Field dependence of the magnetostriction constants of rare-earth iron garnets

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The magnetostriction of holmium-yttrium iron garnets, $Ho_xY_{3-x}Fe_5O_{12}$ ($x = 3$ and 1.05), has been measured in pulsed magnetic fields up to 200 kOe at 78 K. It is shown that the magnetostriction constants λ_{111} and λ_{100} of these ferrimagnets depend on the magnetic field. The magnetostriction constants of the iron garnet $Ho_3Fe_5O_{12}$ increase, whereas the magnetostriction constants of the iron garnet $Ho_{1.05}Y_{1.95}Fe_5O_{12}$ decrease, on increase of field. It is observed that the constants λ_{111} and λ_{100} of the anisotropic magnetostriction of the iron garnet $Ho_{1.05}Y_{1.95}Fe_5O_{12}$ have significantly different variations with the field: λ_{111} varies quadratically with the total effective field, whereas the variation of λ_{100} with H_{eff} is nearly linear. A theoretical analysis is made of magnetoelastic interaction in rare-earth iron garnets. It is shown that in rare-earth iron garnets, because of the presence of low-symmetry nonequivalent sites of the rare-earth ions and because of the rather large value of the crystalline field, the dependence of the exchange integral on the strain leads to a contribution to the constant λ_{100} of anisotropic magnetostriction.

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1. INTRODUCTION

It was observed earlier[1] that the magnetic anisotropy of rare-earth iron garnets (REIG) depends on the value of the magnetic field. This is due to the fact that the molecular field exerted on the rare-earth (Re) ions by the iron sublattices is comparatively small ($H_M = \lambda M_{Fe} \sim 10^5$ Oe; λ is the molecular-field parameter, M_{Fe} the magnetization of the iron ions). Therefore an external field strongly influences the degree of magnetic ordering of the rare-earth sublattice, which is determined by the joint action of the external and exchange fields; consequently it also strongly influences the value of the magnetic anisotropy, since the magnetic anisotropy depends on the degree of magnetic ordering.

Everything said above can be extended completely to the magneto-striction of REIG. But hitherto, data have been lacking on the magneto-striction constants of REIG in fields comparable in magnitude with the molecular field that acts on the RE ions. In the present paper, an experimental and theoretical investigation is made of the behavior of the magnetostriction constants of REIG in strong magnetic fields.

2. EXPERIMENT

We made measurements of the magnetostriction constants of REIG in magnetic fields up to 200 kOe at temperature 78 K, on the same specimens of holmium-yttrium iron garnets $Ho_xY_{3-x}Fe_5O_{12}$ ($x = 1.05$ and $x = 3.0$) on which the magnetic anisotropy was measured earlier.[1] The magnetostriction measurements were made in pulsed magnetic fields of pulse duration ~10 ms by means of a piezoelectric transducer, a thin quartz plate cemented to the single-crystal iron-garnet specimen under investigation along the appropriate direction of measurement.

On the basis of the general formula for the magnetostriction of a cubic crystal,

$$\Delta l/l = {}^3/{}_2\lambda_{100}(\alpha_1{}^2\beta_1{}^2+\alpha_2{}^2\beta_2{}^2+\alpha_3{}^2\beta_3{}^2-{}^1/{}_3)$$
$$+3\lambda_{111}(\alpha_1\alpha_2\beta_1\beta_2+\alpha_1\alpha_3\beta_1\beta_3+\alpha_2\alpha_3\beta_2\beta_3),$$

where α_i are the direction cosines of the measurement direction and β_i are the direction cosines of the magnetization vector, the magnetostriction constants λ_{111} and λ_{100} were determined from measurements of the magnetostriction along axes ⟨111⟩ and ⟨100⟩. To eliminate the effect of domain structure on the results of the magnetostriction measurements, the values of λ_{111} and λ_{100} were determined from the difference between the longitudinal magnetostriction $\lambda^{\parallel}$ (field parallel to the direction of measurement of the magnetostriction) and the transverse magnetostriction $\lambda^{\perp}$ (field perpendicular to this direction) along the respective axes:

$$^3/_2\lambda_{100}=\lambda^{\parallel}_{\langle 100\rangle}-\lambda^{\perp}_{\langle 100\rangle}, \quad ^3/_2\lambda_{111}=\lambda^{\parallel}_{\langle 111\rangle}-\lambda^{\perp}_{\langle 111\rangle}.$$

