Critical properties of magnets with dislocations and point impurities

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(Submitted 31 October 1980)
Zh. Eksp. Teor. Fiz. 80, 2053–2067 (May 1981)

The critical behavior of a number of models of magnets with dislocations and point impurities is investigated within the framework of the field renormalization group technique. The considered dislocations are linear and produce around them small-scale inhomogeneities. If all the dislocations are parallel and are randomly distributed over the sample volume, then the critical exponents η and ν have longitudinal and transverse components that are connected with one another and with other critical exponents by scaling relations that differ from the usual ones. When point impurities are added to such a magnet, several different variants of critical behavior are possible, depending on the number n of the components. The exponents of this system with a single-component order parameter coincide with the exponents of a magnet with point impurities. At $n \rangle 4$ the system behaves as if it were to contain only dislocations. At intermediate n, the critical exponents take on new values, typical only of the considered magnet. Models are considered in which the dislocations are oriented along several directions. In this case, at $n = 1$, the exponents of an Ising magnet with point impurities are obtained. The values of the critical exponents for other n are obtained.

PACS numbers: 75.40.Dy, 75.30.Hx

1. INTRODUCTION

Both point impurities and dislocations can exert a strong influence on the anomalies of thermodynamic properties of magnets near a phase transition point. However, while the critical behavior of magnets with frozen-in point impurities have already been investigated by the renormalization-group method back in 1975,[1-3] the role of linear defects has been much less studied. The corrections to the mean-field theory for the presence of dislocations in the systems were obtained by Levanyuk *et al.*[4] Bariev[5] obtained a solution for the local magnetization of the planar Ising model near a linear defect. Dubrovskiĭ and Krivoglaz[6] used the local-transition-temperature approximation[7] to describe a magnet with dislocations that produce around themselves large-scale inhomogeneities. The present author considered earlier[8,9] a system of parallel linear defects randomly distributed in space; each defect produces in a transverse direction only a short-scale inhomogeneity in the coefficient of the quadratic term of the Hamiltonian. It is convenient to analyze such a model by the renormalization-group method. It was shown in Refs. 8 and 9 that the fixed point (FP) of the Gell-Mann–Low (GML) equations, which is a characteristic of the considered system, is a stable focus and is located at a finite distance from the origin of the phase plane. In such a magnet, the phase transition is therefore not broadened at low values of the dislocation density and potential. The calculations were performed by using a generalization of the ε-expansion method, and the critical exponents were obtained in the form of expansions in ε and ε_d, where ε_d is the dimensionality of an extended defects. For simplicity, the renormalization of the Green's function was not taken into account, although it will be shown in Sec. 2, that the exponent η differs from zero already in the single-loop approximation. In Sec. 2 we investigate this system within the framework of an approach close to that of Ginzburg and first used for the simple model of the $\lambda\varphi^4$ type.[10] The calculations will be made in a space of dimensionality D with dislocations of dimensionality $d \equiv \varepsilon_d$. Owing to the anisotropy of the renormalization of the Green's function, the exponents η and ν also turn out to be anisotropic and are connected with each other and with the other critical exponents by scaling relations that differ from the usual ones.

The systems investigated so far contained only one of several defect types. In Sec. 3 we consider a magnet in which frozen-in point impurities are added on top of the parallel discloations. It turns out that such a system, depending on the number of components of the order parameter, can exhibit a different critical behavior. In Secs. 4 and 5 we consider two more realistic systems, in which the dislocations can be oriented in different directions. In Sec. 4 there are three such directions and they are mutually perpendicular. In Sec. 5 it is assumed that the number of the dislocation-orientation directions is large.

2. PARALLEL DISLOCATIONS

We introduce first a model of a magnet with parallel dislocations. The effective Hamiltonian that describes a homogeneous magnet is of the form

$$H_0=\sum_{\alpha,\mathbf{k}}(\varkappa_0^2+s^2\mathbf{k}^2)|\varphi^\alpha(\mathbf{k})|^2 + \frac{\gamma}{4!}\sum_{\alpha,\beta}\sum_{\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3+\mathbf{k}_4=0}\varphi^\alpha(\mathbf{k}_1)\varphi^\alpha(\mathbf{k}_2)\varphi^\beta(\mathbf{k}_3)\varphi^\beta(\mathbf{k}_4), \tag{1a}$$

$\alpha=1,\ldots,n$ are the numbers of the spin-variable components and γ is the nonrenormalized interaction constant. If the magnet contains defects, we must add to the Hamiltonian (1a) the term

$$H_{imp}=\sum_i\sum_\alpha\sum_{\mathbf{q},\mathbf{k}}V(\mathbf{q})\exp(i\mathbf{q}\mathbf{r}_i)\varphi^\alpha(\mathbf{k})\varphi^\alpha(\mathbf{q}-\mathbf{k}), \tag{1b}$$

where i is the number of the impurity. If the impurities are pointlike, then the potential $V(q)=V$ is independent

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of the wave vector. It follows from the form of (1b) that all the order-parameter components interact with each impurity in like manner.

Let the point defects be ordered along parallel lines that are randomly distributed in the sample volume. It is assumed that the density c_t of the linear defects in the cross section is much lower than the flow direction, and their potential V is low enough to be able to use the Born approximation.

We shall use the standard diagram technique for impurity systems.[11] As shown in Ref. 8, for the model considered in this section a dashed impurity line corresponds to the following expression:

$$w\prod_j^d \delta(p_j^l)\sim c_t V^2\prod^d \delta(p_j^l), \tag{2}$$

where d is the dimensionality of the extended defect, usually equal to unity, p_j^l are different mutually perpendicular components of the wave vector and lie in the space of the d-dimensional defect. Here and elsewhere, unless specially stipulated, the indices l and t will label components of various quantities that are respectively longitudinal (i.e., parallel to the dislocations) and transverse.

