

Polarization energy losses of a proton in the electron gas of a metal

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The specific energy losses of a proton moving with arbitrary velocity are calculated within the framework of the quantum permittivity of the electron gas of a metal. It is shown that good agreement with experiment is obtained at low velocities, and that in the region of the maximum the polarization losses make a contribution of the order of unity.

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1. An electric charge moving in a medium produces an electromagnetic field that influences in turn the electron motion. The force produced in this interaction is applied to the moving charge and is antiparallel to its velocity v . This deceleration force (sometimes called the radiative friction force) is determined by the electromagnetic properties of the medium, which can be described by the dielectric tensor $\epsilon_{ij}(\mathbf{k}, \omega)$, which is a function of the frequency ω and of the wave vector \mathbf{k} of the oscillations produced when the charged particle moves.

The deceleration force of a singly charged heavy particle (say a proton) can be expressed in the form (see, e.g., Ref. 1)

$$-\frac{dE}{dx} = \frac{e^2}{\pi^2} v^{-1} \operatorname{Re} i \int \frac{d\omega}{\omega} \int \left[\frac{(kv)^2}{k^2 \epsilon^l(\mathbf{k}, \omega)} - \frac{v^2 - (kv)^2/k^2}{k^2 c^2/\omega^2 - \epsilon^{tr}(\mathbf{k}, \omega)} \right] \delta(\omega - kv) dk, \quad (1)$$

where e is the proton charge, $\epsilon^l(\mathbf{k}, \omega)$ and $\epsilon^{tr}(\mathbf{k}, \omega)$ are respectively the longitudinal and transverse permittivities of the isotropic medium. The moving charge excites waves of frequency $\omega = kv$. The first term in the square brackets of the integrand of (1) is determined by the longitudinal permittivity $\epsilon^l(\mathbf{k}, \omega)$ and corresponds to emission of longitudinal potential waves—the polarization energy loss. The second term of this integrand describes the emission of transverse waves. Since the condition for excitation of transverse waves is $(v/c) (\epsilon^{tr})^{1/2} \cos \theta = 1$ (θ is the angle between the particle-motion and wave-propagation directions), and in a plasma $\epsilon^{tr} < 1$, no Cerenkov effect takes place under these conditions, and the second term of (1) must be left out.

As the charged particle moves, the quasifree electrons of the decelerating medium are displaced not only by the action of the polarization forces, but also by the action of the friction force that determines the finite electric conductivity of the medium. Kramers² has shown that the deceleration force of a charged particle, due to the conductivity of the medium, is negligibly small. The specific loss in this case should be calculated using the formula

$$-\frac{dE}{dx} = \frac{e^2}{\pi^2} v^{-1} \operatorname{Re} i \int dk \frac{kv}{k^2 \epsilon^l(\mathbf{k}, kv)}. \quad (2)$$

2. The longitudinal permittivity of a degenerate

Fermi gas of a metal can be calculated using the quantum kinetic equation with an equilibrium function, namely the Fermi-Dirac function (see Ref. 3)

$$\epsilon^l(\xi, \Omega) = 1 + \frac{\alpha}{8\pi\xi^3} \left(4\xi + \varphi \ln \frac{\Omega + \xi + 1}{\Omega + \xi - 1} - \psi \ln \frac{\Omega - \xi + 1}{\Omega - \xi - 1} \right), \quad (3)$$

$$\varphi = 1 - (\Omega + \xi)^2, \quad \psi = 1 - (\Omega - \xi)^2.$$

The notation in (3) is the following: $\Omega = \omega/kv_F$, $\xi = \hbar k/2mv_F$, $\alpha = e^2/\hbar v_F$, v_F is the electron velocity on the boundary of the Fermi surface. The parabolic law $\epsilon_F = mv_F^2/2$ is assumed here and elsewhere.