Figure 1 shows the variation of the magnetostriction constants of holmium iron garnet, $He_3Fe_5O_{12}$, with the field. It is seen that the magnetostriction constants of holmium iron garnet increase in absolute value on increase of the field. This can be explained as follows. In $Ho_3Fe_5O_{12}$ the magnetic compensation point is $T_c = 136$ K; therefore at 78 K the magnetic moment of the rare-earth sublattice M_R is larger than the magnetic moment of the iron sublattice M_{Fe} and is oriented along the field. The total effective field is equal to the sum of the molecular and external fields, $H_{\text{eff}} = H_M + H$, and

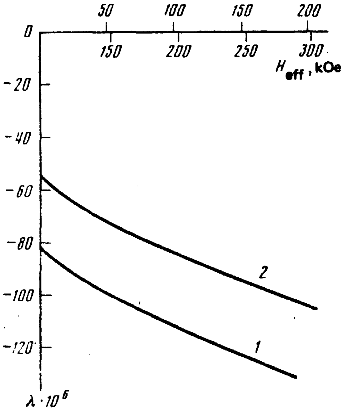

FIG. 1. Variation of the magnetostriction constants of $Ho_3Fe_5O_{12}$ with magnetic field at 78 K: 1, λ_{100}; 2, λ_{111}.

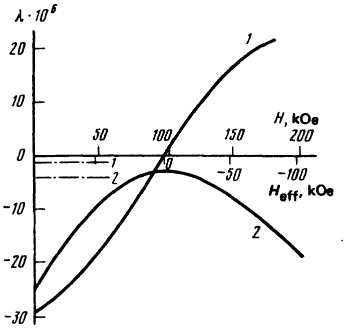

FIG. 2. Variation of the magnetostriction constants of $Ho_{1.05}Y_{1.95}Fe_5O_{12}$ (solid lines) and $Y_3Fe_5O_{12}$ (dashed-dotted lines) with magnetic field at 78 K: 1, λ_{100}; 2, λ_{111}.

increase of the external field leads to increase of the degree of magnetic order in the rare-earth sublattice and consequently to increase (in absolute value) of the magnetostriction constants.

The magnetostriction constants of the holmium-yttrium iron garnet $Ho_{1.05}Y_{1.95}Fe_5O_{12}$ vary in a different manner with the field (Fig. 2). For this ferrite, the magnetic compensation point is $T_c = 36$ K; therefore at 78 K the magnetic moment of the holmium sublattice of this ferrimagnet is less than the magnetic moment of the iron sublattice and is oriented antiparallel to the field. The effective field acting on the holmium sublattice is equal to the difference between the molecular and external fields $H_{eff} = H_M - H$, and therefore in comparatively weak fields ($H < H_M$) increase of the external field leads to decrease of the effective field and of the degree of magnetic order in the holmium sublattice, and consubsequently to decrease (in absolute value) of the magnetostriction constants (Fig. 2). Further increase of the external field ($H > H_M$) leads to increase of the absolute value of H_{eff} and to increase of the degree of magnetic order in the holmium sublattice (when $H > H_M$, the magnetic moment of the holmium sublattice is oriented along the field) and to increase of the absolute value of the magnetostriction constants. The molecular field acting on the holmium sublattice at 78 K is 96 kOe[2]; therefore when $H > 96$ kOe, the absolute value of the magnetostriction constants should increase. In fact, this is observed experimentally: the magnetostriction constants of the iron garnet $Ho_{1.05}Y_{1.95}Fe_5O_{12}$ are smallest in fields ~100 kOe and then increase in absolute value.1)

There is a striking difference between the field variations of the two magnetostriction constants, λ_{111} and λ_{100}, of holmium-yttrium iron garnet $Ho_{1.05}Y_{1.95}Fe_5O_{12}$: whereas λ_{100} changes sign at $H \sim \lambda M_{Fe}$, λ_{111} does not change sign at this field, remaining negative both in fields $H < \lambda M_{Fe}$ and in fields $H > \lambda M_{Fe}$.

