On the theory of nuclear spin relaxation in ferromagnetics at extremely low temperatures

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The various processes of interaction of nuclear spin waves with each other and with magnons, phonons, and impurities are considered. The corresponding lifetimes and scattering probabilities are computed. The system of kinetic equations is solved, and the characteristic relaxation parameters of the nuclear-spin-wave system are found.

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1. INTRODUCTION

Investigations of the high-frequency properties of ferromagnets and antiferromagnets at extremely low temperatures[1,2] have shown that the magnetization $M_n(T)$ of the nuclear spins cannot be described by a Brillouin function for the crystal temperature. Attempts to explain the increase of the deviation of $M_n(T)$ from $\mu_n B_I$ [$B_I(X)$ is the Brillouin function for a spin I and μ_n is the nuclear magneton] with decreasing temperature in terms of the role of spin waves[3] and the contribution of the zero-point vibrations[4] have not been successful. It may be inferred that the cause of the discrepancy between the theoretical predictions and the experimental data[2] lies in the fact that there is not enough time for equilibrium to be established between the nuclear spins and the electron-phonon subsystem in the course of an experiment. This conjecture compels us to analyze the relaxation processes in magnets whose nuclear spins are magnetized by the electrons.

As seems to us, this analysis is of interest, since it allows us, in the first place, to investigate the relaxation in that temperature region in which it has previously not been studied and, in the second place, to explain the characteristics of the nuclear relaxation in magnetically ordered media. As far as we know, only the relaxation processes in magnets at temperatures significantly higher than $\hbar\omega_n$, where ω_n is the precession frequency of the nuclear spins, have thus far been investigated (see Ref. 5, Chap. 5, as well as Refs. 6 and 7).

At low temperatures, not only the electron-shell spins of the atoms, but also the nuclear spins are practically ordered, and the thermal motion is executed by the spin waves (magnons). There are two types of magnons in a single-sublattice ferromagnet with nuclei possessing magnetic moments. Because of the coupling between the electrons and the nuclei, the two types of magnons describe the collective motion of the electronic and nuclear spins. The weakness of the hyperfine interaction between the nuclear and electronic spins is manifested in the fact that one of the amplitudes of the Bogolyubov u–v transformation in the relations connecting the creation, $a^+_{e,n\mathbf{k}}$, and annihilation, $a_{e,n\mathbf{k}}$, operators for the electronic (e) and nuclear (n) spin deviations with the creation operators for the two types of spin waves is close to unity, i.e., one of the spin-excitation branches almost does not differ from the electronic spin wave. We shall call this branch the magnon branch and the electronic excitations, magnons. The other branch describes the motion of the nuclear-spin system, which is magnetized by the ordered electron spins. We shall call these excitations nuclear spin waves (NSW). We denote the magnon- and NSW-creation operators by $\alpha^+_{e\mathbf{k}}$ and $\alpha^+_{n\mathbf{k}}$ respectively.

The magnon dispersion law is practically insensitive to the nuclear subsystem[5]:

$$\omega_e(\mathbf{k}) = \omega_e + \omega_E(ak)^2 + \omega_d \sin^2\theta_{\mathbf{k}}; \tag{1}$$

here $\mathbf{k}$ is the wave vector and $\hbar\omega_e$ is the energy of a magnon with $k=0$:

$$\hbar\omega_e = \mu_e(H+H_a), \quad \hbar\omega_d = 4\pi\mu_e M_0, \tag{2}$$

H is the magnetic field, H_a is the anisotropy field ($\mathbf{H}$ is parallel to the axis of easiest magnetization), μ_e is the Bohr magneton, $M_0 = \mu_e/a^3$ is the electron magnetic moment at $T=0$, $\hbar\omega_E$ is a quantity of the order of the Curie temperature, and a is the interatomic distance.