To calculate the integral (2) we must separate the real and imaginary parts of the permittivity. Recognizing that after integrating with respect to ω in (1) the frequency vanishes and $\mathbf{k} \cdot \mathbf{v}$ appears in the argument ϵ^l , we shall therefore replace Ω in (3) by $\mathbf{k} \cdot \mathbf{v}/kv_F = uy$, where $u = v/v_F$ is the particle velocity referred to the Fermi velocity, and $y = \cos \theta$ varies generally speaking in the range $-1 \leq y \leq 1$. The real part of the permittivity is then

$$\operatorname{Re} \epsilon^l(\xi, uy) = 1 + \frac{\alpha}{8\pi\xi^3} \left(4\xi + \varphi \ln \left| \frac{uy + \xi + 1}{uy + \xi - 1} \right| - \psi \ln \left| \frac{uy - \xi + 1}{uy - \xi - 1} \right| \right), \quad (4)$$

$$\varphi = 1 - (uy + \xi)^2, \quad \psi = 1 - (uy - \xi)^2.$$

The imaginary part $\operatorname{Im} \epsilon^l(\xi, uy)$ is defined in two bands (see Fig. 1):

$$\operatorname{Im} \epsilon^l(\xi, uy) = \begin{cases} 1/8 \alpha \xi^{-3} [1 - (uy - \xi)^2], & \xi - 1 \leq uy \leq \xi + 1 \\ -1/8 \alpha \xi^{-3} [1 - (uy + \xi)^2], & -1 - \xi \leq uy \leq -1 - \xi \end{cases} \quad (5)$$

Recognizing that $\operatorname{Im} \epsilon^l(\xi, uy)$ is an odd function of y while $\operatorname{Re} \epsilon^l(\xi, uy)$ is even, we can write the expression

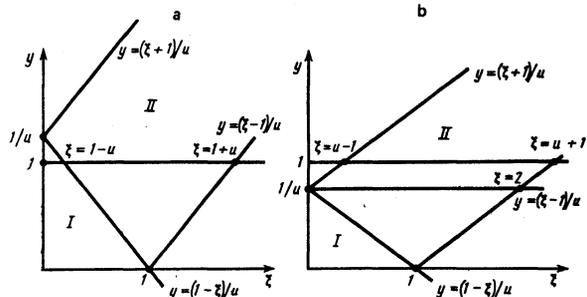


FIG. 1. Region of existence of $\operatorname{Im} \epsilon^l(\xi, uy)$ on the (y, ξ) plane at $y > 0$. In region I we have $\operatorname{Im} \epsilon^l(\xi, uy) = \alpha uy/2\xi^2$, and in region II, $\operatorname{Im} \epsilon^l(\xi, uy) = (\alpha/8\xi^3) [1 - (uy - \xi)^2]$. The integration schemes are shown for $u < 1$ (a) and $u > 1$ (b).

for the specific energy loss (2) in the form

$$-\frac{dE}{dx} = \frac{2}{3\pi} \frac{2Ry}{a_0} \left(\frac{m}{m_*}\right)^2 \frac{1}{\alpha} u g(u, \alpha), \quad (6)$$

where $Ry \approx 13.6$ eV, a_0 is the Bohr radius, m_* is the electron effective mass, and $g(u, \alpha)$ is a dimensionless friction coefficient given, in accord with Fig. 1, by different analytic expressions at $u < 1$ and $u > 1$:

$$g(u < 1, \alpha) = 6 \int_0^{1-u} \frac{d\xi}{\xi} \int_0^1 \frac{dy y^2}{|e'(\xi, uy)|^2} + 6 \int_{1-u}^1 \frac{d\xi}{\xi} \int_0^{(1-\xi)/u} \frac{dy y^2}{|e'(\xi, uy)|^2} + \frac{3}{2u} \int_0^1 dy y \int_{1-uy}^{1+uy} \frac{d\xi}{\xi^2} \frac{1-(uy-\xi)^2}{|e'(\xi, uy)|^2}, \quad (7)$$

$$g(u > 1, \alpha) = 6 \int_0^1 \frac{d\xi}{\xi} \int_0^{(1-\xi)/u} \frac{dy y^2}{|e'(\xi, uy)|^2} + \frac{3}{2u} \int_0^{1/u} dy y \int_{1-uy}^{1+uy} \frac{d\xi}{\xi^2} \frac{1-(uy-\xi)^2}{|e'(\xi, uy)|^2} + \frac{3}{2u} \int_{1/u}^1 dy y \int_{uv-1}^{uv+1} \frac{d\xi}{\xi^2} \frac{1-(uy-\xi)^2}{|e'(\xi, uy)|^2}, \quad (8)$$

with

$$|e'(\xi, uy)|^2 = [\operatorname{Re} \varepsilon'(\xi, uy)]^2 + [\operatorname{Im} \varepsilon'(\xi, uy)]^2.$$

In the first and second integrals of (7) and in the first integral of (8) we used the value of $\operatorname{Im} \varepsilon'(\xi, uy)$ in the region I (see Fig. 1), while in the third integral of (7) and in the second and third of (8) the imaginary part corresponds to $\operatorname{Im} \varepsilon'(\xi, uy)$ in region II.

