Modulation resonance in a spectrally inhomogeneous medium

A. R. Kessel' and R. N. Shakhmuratov

Kazan' Physicotechnical Institute, Academy of Sciences of the USSR
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The response of a spectrally inhomogeneous system of two-level atoms to the modulation of the energy gaps of the atoms in the presence of a strong saturating near-resonance field is theoretically investigated. The case in which the inhomogeneous broadening is much greater than the homogeneous broadening is analyzed. It is found that the response of the system to the modulation has a component that is shifted in phase by $\pi/2$ relative to the phase of the external influence, and that the amplitude of this component has a resonance maximum at a frequency determined by the amplitude of the strong field. This leads to resonant absorption of the field modulating the energy gaps. The width of the resonance is characterized by only the homogeneous broadening.

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INTRODUCTION

A two-level system (TS) acquires completely new properties in the field of a strong electromagnetic wave. The appearance of these properties is due to the fact that, besides the oscillations at the frequency of the influence, there arise oscillations in the TS parameters at the Rabi frequency determined by the amplitude of the strong field. The changes in the properties of the TS are detected with the aid of an auxiliary "test" field that induces transitions between the levels of the system. They manifest themselves in the appearance of new resonance anomalies in the spectrum of the response to the "test" field as the amplitude of the strong field is increased.[1-11]

In our previous papers,[12,13] we proposed to investigate the changes in the properties of the TS by modulating its energy gap with a weak field. We found that resonance anomalies appear at the Rabi frequency in the response of the system to the modulation. The appearance of resonance anomalies in the response to a weak modulating field in the case of the action of a strong resonance field can be described only when the modification of the kinetic equations (KE) for a TS in a strong field is taken into account. Allowance for the modification of the KE becomes unnecessary if the strong field is not in exact resonance with the TS, since this effect then is due to the unmodified KE, and is larger. These results are valid for spectrally homogeneous systems. In the case of a strong spectral inhomogeneity, the description of the resonance under conditions when the TS energy gap is modulated (we shall henceforth call it modulation resonance) should take account of both the contribution due to the modification of the KE and the contribution due to the nonresonant nature of the influence. The purpose of the present paper is to consider the modulation resonance in a medium consisting of two-level atoms in the presence of strong inhomogeneous broadening.

§1. THE KINETIC EQUATIONS FOR A TS SUBJECTED TO THE ACTION OF A STRONG NONRESONANT FIELD

Before considering the modulation resonance, let us derive the modification of the KE necessary for the description of a TS in a strong field in the nonresonant case. For this purpose, let us consider a system consisting of two coupled subsystems: a) A TS subjected to the action of a strong circularly polarized field $\vec{\mathscr{E}}_0(t)$ (the dynamical subsystem) and b) a thermal reservoir (the dissipative subsystem). The Hamiltonian of the system has the form

$$\hat{H}=\hat{E}+\hat{F}+\hat{V},\quad \hat{E}=\hat{H}_0+\hat{D},\quad \hat{H}_0=\frac{\omega_0}{2}(\hat{P}_{22}-\hat{P}_{11}),$$
$$\hat{D}=\frac{\omega_1}{2}(\hat{P}_{12}e^{i\omega t}+\text{a. c.}),\quad \hat{V}=\sum_{m,n}\hat{U}_{mn}\hat{P}_{mn}\quad (m,n=1,2), \tag{1}$$

where $\hat{E}$ is the Hamiltonian of the dynamical subsystem; $\hat{F}$, the Hamiltonian of the dissipative subsystem; $\hat{V}$, the Hamiltonian for the interaction between the subsystems; $\hat{H}_0$, the Hamiltonian of the TS; $\hat{D}$, the Hamiltonian for the interaction between the TS and the strong field; $\hbar=1$; ω_0 is the energy gap of the TS; $\omega_1=2\langle\chi_1|\hat{\mathbf{d}}\vec{\mathscr{E}}_0|\chi_2\rangle$; the χ_m are the eigenfunctions of the Hamiltonian $\hat{H}_0$; $\hat{\mathbf{d}}$ is the electric or magnetic dipole moment operator for the TS (the analysis is valid for both spin $\frac{1}{2}$ in a constant magnetic field and a two-level atom); $\vec{\mathscr{E}}_0$ and ω are the amplitude and frequency, respectively, of the strong alternating field; the $\hat{P}_{mn}$ and the projection operators of the $\hat{H}_0$ representation, which are defined by the following law of action on the χ_m basis: $\hat{P}_{mn}\chi_k=\delta_{nk}\chi_m$; the $\hat{U}_{mn}$ are the operators of the dissipative subsystem.