The NSW-dispersion law is determined by the interaction between the nuclear spins and the magnons (the Suhl-Nakamura interaction: see Refs. 8 and 9)

$$\omega_n(\mathbf{k}) = \omega_n - A^2IS/\hbar^2\omega_e(\mathbf{k}), \tag{3}$$

where S is the electron spin, A is the hyperfine interaction constant and,

$$\hbar\omega_n = AS + \mu_n H. \tag{4}$$

The smallness of the nuclear magneton allows us to neglect the direct effect of the magnetic field on the NSW. The NSW dispersion [the second term in (3)] is so weak that we can neglect it at $T \sim \hbar\omega_n$, setting $\omega_n(\mathbf{k}) \approx \omega_n$. But this dispersion is, as a rule, important in considering the kinetics of the NSW. Let us note that the neglect of the NSW dispersion naturally does not transform the nuclear spins into a system of noninteracting paramagnetic atoms.

The quantities with the dimensions of energy (or frequency) that enter into the formulas (1)–(4) satisfy the strong inequalities

$$A^2IS/\hbar^2\omega_e(\mathbf{k}) = \Delta\omega_n(\mathbf{k}) \ll \omega_n \ll \omega_d, \quad \omega_e \ll \omega_E. \tag{5}$$

The dispersion law (3) is used to compute the prob-

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abilities of the various processes and the collision integrals by the perturbation-theory method. For such an analysis to be valid, it is necessary that the rather rigid inequality $\Delta\omega_n(\mathbf{k})\tau_{\mathbf{k}} \gg 1$, where $\tau_{\mathbf{k}}^{-1}$ is the corresponding NSW attenuation factor, be satisfied. As the formulas obtained below show, it is always satisfied.

The NSW relax as a result of their interaction with each other, with the magnons, and with the phonons. In crystals containing impurities, the NSW are also scattered by the impurities. The present paper is devoted to the investigation of these processes at $T \lesssim \hbar\omega_n$. In choosing the dominant scattering mechanisms, we shall retain only those which do not lead to an exponential temperature dependence of the mean scattering probability with an activation energy significantly higher than $\hbar\omega_n$. This principle significantly restricts the set of relaxation mechanisms (see below).

2. THE NSW–NSW SCATTERING

Allowance for the anharmonicities in the Suhl-Nakamura interaction energy leads to a Hamiltonian that describes the scattering of the NSW by each other[1)3]:

$$H_{nn}^{int}=\frac{A}{2^4\mathfrak{N}^2}\left(\frac{S}{I}\right)^{1/2}\sum_{1,2,3,4}(v_1+v_2+v_3+v_4)\alpha_{n1}^{+}\alpha_{n2}^{+}\alpha_{n3}\alpha_{n4}\Delta(1+2-3-4)+\text{c.c.}, \tag{6}$$

where $\mathfrak{N}$ is the number of atoms in the crystal, $v_i \equiv v(\mathbf{k}_i)$ is the small amplitude of the $u-v$ transformation:

$$v(\mathbf{k})=A(IS)^{1/2}/\hbar\omega_e(\mathbf{k}). \tag{7}$$

Using the standard procedure for computing the relaxation times $\tau(\mathbf{k})$ (see, for example, Ref. 10, Chap. 7), we find the NSW relaxation time:

$$\frac{1}{\tau_{nnk}}=\frac{\pi}{\hbar^2}\frac{A^2S}{2^7I}\frac{1}{\mathfrak{N}^3}\sum_{1,2,3}(v_1+v_2+v_3+v_k)^2\Delta(\mathbf{k}+1-2-3)$$
$$\times\delta(\omega_{nk}+\omega_{n1}-\omega_{n2}-\omega_{n3})[N_1(1+N_2)(1+N_3)-(1+N_1)N_2N_3], \tag{8}$$

where $N_i \equiv N(\hbar\omega_{n\mathbf{k}_i})$ is the equilibrium distribution function for the NSW. In the temperature region of interest to us, we can neglect the wave-vector dependence of the equilibrium distribution functions of the NSW, and take them out from under the summation sign. But even this simplification does not allow an exact computation of the $\tau_{nn\mathbf{k}}^{-1}$.