3. Expressions (7) and (8) were determined by numerical integration. The values of $g(u, \alpha)$ at $0.5 \leq \alpha \leq 3$ and $0.01 \leq u \leq 20$ are listed in Table I. Figure 2 shows a plot of the friction coefficient $g(u, \alpha)$ at two values of α , namely, 1.0 and 2.0.

At small u , the function $g(u, \alpha)$ has same linear specific energy loss velocity dependence as obtained in Ref. 4. Indeed, if we put $u=0$ in (7), then the second and third integrals vanish, and the first integral yields the same value of $g(0, \alpha)$ as in (32) of Ref. 4. Further transition to the "quasiclassical" formula (26) of the cited paper and expansion in terms of $\alpha \ll 1$ yields the known expression obtained by Fermi and Teller.⁵ Unfortunately, this expression is not suitable for the calculation of the specific energy loss, since α in metals is always larger than unity.⁶ At high velocities, $u \gg 1$, the dimensionless friction coefficient can be calculated

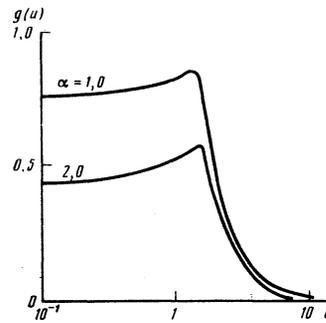


FIG. 2. Dependence of the dimensionless friction coefficient $g(u, \alpha)$ on the velocity u at α equal to 1.0 and 2.0.

by expanding (8) in powers of $1/u$:

$$g(u \gg 1, \alpha) = \frac{1}{u^2} \ln \frac{3\pi u^2}{\alpha}, \quad (9)$$

which corresponds to the Pines formula obtained by the method of collective variables.⁷ Thus, expressions (7) and (8) give correct values of $g(u, \alpha)$ in the limiting cases.

We consider now the behavior of $g(u, \alpha)$ at $u \approx 1$. As seen from Fig. 2, the dimensionless friction coefficient reaches a clearly pronounced maximum in the region $u \approx 1$. This leads to deviation from linearity of the specific energy loss, which increases somewhat faster with increasing u . This fact is apparently due to the fact that $\operatorname{Re} \varepsilon'(\xi, uy)$ has a minimum at $u \approx 1$, meaning a more intense plasmon emission.

4. We compare now the obtained specific energy loss curves with the experimental data.⁸⁻¹³ By way of example we compare the theoretical results obtained in accord with (6), (7), and (8) with experiment for aluminum (Fig. 3), copper (Fig. 4), and silver (Fig. 5).

We note first the good qualitative agreement between the calculated values of the friction force and the experimental points at low velocities $u \leq 1$, where a linear dependence is observed, with the slope $-dE/dx$ in agreement with the experimental data for the three metals. At $u \approx 2$ the calculated specific loss curve reaches a maximum that is shifted towards lower u ; the maximum agrees with the experiment for Al and is somewhat smaller in the cases of Ag and Cu.

In the velocity region $0.5 \leq u \leq 10$ the energy losses are determined by two processes with overlapping u :

TABLE I.