Let us go over with the aid of the operator $\hat{u}=\exp[(i\omega/2)(\hat{P}_{22}-\hat{P}_{11})t]$ to the generalized interaction representation (GIR) in which the operator $\hat{D}$ does not depend on the time:

$$\hat{E}'=\frac{\Delta}{2}(\hat{P}_{22}-\hat{P}_{11})+\frac{\omega_1}{2}(\hat{P}_{12}+\hat{P}_{21}),\quad \Delta=\omega_0-\omega,$$
$$\hat{V}'=\sum_{m,n}\hat{U}_{mn}\hat{P}_{mn}e^{i\omega(m-n)t}\quad (m,n=1,2).$$

By using the standard procedure for deriving the KE,[14-18] we can obtain in the GIR the relaxation part of the kinetic equations for the following combinations of the averages $\langle\hat{P}_{mn}\rangle$:

$$R(z)=-4K_{12}^{21}(\omega)\left\{z-\left[\cos^4\frac{\theta}{2}\,\text{th}\,p(\omega+2\varkappa)\right.\right.$$
$$\left.\left.+\sin^4\frac{\theta}{2}\,\text{th}\,p(\omega-2\varkappa)+\frac{1}{2}\sin^2\theta\,\text{th}\,p\omega\right]\right\}, \tag{2}$$

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$$R(x)=-[2K_{12}^{21}(\omega)+K_0^0(\omega_1)]x+\sin\theta\left\{2K_{12}^{21}(\omega)\left[-\cos^2\frac{\theta}{2}\operatorname{th}p(\omega+2\varkappa)\right.\right.$$
$$\left.\left.+\cos\theta\operatorname{th}p\omega+\sin^2\frac{\theta}{2}\operatorname{th}p(\omega-2\varkappa)\right]-K_0^0(\omega_1)\operatorname{th}2\varkappa p\right\},$$
$$R(y)=-[2K_{12}^{21}(\omega)+K_0^0(\omega_1)]y,$$

where

$$x=\langle\hat{P}_{12}+\hat{P}_{21}\rangle;\quad y=i\langle\hat{P}_{12}-\hat{P}_{21}\rangle;\quad z=\langle\hat{P}_{11}-\hat{P}_{22}\rangle;$$
$$p=(2kT)^{-1};\quad \operatorname{tg}\theta=\omega_1/\Delta;\quad 2\varkappa=(\omega_1^2+\Delta^2)^{1/2};\quad K_{mn}^{nm}(\omega)$$
$$=\int_0^\infty\cos\omega\tau\,\mathrm{Re}\,A_{mn}^{nm}(\tau)\,d\tau;\quad K_0^0(\omega_1)=K_{11}^{11}(\omega_1)+K_{22}^{22}(\omega_1)-2K_{22}^{11}(\omega_1);$$

$$A_{mn}^{nm}(\tau)=Z^{-1}\sum_{\alpha,\beta}\{\exp(-2pf_\alpha)\exp(if_\alpha\tau)\langle\alpha|\hat{U}_{mn}|\beta\rangle\exp(-if_\beta\tau)\langle\beta|\hat{U}_{nm}|\alpha\rangle\};$$

$Z=\sum_\alpha\exp(-2pf_\alpha)$; $\langle\alpha|$ is the eigenfunction of the dissipative-subsystem Hamiltonian and corresponds to the eigenvalue f_α (here the Greek indices pertain to the dissipative subsystem; the Latin indices, to the dynamical subsystem).