Usually the anisotropic magnetostriction of REIG is explained in the single-ion model by interaction of the anisotropic cloud of 4f electrons of the RE ion with the crystalline field that acts on it. In addition, it is supposed that the anisotropic magnetostriction of REIG may receive a contribution from anisotropic exchange interaction between the RE ions and the iron ions (R–Fe interaction). As regards the isotropic part of the exchange interaction, it has hitherto been assumed that in cubic crystals this interaction leads only to an isotropic volume magnetostriction, independent of the direction of the magnetic moments with respect to the crystallographic axes. This conclusion was drawn for magnets in which all the sites occupied by magnetic ions (of a given type) are equivalent, and it was furthermore supposed that the energy of interaction of the magnetic ions with the crystalline field was much smaller than the energy of isotropic exchange interaction. But for REIG these conditions are not satisfied; as we shall show below, this leads to the result that the isotropic R–Fe exchange interaction also makes a contribution to the anisotropic magnetostriction, proportional to the effective field acting on the RE ions. For this purpose, we shall consider in greater detail the crystallographic structure of REIG and the exchange interactions between the magnetic ions that enter into their makeup.

3. CRYSTALLOGRAPHIC STRUCTURE AND EXCHANGE INTERACTIONS

REIG have a cubic crystal structure and are described by space group O_h^{10}. In the crystal lattice of a garnet (the unit cell contains eight formula units $R_3Fe_5O_{12}$, where R is a rare-earth element or yttrium), the Fe^{3+} ions are located on sites with an octahedral oxygen environment (a sites) and on sites with a tetrahedral oxygen environment (d sites). Between the a and d sublattices of Fe^{3+} ions there is a strong antiferromagnetic exchange interaction (the effective field of this interaction is $H_{a-d} \sim 2 \cdot 10^6$ Oe), which leads to antiparallel orientation of them, so that in fields $H \ll H_{a-d}$ the iron sublattices may be regarded as a single resultant magnetic sublattice, with a magnetic moment M_{Fe} equal to the difference between the magnetic moments of the a and d sublattices ($M_{Fe} = 5\mu_B$ per formula unit at $T = 0$ K).

The RE ions are located on sites with a dodecahedral environment (c sites). The symmetry of the environment of the c sites is determined by the point group D_2 (222). The group D_2 has one-dimensional irreducible representations A, B_1, B_2, B_3. Products of any two quantities transforming according to the same irreducible representation transform according to the representation A, i.e., are invariant in operations of the symmetry group D_2.

The basis functions of the irreducible representations

TABLE I. Basis functions of irreducible representations of the group D_2 [($\hat{\varepsilon}$ is the strain tensor; Y_k^q are the irreducible tensor operators (spherical harmonics with choice of phase in accordance with Condon and Shortley3)].

Irreducible representation	J x	$\varepsilon_{\alpha\beta}$	$(x)^{2n}$
A	—	$\varepsilon_{\alpha\alpha}$	$(K_A)_n^m = \frac{1}{\sqrt{2}}(Y_{2n}^{2m} + Y_{2n}^{-2m}),\ m = 0\ldots n$
B_1	J_z z	ε_{xy}	$(K_{B_1})_n^m = \frac{i}{\sqrt{2}}(Y_{2n}^{2m} - Y_{2n}^{-2m}),\ m = 0\ldots n$
B_2	J_y y	ε_{xz}	$(K_{B_2})_n^m = \frac{1}{\sqrt{2}}(Y_{2n}^{2m-1} - Y_{2n}^{-2m+1}),\ m = 1\ldots n$
B_3	J_x x	ε_{yz}	$(K_{B_3})_n^m = \frac{i}{\sqrt{2}}(Y_{2n}^{2m-1} + Y_{2n}^{-2m+1}),\ m = 1,\ldots, n$

TABLE II. Orientation of crystal axes with respect to a crystallographic system of coordinates.

Local axes	Nonequivalent sites					
	1	2	3	4	5	6
e_x^r	[110]	$[1\bar{1}0]$	[011]	$[01\bar{1}]$	[101]	$[\bar{1}01]$
e_y^r	$[\bar{1}10]$	[110]	$[0\bar{1}1]$	[011]	$[10\bar{1}]$	[101]
e_z^r	[001]	[001]	[100]	[100]	[010]	[010]

of the quantities of interest to us are given in Table I. In the structure of the iron garnets there are six such nonequivalent c sites, differing from one another in the directions of the dodecahedral axes. Table II gives the orientations of the local axes of symmetry for all six nonequivalent sites. In general, therefore, it is necessary to consider six different rare-earth sublattices.