u	alpha									
	0.5	0.8	1.0	1.2	1.4	1.6	1.8	2.0	2.5	3.0
0.01	1.22	0.904	0.770	0.668	0.588	0.524	0.471	0.427	0.341	0.281
0.05	1.22	0.905	0.771	0.669	0.590	0.526	0.472	0.428	0.343	0.283
0.1	1.22	0.907	0.773	0.672	0.592	0.528	0.475	0.431	0.346	0.287
0.3	1.23	0.916	0.785	0.685	0.607	0.544	0.492	0.448	0.364	0.303
0.5	1.23	0.926	0.798	0.701	0.625	0.562	0.511	0.468	0.385	0.325
0.7	1.22	0.930	0.810	0.716	0.642	0.581	0.531	0.489	0.406	0.346
0.9	1.22	0.940	0.821	0.731	0.659	0.600	0.551	0.509	0.428	0.368
1.0	1.22	0.943	0.827	0.738	0.668	0.610	0.561	0.520	0.439	0.379
1.2	1.21	0.954	0.841	0.756	0.689	0.630	0.583	0.542	0.462	0.402
1.4	1.09	0.937	0.863	0.796	0.722	0.661	0.612	0.570	0.489	0.428
1.6	0.828	0.741	0.690	0.654	0.625	0.596	0.573	0.550	0.502	0.460
1.8	0.633	0.570	0.548	0.516	0.499	0.477	0.462	0.446	0.416	0.390
2.0	0.494	0.455	0.427	0.408	0.399	0.383	0.371	0.360	0.338	0.320
2.5	0.285	0.264	0.252	0.242	0.236	0.228	0.223	0.220	0.207	0.198
3.0	0.180	0.166	0.161	0.155	0.150	0.147	0.143	0.141	0.135	0.130
5.0	0.048	0.044	0.044	0.042	0.041	0.040	0.040	0.039	0.038	0.036
10.0	0.008	0.007	0.007	0.007	0.006	0.006	0.006	0.006	0.006	0.006
15.0	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
20.0	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.0009	0.0009	0.0009

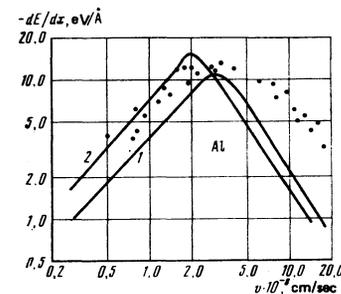


FIG. 3. Specific proton-energy loss vs. proton velocity in aluminum. Curves 1 and 2 correspond to $m_*/m = 1$ and 1.6, respectively. The points mark the experimental data.⁸⁻¹³

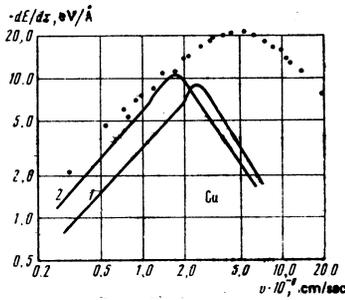


FIG. 4. Specific proton energy loss vs. proton velocity in copper. Curves 1 and 2 correspond to $m_*/m = 1$ and 1.5, respectively. The points mark the experimental data.⁸⁻¹³

polarization of the electron gas of the metal, with characteristic velocity v_F , and ionization of the outer electrons of the lattice ion core of the metal; the velocity of these electrons $v \approx v_0 = e^2/\hbar$ is close to the Bohr velocity.

In real metals (see Ref. 6) we have $\alpha = v_0/v_F \geq 1$, so that the maximum of the total energy loss should indeed be shifted towards velocities $u \geq \alpha$, for when the velocity of the incident proton increases, deeper electrons of the ion core, with large orbital velocities, participate in the ionization process. A quantitative analysis of this fact, however, is quite difficult, since the Bethe-Bloch formula gives a correct result only if $u \gg \alpha$.

Thus, the disparity between the energy-loss maximum calculated with account taken of only the polarization of the electron gas of the metal, on the one hand, and the experimentally obtained value, on the other, is understandable. It can nevertheless be stated that the polarization energy losses are decisive in the low-velocity region up to $u \leq 1$, and near the maximum they contribute a noticeable fraction. At high velocities $u \gg \alpha$, the polarization loss is low, but under channeling conditions (motion in an axial channel, when the loss to ionization becomes much smaller than in an "amorphous" metal, owing to large impact parameters), it may become necessary to take them into account, too.

Extension of the results presented here to include multiply charged ions calls apparently for a special treatment, since motion of such ions in the metal can

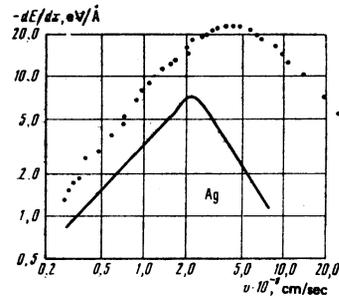


FIG. 5. Specific proton energy loss vs. its velocity in silver at $m_*/m = 1$. The points mark the experimental data.⁸⁻¹³

change their effective charge.

In conclusion, the author thanks A.E. Lenev for the numerical calculations.

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