In deriving the expression (2), we made the following assumptions:

1) The amplitude of the strong field, $\vec{\mathscr{E}}_0(t)$, is bounded from above: $\omega_1\ll\omega$, τ_c^{-1}, where τ_c is the correlation time for the $\hat{U}_{mn}$ operators and is determined by the spectral width of the thermal reservoir: $\omega^*=\tau_c^{-1}$. The spectral functions of the reservoir vary little over frequencies of the order of ω_1:

$$K_{mn}^{nm}(\omega\pm\omega_1)=K_{mn}^{nm}(\omega).$$

2) The relaxation matrix is real, which corresponds to the neglect of the shift of the resonance frequency ω_0 and to the exclusion from consideration of the general shift of the energy levels under the action of the reservoir.

3) $K_{mn}^{nm}(\omega)\ll\omega^*$ (the kinetic limit).

4) Either the condition $\omega>\omega^*$ or the condition $A_{mn}^{kl}(\tau)=A_{mn}^{nm}(\tau)\delta_{ml}\delta_{nk}$ is satisfied. The latter condition follows from the assumption that the system is isotropic, and that the stochastic fields induced by the reservoir on the TS are not correlated. Furthermore, we assume the validity of the relation $[A_{mn}^{nm}(\tau)]^*=A_{mn}^{nm}(-\tau)$, which follows from the allowance for the fact that the stochastic fields are linearly polarized and isotropic.

In practice we are most often faced with the investigation of two opposite cases: 1) the case in which the high-temperature approximation ($p\omega\ll1$), which is normally realized in the RF and microwave bands, is valid and 2) the case in which the low-temperature approximation ($p\omega\gg1$), which is applicable in the optical frequency range, is valid. In either case the condition $p2\varkappa\ll1$ is fulfilled. The relaxation matrix (2) can be written in both approximations as follows:

$$R(z)=-\Gamma_1(z-z_0),\quad R(x)=-\Gamma_2(x-x_0),\quad R(y)=-\Gamma_2 y,\tag{3}$$

where

$$\Gamma_1=4K_{12}^{21}(\omega),\quad \Gamma_2=2K_{12}^{21}(\omega)+K_0^0(\omega_1).$$

In the high-temperature approximation $z_0=p\omega_0$ and $x_0=-p\omega_1$, whereas in the low-temperature approximation $z=1$ and $x_0=-\mu p\omega_1$, where

$$\mu=K_0^0(\omega_1)[2K_{12}^{21}(\omega)+K_0^0(\omega_1)]^{-1}.$$

The obtained modified relaxation matrix (3) differs from the previously known matrix in the presence of the nonzero quantity x_0.

The adopted description of the interaction $\hat{V}$ with the dissipative subsystem is the most general for an ensemble of noninteracting two-level systems. For magnetic systems this form of $\hat{V}$ is possessed by, for example, the spin–lattice interaction in a solid (the thermostat is the phonon gas), by atomic collisions in a gas without resonant exchange of excitations (the thermostat is the thermal motion of the atoms), and by interaction with the thermal-radiation field. For optical systems (electric-dipole transitions), such a form is possessed by the ion-phonon interaction, by collisions in gases, etc. For such relaxation mechanisms the structure of the dissipative part of the KE depends only on the general relations among the Fourier transforms of the correlation functions of the operators $\hat{U}_{mn}$ of the reservoir. The Fourier transforms are themselves determined in the model by the relaxation parameters of the KE (the relaxation times), and should be computed separately for each relaxation mechanism. Since, in deriving the equations, we used the most general relations among the Fourier transforms of the correlation functions, the modification obtained has a general character, and is valid for many systems. It should be noted that, in the optical frequency range, the major role in the modification is played by the spectral functions $K_0^0(\omega_1)$ of the operator $(\hat{U}_{11}-\hat{U}_{22})$, which, in the interaction operator $\hat{V}$, describes the elastic collisions of the system with the particles or quasiparticles of the reservoir.

