Contribution to the theory of the tilt effect in electron-phonon interaction

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A theoretical analysis is presented of the tilt effect, namely the dependence of the absorption and velocity of longitudinal sound on the angle between the magnetic field and the wave vector. It is shown that in the absence of collisions the absorption and velocity of the sound in a classically strong magnetic field are nonanalytic (discontinous) functions of the tilt angle α. The discontinuities arise at a critical value $\alpha_0 = s_{\max}/v$ and are due to the jumplike onset of dissipative collisionless interaction of electrons with the sound, which captures immediately an entire region near the limiting point on the Fermi surface. Owing to the quasi-one-dimensional character of the electron motion in the magnetic field, this interaction is strong and does not contain the small adiabatic parameter s/v. The proposed theory has not made it possible to calculate the angular dependences of the velocity and of the damping of the sound, which turned out to be in good qualitative agreement with the results of experiments on gallium. The influence of the anisotropy of the Fermi surface on the absorption jumps and on the velocity of the longitudinal sound is discussed.

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1. INTRODUCTION

The gist of the tilt effect is that the electronic absorption coefficient and the velocity of sound in a metal change radically as functions of the angle between the magnetic field **H** and the sound propagation direction. This phenomenon exists in the region of strong magnetic fields, thus the cyclotron radius R is much less than the sound wavelength, and at small tilt angles $\alpha \sim s/v$ (s is the speed of sound, v is the velocity of the electrons on the Fermi surface, α is the angle between the vector **H** and the direction perpendicular to the wave vector **q**). The titl effect was first observed by Reneker[1] in sound absorption in bismuth. Its physical nature is connected with the jumplike onset of collisionless absorption of high-frequenvy sound ($\omega t \gg 1$, τ is the time between the collisions), when the frequency ω becomes equal to $gv \sin \alpha$, inasmuch as at $\sin \alpha < \omega/qv$ there is no collisionless absorption.

The tilt effect in the absorption and the velocity of longitudinal sound were subsequently investigated experimentally in gallium by Bezuglyĭ and Burma,[2] and also in bismuth[3] and antimony[4] by Korolyuk and co-authors. There have been relatively few studies of this phenomenon. Spector[5,6] calculated the dependences of the longitudinal-sound velocity on the tilt angle α. However, the sign of the change of the sound velocity with changing angle α was found in Ref. 5 and 6 to contradict the sign obtained in experiment[2] and the results of the present paper. The theoretical description proposed in Refs. 3 and 4 for this phenomenon explain fairly well the results of the experiment at $\omega\tau \lesssim 1-3$, but are inadequate when $\omega\tau \gg 1$ and the tilt effect becomes strong.

The absence of collisionless interaction of electrons with sound at $\mathbf{q} \perp \mathbf{H}$ ($\alpha = 0$) and the quasi-one-dimensional character of the electron motion in a strong magnetic field lead also to an increase of the sound velocity with increasing H. This effect was predicted theoretically by Kulik[7] and later was observed experimentally in the cited Ref. 2. Actually, both this phenomenon and the tilt effect have a common physical cause and are closely connected with each other. Both phenomena are reflections of the essentially nonadiabatic character of the interaction of the electrons with the phonons in the region of small angles $\alpha(\omega/|q_z v| \gtrsim 1$, axis $z \parallel \mathbf{H}$). The fact that the increase of the speed of the longitudinal sound at $\alpha = 0$, predicted by Kulik is actually a strong nonadiabatic effect manifests itself in the absence of the small parameter s/v in the observed change of $s(H)$:

$$s^2(H) - s^2(0) = nmv^2/15\rho, \tag{1.1}$$

where n is the electron density, m is their effective mass, and ρ is the density of the crystal. It turns out that the additional increase of the speed of sound with increasing α in the tilt effect likewise does not contain the small adiabatic parameter s/v, and its order of magnitude is that of (1.1). This conclusion agrees with the experimental data,[2] which have so far found no theoretical explanation.

The purpose of the present paper is a theoretical description of the tilt effect as an essentially nonadiabatic phenomeon, in which the electron-phonon interaction becomes very strong. Owing to the existence of an abrupt boundary of the Fermi distribution in velocity the strong collisionless interaction of the electrons with the phonons is turned on jumpwise when the angle α exceeds a threshold value $\alpha_0 = s_{\max}/v$. This leads to a nonanalytic dependence of the absorption and of the speed of sound on the angle α: both quantities undergo a first-order discontinuity at $\alpha = \alpha_0$ and as $\omega\tau \to \infty$. It turns out then that the collisionless absorption of sound at the maximum is determined by an entire region on the Fermi surface and not only by the resonant electrons, therefore the damping and the change of the velocity do not contain the small quantity s/v and turned out to be large in terms of this parameter. It must be emphasized that under conditions of strong electron-phonon interaction the role of the electromag-

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netic fields that accompany the sound in the metal increases sharply. It will be shown below that the contribution of the solenoidal fields to the spectrum and damping of the phonons cancel out almost completely the singularity due to the direct deformation interaction. As a result of this cancellation, the electronic-renormalization singularity due to the resonant electrons turns out to be not in the numerator but in the denominator of the right-hand side of the dispersion equation. This conclusion is of fundamental importance, since a similar increase of the role of the electromagnetic fields should always take place whenever the electron-phonon interaction becomes strong and essentially nonadiabatic.