In the case of point impurities, the simplest mass-operator single-loop diagram that contains a dashed line is independent of the wave vector. In our model, however, this diagram contains the δ-function (2), the presence of which causes the mass operator Σ to become dependent on the square of the longitudinal component of the wave vector even in first order. Since the calculations will be performed only in first order, the propagator can be represented in the form

$$G^{-1}=\mathbf{p}^2+\varkappa^2-\Sigma(\mathbf{p}_l^2)\approx\left(1-\frac{\partial\Sigma}{\partial\mathbf{p}_l^2}\bigg|_{\mathbf{p}_l^2=0}\right)\mathbf{p}_l^2+\mathbf{p}_t^2+\varkappa_0^2-\Sigma(0). \tag{3}$$

We introduce now the Green's-function renormalization constant

$$Z_l^{-1}=1-\frac{\partial\Sigma}{\partial\mathbf{p}_l^2}\bigg|_0, \tag{4}$$

as well as the transverse and longitudinal masses

$$\varkappa_t^2=\varkappa_0^2-\Sigma(0),\qquad \varkappa_l^2=Z_l\varkappa_t^2. \tag{5}$$

We can now use in the theory the propagator

$$\begin{aligned}G^{-1}&=Z_l^{-1}\mathbf{p}_l^2+\mathbf{p}_t^2+\varkappa_t^2\\&=Z_l^{-1}\mathbf{p}_l^2+\mathbf{p}_t^2+Z_l^{-1}\varkappa_l^2,\end{aligned} \tag{6}$$

therefore

$$G^{-1}(\mathbf{p}_l^2=0)=\mathbf{p}_t^2+\varkappa_t^2, \tag{7a}$$

$$G^{-1}(\mathbf{p}_t^2=0)=Z_l^{-1}(\mathbf{p}_l^2+\varkappa_l^2). \tag{7b}$$

Some complication is now involved in the determination of the invariant charges or the renormalized vertices. The point is that, as will be shown below, when a propagator with $\varkappa_t$ (6a) is used, the invariant charges must be defined differently than when (6b) is used. Therefore the GML equations and the expression for their fixed points have different forms in these two approaches. This is not dangerous, since the FP are not directly physical quantities. The expressions that connect the critical exponents with the values of the invariant charges in the PF of the GML equations are different in the case of calculations with $\varkappa_t$ than in calculations with $\varkappa_l$. We shall therefore have to consider both approaches and verify that both lead to the same values of the critical exponents.

We determine the invariant charges and require that in each perturbation-theory order the vertices be renormalized in like fashion. We use first the theory variant with the propagator (6a). The invariant charges are then of the form

$$\begin{aligned}\tilde{\gamma}_t&=J(D)\varkappa_t^{D-4}Z_l^{x}\Gamma(0),\\ \tilde{w}_t&=\frac{1}{(2\pi)^d}J(D-d)\varkappa_t^{D-4-d}Z_l^{-y}W(0),\end{aligned} \tag{8}$$

where Γ is a four-point vertex not cut along the dashed lines, and W is the impurity vertex. The external momenta of the vertices are set equal to zero. The label t in the symbols for the invariant charges means that the theory considered is constructed with the propagator (6a). The numerical factors were introduced for convenience. The powers of $\varkappa_t$ make it possible to make the GML equations dimensionless. The function $J(D)$ is expressed in terms of the derivative of the integral

$$I=(2\pi)^{-D}\int d^D\mathbf{k}(\mathbf{k}^2+\varkappa^2)^{-2}:$$

$$\varkappa^{6-D}\frac{\partial I}{\partial\varkappa^2}=-J(D)=\frac{\Gamma(3-D/2)}{2^{D+1}\pi^{D/2}}. \tag{9}$$

The powers x and y can be easily obtained by considering the single-loop diagrams included among the diagrams of Fig. 1. Substituting (6a) and (8) in the expression for the simplest single-loop diagram with two γ interactions, we obtain

$$Z_l^{-2x}Z_l^{d/2}=Z_l^{-x}. \tag{10}$$

The second factor in the left-hand side of this expression appeared in the calculation of the integral with the propagator (6a). Therefore $x=d/2$. Analogously, estimating a single-loop diagram with two impurity lines, we obtain

$$Z_l^{-2y}=Z_l^{-y}. \tag{11}$$

In contrast to (10), the left-hand side of (11) does not contain a second factor, inasmuch the term $Z_l p_l^2$ drops out of the propagator (6a), owing to the δ function (2) under the integral sign. Thus, $y=0$. It is easy to verify that equal powers x and y would be obtained by considering any other diagram. In the upshot

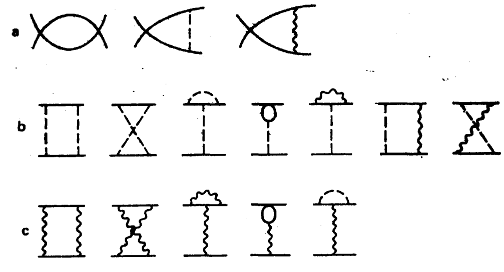

FIG. 1. One-loop diagrams that yield the contributions to the GML functions of a magnet with defects of two types: a) expansion terms of the vertex functions Γ, b and c) terms of the expansions of U and W.