Magnetic ordering of the RE ions is brought about by negative exchange interaction with the iron sublattice, which we shall for simplicity consider isotropic. The effective field of the R-Fe exchange interaction has the order of magnitude 10^5 Oe. We shall neglect exchange interaction between Re ions, since its value is an order of magnitude smaller than the value of the R-Fe exchange interaction. The RE ions in garnets are subject to significant crystalline fields, whose energy of interaction with the Re ions in most cases exceeds the energy of the R–Fe exchange interaction.[4]

Because of the large value of the spin-orbit interaction in RE ions (the splitting of the multiplets $\sim 10^4$ cm^{-1}), we may restrict ourselves to consideration only of the ground multiplet with a given value of J. This multiplet is split under the influence of the crystalline field and of the field of the exchange forces exerted on the RE ions by the iron ions. This field of the exchange forces is directed opposite to the orientation of the magnetization $\underset{\sim}{\mathbf{M}}_{\rm Fe}$ of the iron sublattice.

4. THE HAMILTONIAN

Since we have neglected exchange interaction between RE ions, the contribution of the RE ions to the Hamiltonian of the system is an additive sum of Hamiltonians of individual RE ions. We shall represent the Hamiltonian of an RE ion, located at the r-th nonequivalent site (in the local axes of symmetry of the given site), in the form

$$\mathscr{H}_r = -\beta g_J \mathbf{J}_r \mathbf{H} + \lambda_r \mathbf{J}_r \mathbf{M}_{\rm Fe} + V_c, \tag{1}$$

where β is the Bohr magneton, g_J is the Landé factor of the multiplet under consideration, and $\underset{\sim}{J}$ is the total angular momentum operator. The first term on the right side of the expression (1) is the Hamiltonian of interaction of the RE ion with the external field; the second term is the Hamiltonian of R–Fe exchange interaction, in the molecular-field approximation with molecular-field parameter λ; and V_c is the Hamiltonian of the crystalline field.

The occurrence of magnetoelastic interaction is due first to the fact that in (1) the molecular-field parameter depends on the strain (the exchange mechanism), and second to interaction of the anisotropic cloud of $4f$ electrons with distortion of the crystalline field by the strain (the single-ion mechanism). For each nonequivalent site r ($r = 1, 2, \ldots, 6$), we express the molecular-field parameter as a series in the invariants of the group D_2 (see Table I) formed by the elements of the strain tensor[2] $\hat{\varepsilon}$ with respect to the local axes of symmetry of the given site,

$$\lambda = \lambda^r = \lambda_0 + q_1 \varepsilon_{xx}{}^r + q_2 \varepsilon_{yy}{}^r + q_3 \varepsilon_{zz}{}^r, \qquad \hat{\varepsilon}^r = \hat{T}^r \hat{\varepsilon} (\hat{T}^r)^{-1}. \tag{2}$$

$\hat{T}^r$ is the matrix of transformation of the crystallographic system of coordinates to the local for the r-th site, which can be written in the form

$$T_{\alpha\beta}{}^r = \mathbf{e}_\alpha \mathbf{e}_\beta{}^r, \quad \alpha, \beta = x, y, z; \tag{3}$$

$\underset{\sim}{e}_x = (100)$, $\underset{\sim}{e}_y(010)$, $\underset{\sim}{e}_z = (001)$, $\underset{\sim}{e}_\beta^r$ are defined in Table II; q_1, q_2, q_3 are numerical coefficients.

We now write the complete Hamiltonian of a RE ion located on the r-th nonequivalent site, in the form (in local axes)

$$\mathscr{H} = \mathscr{H}_0 + \mathscr{H}_{\rm me}; \tag{4}$$

$$\mathscr{H}_0 = V_c{}^0 - \beta g_J \mathbf{J} \mathbf{H}_{\rm eff}; \tag{5}$$

$$\mathbf{H}_{\rm eff} = \mathbf{H} - \lambda_0 \mathbf{M}_{\rm Fe}, \tag{6}$$

where V_c^0 is the Hamiltonian of the crystalline field acting on the RE ion in the absence of strain; $\mathscr{H}_{\rm me}$ is the magnetoelastic Hamiltonian. The magnetoelastic Hamiltonian consists of two parts,

$$\mathscr{H}_{\rm me} = \mathscr{H}_{\rm me}^{\rm exc} + V_{\rm me}.$$

The exchange magnetoelastic Hamiltonian $\mathscr{H}_{\rm me}^{\rm exc}$, which originates because of the dependence of the molecular-field parameter on the strain [see (2)], has the form

$$\mathscr{H}_{\rm me}^{\rm exc} = (q_1 \varepsilon_{xx}{}^r + q_2 \varepsilon_{yy}{}^r + q_3 \varepsilon_{zz}{}^r) \beta g_J \mathbf{J} \mathbf{M}_{\rm Fe}. \tag{7}$$