In the next section we present the initial equations of the theory. In Sec. 3, a dispersion equation is derived for compensated and uncompensated metals. The concluding fourth section is devoted to an analysis of the dependences of the absorption coefficient and of the speed of sound on the tilt angle, and also to a comparison of the results of the theory with experiment.

2. INITIAL EQUATIONS

The complete system of equations describing the propagation of sound waves in metals consists, as is well known, of the elasticity-theory equations for the displacements $\mathbf{u}(\mathbf{r}, t)$, Maxwell's equations for the electromagnetic fields accompanying the sound wave, and the kinematic equation for the conduction electrons. The problem of deriving and proving such a system of equations has been actively discussed for many years up to recently. Its solution was the result of efforts of many investigators (see Refs. 7–12 and the literature cited therein). The form of the kinetic equation in the presence of a sound wave was established in Refs. 8, 9, and 11.[1)] The most complicated is the question of obtaining an equation for the force exerted by the electrons on the lattice. This force was first obtained by Cohen and the Harrisons in the case of free electrons,[12] and by Silin for an arbitrary dispersion law of the electrons, but without allowance for collisions,[13] The correct formula for the force, in the free-electron approximation with allowance for collisons and small terms from the Stuart-Tolman effect, was given in the first paper of Ref. 14 and used in Ref. 7. For an arbitrary dispersion law, the total expression for this force, with allowance for collisions and for the Stuart–Tolman effect, was obtained by Kontorovich[15] and independently by Vlasov and Filippov in the second paper of Ref. 14, and by Skobov and Kaner[16] without allowance for the Stuart-Tolman effect. The theory was further developed in Refs. 17–23. The role of the dragging of the electrons by the lattice, which makes a substantial contribution to the interaction of the electrons with the sound, was analyzed in Refs. 20 and 17–19. A microscopic derivation of this system of equations, with allowance of interband transitions of the conduction electrons in a metal, was published in a recent paper.[22] In that paper, correct expressions weee obtained for the adiabatic elastic moduli, and all the quantities that enter in the equations could be expressed in terms of microscopic interactions of the electrons and ions in the metal; by the same token, the validity of these equations for the description of strong nonadiabatic effects could be corroborated.

In accordance with the established approach, we write down the kinetic equation for the nonadiabatic part $f(\mathbf{p}, \mathbf{r}, t)$ of the complete distribution function

$$F(\mathbf{p}, \mathbf{r}, t)=f_0(\varepsilon')+f(\mathbf{p}, \mathbf{r}, t), \tag{2.1}$$

where $f_0(\varepsilon')$ is the equilibrium Fermi function, which depends on the electron energy in the co-moving reference frame:

$$\varepsilon'=\varepsilon_0+\lambda_{ik}u_{ik}-m_0\mathbf{v}\dot{\mathbf{u}}-|e|\varphi.$$

Here $\varepsilon_0=\varepsilon_0(\mathbf{p})$ is the law of electron dispersion in the absence of sound, u_{ik} is the strain tensor, φ is the scalar potential of the electromagnetic field in the metal, m_0 is the mass of the free electron, $-|e|$ is its charge, $\mathbf{v}=\partial\varepsilon_0/\partial\mathbf{p}$ is the velocity of the conduction electron in the metal, the superior dot denotes partial differentiation with respect to time, and $\lambda_{ik}(\mathbf{p})$ is the strain-potential tensor for which a microscopic expression was obtained in Ref. 22, The kinetic equation is of the form

$$\frac{\partial f}{\partial t}+(\mathbf{v}\nabla)f-\frac{|e|}{c}[\mathbf{v}\times\mathbf{H}]\frac{\partial f}{\partial \mathbf{p}}+\nu\{f\}=-\frac{\partial f_0}{\partial\varepsilon}(-|e|\tilde{\mathbf{E}}\mathbf{v}+\Lambda_{ik}\dot{u}_{ik}). \tag{2.2}$$

Here $\nu\{f\}$ is the collision integral, $\Lambda_{ik}(\mathbf{p})$ is the renormalized strain potential:

$$\Lambda_{ik}(\mathbf{p})=\lambda_{ik}(\mathbf{p})-\langle\lambda_{ik}\rangle, \tag{2.3}$$

the angle brackets denote averaging over the Fermi surface:

$$\langle\psi\rangle=\frac{1}{Q(\varepsilon_F)}\sum_{\mathbf{p}}\psi\delta(\varepsilon_{\mathbf{p}}-\varepsilon_F),\quad \sum_{\mathbf{p}}(\ldots)=\frac{2}{(2\pi\hbar)^3}\sum\int d^3p(\ldots), \tag{2.4}$$

the symbol $\sum$ denotes summation over different groups in the case of a multiply connected Fermi surface, and $Q(\varepsilon_F)$ is the density of the electronic state on the Fermi surface.