$$\tilde{\gamma}_l = J(D)\varkappa_l^{D-4} Z_l^{d/2}\,\Gamma(0), \quad \tilde{w}_l = (2\pi)^{-d} J(D-d)\varkappa_l^{D-4-d}\, W(0). \tag{12}$$

To obtain similar results in the variant of the theory with $\varkappa_l$, it is convenient to rewrite the propagator (6b) as follows:

$$G^{-1} = Z_l^{-1}(\mathbf{p}_l^2 + Z_l\mathbf{p}_t^2 + \varkappa_l^2). \tag{13}$$

We then obtain in place of (10) and (11)

$$Z_l^{-2x} Z_l^{2-(D-d)/2} = Z_l^{-x} \tag{14}$$

and a second such equation with y in place of x. Therefore

$$\begin{gathered}\tilde{\gamma}_l = J(D)\varkappa_l^{D-4} Z_l^{-(4-D+d)/2}\,\Gamma(0),\\ \tilde{w}_l = (2\pi)^{-d} J(D-d)\varkappa_l^{D-4-d} Z_l^{-(4-D+d)/2}\, W(0).\end{gathered} \tag{15}$$

Before deriving the GML equations, we obtain first expressions for the critical exponents η and γ. It follows directly from (7a) that $\eta_t=0$. To find the longitudinal component η_l we must calculate a one-loop diagram with a dashed line, which contributes to the mass operator. Using the standard expressions,[12] we obtain

$$Z_l \approx 1 + \left.\frac{\partial\Sigma}{\partial \mathbf{p}_l^2}\right|_0 = 1 - \frac{2}{4-D+d}\tilde{w}_{l,l}. \tag{16}$$

Since the GML equations yield in zeroth order

$$\frac{\partial \tilde{w}_{l,l}}{\partial \ln \varkappa_{t,l}^{-2}} = \frac{4-D+d}{2}\tilde{w}_{l,l}, \tag{17}$$

it follows that

$$\frac{\partial \ln Z_l}{\partial \ln \varkappa_{t,l}} = \frac{\partial \ln Z_l}{\partial \tilde{w}_{t,l}}\,\frac{\partial \tilde{w}_{t,l}}{\partial \ln \varkappa_{t,l}} = 2\tilde{w}_{t,l}. \tag{18}$$

It follows from (7b) that[10]

$$Z_l \sim \varkappa_l^{\eta_l}, \tag{19}$$

and in the second variant of the theory we have

$$\eta_l = 2\tilde{w}_l. \tag{20}$$

To calculate the exponent η_l it is necessary to substitute in (20) the value of $\tilde{w}_l$ in the FP of the GML equations.

On the other hand, (7a) leads to

$$G^{-1}(\mathbf{p}_t^2=0) = Z_l^{-1}(\mathbf{p}_l^2 + Z_l\varkappa_t^2), \tag{21}$$

therefore

$$Z_l \sim (Z_l^{1/2}\varkappa_t)^{\eta_l}, \tag{22}$$

$$Z_l \sim \varkappa_t^{\eta_l/(1-\eta_l/2)} \tag{23}$$

From (23) and (18) it follows that the longitudinal component of the Fisher exponent

$$\eta_l = 2\tilde{w}_t/(1+\tilde{w}_t). \tag{24}$$

Since it is seen from the form of the Hamiltonian that all the components of the field φ interact with the dislocations in the same manner, it is clear that the critical exponent γ should be isotropic. As usual, to calculate γ we use the Ward identity

$$\partial G^{-1}(0)/\partial\tau = \mathcal{T}(0), \tag{25}$$

where $\tau(0)$ is a renormalized three-point vertex calculated at zero values of the outer momenta, and $\tau \equiv (T-T_c)/T_c$. Therefore

$$G^{-1} \sim \tau^{\gamma} \sim \varkappa_t^2. \tag{26}$$

It follows from (25) and (26) that

$$\partial \ln \mathcal{T}(0)/\partial \ln \varkappa_t^2 = g_t = (\gamma-1)/\gamma. \tag{27}$$

At the same time

$$G^{-1} \sim \varkappa^{2-\eta_l}, \tag{28}$$

therefore

$$\frac{\partial \ln \mathcal{T}(0)}{\partial \ln \varkappa_l^2} = g_l = \left(1 - \frac{\eta_l}{2}\right)\frac{\gamma-1}{\gamma}. \tag{29}$$

There are therefore two expressions for the exponent γ:

$$\gamma = (1-g_t)^{-1}, \tag{30}$$

$$\gamma = \left[1 - \frac{g_l}{1-\eta_l/2}\right]^{-1}. \tag{31}$$

In the single-loop approximation

$$g_{l,t} = (n+2)\tilde{\gamma}_{l,t}/6 - \tilde{w}_{l,t}. \tag{32}$$

The only critical exponents that are anisotropic are η and ν. They are connected with each other and with the other exponents by scaling relations that differ from the usual ones:

$$(D-d)\nu_t + d\nu_l = 2-\alpha, \quad (2-\eta_t)\nu_t = (2-\eta_l)\nu_l = \gamma. \tag{33}$$

The remaining relations coincide with the standard ones. Similar relations were obtained in the investigation of the Lifshitz critical point.[13]