We represent the single-ion magnetoelastic Hamiltonian $V_{\rm me}$, for a RE ion located on the r-th nonequivalent site, in the form

$$V_{\rm me} = \sum_{\alpha = x,y,z} \sum_{n=1}^{3} \sum_m (C_\alpha)_n{}^m \varepsilon_{\alpha\alpha}{}^r (K_A)_n{}^m + \sum_{i,n=1}^{3} \sum_m (C_{B_i})_n{}^m \varepsilon_{B_i}^r (K_{B_i})_n{}^m, \tag{8}$$

where the operators $[K_{A,B_i}(J)]_n^m$ are linear combinations of the irreducible tensor operators (spherical harmonics) that transform according to the irreducible representations A, B_i ($i = 1, 2, 3$) of group D_2 (see Table I), ε_{B_i} are elements of the strain tensor that transform according to the irreducible representation B_i (see Table I), and $(C_{\alpha,B_i})_n^m$ are magnetoelastic constants. In the expansion (8), because of time-reversal symmetry, we have kept only terms of even order in J up to the sixth power. The latter circumstance is due to the fact that terms of higher degree make no contribution to the matrix elements.[3]

5. EXCHANGE MAGNETOSTRICTION

We shall investigate the exchange contribution of the RE ions to the magnetostriction of REIG. On averaging the Hamiltonian $\mathscr{H}_{\rm me}^{\rm exc}$ [see (7)] over the eigenfunctions of the Hamiltonian $\mathscr{H}_0$ [see (5)] (with equilibrium density matrix ρ_0), we find, after summation over all the nonequivalent sites, that the magnetoelastic addition to the free energy, resulting from R–Fe exchange interaction, takes the form (per $R_xY_{3-x}Fe_5O_{12}$ molecule)

$$\Phi_{\mathrm{me}}^{\mathrm{exc}}=\frac{x}{6}\sum_{r=1}^{6}\mathrm{Sp}(\mathscr{H}_{\mathrm{me}}^{\mathrm{exc}}\rho_0)=-\frac{x}{6}\sum_{r=1}^{6}(q_1\varepsilon_{xx}{}^r+q_2\varepsilon_{yy}{}^r+q_3\varepsilon_{zz}{}^r)\cdot$$
$$\cdot\sum_n\langle n_r|-\beta g_JJ|n_r\rangle\exp\left(-\frac{E_n{}^r}{T}\right)\mathbf{M}_{\mathrm{Fe}}{}^r\Big/\sum_n\exp\left(-\frac{E_n{}^r}{T}\right)$$
$$=-\frac{x}{6}\sum_{r=1}^{6}(q_1\varepsilon_{xx}{}^r+q_2\varepsilon_{yy}{}^r+q_3\varepsilon_{zz}{}^r)\mathbf{M}_R{}^r\mathbf{M}_{\mathrm{Fe}}{}^r,\tag{9}$$

where $|n_r\rangle$ and E_n^r are the wave functions and energy levels of a RE ion located on the r-th nonequivalent site, determined by the Hamiltonian $\mathscr{H}_0$; $\mathbf{M}_R$ is the magnetic moment of a RE ion located on the r-th site. Thus in order to calculate the exchange contribution of the RE ions to the magnetostriction, it is necessary to know the distribution of the magnetic moments of the RE ions over the nonequivalent sites (this, in turn, requires knowledge of the eigenfunctions and energy levels of the RE ions in the crystalline and effective fields).

We note that if the magnetic moments of the RE ions located on the various sites were oriented exactly parallel to one another, then the expression (9) would make no contribution to the anisotropic magnetostriction of the crystal. In fact, in this case (9) reduces to an expression containing the sum of the diagonal elements of the tensor $\hat{\varepsilon}$ (the dilatation),

$$\Phi_{\mathrm{me}}^{\mathrm{exc}}=-\frac{x}{6}\mathbf{M}_{\mathrm{Fe}}\mathbf{M}_R\sum_{r=1}^{6}(q_1\varepsilon_{xx}{}^r+q_2\varepsilon_{yy}{}^r+q_3\varepsilon_{zz}{}^r)\sim\mathbf{M}_{\mathrm{Fe}}\mathbf{M}_R(\varepsilon_{xx}+\varepsilon_{yy}+\varepsilon_{zz}),$$

which leads only to volume striction. Such a situation occurs in the model usually used, according to which the potential V_c of the crystalline field is much smaller than the interaction of the ion with the effective field. But in REIG, the interaction of a RE ion with the crystalline field is in most cases larger than the R-Fe exchange interaction.[4] Therefore we shall consider a model according to which the energy of interaction of the RE ions with the effective field is assumed to be small in comparison with the minimum distances between levels of the multiplet of the RE ion split by the crystalline field.