We give only the band-averaged value:

$$\frac{1}{\tau_{nn}}=\left\langle\frac{1}{\tau_{nnk}}\right\rangle=\sum_{\mathbf{k}}\frac{N_{\mathbf{k}}}{\tau_{nnk}}\Big/\sum_{\mathbf{k}}N_{\mathbf{k}}=\frac{a^3}{(2\pi)^3}\int\frac{d\mathbf{k}}{\tau_{nnk}}. \tag{9}$$

Analysis of the energy and quasimomentum conservation laws, as well as of the form of $v(\mathbf{k})$ [see (7)], shows that the major role in NSW relaxation due to the process under investigation is played by the collisions of the quasiparticles with large quasimomenta [$ak \gg (\omega_e/\omega_E)^{1/2}$]. This allows us to neglect ω_e in the expression for $\omega_e(\mathbf{k})$, and take out all the dimensional factors from under the δ-function sign. Since as a result the corresponding integral (over nine-dimensional space) does not depend on any parameter, we can obtain the value of $\langle\tau_{nn\mathbf{k}}^{-1}\rangle$ up to a numerical factor:

$$\left\langle\frac{1}{\tau_{nnk}}\right\rangle\sim\frac{A^2S}{\hbar^2\omega_E I}N(1+N). \tag{10}$$

As we shall see, the obtained expression (10) exceeds by many orders of magnitude the relaxation times describing the interaction with the phonons and magnons. This gives us grounds for believing that the "bosonization" of the NSW gas is the fastest of the investigated relaxation processes.

The relaxation time in a system of nuclear spins is determined by the slow processes. The estimate, made here, of the relaxation time for a NSW system allows us to choose a method for solving the kinetic equation (see Sec. 6). To compute the relaxation times, we do not need to know the H_{nn} -governed lifetime of the NSW as a function of $\mathbf{k}$. Richards[6] has computed the lifetime of the NSW in the high-temperature limit (i.e., for $T \gg \hbar\omega_n$). The expression obtained by him [see Ref. 6, the formula (20)] shows that at $T \gg \hbar\omega_n$

$$\frac{1}{\tau_{nnk}}\approx\frac{A}{4\pi\hbar\omega_E}\frac{T}{\hbar}ak.$$

Comparing the last expression with our formula (10) we see that τ_{nn}^{-1} should tend to zero appreciably faster as the temperature is lowered (i.e., as $T \to 0$) than the Richards formula predicts.

3. THE NSW LIFETIME DUE TO COLLISIONS WITH THE MAGNONS

The NSW—magnon interaction Hamiltonian containing the least number of creation and annihilation operators describes the production (absorption) of NSW by a magnon:

$$H_{ne}^{int}=\frac{1}{\mathfrak{N}^{1/2}}\sum_{1,2,3}\Phi_{ne}(1,2,3)\alpha_{n1}\alpha_{e2}^{+}\alpha_{e3}\Delta(1-2+3)+\text{c.c.}, \tag{11}$$

here

$$\Phi_{ne}(1,2,3)=-8\pi(2S)^{1/2}\mu_e M_0 v_1\frac{k_{1x}k_1^{-}}{k_1^2}. \tag{12}$$

We have, using the fact that μ_n is small, retained only the term proportional to $v(\mathbf{k})$.

The NSW lifetime due to the process in question is given by the following formula:

$$\frac{1}{\tau_{nek}}=\frac{2^8\pi^3S}{\hbar^2\mathfrak{N}^2}(\mu_e M_0)^2\sum_{1,2}v_k^2\left|\frac{k_x k^{-}}{k^2}\right|^2\Delta(2-1-\mathbf{k})\delta(\omega_{e2}-\omega_{e1}-\omega_n)[N_1-N_2]. \tag{13}$$

Since $\omega_n \ll \omega_e(\mathbf{k})$, the difference between the equilibrium distribution functions of the magnons should be written as:

$$N_1-N_2\approx N_1^2\exp\{\hbar\omega_{e1}/T\}\{\exp[\hbar\omega_n/T]-1\}. \tag{14}$$

Integrating with allowance for the δ function, we find the lifetime of the NSW as a function of the wave vector $\mathbf{k}$.