The electric field $\tilde{\mathbf{E}}$ is given by

$$\tilde{\mathbf{E}}=\mathbf{E}'+\frac{1}{c}[\dot{\mathbf{u}}\times\mathbf{H}]+\frac{m_0}{|e|}\ddot{\mathbf{u}}, \tag{2.5}$$

where $\mathbf{E}'=\mathbf{E}+\nabla\varphi_0$ is the nonadiabatic part of the electric field in the metal, and φ_0 is the adiabatic electrostatic potential,

$$|e|\varphi_0=\langle\lambda_{ik}\rangle u_{ik}. \tag{2.6}$$

The equation of motion of the lattice, with allowance for the electron contribution to the elasticity of the metal, is of the form[13-16,21]

$$\rho\ddot{u}_i=K_{iklm}\frac{\partial^2u_m}{\partial x_k\partial x_l}+\frac{\partial}{\partial x_k}\sum_{\mathbf{p}}\Lambda_{ik}f+\frac{1}{c}[\mathbf{j}\times\mathbf{H}]_i+\frac{m_0}{|e|}\frac{\partial j_i}{\partial t}. \tag{2.7}$$

Here ρ is the mass density of the crystal, and K_{iklm} is the tensor of the adiabatic elastic moduli of the metal, as obtained in Ref. 22. The electric current density j is

$$\mathbf{j}=-|e|\sum_{\mathbf{p}}\mathbf{v}f. \tag{2.8}$$

Finally, we write Maxwell's equations in the form

$$\text{rot rot}\,\mathbf{E}' = -\frac{4\pi}{c^2}\frac{\partial \mathbf{j}}{\partial t}. \qquad (2.9)$$

This equation contains in fact the electron quasineutrality condition div $\mathbf{j}=0$.

An essential role in the derivation of the dispersion equation for the sound oscillations is the relation between the left-hand side of Eq. (2.9) and the terms with $\sigma\mathbf{E}'$ in its right-hand side. It is quite obvious that there should exist a frequency region in which $q^2c^2 \ll 4\pi\omega\,|\sigma_{ik}^{\perp}|$ (ω and $\mathbf{q}$ are the frequency and wave vector, and the index $\perp$ labels the conductivity-tensor components that are transverse to the wave vector $\mathbf{q}$). In this case we can neglect the left-hand side of (2.9) when finding the connection between the electric field and the displacement vector u, i.e., we can use in place of (29) the equation[2)] $\mathbf{j}=0$. The conditions under which this equation can be used include the region of ultrasonic and hypersonic frequencies which is of most interest and of greatest practical importance, as well as all the attainable magnetic fields $H < (4\pi nmv^2)^{1/2} \lesssim 10^5-10^6$ Oe.

3. DISPERSION EQUATION FOR THE SOUND OSCILLATIONS

We consider the propagation of longitudinal high-frequency sound in a classically strong magnetic field, for the following inequalities hold

$$\nu \ll \omega \ll qv \ll \Omega \qquad (3.1)$$

(ν is the collision frequency and Ω is the cyclotron frequency).

We begin with the case of a metal with unequal electron and hole densities. Simple estimates show that the main contribution to the electronic elasticity is made by the solenoidal electric field component $\tilde{E}_z$ directed along the external magnetic field $\mathbf{H}$. The dispersion equation for the longitudinal sound is of the form

$$\omega^2 - s^2q^2 = \frac{q^2}{2\pi^2\hbar^3\rho}\left\{\int dp_z|m|\overline{\Lambda}^2\frac{\omega}{\omega_*} - \frac{(\int dp_z|m|\overline{\bar{v}_z\Lambda}\omega/\omega_*)^2}{\int dp_z|m|\bar{v}_z^2\omega/\omega_*}\right\}. \qquad (3.2)$$

Here $\omega_* = \omega + i\nu - q_z\bar{v}_z$, m is the cyclotron mass, $\Lambda = \Lambda_{qq}$, and

$$s^2 = K_{qqqq}/\rho \qquad (3.3)$$

is the square of the adiabatic sound velocity in the metal. Formula (3.2) was written for a singly connected Fermi surface; in the case of a multiply connected surface, formula (3.2) retains the same form if the integrals with respect to p_z are taken to mean the sums of analogous integrals over the different groups. We note that the second term in the right-hand side of (3.2) is just the one that describes the contribution of the solenoidal field to the electronic renormalization of the spectrum and to the damping of the sound oscillations.