For RE ions with an even number of electrons in an unfilled $4f$ shell, the energy levels in a crystalline field of symmetry D_2 are singlets, whose eigenfunctions transform under symmetry operations according to the irreducible representations A, B_1, B_2, B_3. In order to explain the distribution of the magnetic moments of the RE ions, it is necessary to know the eigenfunctions and energy levels of the ions in the crystalline and effective fields. Within the framework of perturbation theory, it is easy to show that the corrections to the eigenfunctions and energy levels of a RE ion located on the r-th site, because of the action of the effective field, are (in local axes)

$$|n_r\rangle-|n_0\rangle=\sum_\alpha\sum_m{}'(H_{\mathrm{eff}}^r)_\alpha\frac{\langle m_0|-\beta g_JJ_\alpha|n_0\rangle}{E_n{}^0-E_m{}^0}|m_0\rangle$$
$$+\sum_{\alpha,\delta=x,y,z}\sum_m{}'\sum_k{}'(H_{\mathrm{eff}}^r)_\alpha(H_{\mathrm{eff}}^r)_\delta\frac{\langle m_0|J_\alpha|k_0\rangle\langle k_0|J_\delta|n_0\rangle(\beta g_J)^2}{(E_n{}^0-E_k{}^0)(E_k{}^0-E_m{}^0)}|m_0\rangle$$
$$-\frac{|n_0\rangle}{2}\sum_\alpha\sum_m{}'(H_{\mathrm{eff}}^r)_\alpha{}^2\frac{|\langle n_0|\beta g_JJ_\alpha|m_0\rangle|^2}{(E_n{}^0-E_m{}^0)^2};\tag{11a}$$

$$E_n{}^r-E_n{}^0=\sum_\alpha\sum_m{}'(H_{\mathrm{eff}}^r)_\alpha{}^2\frac{|\langle n_0|\beta g_JJ_\alpha|m_0\rangle|^2}{E_n{}^0-E_m{}^0}+\ldots,\tag{11b}$$

where E_k^0 and $|k_0\rangle$ are the energy levels and eigenfunctions of the RE ion in the crystalline field and where $(H^r_{\mathrm{eff}})_\alpha$ is a component of the effective field ($\alpha=x,y,z$).

Using the eigenfunctions and energy levels determined by formulas (11a) and (11b), we find that in the linear approximation in H_{eff}, the magnetic moment of a RE ion located on the r-th nonequivalent site can be represented in the form (in local axes of symmetry of the given site)

$$(M_R{}^r)_\alpha=\chi_{\alpha\alpha}(T)(H_{\mathrm{eff}}^r)_\alpha,\tag{12}$$

$$\chi_{\alpha\alpha}(T)=\sum_{k_0}\sum_{m_0\neq k_0}\frac{|\langle k_0|\beta g_JJ_\alpha|m_0\rangle|^2\exp(-E_k{}^0/T)}{(E_k{}^0-E_m{}^0)\sum\limits_{n_0}\exp\left(-\frac{E_n{}^0}{T}\right)}.\tag{13}$$

Thus the exchange contribution to the magnetoelastic free energy takes the form

$$\Phi_{\mathrm{me}}^{\mathrm{exc}}=-\frac{x}{3}\sum_{r=1}^{6}(q_1\varepsilon_{xx}{}^r+q_2\varepsilon_{yy}{}^r+q_3\varepsilon_{zz}{}^r)\sum_{\alpha=x,y,z}\chi_{\alpha\alpha}(T)(H_{\mathrm{eff}}^r)_\alpha(M_{\mathrm{Fe}}{}^r)_\alpha.\tag{14}$$