Now, remembering the different definitions of the invariant charges, it is easy to derive the GML equations for both variants of the theory. We begin with the variant with $\varkappa_t$. In this case, using (12), (18), and (25) with the Green's function (6a), we obtain

$$\begin{aligned}\frac{\partial\tilde{\gamma}_t}{\partial t} &= \frac{4-D-d\tilde{w}_t}{2}\tilde{\gamma}_t - \frac{n+8}{6}\tilde{\gamma}_t^2 + 6\tilde{\gamma}_t\tilde{w}_t,\\ \frac{\partial\tilde{w}_t}{\partial t} &= \frac{4-D+d}{2}\tilde{w}_t - \frac{n+2}{3}\tilde{\gamma}_t\tilde{w}_t + 4\tilde{w}_t^2,\end{aligned} \tag{34}$$

where $t \equiv \ln\varkappa_t^{-2}$. The pictures of the phase trajectories of the system (34) take the same form as in Refs. 8 and 9. There are four FP of Eqs. (34) on the $(\tilde{\gamma}_t, \tilde{w}_t)$ plane: Gaussian $(0,0)$, Heisenberg $(3(4-D)/(n+8), 0)$, unphysical $(0, -(4-D+d)/8)$, and the focus-type FP of interest to us:

$$\begin{aligned}\tilde{\gamma}_t &= \frac{3}{2}\,\frac{16-4D+8d+Dd-d^2}{(8-d)n-2(4+d)},\\ \tilde{w}_t &= \frac{1}{2}\,\frac{(D+d-4)n+4(4-D+2d)}{(8-d)n-2(4+d)}.\end{aligned} \tag{35}$$

It follows therefore that random degeneracy of the system (34) takes place at $n=1+3d/(8-d)$. The FP (35) is stable and is located in the positive quadrant of the phase plane at

$$\begin{gathered} n > 1+3d/(8-d), \quad D+d \geqslant 4,\\ 1+3\frac{d}{8-d} < n < 4\left(1+\frac{3d}{4-D-d}\right), \quad D+d<4.\end{gathered}$$

It is assumed here that $D \leqslant 4$. We need only the upper of these two inequalities, since in our case it is meaningful to consider exclusively the dimensionalities $D=3$ and 4 and $d=1$.

In the case $D=4$, $d=1$ our model is equivalent to the model of a pressure phase transition in a magnet with point impurities at zero temperature, which was introduced by Khmel'nitskiĭ.[14] Indeed, the role of Kmel'nitskiĭ's fourth coordinate is played by the frequency ω, and the δ functions of the type (2) correspond in a number of diagrams of our problem to the absence of the integral $\int d\omega$ from these diagrams in Ref. 14.

Substituting (35) in (24) and in (30), we obtain

$$\eta_l=2\frac{(D+d-4)n+4(4-D+2d)}{(12+D-d)+4d-4D}, \tag{36}$$

$$\gamma=4\frac{(8-d)n-2(4+d)}{(8+6D-10d-Dd+2d^2)-32-8d-2Dd+2d^2}. \tag{37}$$

The remaining exponents can be easily obtained by using the scaling relations (33).

The GML equations for the theory with $\varkappa_l$ can be derived by using (15), (18), and (25) with the Green's function (6b):

$$\begin{aligned}\frac{\partial\tilde{\gamma}_l}{\partial t}&=\frac{4-D-(4-D+d)\tilde{w}_l}{2}\tilde{\gamma}_l-\frac{n+8}{6}\tilde{\gamma}_l^2+6\tilde{\gamma}_l\tilde{w}_l,\\ \frac{\partial\tilde{w}_l}{\partial t}&=\frac{4-D+d-(4-D+d)\tilde{w}_l}{2}\tilde{w}_l-\frac{n+2}{3}\tilde{\gamma}_l\tilde{w}_l+4\tilde{w}_l^2,\end{aligned} \tag{38}$$

where now $t\equiv\ln\varkappa_l^{-2}$. From among the FP of the system (38) on the $(\tilde{\gamma}_l,\tilde{w}_l)$ plane only the unphysical point $(0, -(4-D+d)/(4+D-d))$ and the focus

$$\begin{aligned}\tilde{\gamma}_l&=3\frac{16-4D+8d+Dd-d^2}{(12+D-d)n-4D+4d}\\ \tilde{w}_l&=\frac{(d+D-4)n+4(4-D+2d)}{(12+D-d)n-4D+4d}\end{aligned} \tag{39}$$

have coordinates other than the corresponding FP of the system (34). Substituting (39) in (30) and (31) we obtain expressions for η_l and γ, which agree fully with (36) and and (37). Thus, two different systems of GML equations have led to the same formulas for the critical exponents.[1]

3. DISLOCATIONS AND POINT IMPURITIES

Assume that the magnet contains, besides the parallel dislocations considered in Sec. 2, also frozen-in point impurities. We introduce into the employed diagram technique two sorts of impurity lines. The dashed lines correspond to averaging over the point defects, and the wavy lines to averaging over the extended defects. The impurity lines can also be made to correspond to the correlator of the coefficients in front of the quadratic terms in the Hamiltonian. In our case, the Fourier transform of such an operator takes the form

$$u+w\prod_j^d\delta(p_j^l),$$

which corresponds to the sum of the dashed and wavy lines.

Considering the diagrams in different perturbation-theory orders, it is easy to verify that after "dressing" the impurity lines the structure of the correlator remains the same as in the zeroth order, namely:

$$U+W\prod_j^d\delta(p_j^l).$$

We can now separate the diagrams that contribute to the first term of this expression from the diagrams that contribute to the second term. It is then easy to obtain equations for the vertex functions Γ, U, and W. The right-hand sides of these equations correspond to the diagrams shown in Fig. 1.