If we neglect the dipole term in (1), then there is no restriction on $\mathbf{k}$, and

$$\frac{1}{\tau_{nek}}=2^5S\frac{T}{\hbar\omega_E}\left(\frac{\mu_e M_0}{\hbar\omega_e}\right)^2\frac{A^2IS}{\hbar^2\omega_E}\exp\left(-\frac{\hbar\omega_e}{T}\right)\sin^2\theta_k\varphi_k,$$
$$\varphi_k=\frac{1}{ak}\exp\left\{-\frac{\hbar\omega_E}{T}\left[\frac{(ak)^2-\omega_n/\omega_E}{2ak}\right]^2\right\}. \tag{15}$$

For $ak \approx (\omega_n/\omega_E)^{1/2}$ the function φ_k has in the temperature region ($T \ll \hbar\omega_e$) of interest to us a sharp peak with

a temperature-independent height approximately equal to $(\omega_E/\omega_n)^{1/2}$. But the mean probability for the inverse lifetime of the NSW is exponentially small:

$$\frac{1}{\tau_{ne}}=\left\langle\frac{1}{\tau_{nek}}\right\rangle\approx 4\left(\frac{\mu_e M_0}{\hbar\omega_e}\right)^2\frac{A^2IS}{\hbar^2\omega_E}\exp\left(-\frac{\hbar\omega_e}{T}\right). \tag{16}$$

The probability τ_{enk}^{-1} for a magnon to emit or absorb a NSW is significantly higher than τ_{nek}^{-1}. Indeed, according to (11)

$$\frac{1}{\tau_{enk}}=\frac{2\pi}{\hbar^2\mathfrak{N}^2}\sum_{1,2}|\Phi_{ne}(1,2,\mathbf{k})|^2\Delta(\mathbf{k}-1-2)\delta(\omega_{ek}-\omega_{e1}-\omega_n)(1+N_1+N_2)$$
$$+|\Phi_{ne}(2,1,\mathbf{k})|^2\Delta(\mathbf{k}-1+2)\delta(\omega_{ek}-\omega_{e1}+\omega_n)(N_2-N_1). \tag{17}$$

Allowance for the dipole interaction in the magnon-dispersion law (in the case in which $\omega_e\sim\omega_d$) can apparently affect the dependence of the relaxation time on $\mathbf{k}$. But because of the complexity of the resulting integrals the computation of the probability is not possible when allowance is made for the dipole-dipole interaction. Therefore, below we restrict outselves to the consideration of the case in which $\omega_d\ll\omega_e$. It follows from the conservation laws that NSW can be emitted by magnons with wave vectors greater than $a^{-1}(\omega_n/\omega_e)^{1/2}$ and be absorbed by magnons with any wave vector.

According to (17), the NSW-emission probability is equal to

$$\frac{1}{\tau_{enk}^{em}}=2^6\pi S(1+N)\frac{(\mu_e M_0)^2}{\hbar^2\omega_E}\frac{A^2IS((ak)^2-\omega_n/\omega_E)^{1/2}}{\hbar^2\omega_e(\omega_e+4\omega_E(ak)^2)}, \tag{18}$$
$$ak\geqslant(\omega_n/\omega_E)^{1/2}.$$

The probability for absorption of NSW is

$$\frac{1}{\tau_{enk}^{ab}}=2^6\pi SN\frac{(\mu_e M_0)^2}{\hbar\omega_E}\frac{A^2IS((ak)^2+\omega_n/\omega_E)^{1/2}}{\hbar\omega_e(\omega_e+4\omega_E(ak)^2)}. \tag{19}$$

The mean magnon lifetime, τ_{en}, due to the emission and absorption of NSW is computed by summing and averaging the expressions (18) and (19):

$$\frac{1}{\tau_{en}}=\left\langle\frac{1}{\tau_{enk}}\right\rangle=(1+2N)2^7\pi^{1/2}S\frac{(\mu_e M_0)^2}{\hbar^2\omega_E}\frac{A^2IS}{\hbar^2\omega_e^2}\left(\frac{\omega_n}{\omega_E}\right)^{1/2}. \tag{20}$$

4. SCATTERING OF NSW BY PHOTONS

Formally, there are two types of single-phonon processes: a) the emission (absorption) of a phonon by a nuclear spin wave and b) the annihilation (creation) of two NSW with the emission (absorption) of a phonon. It can be shown that, owing to the weakness of the NSW dispersion, the probability for the processes of the first type is equal to zero. Therefore, we shall discuss the processes of the second type. The corresponding interaction Hamiltonian is derived from the magnetostriction energy with allowance for the NSW-excitation impurity in the electronic spin deviations:

$$H_{nph}^{int}=i\frac{\gamma}{2}\left(\frac{\hbar}{2\rho a^3c_s}\right)^{1/2}\mu_e M_0\sum_{1,2,3,j}v_1v_2$$
$$\times\frac{e_j^+k_3^+}{k_3^{1/2}}\alpha_{n1}^+\alpha_{n2}^+b_3\Delta(1+2-3)+\text{c.c.}, \tag{21}$$

where c_s is the speed of sound, γ is the magnetostriction constant, ρ is the density of the magnet, and the index j (= 1, 2, 3) numbers the phonon polarizations. Naturally, only the acoustic phonons are taken into account: the role of the optical phonons at so low temperatures is quite unimportant.