It is of interest to discuss different limiting cases contained in Eq. (3.2). This equation admits of a transition to strictly longitudinal propagation ($\mathbf{q} \parallel \mathbf{H}$, $\alpha = \pi/2$). In this geometry, the numerator of the second term in (3.2) vanishes in the principal approximation in ω/qv, in view of the fact that $\langle\Lambda\rangle = 0$, i.e., it is necessary to retain in the right-hand side of (3.2) only the first term. When this term is calculated, ω_*^{-1} is replaced by $-\pi i\delta(\omega - q\bar{v}_z)$, after which the spectrum and the damping of the longitudinal sound are determined by the formula

$$\omega = qs\left\{1 - i\frac{\omega}{4\pi\rho s^2\hbar^3}\int dp_z|m|\overline{\Lambda}^2\delta(\omega - q\bar{v}_z)\right\}. \qquad (3.4)$$

It is seen from it that the sound absorption is determined only by the electrons on the Fermi surface "belt" $\bar{v}_z - s = 0$. In particular, in the case of an isotropic quadratic dispersion law we obtain for the relative damping decrement the known expression[12]

$$\Gamma = \frac{\omega''}{\omega'} = \frac{\pi}{12}\frac{nmv}{\rho s} \sim \frac{s}{v} \quad (\omega = \omega' - i\omega''). \qquad (3.5)$$

The next terms of the expansion in ω/qv, including also of the second term in the right-hand side of (3.2) lead to a small nonadiabatic correction to the sound velocity $\Delta s/s \sim (s/v)^2$. We emphasize that in this geometry the role of the solenoidal fields is negligibly small in terms of the parameter s/v.

We discuss now the limiting case of longitudinal sound propagation across $\mathbf{H}$ ($\mathbf{q} \perp \mathbf{H}$, $\alpha = 0$). We assume for simplicity the collision frequency to be constant for each group. In this situation, the contribution of the solenoidal fields vanishes identically because the function $\bar{v}_z\overline{\Lambda}$ is odd in p_z, as a result of which the spectrum and the damping of the longitudinal sound are determined by the formula[23]

$$\frac{\omega^2}{q^2} = s^2 + \frac{1}{4\pi^2\hbar^3\rho}\sum\frac{\omega}{\omega + i\nu}\int dp_z|m|\overline{\Lambda}^2. \qquad (3.6)$$

This formula generalizes, to an arbitrary dispersion law, the result previously obtained[7] for free electrons, when

$$\Lambda_{ik} = 2\varepsilon_F(\delta_{ik} - 3n_in_k)/3, \quad n_i = v_i/v. \qquad (3.7)$$

Equation (3.6) then yields precisely Kulik's result[7]

$$\frac{\omega^2}{q^2} = s^2 + \frac{nmv^2}{15\rho}\left(1 - \frac{i\nu}{\omega}\right). \qquad (3.8)$$

We turn now to an analysis of Eq. (3.2) at finite but small α, when $q_zv \sim \omega$. In this case the contribution of the solenoidal fields turns out to be quite substantial. All the integrals in (3.2) have singuarities at those tilt angles at which $\alpha = \omega/q\bar{v}_{z\,\text{ext}}$. In other words, singulatities in the integrals can result from the limiting points and from those Fermi-surface sections at which v_z has an extremum as a function of p_z. Thus, e.g., in the case of a convex Fermi surface, the singularity is connected only with the limiting point. The structure of the right-hand side of (3.2) is such that the singular terms in it cancel each other (subtraction of two infinities takes place). As a result of this cancellation, a noticeable nonadiabaticity of the spectrum renormalization appears, and the remaining singular term contains a singularity no longer in the numerator but in the denominator. In other words, at the singularity point the contributions of the singular term to the spectrum and damping becomes equal to zero and not to infinity, as would have been the case if the solendoial fields were not taken into account and only the direct deformation interaction retained [the first term in the right-hand side of (3.2)].

For an isotropic (alkali) metal, when the effective mass m differs from m_0, the tensor Λ_{ik} is determined as before by formula (3. 7), in which it is necessary to introduce a dimensionless factor β connected with the fact that the electrons are not free. In this case the disperison equation (3. 2) can be represented in the form

$$\frac{\omega^2}{q^2}-s^2=\frac{n\varepsilon_F}{2\rho}\frac{\beta^2\omega(\omega+i\nu)}{(q_zv)^2}\left\{1-\frac{(q_zv)^2}{3(\omega+i\nu)^2}+\frac{(q_zv)^2}{3(\omega+i\nu)^2}\frac{1}{\mathscr{P}(\omega,q_z)}\right\}, \tag{3.9}$$

where the function $\mathscr{P}(\omega, q_z)$ is given by

$$\mathscr{P}(\omega,q_z)=\frac{1}{4\pi}\oint do_v\frac{q_zv_z}{q_zv_z-\omega-i\nu}=1-\frac{\omega+i\nu}{2q_zv}\ln\frac{(\omega+q_zv)+i\nu}{(\omega-q_zv)+i\nu}. \tag{3.10}$$

The logarithm in (3. 2) is so defined that it is real when the argument is positive. At real q and complex $\omega=\omega'-i\omega''$ $(\omega''>0)$ the imaginary part of the logarithm is determined by the Landau bypass rule and is equal to

$$\operatorname{Im}\ln\left[\frac{\omega'+q_zv+i(\nu-\omega'')}{\omega'-q_zv+i(\nu-\omega'')}\right]=\operatorname{arctg}\frac{\omega'-q_zv}{\nu-\omega''}-\operatorname{arctg}\frac{\omega'+q_zv}{\nu-\omega''}-2\pi\chi(q_zv-\omega')\chi(\omega''-\nu)\quad(\omega'>0,\quad q_z>0), \tag{3.11}$$

where $\chi(x)=1$ at $x>0$ and $\chi(x)=0$ at $x>0$.