We use next the variant of the theory with $\varkappa_t$. For brevity we shall omit the subscript t of $\varkappa$, u, and w in this and following sections. In addition to $\tilde{\gamma}$ and $\tilde{w}$ (12) we introduce also the invariant charge

$$\tilde{u}=J(D)\varkappa^{D-4}Z_l^{d/2}U(0). \tag{40}$$

The system of GML equations now takes the form

$$\begin{aligned}\frac{\partial\tilde{\gamma}}{\partial t}&=\tilde{\gamma}\left[\frac{4-D}{2}-\frac{n+8}{6}\tilde{\gamma}+6\tilde{u}+\left(6-\frac{d}{2}\right)\tilde{w}\right],\\ \frac{\partial\tilde{u}}{\partial t}&=\tilde{u}\left[\frac{4-D}{2}-\frac{n+2}{3}\tilde{\gamma}+4\tilde{u}+\left(6-\frac{d}{2}\right)\tilde{w}\right],\\ \frac{\partial\tilde{w}}{\partial t}&=\tilde{w}\left[\frac{4-D+d}{2}-\frac{n+2}{3}\tilde{\gamma}+2\tilde{u}+4\tilde{w}\right].\end{aligned} \tag{41}$$

If we now put $\tilde{w}=0$ in the system (41), then we obtain the GML equations for a magnet with point impurities only. On the other hand if we put $u=0$, then we arrive at the case of parallel dislocations. It must be noted that, generally speaking, one cannot put $d=0$ in the system (41). The solutions of the GML equations with $d=0$ do not describe the critical behavior of a magnet with point impurities. The point is that the diagrams for the vertex W can be selected only in the presence of a term proportional to

$$\prod_j^d\delta(p_j^l).$$

If $d=0$, then $W=0$ and the system (41) assumes a convoluted form with $\tilde{w}=0$.

The longitudinal component of the Fisher exponent is calculated with the aid of (24), and the exponent γ is now expressed in terms of $\tilde{\gamma}$, $\tilde{u}$, and $\tilde{w}$:

$$\gamma\approx\left[1-\frac{n+2}{6}\tilde{\gamma}+\tilde{u}+\tilde{w}\right]^{-1}\approx1+\frac{n+2}{6}\tilde{\gamma}-\tilde{u}-\tilde{w}. \tag{42}$$

We shall not write out all the fixed points of Eqs. (41). Their relative position in the space of the invariant charges $(\tilde{\gamma},\tilde{u},\tilde{w})$ is shown in Fig. 2. From the directions of the phase trajectories near each FP on the figure, it is easy to determine its type. The initial representative point of the system is located near the unstable Gaussian FP (0, 0, 0) and has positive coordinates.[2,3] The GML-

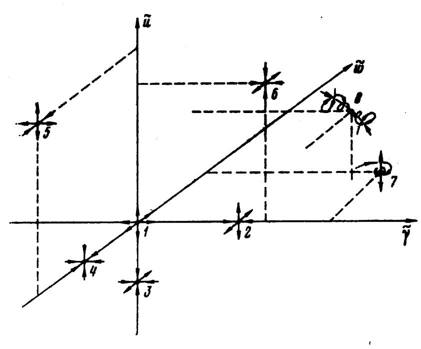

FIG. 2. Disposition of the fixed points in the space of the invariant charges at $4(4-D+d)/(4-D+4d)<n<4$. The arrows show the directions of the phase trajectories near the FP. When n changes from $4(4-D+d)/(4-D+4d)$ to 4, the FP 8 shifts from FP 6 to FP 7.

equation phase trajectories shown schematically in Fig. 2 are such that for any $t \geq 0$ the representative point does not reach the FP 3, 4, or 5. We are therefore interested only in the Gaussian point 1, the Heisenberg point 2, and the impurity points 6, 7, and 8. The coordinates of the Khmel'nitskii point 6, which is peculiar only to a point with only point impurities,[2,3] are

$$\left(\frac{3}{4}\frac{4-D}{n-1},\ \frac{1}{16}\frac{4-n}{n-1}(4-D), 0\right).$$

The FP 7 was investigated in Sec. 2, and the FP 8, possessed by a system with defects of two types, has the coordinates

$$\begin{gathered}\tilde{\gamma}=3\frac{(4-D)(4-d)+12d-d^2}{(4-3d)n+32},\\ \tilde{u}=\frac{1}{4}\frac{4-n}{(4-3d)n+32}[(4-D)(4-d)+12d-d^2],\\ \tilde{w}=\frac{(4-D+4d)n-4(4-D+d)}{(4-3d)n+32}.\end{gathered}\tag{43}$$

At $1+3(4-D)/(4-D+d)<n<4$ (see Fig. 2), this FP is stable. The focus 7 is in this case unstable for the direction of $\tilde{u}$, while the FP 6 and the Heisenberg points are saddles. The representative point therefore reaches ultimately the vicinity of the FP 8. The critical exponents of the considered system differ then from the exponents of a magnet with only one type of defect.

At $n>4$ the coordinate $\tilde{u}_8$ of the eighth FP reverses sign and it becomes unstable, FP 7 is transformed into a stable focus, and FP 6 goes off below the Heisenberg point which, however, remains unstable. Thus at $n>4$ the representative point arrives at the vicinity of FP 7 and the critical exponents of a magnet with a mixture of defects of two kinds coincide with the exponent of a material having dislocations only.

At $n \leq 1+3(4-D)/(4-D+4d)$, i.e., at $n=1$, $\tilde{w}_8<0$ and FP 6 is a stable node. Therefore the exponents of the investigated system coincide with those of a magnet having only point impurities.