Proceeding in much the same way as described above, we have

$$\frac{1}{\tau_{nphk}}=\frac{\pi\gamma^2}{2}\frac{\hbar}{\rho a^3c_s}(\mu_e M_0)^2v_k^2\frac{1}{\mathfrak{N}^2}\sum_{1,2,j}v_1^2\frac{|k_2^+e_j^+|^2}{k_2}(N_1-N_2)$$
$$\times\Delta(\mathbf{k}+1-2)\delta(\omega_{nk}+\omega_{n1}-\omega_{p2}). \tag{22}$$

Assuming for simplicity that the phonon dispersion law is isotropic, we find after integration that

$$\frac{1}{\tau_{nphk}}=\frac{8}{9\pi}\frac{(\gamma\mu_e M_0)^2}{\hbar\Theta_D}\left(\frac{\hbar\omega_n}{\Theta_D}\right)^3\left(\frac{A^2IS}{\hbar^2\omega_e^2(\mathbf{k})}\right)^2\operatorname{sh}^{-1}\left(\frac{\hbar\omega_n}{T}\right)\frac{\hbar}{\rho a^4c_s}. \tag{23}$$

The process considered occurs at any value of the wave vector, $\mathbf{k}$, of the NSW, the dependence on $\mathbf{k}$ being such that we can, in averaging $\tau_{nph\mathbf{k}}^{-1}$, extend the integration over $\mathbf{k}$ to infinity:

$$\frac{1}{\tau_{nph}}=\left\langle\frac{1}{\tau_{nphk}}\right\rangle=\left(\frac{I}{S}\frac{\gamma}{12\pi}\right)^2$$
$$\times\frac{(\mu_e M_0)^2}{\hbar\Theta_D}\left(\frac{\hbar\omega_n}{\Theta_D}\right)^3\frac{\hbar}{\rho a^4c_s}\frac{\omega_n^4}{\omega_e^{3/2}\omega_E^{1/2}}\operatorname{sh}^{-1}\left(\frac{\hbar\omega_n}{T}\right). \tag{24}$$

Estimates of the NSW lifetimes due to the two-phonon processes, as well as the magnon-phonon interaction processes involving the emission (absorption) of NSW, show that these processes are less probable than the single-phonon processes. But from the point of view of the magnons (see Sec. 6), there is among the processes in which the magnons, phonons, and NSW participate one that plays an important role in the relaxation. This is the decay of a magnon into a phonon and a NSW. The corresponding interaction Hamiltonian has the form

$$H_{enph}^{int}=\frac{1}{\mathfrak{N}^{3/2}}\sum_{1,2,3,j}\psi_{enph}(1,2,3)\alpha_{e1}\alpha_{n2}^+b_3^+\Delta(1-2-3)+\text{c.c.}, \tag{25}$$

$$\psi_{enph}(1,2,3)=i\gamma\mu_e M_0\left(\frac{\hbar}{2\rho a^3c_s}\right)^{1/2}v_2\frac{\mathbf{e}_j^+\mathbf{k}_3^+}{(k_3)^{1/2}}. \tag{26}$$

From this we compute in the limiting cases, using the fact that the NSW energy is small, the magnon lifetime with respect to the decay processes ($\tau^{-1}\sim 1+N$) and the coalescence process ($\tau^{-1}\sim N$):

$$\frac{1}{\tau_{enphk}}=\binom{1+N}{N}\frac{4}{3\pi\hbar^2}\frac{(\gamma\mu_e M_0)^2}{\Theta_D}\frac{\hbar}{\rho a^4c_s}$$
$$\times\begin{cases}\dfrac{A^2IS}{\hbar\omega_e^2}\left(\dfrac{\hbar\omega_e}{\Theta_D}\right)^3, & ak\ll\left(\dfrac{\omega_e}{\omega_E}\right)^{1/2}\\ \dfrac{A^2IS}{\hbar\omega_E^2}\left(\dfrac{\hbar\omega_E}{\Theta_D}\right)^3(ak)^2, & \left(\dfrac{\omega_e}{\omega_E}\right)^{1/2}\ll ak\ll 1.\end{cases} \tag{27}$$