Formula (3. 9) illustrates the already noted cancellation of the singular terms: the function $\mathscr{P}(\omega, q_z)$, which has a singularity at $\omega=\pm q_zv-i\nu$, turned out to be in the denominator of the right-hand side of the dispersion equation (3. 9). A numerical analysis of this equation is given in the next section.

We point out that a singularity in an electron-phonon interaction can occur not only in the tilt effect, but also in a number of other cases. In such a situation, the appearance of a singularity in direct interaction between electrons and sound is always accompanied by an increasing role of the electromagnetic fields (longitudinal or vortical) and by a corresponding cancellation of the produced singularity. This is precisely why one cannot neglect the contribution of these alternating fields, which lead to terms similar to the second term on the right-hand side of (3. 2). Yet in a number of papers (see, e.g., Refs. 24–26) the dispersion equation was analyzed without such terms. As a result, a large part of the results of Refs. 24–26 calls for a review with allowance, in particular, for the possibility of the appearance of waves of the zero-sound type.

We obtain now a dispersion equation for sound in compensated metals. The need for a separate consideration of these metals is brought about by the fact that the role of the solenoidal electromagnetic fields in the electron-phonon interaction depends substantially on the Hall conductivity that vanishes in compensated metals. In contrast to the case of unequal electron and hole densities, a substantial contribution to the electron elasticity is made now not only by the field component E_z, but also by $\tilde{E}_x$ (the x axis is perpendicular to the vectors $\mathbf{q}$ and $\mathbf{H}$). Simple calculations lead to the following form of the dispersion equation

$$\rho\left(\frac{\omega^2}{q^2}-s^2\right)=\left\langle\overline{\Lambda}^2\frac{\omega}{\omega^*}\right\rangle-\left[\left\langle\left(\frac{S}{m}\right)^2\frac{\omega}{\omega_*}\right\rangle\left\langle\bar{v}_z^2\frac{\omega}{\omega^*}\right\rangle-\left\langle\frac{S}{m}\bar{v}_z\frac{\omega}{\omega_*}\right\rangle^2\right]^{-1}\left\{\left\langle\frac{S}{m}\overline{\Lambda}\frac{\omega}{\omega_*}\right\rangle^2\left\langle\bar{v}_z^2\frac{\omega}{\omega_*}\right\rangle+\left\langle\bar{v}_z\overline{\Lambda}\frac{\omega}{\omega_*}\right\rangle^2\times\left\langle\left(\frac{S}{m}\right)^2\frac{\omega}{\omega_*}\right\rangle-2\left\langle\frac{S}{m}\overline{\Lambda}\frac{\omega}{\omega_*}\right\rangle\left\langle\bar{v}_z\overline{\Lambda}\frac{\omega}{\omega_*}\right\rangle\left\langle\frac{S}{m}\bar{v}_z\frac{\omega}{\omega_*}\right\rangle\right\}. \tag{3.12}$$

Here S is the area of the intersection of the Fermi surface by the plane $p_z=\mathrm{const}(2\pi\overline{p_xv_x}=S/m, \overline{p_xv_z}=0)$.

The structure of the dispersion equation (3. 12), just as of Eq. (3. 9), is such that the principal singular terms cancel each other, and the singularity is contained in fact only in the denominator of its right-hand side.

In particular, at $\alpha=0$ the mean values that are linear in the projection of the velocity $\bar{v}_z$ vanish identically, and the spectrum and damping of the sound are determined from the equation

$$\rho\left(\frac{\omega^2}{q^2}-s^2\right)=\left\langle\overline{\Lambda}^2\frac{\omega}{\omega+i\nu}\right\rangle-\left\langle\frac{S}{m}\overline{\Lambda}\frac{\omega}{\omega+i\nu}\right\rangle^2\Big/\left\langle\left(\frac{S}{m}\right)^2\frac{\omega}{\omega+i\nu}\right\rangle. \tag{3.13}$$

This equation shows that in the case of transverse propagation the damping is due to collisions, and the renormalization of the speed of sound at $\omega\gg\nu$ is of the same order as in (3. 8). Formula (3. 13) generalizes the result of Ref. 7 to include the case of a compensated metal.