The phase trajectory that leads to a stable FP is determined by the initial position of the representative point of the system of the GML equations (41) $(\tilde{\gamma}_0, \tilde{u}_0, \tilde{w}_0)$, i.e., by the nonrenormalized constants of the Hamiltonian and by the densities of the defects. Without writing down the concrete estimates for these quantities, since usually the densities and the potentials of the defects are unknown, we consider, for example, the case $\tilde{\gamma}_0 \gg \tilde{w}_0 \gg \tilde{u}_0$. Let $4(4-D+d)/(4-D+4d) \leq n < 4$ (see Fig. 2). Then the representative point from the vicinity of the Gaussian FP lands in the vicinity of the Heisenberg FP, approaches next FP 7, and only then does it come near the stable FP 8. Thus, as T_c is approached, the critical exponents occupy in succession values corresponding to FP 1 (the exponents of the mean-field theory), FP 2 (the exponent of a pure isotropic magnet), FP 7 (the exponent of a magnet with parallel dislocations), and FP 8 (the exponents of a magnet with defects of both types). The relative magnitude of each temperature interval in which exponents have particular values are determined by the relations between $\tilde{\gamma}_0, \tilde{u}_0$, and $\tilde{w}_0$. The corrections that must be introduced in the scaling laws because of the presence of focus FP are oscillating functions of the temperature.[14,8,9]

It is easy to understand how the system approaches T_c also at other relations between $\tilde{\gamma}_0$, $\tilde{u}_0$, and $\tilde{w}_0$. At $\tilde{\gamma}_0 \approx \tilde{u}_0 \approx \tilde{w}_0$ the representative point shifts from the Gaussian FP directly to FP 8. The expressions for the critical exponents corresponding to this FP will be given in Sec. 6.

4. DISLOCATIONS ORIENTED IN THREE DIRECTIONS

Assume that the dislocations in the magnet can have not one direction, as in Secs. 2 and 3, but three mutually perpendicular directions. It can be shown that in this case the correlator of the variations of the coefficients in front of the quadratic term of the Hamiltonian is of the form $u+w_1\delta(k_x)+w_2\delta(k_y)+w_3\delta(k_z)$. The first term in the sum corresponds to point impurities. In this and following sections we put $D=3$ and $d=1$. Fortunately we shall not have to deal with the five-charge theory. It is easily seen that the GML equations are symmetrical with respect to permutations of $\tilde{w}_1$, $\tilde{w}_2$, and $\tilde{w}_3$. We are interested only in FP of these equations with positive coordinates. It follows from the indicated property of the GML equations that at this FP $\tilde{w}_1=\tilde{w}_2=\tilde{w}_3$. The FP is here either stable along each of the three axes $(\tilde{w}_1, \tilde{w}_2, \tilde{w}_3)$ of the phase space, or, conversely, is unstable in all these directions. We can therefore seek this FP by using the correlator[2] $u+w(\delta(k_x)+\delta(k_y)+\delta(k_z))$. The theory now involves three and not five charges.

The exponents ν and η of the considered magnet, in contrast to the systems investigated in Secs. 2 and 3, are isotropic. In accord with Eqs. (12), (15), and (40), the invariant charges are determined in the following manner:

$$\tilde{\gamma}=\frac{1}{16\pi}\varkappa^{-1+2\eta}\Gamma,\quad \tilde{u}=\frac{1}{16\pi}\varkappa^{-1+2\eta}U,\quad \tilde{w}=\frac{1}{8\pi^2}\varkappa^{-2+2\eta}W.\tag{44}$$

In the employed diagram technique, we relate to the correlator introduced above the sum of four impurity lines: one dashed and three wavy ones with exponents x, y, and z. Considering single-loop diagrams that contribute to the three-point vertex, we obtain an expression for the exponent

$$\gamma \approx 1+\frac{n+2}{6}\tilde{\gamma}-\tilde{u}-3\tilde{w}.\tag{45}$$

The sum of the three single-loop diagrams with wavy lines x, y, and z determines the Fisher exponent:

$$\eta=2\tilde{w}.\tag{46}$$

We write down now the system of GML equations:

$$\begin{gathered}\frac{\partial\tilde{\gamma}}{\partial t}=\frac{1}{2}\tilde{\gamma}-\frac{n+8}{6}\tilde{\gamma}^2+6\tilde{\gamma}\tilde{u}+16\tilde{\gamma}\tilde{w},\\ \frac{\partial\tilde{u}}{\partial t}=\frac{1}{2}\tilde{u}-\frac{n+2}{3}\tilde{\gamma}\tilde{u}+4\tilde{u}^2+16\tilde{u}\tilde{w}+\frac{9}{2}\pi\tilde{w}^2,\\ \frac{\partial\tilde{w}}{\partial t}=\tilde{w}-\frac{n+2}{3}\tilde{\gamma}\tilde{w}+2\tilde{u}\tilde{w}+6\tilde{w}^2,\end{gathered}\tag{47}$$

where $t \equiv \ln\varkappa^{-2}$. The single-loop diagrams that contribute to the GML functions are shown in Fig. 3. Particular interest attaches to diagrams that are cut through wavy lines having different exponents. The final expres-

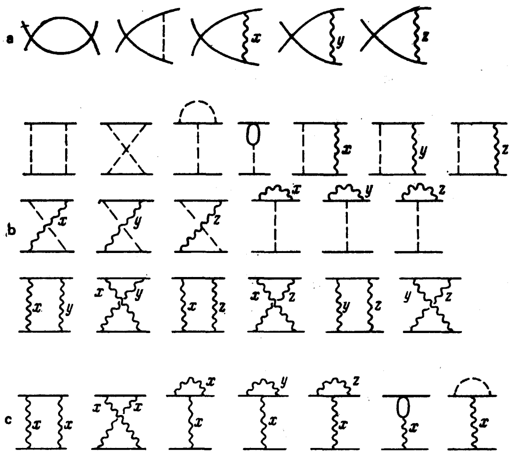

FIG. 3. One-loop contributions to the GML functions of a magnet with linear defects oriented along three directions. a, b, c) diagrams whose sums yield respectively $\partial\tilde{\gamma}/\partial t$, $\partial\tilde{u}/\partial t$, and $\partial\tilde{w}/\partial t$.

sions for them do not contain δ functions, and contribute therefore to the right-hand side of the equation for[3)] $\partial\tilde{u}/\partial t$. This contribution leads to the appearance of $\tilde{u}\neq 0$ even if the unrenormalized $u=0$.