§2. PROPERTIES OF THE TS IN A STRONG VARIABLE FIELD

In the GIR the modified kinetic equations for a TS subjected to the action of a field $\mathscr{E}_0(t)$ have the form

$$\frac{dz}{dt}=-\omega_1 y-\Gamma(z-z_0),\quad \frac{dy}{dt}=\Delta x+\omega_1 z-\Gamma y,$$
$$\frac{dx}{dt}=-\Delta y-\Gamma(x-x_0).\tag{4}$$

Here we have, for simplicity of analysis, set the relaxation rates Γ_1 and Γ_2 equal: $\Gamma_1=\Gamma_2=\Gamma$. Equality of the relaxation rates is possible under the condition that $K_0^0(\omega_1)=2K_{12}^{21}(\omega)$, a situation which is realized in the short-correlation-time limit ($\omega\tau_c\ll1$) for relaxation mechanisms satisfying the condition

$$\langle|U_{11}-U_{22}|^2\rangle_q=4\langle(\mathrm{Re}\,U_{12})^2\rangle_q=4\langle(\mathrm{Im}\,U_{12})^2\rangle_q,$$

where $\langle\ \rangle_q$ denotes averaging over the states of the reservoir with the distribution $I=Z^{-1}\exp(-2p\hat{F})$. The steady-state solution to Eq. (4) has the form

$$z_{st}=[-\omega_1\Delta x_0+(\Gamma^2+\Delta^2)z_0]S,$$
$$x_{st}=[(\Gamma^2+\omega_1^2)x_0-\omega_1\Delta z_0]S,\tag{5}$$
$$y_{st}=\Gamma(\Delta x_0+\omega_1 z_0)S,\ S^{-1}=\Gamma^2+\omega_1^2+\Delta^2.$$

The only difference between it and the solution to the unmodified KE is that all its three components, x_{st}, y_{st}, and z_{st}, contain additional additive terms proportional to x_0. Because of this, the component x_{st} for $\Delta=0$ is nonzero. Since it is responsible for the χ' dispersion, it may be said that allowance for the modification leads to nonzero dispersion at exact resonance. It is shown in Ref. 19 that, in the description of a laser, this fact yields a shift in the generation frequency. The appearance of the additional term in z_{st} leads to a situation in which the quantity z_{st} becomes, when the condition $\frac{1}{4}\omega_1^2(x_0/z_0)^2 > \Gamma^2$ is fulfilled, negative for Δ satisfying the inequality

$$\frac{1}{2}\omega_1\frac{x_0}{z_0}-T<\Delta<\frac{1}{2}\omega_1\frac{x_0}{z_0}+T,$$

where $T=[\frac{1}{4}\omega_1^2(x_0/z_0)^2-\Gamma^2]^{1/2}$, i.e., the atoms with such detuning Δ will have inverted populations. The atoms for which the inequality $\Delta > z_0/\mu p$ is satisfied will intensify the field, since for them $y_{st}<0$ (y_{st} is responsible for the absorption, and is proportional to χ''). The enumerated anomalies in the behavior of the variables x_{st}, y_{st}, and z_{st} indicate the allowance for the modification of the KE reveals completely new properties of a TS in a strong field.

§3. SOLUTION OF THE MODIFIED KE FOR A TS WITH AN ENERGY GAP UNDERGOING A SLIGHT MODULATION

Let the TS be acted upon by, besides the field $\vec{\mathscr{E}}_0(t)$, a field $\vec{\mathscr{E}}_1(t)$ that causes modulation of its energy gap. The Hamiltonian for the interaction with this field has the form

$$\hat{H}_m=\frac{\omega_2}{2}(\hat{P}_{22}-\hat{P}_{11})\cos\Omega t, \tag{6}$$

where ω_2 and Ω are the modulation amplitude and frequency, respectively. The action of the field $\vec{\mathscr{E}}_1(t)$ leads to the appearance in the KE of the additional terms:

$$\left(\frac{dx}{dt}\right)_m=-\omega_2\cos\Omega t\,y,\quad \left(\frac{dy}{dt}\right)_m=\omega_2\cos\Omega t\,x. \tag{7}$$

Since ω_2 is assumed to be small in comparison with ω_1 and Γ (weak modulation), the solution to the KE(4) with the additional terms (7) can be sought in the linear approximation in ω_2. We are interested in the behavior of the component $z=z_{st}+\bar{z}$, where the quantity $\bar{z}$ is determined up to the first power of ω_2. The solution to the equation yields for it the following expression:

$$\bar{z}(t)=\omega_2(\chi'\cos\Omega t+\chi''\sin\Omega t), \tag{8}$$
$$\chi'=(AC_1-BC_2)(A^2+B^2)^{-1};\quad \chi''=(AC_2+BC_1)(A^2+B^2)^{-1};$$
$$A=\Gamma(\Gamma^2+\omega_1^2-3\Omega^2+\Delta^2);\quad B=\Omega(3\Gamma^2+\omega_1^2-\Omega^2+\Delta^2);$$
$$C_1=\omega_1 S\Gamma[2\Delta\omega_1 z_0-(\Gamma^2+\omega_1^2-\Delta^2)x_0];\quad C_2=\omega_1 S\Omega[-\Delta\omega_1 z_0+(\Gamma^2+\omega_1^2)x_0].$$