For $T\lesssim\hbar\omega_n\ll\hbar\omega_e$, the mean value of the probability for the process in question is naturally given by the value of $\tau_{enph\mathbf{k}}^{-1}$ for $ak\ll(\omega_e/\omega_E)^{1/2}$, i.e., by

$$\frac{1}{\tau_{enph}}=\left\langle\frac{1}{\tau_{enphk}}\right\rangle\approx\frac{4}{3\pi\hbar}\frac{(\gamma\mu_e M_0)^2}{\Theta_D}\frac{\hbar}{\rho a^4c_s}(1+2N)\frac{A^2IS}{\Theta_D^2}\frac{\hbar\omega_e}{\Theta_D}. \tag{28}$$

5. SCATTERING OF THE NSW BY THE IMPURITIES

Naturally, any local defect in the crystal serves as a scattering center for the NSW (as for any quasiparticle). Evidently, the largest cross sections should be possessed by foreign atoms having magnetic moments (paramagnetic impurities). For definiteness we shall assume that all the impurity atoms are of the same type, their concentration is equal to $c=\mathfrak{N}_{im}/\mathfrak{N}\ll 1$, the electronic spin is equal to S' (S' is, generally speaking, not equal to S), the nuclear spin is equal to I', and the constant characterizing the hyperfine interaction in the defect

atom is $A' \neq A$. The NSW—impurity atom interaction Hamiltonian has the form standard for problems of this type

$$H_{ni}^{int}=(A'S'-AS)\sum_{i}{}' a_{ni}^{+}a_{ni}$$
$$\cong(A'S'-AS)\frac{1}{\mathfrak{N}}\sum_{i_1 i_2}{}'\exp(-i(\mathbf{k}_1-\mathbf{k}_2)\mathbf{R}_i)\alpha_{n_1}^{+}\alpha_{n_2}, \qquad (29)$$

the summation being performed over the impurity sites (this is indicated by the prime on the summation sign).

Computing the scattering probability, and averaging it over the impurity distribution, we obtain

$$\frac{1}{\tau_{n\mathbf{k}}}=\frac{2\pi c}{\hbar^2}(A'S'-AS)^2\frac{1}{\mathfrak{N}}\sum_{\mathbf{k}'}\delta(\omega_{n\mathbf{k}'}-\omega_{n\mathbf{k}})$$
$$=2\pi c\hbar\left(\frac{A'S'-AS}{A(IS)^{1/2}}\right)^2\omega_e^{\,2}(\mathbf{k})\sum_{\mathbf{k}'}\delta(\omega_e(\mathbf{k}')-\omega_e(\mathbf{k})). \qquad (30)$$

Going over from summation to integration, and using the δ function, we have

$$\frac{1}{\tau_{n\mathbf{k}}}=\frac{ca^3}{(2\pi)^2}\left(\frac{A'S'-AS}{A(IS)^{1/2}}\right)^2\omega_e^{\,2}(\mathbf{k})\oint_{\omega_e(\mathbf{k})=\text{const}}\frac{dS_{\mathbf{k}}}{|\mathbf{v}_{e\mathbf{k}}|}, \qquad (31)$$

where $\mathbf{v}_{e\mathbf{k}}=\partial\omega_e(\mathbf{k})/\partial\mathbf{k}$ is the magnon velocity.