The coordinates of the FP of interest to us, that of Eqs. (47), is determined by the expressions

$$\tilde{w}=\frac{\{a^2+12b(5n-8)\}^{1/2}-a}{2b},$$
$$a=65n^2+80n+80, \qquad (48)$$
$$b=(225\pi-624)n^2+(360\pi-1008)n+144\pi-384,$$
$$\tilde{\gamma}=3\frac{5+4\tilde{w}}{5n+4}, \quad \tilde{u}=-\frac{13\tilde{w}n+8\tilde{w}-3}{5n+4}.$$

This FP is located in the region of positive $\tilde{\gamma}$, $\tilde{u}$, and $\tilde{w}$ and is stable at $n>\frac{8}{5}$ (we do not consider very large n). At $n<\frac{8}{5}$ it drops below the Khmel'nitskiĭ point that is typical of systems with point impurities, and which becomes stable. Therefore at $n=1$ the critical exponents of our system coincide with the exponents of a magnet with point impurities. The numerical values of the invariant charges in the FP (48) at certain values of n are listed in Table I. The η corresponding to them are listed there, too. To obtain the exponent γ, it is in principle not necessary to calculate $\tilde{w}$. It suffices to substitute the expressions obtained for $\tilde{\gamma}$ and $\tilde{u}$ from (48) and (45). Through a fortunate cancellation we obtain $\gamma=\frac{3}{2}$ for all $n>\frac{8}{5}$.

TABLE I. Values of the invariant charges and of the critical exponent η at the FP (42).

n	$\tilde{\gamma}$	$\tilde{u}$	$\tilde{w}$	η
2	1.082	0.186	0.012	0.024
3	0.804	0.102	0.023	0.045
4	0.637	0.064	0.024	0.048

5. RANDOMLY ORIENTED DISLOCATIONS

It is assumed in this section that the number m of directions along which the dislocations are disposed in the magnet is large. The expression for the correlator of the variation of the coefficient that precedes the quadratic term of the Hamiltonian is now of the form

$$u+w\sum_{i}^{m}\delta(\mathbf{k}\mathbf{n}_i),$$

where $\{\mathbf{n}_i\}$ is a system consisting of a large number m of unit vectors whose end points are uniformly distributed over the surface of the unit sphere.[4]

In contrast to the preceding section, the diagram technique involves now not three but m different wavy lines with exponents $\{i\}$. For convenience we replace in the formulas that follow $m\tilde{w}$ by $\tilde{w}$. Then

$$\eta\approx{}^2/_3\tilde{w}, \quad \gamma\approx 1+(n+2)\tilde{\gamma}/6-\tilde{u}-\tilde{w}. \qquad (49)$$

An examination of Fig. 3 shows readily which diagrams determine in the single-loop approximation the GML functions. We retain in the right-hand sides of the GML equations only the principal terms in m. The contribution to $\partial\tilde{u}/\partial t$ from diagrams with two wavy lines can be calculated by replacing the sum of the diagrams by an integral over the angles swept by the unit vectors $\{\mathbf{n}_i\}$. The result is the following system of GML equations

$$\frac{\partial\tilde{\gamma}}{\partial t}=\frac{1}{2}\tilde{\gamma}-\frac{n+8}{6}\tilde{\gamma}^2+6\tilde{\gamma}\tilde{u}+\frac{16}{3}\tilde{\gamma}\tilde{w},$$
$$\frac{\partial\tilde{u}}{\partial t}=\frac{1}{2}\tilde{u}-\frac{n+2}{3}\tilde{\gamma}\tilde{u}+4\tilde{u}^2+\frac{16}{3}\tilde{u}\tilde{w}+\frac{3}{8}\pi^2\tilde{w}^2, \qquad (50)$$
$$\frac{\partial\tilde{w}}{\partial t}=\tilde{w}-\frac{n+2}{3}\tilde{\gamma}\tilde{w}+2\tilde{u}\tilde{w}+\frac{4}{3}\tilde{w}^2.$$

These equations do not contain m explicitly, since it has been successfully combined with the invariant charge $\tilde{w}$.

The coordinates of the needed FP of the GML equations (50) are determined from the following equations:

$$\tilde{w}=\frac{\{a^2+(216n-345.6)b\}^{1/2}-a}{2b},$$
$$a=84n^2+192n-38.4, \qquad (51)$$
$$b=[67.5\pi^2-358.4]n^2+[108\pi^2-588.8]n+43.2\pi^2-204.8,$$
$$\tilde{\gamma}=(15-8\tilde{w})/(5n+4), \quad \tilde{u}=(9-14\tilde{w}n-16\tilde{w})/3(5n+4).$$

At $n<\frac{8}{5}$ the Khmel'nitskiĭ point is stable, and the investigated system has the same exponents as a magnet with point impurities. At $n>\frac{8}{5}$, the FP (51) is in the region of positive invariant charges and is stable. Table II gives the coordinates and critical exponents η and γ corresponding to the FP (51) for certain n.