It is necessary to average it over the frequency spread $\Delta=\omega_0-\omega$. The averaging amounts to integration over the frequency ω_0 with the distribution function $g(\omega_0-\omega_c)$, where ω_c is the central frequency. We shall consider a system with a strong spectral inhomogeneity: $\delta\gg\Gamma$, ω_1 (δ is the width of the distribution function), $g(\pm\Gamma)\approx g(\pm\omega_1)\approx g(0)$. The function $z(\Delta)$ varies significantly in the interval $(-e,e)$, where the quantity e is of the order of Γ and ω_1. The variation of the function $g(\Delta+\omega-\omega_c)$ in this interval can be neglected. Therefore, under the condition that $\omega-\omega_c\ll\delta$, the integration can be performed as follows:

$$\int\bar{z}(\Delta)g(\Delta+\omega-\omega_c)d\Delta=g(\Delta_0)\int\bar{z}(\Delta)d\Delta.$$

Here Δ_0 is that value of the argument of the function $\bar{z}(\Delta)$ at which the function makes the dominant contribution to the integral. The quantity $g(\Delta_0)$ can be estimated from the normalization condition: $g(\Delta_0)\delta\approx 1$, i.e., $g(\Delta_0)\approx\delta^{-1}$.

Thus, the averaging operation reduces to a simple integration with respect to Δ from $-\infty$ to $+\infty$. As can be seen from the expression (8), the functions A and B are even functions of Δ. In the functions C_1 and C_2, the terms with the factor x_0 are even functions of Δ in both the low- and high-temperature approximations. Therefore, their contribution to the averaged response of the system is nonzero (the functions C_1 and C_2 enter only into the numerator of the averaged function). In the low-temperature approximation the terms with the factor z_0 are odd functions of Δ, and, consequently, their contribution vanishes in the averaging. In the high-temperature approximation, since $z_0=p\omega_0$, these terms make a nonzero contribution. Let us consider the two approximations separately.

A. The low-temperature approximation

In this case only the terms with the factor x_0 make a nonzero contribution. Therefore, we should obtain zero response to the modulation (6) if we do not allow for the modification of the KE. Performing the integration

$$\bar{\chi}=\frac{1}{\delta}\int_{-\infty}^{\infty}\chi(\Delta)d\Delta,$$

we obtain

$$\bar{\chi}''=\varphi_1\psi,\quad \bar{\chi}'=\varphi_2\psi,\quad \psi^{-1}=\delta(\Gamma^2+\Omega^2)bc(a^2+ac+b),$$
$$\varphi_2=\mu\pi p\omega_1^2\{b[\Omega^2(a^2+3\Gamma^2)-\Gamma^2(b+ac)]+(a+c)ad(\Omega^2+\Gamma^2)\},$$
$$\varphi_1=-\mu\pi p\omega_1^2\Omega\Gamma[b(2\Gamma^2+d+b+ac)-2a(\Gamma^2+\Omega^2)(a+c)], \tag{9}$$
$$a=(\Gamma^2+\omega_1^2)^{1/2},\quad b=(d^2+4\Gamma^2\Omega^2)^{1/2},\quad c=(2(d+b))^{1/2},\quad d=a^2-\Omega^2.$$