According to (31)

$$\frac{1}{\tau_{n\mathbf{k}}}=\begin{cases}\dfrac{c}{2\pi S}\left(\dfrac{A'S'-AS}{A(IS)^{1/2}}\right)^2\dfrac{\omega_e^{\,2}}{\omega_E}ak, & ak\ll\left(\dfrac{\omega_e}{\omega_E}\right)^{1/2}\\[2ex] \dfrac{c}{2\pi S}\left(\dfrac{A'S'-AS}{A(IS)^{1/2}}\right)^2\omega_E(ak)^5, & \left(\dfrac{\omega_e}{\omega_E}\right)^{1/2}\ll ak\ll 1.\end{cases} \qquad (32)$$

As we can see, the inverse relaxation time increases rapidly with increasing wave vector. Therefore, the mean (over the NSW band) probability for scattering by the impurities can be estimated only in order of magnitude:

$$\frac{1}{\tau_{ni}}=\left\langle\frac{1}{\tau_{n\mathbf{k}}}\right\rangle\sim c\left(\frac{A'S'-AS}{A(IS)^{1/2}}\right)^2\omega_E. \qquad (33)$$

A comparison of (28) with $\Delta\omega_{n\mathbf{k}}$ shows that the condition $\Delta\omega_{n\mathbf{k}}\tau_{ni\mathbf{k}}\gg 1$ is always valid when $k< a^{-1}(\omega_n/\omega_E)^{2/7}c^{-1/7}$ for concentrations c lower than 10^{-3}, which are the concentrations that usually obtain in experiments. Although at reasonable impurity concentrations the mean probability for NSW scattering by the impurities exceeds the probability for the other processes, the role of impurity scattering is limited by the isotropization of the distribution function. This process cannot lead to a change in the mean NSW energy, and is therefore not of much importance in the relaxation process (see below).

6. RELAXATION IN A NSW SYSTEM

We shall, in investigating the relaxation process in a NSW system, proceed from the assumption (usually justified in practice) that the phonon-relaxation time τ_{ph} is significantly shorter than the other characteristic relaxation times. In other words, the phonons will be regarded as a thermostat the interaction with which leads to the establishment of the equilibrium states of the magnon and NSW systems (T is the phonon temperature). A comparison of the above-computed relaxation times shows that

$$\tau_{ni}^{-1}\gg\tau_{nn}^{-1}\gg\tau_{en}^{-1}\gg\tau_{enph}^{-1}\gg\tau_{nph}^{-1}. \qquad (34)$$

Let us emphasize that all the computed probabilities in (34) (they pertain to the NSW and the magnons) do not contain exponential factors with a large ($\sim\hbar\omega_e$) activation energy. Let us, using this fact, write down the kinetic equations describing the nonactivation relaxation:

$$\dot{N}_{\mathbf{k}}=L_{nph}(N_{\mathbf{k}},\bar{f}_{\mathbf{k}}), \qquad (35')$$
$$\dot{n}_{\mathbf{k}}=L_{en}(n_{\mathbf{k}},N_{\mathbf{k}})+L_{enph}(n_{\mathbf{k}},N_{\mathbf{k}},\bar{f}_{\mathbf{k}}). \qquad (35'')$$

The expressions for the collision operators L_{nph}, L_{en}, and L_{enph} can be written out on the basis of the Hamiltonians (11), (21), and (25) respectively (see, for comparison, Ref. 10, §§25 and 26). We have dropped the operator L_{nn}, since we are interested in the phase of the relaxation following the bosonization of the NSW distribution function. The relaxation rate is given by the slowest process; it is given by the Eq. (35′) in which $\bar{f}_k$ is the equilibrium phonon distribution function. Equation (35″) shows that the magnon distribution function $n_{\mathbf{k}}(t)$ follows the NSW distribution function $N_{\mathbf{k}}(t)$. According to (35″), the magnon temperature T_e coincides with the NSW temperature T_n:

$$T_e(t)=T_n(t), \qquad (36)$$

the lagging of T_e behind T_n being determined by the time τ_{en}, the shortest of the long times ($\tau_{en}\ll\tau_{enph}\ll\tau_{nph}$). The dependence of the nonequilibrium chemical potential, ξ_e, of the magnons on time is determined by the second term in Eq. (35″):

$$\xi_e(t)=\frac{\hbar\omega_e}{T}(T-T_n(t))+\frac{3}{2}T_n(t)\ln\frac{T}{T_n(t)}. \qquad (37)$$