TABLE II. Coordinates of FP (46) and circtical exponents η and γ.

n	$\tilde{\gamma}$	$\tilde{u}$	$\tilde{w}$	η	γ
2	1.055	0.184	0.029	0.019	1.49
3	0.768	0.107	0.050	0.033	1.48
4	0.607	0.072	0.053	0.035	1.48

6. DISCUSSION

In Secs. 2 and 3 the values of D and d were in general not fixed. The expressions for the critical exponents, other than η_l and ν_l, of the system investigated in Sec. 2 go over at $d=0$ into the formulas for the exponents of a magnet with point impurities. Magnets with parallel linear defects were investigated in Refs. 8 and 9 by the method of ε and ε_d expansions. These expansions can be obtained for the critical exponents also from the formulas of the present paper, by expanding the corresponding expressions in terms of $\varepsilon \equiv 4-D$ and $\varepsilon_d \equiv d$. For example, for the model introduced in Sec. 3 we have at $4(\varepsilon+\varepsilon_d)/(\varepsilon+4\varepsilon_d)<n<4$

$$\eta_l=\frac{1}{2}\frac{(n-4)\varepsilon+4(n-1)\varepsilon_d}{n+8},\qquad \gamma=1+\frac{n+2}{2(n+8)}\varepsilon+\frac{5n+4}{4(n+8)}\varepsilon_d. \tag{52}$$

Since some of our systems degenerate randomly at $n\sim1$, the critical exponents of these models as obtained in the one-loop approximation are too large. We can then take seriously only the qualitative conclusions of the theory. It is possible that in the higher orders the FP will move closer to the origin of the phase plane and the results will improve. Allowance for the higher orders can also shift somewhat the limiting n at which a transition takes place from one critical behavior to another. Unfortunately, calculations in the multiple-charge theory become difficult on going beyond the single-loop approximation.

Let us touch upon briefly on two features that are characteristic of all the models investigated above, as well as for a magnet with point impurities. First, the exponent $\alpha<0$, as can be easily verified by substituting the expressions of this paper for the exponents η and γ into the corresponding scaling relations. Second, if the unrenormalized charges are in the regions $\tilde{\gamma}\ll\tilde{u}$ or $\tilde{\gamma}\ll\tilde{w}$ of the phase space of the GML equations, it is usually assumed[3,15] that the phase transition is smeared out.

In addition to the defects studied above, one can consider also linear defects produced by random fields. In this case a term linear in φ is added to the Hamiltonian. Proceeding in this manner it can be shown, as in Refs. 16 and 17, that the upper critical dimensionality of the model increases to 7. In the tricritical point, the logarithmic dimensionality of such a model is 6.

We now discuss, in conclusions, the systems to which the results of the present article can be applicable. Unfortunately, the author knows so far of no experiments in which a change in the critical behavior of magnets was observed and which might be quite reliably attributed to the presence in fact of extended defects. In some studies (see, e.g., Ref. 20), they investigated experimentally the influence of dislocations on a first-order phase transition close to second-order in ferroelectrics. Condensation of the asymmetrical phase on dislocations was observed there somewhat above the transition temperature of the pure material. A theory that describes such systems should be constructed in analogy with the theory of Dubrovskiĭ and Krivoglaz,[6] in which the dislocation produce around themselves large-scale inhomogeneities. Linear defects with short-scale inhomogeneity in the transverse direction might be produced by bombarding a thin magnetic plate with a beam of heavy ions from an accelerator. This would produce a high density of linear defects oriented in one or several directions. In the former case it is seen from the results of Secs. 2 and 3 that the critical exponent $\nu_t<\nu_l$ and the anisotropy of the system increase as T_c is approached. This increase of the degree of anisotropy might be observed in neutron-scattering experiments or in measurements of the electric conductivity. We note finally that a numerical computer experiment for the system considered in Sec. 1, at $n=1$ and $D=3$, is even easier to perform than for a three-dimensional Ising model with point defects. In this case the linear defects can constitute lines made up of spins that differ in magnitude from the spins in the other latter sites, but not smaller than zero. Also possible are lines of bonds that differ by a certain amount from the remaining ones (the signs of the bonds must not change). Such a computer experiment is of great interest. After all, it was computer simulation that has clearly confirmed (see, e.g., Ref. 21) that, according to Refs. 1–3, frozen-in point impurities do not smear out a second-order phase transition.

The approach used in the present paper can be generalized to include also other systems in which only effects due to the presence of point defects have been investigated so far. For example, a corresponding generalization is possible for the problem of the conductivity of a metal with point impurities.[18,19]

The author thanks S. L. Ginzburg, A. L. Korzhenevskiĭ, V. N. Prigodin, A. I. Sokolov, Yu.A. Firsov, D. E. Khmel'nitskiĭ, B. N. Shaleev for discussions.

1) The critical exponents can be calculated in principle not from formulas (24), (30), and (31) but from simpler expressions obtained by them by expanding η_l and γ in terms of the invariant charges. Then $\eta_l\approx2\tilde{w}_{l,t}$ and $\gamma\approx1+g_{l,t}$. The exponent values obtained in different approaches will then differ somewhat. The smaller the invariant charges, however, the less the relative differences between the results of the two variants of the theory.

2) This can be verified also without resorting to such arguments, and by simply checking that the stability of the FP $\tilde{w}_1=\tilde{w}_2=\tilde{w}_3$ is not altered by small additions $\delta\tilde{w}_i$ to $\tilde{w}$.

3) We note that the appearance of numerical coefficients such as π even in the one-loop approximation is very rare in the case of the GML functions.

4) The vector directions can also be random.

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Translated by J. G. Adashko