Exchange effects at the boundaries of magnets

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Boundary conditions at the surface of an antiferromagnet are considered, neglecting relativistic effects. The surface spin-wave spectrum can either be linear or vary as $\omega \propto k^{3/2}$. It is shown that at T < 1 mK, heat exchange at the solid – liquid He³ interface is produced mainly by magnons. In this case, the Kapitza jump is not great.

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1. SURFACES OF ANTIFERROMAGNETS

In the investigation of the surfaces of magnets, effects associated with the anisotropy or demagnetization are usually discussed. This paper is devoted to clarification of the magnet-surface macroscopic properties due to exchange forces.

The Lagrangian of a collinear antiferromagnet has the form¹

$$\frac{\chi_{\perp}}{2\gamma^2}\int\left\{\dot{\mathbf{i}}^2-c^2\left(\frac{\partial\mathbf{l}}{\partial x_i}\right)^2\right\}dV,$$
(1)

where l is a unit vector in the direction of the antiferromagnetism, χ_{\perp} is the magnetic susceptibility, γ is the gyromagnetic ratio, and c is the spin-wave velocity. Varying Eq. (1) with respect to 1, we obtain the equations of motion (see Ref. 1)

$$[\mathbf{l} \times \ddot{\mathbf{i}} - c^2 \Delta \mathbf{l}] = 0 \tag{2}$$

and the natural boundary conditions on the crystal surface

$$\left[1 \times \frac{\partial l}{\partial z}\right] = 0, \tag{3}$$

the z axis is normal to the surface. However, the conditions (3) are violated in most cases. In fact, assume that no elements that change the antiferromagnetic vector 1 are present among the crystallographic exchange symmetry elements¹ which transform a given crystal face into itself. In other words, 1 transforms according to a unit representation of the exchange symmetry group of the surface. In this case, the surface should be characterized by nonzero magnetization M parallel to 1. Then we must add to the Lagrangian in Eq. (1) the surface contribution [cf., e.g., Eq. (41) of Ref. 1]

$$\frac{1}{\gamma}\int M\Omega dS,$$

where Ω is the rotational frequency. Instead of Eq. (3), we obtain

$$\gamma \dot{\mathbf{M}} + \chi_{\perp} c^2 \left[\mathbf{1} \times \frac{\partial \mathbf{1}}{\partial z} \right] = 0.$$

On reflection from such a boundary, the spin waves change polarization. Along the boundary, surface spin waves will also propagate with velocity $s=c\{1+(\gamma M/\chi_{\perp}c)^{2}\}^{-\frac{1}{2}}$

Obviously, the magnetic structure of the surface may differ in symmetry from the state due to symmetry within the volume of the crystal. In collinear magnets, a new degree of freedom appears in this case: rotations about 1. Let us consider a symmetric face on which magnetization parallel to 1 is forbidden. According to exchange-symmetry theory,¹ in this case two fundamentally different cases are possible: the ferrimagnetic case $M \perp l$ (or $M \perp l_1 \perp l$) and the antiferromagnetic case $l_1 \perp l$ (or $l_1 \perp l_2 \perp l$). In the second case, for the volume problem, accurate to terms of the next order with respect to the derivatives, the conditions (3) remain, while surface spin waves with a line spectrum correspond to the new degree of freedom. In the first case, taking into account in the surface part of the Lagrangian the inhomogeneity energy for the new degree of freedom

$$2a\int \left(1\frac{\partial\varphi}{\partial x_{v}}\right)^{2}dS, \quad v=1,2$$

(φ is the angle of rotation of the spin space¹), we obtain the linearized boundary condition

$$\gamma \dot{\mathbf{M}} - 2a\gamma^2 \mathbf{l}_0 \left(\mathbf{l}_0 \cdot \Delta \varphi \right) + \chi_{\perp} c^2 \left[\mathbf{l}_0 \frac{\partial \mathbf{l}}{\partial z} \right] = 0.$$

The spectrum of surface spin waves on a ferrimagnetic surface has the unusual form:

$$\omega^2 = (\chi_\perp c^2 a/M^2) k^3.$$

In more complicated noncollinear magnets and ferromagnets, such surface phenomena are also possible. We will not consider the ensuing possibilities. In each specific magnetic system such a study (even taking into account relativistic effects) is easily performed in a completely analogous manner to the collinear antiferromagnetic case, using the approach developed previously.¹ Here we only note that surface spin waves should also exist on the surface of a so-called disordered antiferromagnet² (see also Ref. 1) or in the Bphase of superfluid He³, whose spin dynamics equations are the same.³ If we replace the rotation angle by the displacement vector, the linearized equations of motion in this case formally coincide with the equations of elasticity theory for an isotropic solid. Therefore, if the surface magnetization is absent, then the spectrum of surface spin waves completely coincides with the spectrum of elastic Rayleigh waves.

2. MAGNON HEAT TRANSFER AT A SOL'ID-LIQUID He³ INTERFACE AT T < mK

There is every reason to expect that the interface between solid and liquid He³ at a temperature less than 1 mK will have the quantum properties⁴ observed recently⁵ in He⁴. Phonon heat transfer on such interfaces is significantly hindered⁶; therefore, it is of interest to consider heat transfer by passage of magnons.

Solid He³ at a temperature below 1 mK is⁷ a collinear antiferromagnet, the spin dynamics of which are described by Eq. (2) (strictly speaking, the velocity should be anisotropic). The linearized equations for He³-B have the form

$$\ddot{\boldsymbol{\varphi}} - c_t^2 \Delta \boldsymbol{\varphi} - (c_t^2 - c_t^2) \nabla \operatorname{div} \boldsymbol{\varphi} = 0.$$
(4)

Clearly, the rotation angle of the spin space in the solid and the liquid are equal at the interface, since there is no reason for the exchange interaction energy between atoms of the crystal and the liquid to be small compared with the exchange energy in the crystal. The linearized boundary conditions, when account is taken of the surface magnetization parallel to 1 (see Sec. 1), turn out to be

$$\frac{\chi c_i^2}{\chi_{\perp} c^2} \left\{ \frac{\partial \varphi}{\partial z} + \left(\frac{c_i^2}{c_i^2} - 1 \right) \mathbf{n} \left(\nabla \varphi \right) \right\} \Big|_{+ \circ}$$

$$= \left\{ \frac{\partial \varphi}{\partial z} - \mathbf{l}_{\circ} \left(\mathbf{l}_{\circ} \frac{\partial \varphi}{\partial z} \right) \right\} \Big|_{- \circ} + \frac{\chi M}{\chi_{\perp} c^2} [\dot{\varphi} \mathbf{l}_{\circ}].$$

$$(5)$$

By way of example, we give here the expressions for the probability of reflection of a plane-polarized magnon, incident normally on the boundary from the crystal, with the polarization becoming perpendicular to the initial direction:

$$W = 4\eta \{ (1+Z)^2 + \eta \}^{-2}, \ \eta = (\gamma M / \chi_{\perp} c)^2, \ Z = \chi c_t / \chi_{\perp} c, \tag{6}$$

and for the total probability that the magnon goes into the liquid:

$$W_{t} = 4Z/[(1+Z)^{2} + \eta].$$
(7)

The parameter Z is thus the magnon impedance (in analogy with acoustic impedance). Experimental data on the susceptibility,⁸ and on the spin-wave velocity in the solid⁹ and liquid¹⁰ states indicate that Z does not differ substantially from unity. There is also no reason for η to be anomalously large. Therefore, the heat transfer due to magnon exchange should be effective.

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