The quantities b and c in the expression (9) assume minimum values when $d=0$, i.e., when $\Omega=(\omega_1^2+\Gamma^2)^{1/2}$. We can, on the basis of this fact, conclude that $\bar{\chi}'$ and $\bar{\chi}''$ have extrema in the vicinity of $d=0$. Figure 1a shows the plots of $\bar{\chi}'\xi$ and $\bar{\chi}''\xi$ as functions of Ω/Γ for $\omega_1=10\Gamma$, where $\xi^{-1}=\mu\pi p\omega_1(10\delta)^{-1}$. As can be seen from the figure, $\bar{\chi}'$ and $\bar{\chi}''$ have resonance anomalies in the vicinity of the frequency ω_1. The resonance anomalies arise under the condition that $\omega_1>\Gamma$, since $(\bar{\chi}',\bar{\chi}'')\sim(\omega_1/\Gamma)^{1/2}\omega_1$. The resonance-line widths are determined by the homogeneous width Γ. Therefore, the modulation resonance in a medium with a spectral inhomogeneity can be used to investigate the system's spectrum masked by a strong inhomogeneous broaden-

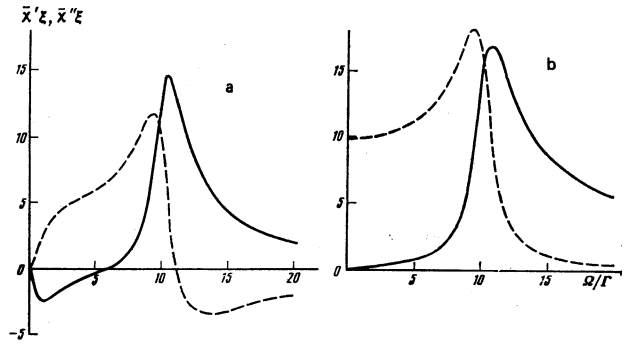

FIG. 1. Dependence of the response components $\bar{\chi}''$ (continuous curve) and $\bar{\chi}'$ (dashed curve) on the external-field frequency in (a) the low-temperature approximation and (b) the high-temperature approximation (in the latter case the quantities $\bar{\chi}\tilde{\xi}$ and $\bar{\chi}''\tilde{\xi}$ have been plotted along the axis of ordinates).

ing. Since there arises in the system a response, $\bar{\chi}''$, shifted in phase by $\pi/2$ with respect to the phase of the influence, the system absorbs the field $\vec{\mathscr{E}}_1(t)$, and the absorption is proportional to the quantity $\Omega\omega_2^2 N\bar{\chi}''$, where N is the number of two-level atoms in the medium.

The appearance of the resonance can be qualitatively explained as follows. The strong field $\vec{\mathscr{E}}_0(t)$ requantizes the states of the system in such a way that new states [the so-called quasi-energy (QE) states] arise in the alternating-field representation (i.e., in the representation in which the Hamiltonian $\hat{E}'$ is diagonal).[12,20] The Hamiltonian for a TS interacting with a strong field $\vec{\mathscr{E}}_0(t)$ and with a weak modulating field $\vec{\mathscr{E}}_1(t)$ has in the alternating-field representation (AFR) the form

$$\hat{E}+\hat{H}_m=\hat{W}(\hat{E}'+\hat{H}_m)\hat{W}^+=\varkappa(\hat{\mathscr{P}}_{22}-\hat{\mathscr{P}}_{11})+\frac{\omega_2}{2}[\cos\theta(\hat{\mathscr{P}}_{22}-\hat{\mathscr{P}}_{11})-\sin\theta(\hat{\mathscr{P}}_{12}+\hat{\mathscr{P}}_{21})]\cos\Omega t,$$

where $\hat{W}=\exp[-\frac{1}{2}\theta(\hat{P}_{12}-\hat{P}_{21})]$ is the transition operator in the AFR and the $\hat{\mathscr{P}}_{ij}$ are the projection operators of the AFR. Under conditions of exact resonance with respect to the strong field, $\cos\theta=0$ ($\Delta=0$), and the system Hamiltonian $\hat{E}+\hat{H}_m$ corresponds to the Hamiltonian of a TS acted upon by a field inducing only transitions between its states (the quasi-energy states): the strong field $\vec{\mathscr{E}}_0(t)$ creates new states, and the field $\vec{\mathscr{E}}_1(t)$ induces transitions between them. It is clear that the weak field in this case should be resonantly absorbed.