4. THEORETICAL ANALYSIS OF THE TILT EFFECT

We proceed now to a solution of the dispersion equation (3. 9). Our purpose is to determine the velocity and the damping of the longitudinal sound as a function of the tilt angle α. Introducing the notation

$$z=\frac{\omega}{qs},\quad\theta=\frac{v}{s}\sin\alpha,\quad\xi=\frac{\nu}{\omega},\quad A=\frac{n\varepsilon_F\beta^2}{2\rho s^2}, \tag{4.1}$$

we write down Eq. (3. 9) in dimensionless variables:

$$z^2-1=A\left\{\frac{z^2}{\theta^2}(1+i\xi)-\frac{1}{3(1+i\xi)}+\frac{1}{3(1+i\xi)}\frac{1}{\mathscr{P}(z,\theta)}\right\}, \tag{4.2}$$

where

$$\mathscr{P}(z,\theta)=1-\frac{1+i\xi}{2}\frac{z}{\theta}\ln\frac{z(1+i\xi)+\theta}{z(1+i\xi)-\theta}. \tag{4.3}$$

The tilt effect of interest to us is formally connected with the reversal of the sign of the difference $z-\theta$ and manifests itself most distinctly in the region of high frequencies, when the parameter $\xi\ll1$. We confine ourselves therefore to an investigation mainly of the collisionless regime, putting $\xi=0$. In the region of angles α which is of importance for the tilt effect, the values of z and θ turn out to be approximately equal and comparable with unity. For this reason Eq. (4. 2), as can be easily seen, does not contain the small adiabatic parameter s/v. Nonetheless Eq. (4. 2) can be used for the description of a strong nonadiabatic interaction of electrons with phonons under conditions of the tilt effect, inasmuch as in the derivation[22] of the initial system of equations (2. 1)–(2. 9) the adiabatic approximation was not used.

We consider first values of θ that are small compared with unity. In this angle region, the function $\mathscr{P}(z,\theta)$ at $\xi=0$ is real and positive, and its expansion in powers of θ is of the form

$$\mathscr{P}(z,\theta)=-\frac{\theta^2}{3z^2}\left(1+\frac{3}{5}\frac{\theta^2}{z^2}+\frac{3}{7}\frac{\theta^4}{z^4}+\ldots\right),\ \theta\ll z. \tag{4.4}$$

From (4.2) we easily find the solution in the form

$$z^2=1+\frac{4A}{15}+\frac{138}{175}\theta^2\frac{A}{1+4A/15}. \tag{4.5}$$

This formula shows that at small tilt angles $\alpha\ll s/v$ the speed of the high-frequency longitudinal sound increases with increasing α. The reason why the solution (4.5) does not contain an imaginary frequency increment is that there is no collisionless absorption at $\xi=0$ and small α. The increase of the speed of sound continues until collisionless absorption sets in. It appears at $z=\theta$, when the last term in the right-hand side of (4.2) vanishes. This point corresponds to

$$z=\theta_0=(1+2A/3)^{1/2},\ \alpha_0=s\theta_0/v=s_{max}/v. \tag{4.6}$$

At this point the speed of sound $s(\alpha)$ reaches a maximum, with

$$s_{max}=s(\alpha_0-0)=s(0)\left(\frac{15+10A}{15+4A}\right)^{1/2}. \tag{4.7}$$

It is seen that s_{max} exceeds the speed of sound $s(0)$ at $\alpha=0$, and the difference between them does not contain the small adiabatic parameter s/v. The spectrum of the wave near $\alpha=\alpha_0$, on the side of the smaller angles, is described by the expression

$$z=\theta+2\exp\left\{-\frac{A}{3(\theta_0-\theta)\theta_0}-2\right\},\ \theta<\theta_0. \tag{4.8}$$

At $\alpha=\alpha_0$, collisionless damping due to the reversal of the sign of the different $z-\theta$ is turned on abruptly, and this leads likewise to a jump in the speed of sound at this point. To determine the jumps of the velocity and damping on going though the value $\theta=\theta_0$, we write down the dispersion equation (4.2) for the complex quantity

$$\Delta=1-z/\theta=\Delta_1+i\Delta_2=|\Delta|e^{i\varphi}. \tag{4.9}$$

As we shall see, Δ_1 and Δ_2 turn out to be small in absolute magnitude compared with unity. Using this circumstance, we represent Eq. (4.2) at $\theta=\theta_0+0$ in the form

$$|\Delta|e^{i\varphi}=(B/3)\left[\ln\frac{C}{|\Delta|}-i(\pi+\varphi)\right]^{-1},\ B=\frac{3A}{3-A},\ C=2e^{-2}\approx0{,}27067. \tag{4.10}$$

Equation (4.10) determines the jumps of the sound velocity and of its damping on going through the critical angle α_0. The reason why Δ_1 and Δ_2 are finite is that the logarithm in (4.3) acquires a phase $\pi+\varphi$ at $\alpha<\alpha_0$.