On carrying out the summation over nonequivalent sites in (14) and omitting terms that lead only to volume magnetostriction, we find that the magneto-elastic energy can be represented in the form

$$\Phi_{\mathrm{me}}^{\mathrm{exc}}=\varphi(T)M_{\mathrm{Fe}}|\mathbf{H}_{\mathrm{eff}}|\sum_\alpha\varepsilon_{\alpha\alpha}(\gamma_R{}^\alpha\gamma_{\mathrm{Fe}}{}^\alpha-\gamma_R\gamma_{\mathrm{Fe}}/3)$$
$$+\frac{\varphi'}{2}M_{\mathrm{Fe}}|\mathbf{H}_{\mathrm{eff}}|\sum_{\alpha\neq\beta}\varepsilon_{\alpha\beta}(\gamma_R{}^\alpha\gamma_{\mathrm{Fe}}{}^\beta+\gamma_R{}^\beta\gamma_{\mathrm{Fe}}{}^\alpha),\tag{15}$$
$$\varphi'=(q_2-q_1)(\chi_{xx}-\chi_{yy})x/6,\quad\gamma_R=\mathbf{H}_{\mathrm{eff}}/|\mathbf{H}_{\mathrm{eff}}|,\quad\gamma_{\mathrm{Fe}}=\mathbf{M}_{\mathrm{Fe}}/M_{\mathrm{Fe}},$$
$$\varphi(T)=-\frac{x}{3}\left(\frac{q_1+q_2}{2}-q_3\right)\left(\frac{\chi_{xx}(T)+\chi_{yy}(T)}{2}-\chi_{zz}(T)\right).\tag{16}$$

Adding to (15) the elastic energy, written in the usual form

$$E_y={}^1\!/_2c_{11}(\varepsilon_{xx}{}^2+\varepsilon_{yy}{}^2+\varepsilon_{zz}{}^2)+{}^1\!/_2c_{44}(\varepsilon_{xy}{}^2+\varepsilon_{yz}{}^2+\varepsilon_{xz}{}^2)$$
$$+c_{12}(\varepsilon_{xx}\varepsilon_{yy}+\varepsilon_{xx}\varepsilon_{zz}+\varepsilon_{yy}\varepsilon_{zz}),\tag{17}$$

where c_{ik} are the elastic constants, and minimizing with respect to the $\varepsilon_{\alpha\beta}$, we find that their equilibrium values have the form

$$\varepsilon_{\alpha\alpha}^{\mathrm{exc}}=\frac{\varphi(T)M_{\mathrm{Fe}}}{c_{12}-c_{11}}|\mathbf{H}_{\mathrm{eff}}|\left(\gamma_{\mathrm{Fe}}{}^\alpha\gamma_R{}^\alpha-\frac{1}{3}\gamma_{\mathrm{Fe}}\gamma_R\right),$$
$$\varepsilon_{\alpha\beta}=\frac{\varphi'}{c_{44}}M_{\mathrm{Fe}}|\mathbf{H}_{\mathrm{eff}}|(\gamma_{\mathrm{Fe}}{}^\alpha\gamma_P{}^\beta+\gamma_{\mathrm{Fe}}{}^\beta\gamma_R{}^\alpha)\quad\text{at}\quad\alpha\neq\beta.\tag{18}$$

Thus in contrast to the generally accepted assumptions (see, for example, Ref. 5), according to which the strain dependence of the exchange integral leads, in cubic crystals in the case of isotropic exchange, only to dilatation, in our model the mechanism considered leads to anisotropic magnetostriction, which in collinear phases varies linearly with the field. The magnetic moment of the Ho^{3+} ion in REIG is oriented principally along the local z axis, i.e., $|\chi_{xx},\chi_{yy}|$

$\ll |\chi_{zz}|$; in this case the exchange mechanism makes a contribution only to λ_{100}.

6. SINGLE-ION MAGNETOSTRICTION

The single-ion contribution to the magnetoelastic free energy of the system results from summation over nonequivalent sites of the temperature-averaged Hamiltonians V_{me} [see (8)]:

$$\Phi_{\mathrm{me}}^{\mathrm{si}}=-\frac{x}{6}\sum_{r}\sum_{n}\langle n_r|V_{\mathrm{me}}|n_r\rangle\exp\left(-\frac{E_n^0}{T}\right)\Big/\sum_{k}\exp\left(-\frac{E_k^0}{T}\right). \tag{19}$$

Analysis of the expression (19), with use of the wave functions determined by formula (11a), shows that the equilibrium values of the $\varepsilon_{\alpha\beta}$ dependent on the single-ion mechanism have the form

$$\varepsilon_{\alpha\alpha}^{\mathrm{si}}=x\psi_1(T)H_{\mathrm{eff}}^2[(\gamma_R^\alpha)^2-{}^1/_3],\quad \varepsilon_{\alpha\beta}^{\mathrm{si}}=x\psi_2(T)H_{\mathrm{eff}}^2\gamma_R^\alpha\gamma_R^\beta(1-\delta_{\alpha\beta}), \tag{20}$$

where $\psi_1(T)$ and $\psi_2(T)$ are functions of temperature, dependent on the magnetoelastic constants and the spectrum of the RE ion in the crystalline field; because of their unwieldiness, we shall not write them out.