But without allowance for the modification of the KE the difference between the populations of the QE states is equal to zero, since $\langle\hat{\mathscr{P}}_{11}-\hat{\mathscr{P}}_{22}\rangle_{st}^{AFR}=-\langle\hat{P}_{12}+\hat{P}_{21}\rangle_{st}^{GIR}=0$ [see the expression (5)], where the symbols $\langle\ \rangle_{st}^{AFR}$ and $\langle\ \rangle_{st}^{GIR}$ denote averaging with the steady-state density matrix in the AFR and GIR respectively. Therefore, the field $\vec{\mathscr{E}}_1(t)$ is not, in the first approximation, absorbed. Only the consideration of the modification of the KE can lead in the present case to a nonzero difference between the populations of the QE states. But in many cases the appearance of this difference does not lead to the establishment of a canonical distribution over the QE states,[12] since, as a rule, the inequality $|\langle\hat{\mathscr{P}}_{12}\rangle_{st}|\gg\langle\hat{\mathscr{P}}_{11}-\hat{\mathscr{P}}_{22}\rangle_{st}$ will be satisfied [this inequality can be derived from the expression (5), using the relation between the averages $\langle\ \rangle_{st}^{AFR}$ and $\langle\ \rangle_{st}^{GIR}$, which relation is determined by the operator $\hat{W}$]. Thus, this qualitative explanation rests on the knowledge of the steady-state solution to the modified equations. Therefore, it is more natural to use only the change in the structure of the equations in explaining the effect.

A difference between the populations of the QE states can also arise as a result of the nonresonance character of the influence, since

$$\langle\hat{\mathscr{P}}_{11}-\hat{\mathscr{P}}_{22}\rangle_{st}^{VFR}=\cos\theta\langle\hat{P}_{11}-\hat{P}_{22}\rangle_{st}^{GIR}-\sin\theta\langle\hat{P}_{12}+\hat{P}_{21}\rangle_{st}^{GIR}.$$

But since only the contribution from the terms with the factor $\varkappa_0$ (which arise as a result of the modification) remains when the response is averaged in the low-temperature approximation, the interaction via the "nonresonance" population difference does not play any role.

B. The high-temperature approximation

In the high-temperature approximation the terms with the factor $\Delta z_0=p(\Delta^2+\Delta\omega)$ yield nonzero contributions in the averaging. Averaging the expression (8) with allowance for this fact, we obtain:

$$\chi''=\Phi_1\tilde{\Psi},\quad \chi'=\Phi_2\tilde{\Psi},\quad \tilde{\Psi}=\frac{\pi p\omega_1^2}{\delta bc},\quad \Phi_1=2\Omega\Gamma,\quad \Phi_2=d+b.$$

Figure 1b shows the plots of $\bar{\chi}'\tilde{\xi}$ and $\bar{\chi}''\tilde{\xi}$ as functions of Ω/Γ for $\omega_1=10\Gamma$, where $\tilde{\xi}=\mu\xi$. The resonance curves in this figure have roughly the same widths and peak heights as in the low-temperature approximation. The main difference is that $\bar{\chi}'(0)\approx\frac{1}{2}\chi'(\Omega_{max})$, where Ω_{max} is the frequency at which $\bar{\chi}'$ assumes its maximum value.

CONCLUSION

For the modulation resonance to be realized, the following conditions must be fulfilled. Firstly, the amplitude of the strong field acting on the TS should satisfy the inequality $\mathscr{E}_0>\Gamma d_{12}^{-1}\hbar$, i.e., all the Rabi frequencies of the system should be greater than the homogeneous line width. Secondly, it must be feasible to slightly modulate the energy gap of the TS by some field. For magnetic systems (the operator $\hat{D}$ describes the magnetic dipole transition) the second condition is realized by modulating the magnetic field producing the ω_0 splitting. For optical systems (the operator $\hat{D}$ describes the electric dipole transition) it can be realized with the aid of a modulated electric field on account of the Stark effect. In solids alternating electric field can be produced on the atoms with the aid of sound. And these fields can be significantly stronger than those produced by apparatus. The modulation of ω_0 for optical systems can also be achieved by modulating the splitting of the magnetic sublevels of the atom in a magnetic field.

The modulation resonance can be used to measure the amplitude of the coherent light field and the matrix elements of the dipole transition. Since its width is

determined by the homogeneous line width, the modulation resonance can be used to investigate the system spectrum masked by a strong inhomogeneous broadening, e.g., the Doppler broadening, which is characteristic of gases.

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