From the two equations for the real and the imaginary parts of Δ we can eliminate $|\Delta|$ with the aid of the relation

$$\ln(C/|\Delta|)=(\pi+\varphi)\operatorname{ctg}\varphi. \tag{4.11}$$

We then obtain an equation for φ:

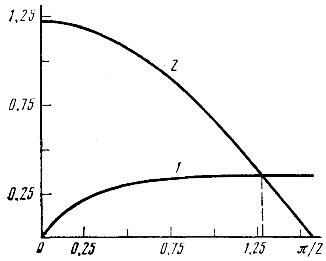

FIG. 1. Graphic solution of Eq. (4.12) at $\varkappa=0.812$ ($B=1$); 1—the function $[(\sin\varphi)/\varkappa(\pi+\rho)]\ln[\varkappa(\pi+\varphi)/\sin\varphi]$, 2—the plot of $(1/\varkappa)\cos\varphi$.

$$\cos\varphi=\frac{\sin\varphi}{\pi+\varphi}\ln\left[\frac{\varkappa(\pi+\varphi)}{\sin\varphi}\right],\quad \varkappa=\frac{3C}{B}. \tag{4.12}$$

It is seen from Fig. 1 that Eq. (4.12) has a single root $\varphi_0=1.28048$ at $B=1$. Hence $|\Delta|=0.072225$, $\Delta_1=0.020675$, and $\Delta_2=0.069203$. Accordingly, the jumps of the absorption and of the relative velocity on going through $\alpha=\alpha_0$ are

$$\frac{\omega''}{qs}=0.084753,\ \frac{s(\alpha_0-0)-s(\alpha_0+0)}{s}=0.025321. \tag{4.13}$$

Thus, the speed and damping of the longitudinal sound in the collisionless regime are discontinuous functions, i.e., they have a non-analytic dependence on the tilt angle α. It is remarkable that despite the absence of the small parameter s/v, the discontinuities are nevertheless numerically small compared with unity. The physical reason of the obtained non-analytic dependence is the existence of an abrupt boundary in the Fermi distribution of the electrons in velocity and the jump-like onset of a collisonless interaction of electrons with the sound when the critical tilt angle $\alpha=\alpha_0$ is reached. As for the smallness of the speed and absorption discontinuties, it can be attributed to the fact that near the turning point on the Fermi surface the number of electrons which become involved in the collisionless interaction at the critical angle α_0 is small. A measure of this smallness is the ratio of $\ln(C/|\Delta|)$ to $\pi+\varphi$ in Eq. (4.10), which equals 0.29875. Inasmuch as at the critical value α_0 the interaction of the electrons with the sound is strong and essentially nonadiabatic, it immediately affects not only the resonant electrons with $v_z=v_{z\,max}$, but also the electrons on the end sections of the Fermi surface near the limiting points. By the same token, if the interaction is strong, not only the known resonant "belts" and points become involved, as in the case of a weak (in terms of the parameter s/v) nonadiabatic interaction, but also entire regions of the Fermi surface near them. This manifests itself formally in the fact that the imaginary part of the integral (3.10) for $\mathscr{P}(z,\theta)$ exceeds π considerably even as $\nu\to0$, and its real part is only numerically smaller than $\operatorname{Im}\mathscr{P}(z,\theta)$.

We point out that the logarithmic character of the singularity of the function $\mathscr{P}(z,\theta)$ is due to the onset of the collisionless interaction of the sound with the electrons in the elliptical turning point. For this reason, the as-

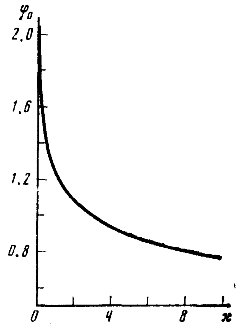

FIG. 2. Dependence of the root of Eq. (4.12) on ϰ.

sumption that the Fermi surface as a whole is isotropic is of no fundamental significance: $\mathscr{P}(z, \theta)$ acquires a logarithmic singularity also in the anisotropic case, if the turning point is elliptic. Anisotropy of the Fermi surface, first affects the value of A, and second, can lead to the appearance of an additional factor $1/\lambda$ in front of the logarithm in (4.3). The most substantial is the second circumstance, since it is precisely the one that changes mainly the constant C, i.e., influences the ratio of Δ_1 and Δ_2, and by the same token the relative fraction of the electrons that enter in collisionless interaction with the sound at the critical value of the angle α. It can be shown that the elliptic limiting point, for an arbitrary dispersion law, we have

$$C=2e^{-2\lambda},\quad \lambda=\frac{1}{2}\left\{\frac{\mathcal{K}^{1/2}}{m_{zz}}+\frac{1}{m^2}\frac{\partial|m|}{\partial p_z}\right\}_0 \int_{\varepsilon=\varepsilon_F} |m|\,dp_z, \qquad (4.14)$$

where $\mathcal{K}$ is the Gaussian curvature of the Fermi surface, m_{zz} is a component of the effective-mass tensor, m is the cyclotron mass, the zero subscript designates the turning point, and the integral $\int |m|\,dp_z$ is proportional to the density of the electronic states $Q(\varepsilon_F)$. It stems from the regular part of the integral in the denominator of the second term of (3.2). It follows from expression (4.14) that when λ is increased the role of the angular part of $\mathscr{P}(z, \theta)$ decreases and this corresponds to a decrease of the fraction of the electrons that interact with the sound at $\alpha = \alpha_0$. We note that λ is equal to unity even for an anisotropic quadratic electron dispersion law, and the deviation of λ from unity is evidence of an essentially nonquadratic $\varepsilon(\mathrm{p})$ dependence.