Thus in contrast to the exchange mechanism, the single-ion mechanism of magnetostriction makes contributions both to λ_{100} and to λ_{111}; they are quadratic functions of H_{eff}.

7. CONCLUSION

The treatment carried out above has shown that the field (and temperature) dependences of the magnetostriction constants λ_{111} and λ_{100} have different forms. They can be put into the form

$$\lambda_{100}=A\varphi(T)H_{\mathrm{eff}}+B\psi_1(T)H_{\mathrm{eff}}^2,\quad \lambda_{111}=C\psi_2(T)H_{\mathrm{eff}}^2, \tag{21}$$

where the term linear in H_{eff} in (21) is dependent on the exchange mechanism, while the terms quadratic in H_{eff} are the single-ion contributions. It follows from (21) and (22) that at small value of H_{eff}, the constant λ_{100} is determined primarily by the exchange mechanism of magnetostriction and is larger in absolute value than the constant λ_{111}.

Comparison of the calculated field dependences of the magnetostriction constants with those observed experimentally, as the analysis has shown, revealed good agreement of them at small values of H_{eff} (see Fig. 2). At large values of H_{eff} (for $T < T_c$, when $H_{\mathrm{eff}} = H_{\mathrm{eff}} + \lambda M_{\mathrm{Fe}}$), the experimentally observed increase of the magnetostriction constants with field (Fig. 1) is considerably smaller than follows from the theory. This discrepancy is attributable to the fact that at large values of H_{eff}, the ratio of the magnitude of the energy of interaction of a RE ion with the effective field to the distance between levels of the multiplet of the RE ion, split by the crystalline field, ceases to be a good smallness parameter of the theory, and this leads to enhancement of the role of terms of higher order in H_{eff} and to a slowing of the increase of the magnetostriction constants with field.

On further increase of the field, when the energy of interaction of the RE ions with the effective field becomes comparable with or exceeds the distances between the levels of the RE ion in the crystalline field, the RE subsystem becomes isotropic, and the magnetic moments of the RE ions on different nonequivalent sites are oriented parallel to one another; this, as was shown above, leads to disappearance of the mechanism under consideration for an exchange contribution to the anisotropic magnetostriction. Thus one can explain the experimentally observed (Fig. 1) tendency toward identical field dependences of the constants λ_{100} and λ_{111} at large values of H_{eff}.

Thus the measurements of the magnetostriction of holmium-yttrium iron garnets in strong magnetic fields have revealed qualitatively different field dependences of the constants of anisotropic magnetostriction λ_{111} and λ_{100}: λ_{111} varies quadratically with the total effective field H_{eff}, acting on the RE ions, whereas the variation of λ_{100} with H_{eff} is nearly linear.

A theoretical treatment has shown that the magnetostriction quadratic in H_{eff} is dependent on the single-ion mechanism. The contribution to the magnetostriction that is linear in H_{eff} has an exchange origin. It has usually been supposed that in a cubic crystal the exchange magnetostriction is isotropic. In our case, however, in consequence of the presence of nonequivalent low-symmetry sites and of a rather large crystalline field, the strain dependence of the exchange integral leads to anisotropic magnetostriction.

We note that this conclusion is general and can be extended to other magnetic materials in which the conditions enumerated above are satisfied, for example to rare-earth intermetallic compounds.

[1] The nonzero magnetostriction at field $H = H_M$ is explained by the contribution to the measured magnetostriction of the system $\mathrm{Ho_{1.05}Y_{1.95}Fe_5O_{12}}$ from the iron sublattices, as follows from a comparison with the magnetostriction of yttrium iron garnet.

[2] We neglect the antisymmetric components of the distortion tensor.

[3] The RE ions have an unfilled 4f shell ($l = 3$), whose one-electron eigenfunctions transform according to the irreducible representation D_3 of the rotation group. But terms of degree k in the operator J transform according to the representation D_k, so that the direct product $D_3 \times D_k \times D_3$, which represents the matrix elements of the operator J of degree k, does not contain the unit representation when $k > 6$; therefore its matrix elements are zero.

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