For $T\sim\hbar\omega_n$ we can neglect the NSW dispersion. Then the kinetic equation (35′) for $N_{\mathbf{k}}(t)$ allows us to derive the equation describing the evolution of the NSW number[2)]—the function $N=N(t)$:

$$\dot{N}=\lambda\{(1+N)^2\bar{f}(2\hbar\omega_n)-N^2(1+\bar{f}(2\hbar\omega_n))\}, \qquad (38)$$

where

$$\lambda=\frac{\pi^{-2}}{144}\frac{\hbar}{\rho a^4c_s}\frac{(\gamma\mu_eM_0)^2}{\hbar\Theta_D}\left(\frac{\hbar\omega_n}{\Theta_D}\right)^3\frac{(A^2IS)^2}{\hbar^4\omega_e^{5/2}\omega_E^{3/2}}. \qquad (39)$$

Equation (38) was derived through integration over the band, and since the quantity $v_{\mathbf{k}}$ entering into the NSW—phonon interaction probability is proportional to $[\omega_e+\omega_E(ak)^2]^{-1}$, the integration over $\mathbf{k}$ can be extended to infinity. It is convenient to rewrite Eq. (38) in the form of an equation for the inverse NSW temperature introduced by the equality

$$N(t)=\left[\exp\left(\frac{\hbar\omega_n}{T_n(t)}-1\right)\right]^{-1}. \qquad (40)$$

Setting

$$\frac{\hbar\omega_n}{T_n(t)}=y,\quad \Lambda t=x,\quad \Lambda=\lambda\,\mathrm{sh}^{-1}\left(\frac{\hbar\omega_n}{T}\right),$$

we obtain

$$dy/dx=-\mathrm{sh}\,(y-y_\infty),\quad y_\infty=\hbar\omega_n/T,$$

from which we find that

$$\mathrm{th}\left(\frac{y-y_\infty}{2}\right)=\mathrm{th}\left(\frac{y_0-y_\infty}{2}\right)e^{-x}. \qquad (41)$$

It can be seen from this that the relaxation time of the system in question is $\frac{1}{2}\Lambda$, i.e.,

$$\tau_n=72\pi^2\left(\frac{\rho a^4c_s}{\hbar}\right)\frac{\hbar\Theta_D}{(\gamma\mu_eM_0)^2}\left(\frac{\Theta_D}{\hbar\omega_n}\right)^3\frac{\hbar^4\omega_e^{5/2}\omega_D^{3/2}}{(A^2IS)^2}\,\mathrm{sh}\left(\frac{\hbar\omega_n}{T}\right). \qquad (42)$$

Thus, the analysis carried out here shows that, at $T \lesssim \hbar\omega_n$, the relaxation in a system of NSW and magnons occurs in the following manner:

1. The collisions with the impurities isotropizes the NSW and magnon distribution functions (apparently, under real conditions this is the fastest process).

2. The next stage is the bosonization of the NSW and magnon gases.

3. Owing to the interaction with the phonons, the number of NSW tends to its equilibrium value [this phase of the relaxation is described by Eqs. (38)–(42)]. The temperature and chemical potential of the magnons follow the instantaneous value of the nuclear temperature $T_n(t)$ [see the formulas (20) and (28)].

The cause of such a relaxation picture is the nonactivation character of the phonon spectrum, the weak activation in the NSW spectrum, and the negligibility of the dispersion in the NSW spectrum.

Although the relaxation time obtained is very long [~28 h if we use the following values for the parameters entering into the formula (42): $T \approx \hbar\omega_n = 3 \cdot 10^{-2} K$, $\Theta_D = 10^2$ K, $\hbar\omega_E = 10$ K, $\hbar\omega_e = 0.1$ K, $\mu_e M_0 = 1$ K, $\gamma = 10$], it is many orders of magnitude shorter than the activation times. The shortest of them is proportional to $\exp(2\hbar\omega_e/T)$, and in the case under consideration $\exp(2\hbar\omega_e/T) \approx e^{60}$. In conclusion, we wish to emphasize the great role played by the NSW in the relaxation of magnons at temperatures low compared to $\hbar\omega_e$.

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[1)] The terms containing three α_{nk} operators and arising as a result of the dipole interaction have been discarded, since the corresponding processes do not occur because of the weakness of the dispersion,

[2)] In the case in which the dispersion is neglected the equation using the energy conservation law is identical to Eq. (35').

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