eqs. (A.1)–(A.3) to variables of one-dimensional diffusion

$$\mu=\mu,\quad q=u-(\omega/\Omega_L)\mu, \tag{A.4}$$

we find that

$$\frac{\partial f}{\partial t}=\frac{\partial}{\partial\mu}\mu D(t)\frac{\partial f}{\partial\mu}+\langle j\rangle, \tag{A.5}$$

$$\langle\gamma\rangle=\frac{16\pi^3e^2\omega}{mc^2\Omega_Lk}\int_{\tilde q}^{\infty}dq\int_{\mu_k}^{\mu_m}\mu\frac{\partial f}{\partial\mu}d\mu, \tag{A.6}$$

$$D=\frac{8\pi^2e^2(2q)^{1/2}}{m^2c^2}\int W_k\,kdk,\quad \mu_m=\frac{q}{1-\omega/\Omega_L}-\frac{(\Omega_L-\omega)\Omega_L}{2k^2},$$

$$q=(\Omega_L-\omega)^2/2k^2,\ \sigma=B_{max}/B,\ k^2=\omega_p^2\omega/c^2(\Omega_e-\omega).$$

In the case of a sufficiently short trap,[3] Eq. (A.5) should be solved subject to the following initial and boundary conditions:

$$t=0,\ f=f_0(\mu,q),\ \mu=\mu_k(q),\ f=0,\ \mu=\mu_m,\ \partial f/\partial\mu=0, \tag{A.7}$$

where $\mu_k=u/\sigma\equiv q/(\sigma-\omega/\Omega_L)$ is a boundary with a loss cone. The general solution of Eq. (A.5) can be represented as a sum of eigenfunctions Z_s of the diffusion operator $(d/d\mu)\mu(d/d\mu)$. If this sum is confined to the function with the smallest eigenvalue Z_{s0}, we can write the solution of Eq. (A.5) in the form

$$f(t,\mu,q)=N(t)\Phi(q)Z_{s_0}(\mu,q), \tag{A.8}$$

where $N(t)$ is the number of particles in a force tube with a unit cross section at the end of f satisfies the normalization condition

$$2\pi\sigma\int\tau_Bdqd\mu\Phi(q)Z_{s_0}(\mu,q)=1. \tag{A.9}$$

The function $Z_{s0}(\mu,q)$ is

$$Z_{s_0}(\mu,q)=\pi s_0^2[J_0(2s_0\mu_k^{1/2})N_0(2s_0\mu^{1/2})-N_0(2s_0\mu_k^{1/2})J_0(2s_0\mu^{1/2})], \tag{A.10}$$

$$\int_{\mu_k}^{\mu_m}Z_{s_0}d\mu=1$$

where J_0 and N_0 are, respectively, zeroth-order Bessel and Neumann functions, and the eigenvalue is

$$s_0=0{,}15\pi(\mu_m^{1/2}-\mu_k^{1/2})^{-1}. \tag{A.11}$$

Assuming that

$$\Phi(q)=(\pi^{1/2}\sigma lq_0^{1/2})^{-1}\exp(-q/q_0)$$

and using (A.10), we find the approximate value of the increment

$$\langle\gamma\rangle\approx\frac{\Delta N\delta^{-2/3}}{n_{xL}\sigma l}\Omega_Ly^{1/2}\exp(-y^3), \tag{A.12}$$

where $\delta=\Omega_L^2/\beta^2\omega_{pL}^2$, $\beta^2=2q_0/c^2$, $y=\delta^{1/3}(1-\omega/\Omega_L)$, and the coefficient Δ is given by

$$\Delta=0{,}3\frac{\mu_m^{1/2}+\mu_k^{1/2}}{\mu_m^{1/2}-\mu_k^{1/2}}=0{,}3\frac{(\sigma-\omega/\Omega_L)^{1/2}+(1-\omega/\Omega_L)^{1/2}}{(\sigma-\omega/\Omega_L)^{1/2}-(1-\omega/\Omega_L)^{1/2}}.$$

The expression (A.12) has a maximum in its dependence on ω:

$$\gamma_m=0.6\Delta_m(N/\sigma ln_{xL})\delta^{-2/3}\Omega_L, \tag{A.13}$$

$$y_m=0.55,\quad 1-\omega/\Omega_L=0.55\delta^{-1/3}. \tag{A.14}$$

It follows that the inequality $1-\omega/\Omega_L\ll1$ assumed above can be satisfied if $\delta\gg1$.

Bearing in mind the small width of the spectrum of the excited waves, and also using Eqs. (A.8)–(A.13), we shall write Eqs. (A.1) and (A.2) in the form

$$\frac{dN}{dt}=-\frac{\sigma l}{\eta\mu}\gamma_mW+J, \tag{A.15}$$

$$\frac{dW}{dt}=(\gamma_m-\nu-\tau_g^{-1}|\ln R|)W, \tag{A.16}$$

where

$$\eta=\omega_m(\bar\mu-\mu_m)/\Omega_L\bar\mu,\quad \int\gamma k^2W_kdk\approx\gamma_mW,$$

$$N\bar\mu=\int_{\tilde q}^{\infty}(2q)^{-1/2}dq\int_{\mu_k}^{\mu_m}\mu f\,d\mu,\quad J=2\pi\sigma\int\tau_B\langle j(\mu,q)\rangle dq\,d\mu.$$

[1)] Strictly speaking, weak transfer of radiation to vacuum is possible because of the linear and nonlinear interactions between the waves (see below, Sec. 3b).

[2)] Under these conditions we can expect also a convective instability of electrostatic waves with frequencies close to harmonics of the electron gyrofrequency and of the upper hybrid resonance, but in the $\omega_p\leqslant\Omega_e$ case the increment of this instability is less than for electromagnetic waves (see Ref. 5).

[1] A. V. Gaponov, M. I. Petelin, and V. K. Yulpatov, Izv. Vyssh. Uchebn. Zaved. Radiofiz. **10**, 1414 (1967); A. A. Andronov, V. A. Flyagin, A. V. Gaponov, A. L. Goldenberg, M. I. Petilin, V. G. Usov, and V. K. Yulpatov, Izv. frared Phys. **18**, 385 (1978).
[2] V. L. Ginzburg, Rasprostranenie élektromagnitnykh voln v plazme, Nauka, M., 1967, p. 462 (The Propagation of Electromagnetic Waves in Plasmas, 2nd ed., Pergamon Press, Oxford, 1970).
[3] P. A. Bespalov and V. Yu. Trakhtengerts, Vopr. Teor. Plazmy **10**, 88 (1980).
[4] R. Z. Sagdeev and V. D. Shafranov, Zh. Eksp. Teor. Fiz. **39**, 181 (1960) [Sov. Phys. JETP **12**, 130 (1961)].
[5] V. V. Zheleznyakov and E. Ya. Zlotnik, Solar Phys. **43**, 431 (1975).
[6] A. A. Vedenov, E. P. Velikhov, and R. Z. Sagdeev, Nucl. Fusion Suppl. Pt. II, 465 (1962).
[7] Ya. I. Khanin, Dinamika kvantovykh generatorov (Laser Dynamics), Sovet-skoe Radio, M., 1975, p. 74.
[8] V. V. Alikaev, V. M. Glagolev, and S. A. Morozov, Plasma Phys. **10**, 753 (1968).

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