Thus, the influence of the anisotropy of the Fermi surface in Δ_1 and Δ_2 manifests itself through a change

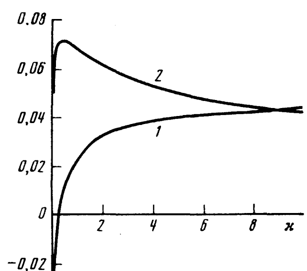

FIG. 3. Dependence on ϰ of the relative jumps of the speed (curve 1—Δ_1) and of absorption (curve 2—Δ_2) of the longitudinal sound at the critical angle α_0.

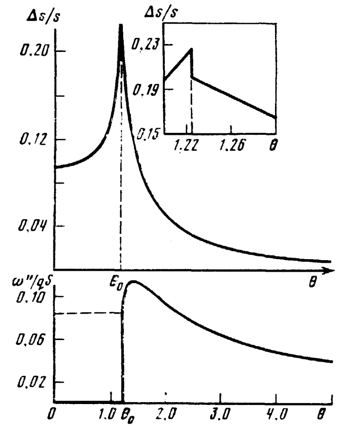

FIG. 4. Dependence of the change of the relative speed of sound $\Delta s/s = (s(\alpha) - s)/s$ and of the relative absorption on $\theta = \alpha v/s$ for an isotropic metal ($B = 1$); the inset shows the vicinity of the maximum.

in the constants B and C. Therefore Eq. (4.10) can be used to analyze the dependence of the jumps of the absorption and speed of sound on the anisotropic-Fermi surface characteristics that determine the value of the parameter $\varkappa = 3C/B$. Figure 2 shows the dependence of the root of Eq. (4.2) on $\varkappa$; as $\varkappa \to 0$ the root φ_0 tends to π. Figure 3 shows the dependence of Δ_1 and Δ_2 on $\varkappa$. It is curious that the jump in the speed of sound reverses sign at $\varkappa = \varkappa_0 = 3/2\pi$. When $\varkappa < \varkappa_0$ and $\varkappa$ tends to zero, the absorption jump Δ_2 decreases monotonically to zero, while the jump Δ_1 of the speed of sound goes through a minimum (in the region of very small $\varkappa \sim 10^{-3}$), and then likewise tends to zero.

Beyond the critical angle $\alpha > \alpha_0$, the speed of sound decreases monotonically, and far from α_0 we have

$$\frac{s(\alpha)-s}{s}=\frac{2}{3}A\left(\frac{s}{v\alpha}\right)^2\left[1-\frac{\pi^2}{16}\left(1+\frac{A}{12}\right)\right],\quad \frac{s}{v}\ll\alpha\ll1. \qquad (4.15)$$

At $\alpha > \alpha_0$ the damping of the sound first increases, reaches a maximum, after which it decreases like

$$\frac{\omega''}{qs}=\frac{\pi}{12}A\frac{s}{v\alpha},\quad \frac{s}{v}\ll\alpha\ll1. \qquad (4.16)$$

The dependence of the speed and absorption of the sound on the angle α is illustrated in Fig. 4, which shows the results of a numerical solution of the dispersion equation (4.2) in the collisionless regime ($\xi = 0$) for a Fermi sphere ($A = 3/4, B = 1$). The maximum of the relative damping is located at $\theta = \alpha v/s \approx 1.4$ and is equal to 0.114, i.e., it exceeds by approximately 1.35 times the damping jump. The onset of a maximum in the angular dependence of the damping is due to the decrease of the phase φ of the complex parameter Δ in (4.9) and to the relatively smooth increase of the relative number of electrons that participate in the collisionless absorption of the sound.

Allowance for the finite relaxation times and temperature leads to an obvious smearing and smoothing of the singularities in the absorption and in the speed of the sound. We, however, shall not discuss here in detail the role of the collisions and of the temperature.

Violation of the condition $j=0$ on account of the term curl curl $\mathbf{E}'$ in (2.9) has no fundamental effect on the considered picture and can lead only to a change in the values of Δ_1 and Δ_2. An investigation of the role of all these factors in the tilt effect will be reported separately.

In conclusion, we refer to the experimental plot of the speed of longitudinal sound in gallium, obtained by Bezuglyĭ and Burma (see Fig. 3 of Ref. 2). The theoretical analysis above is in good qualitative agreement with the experimental data. In particular, the increase of the speed of sound with increasing α and its abrupt decrease beyond the threshold are distinctly observed. The quantitative differences between the experimental and calculated results are due to the fact that the model of an isotropic uncompensated metal does not fully apply to gallium, which is a compensated metal with a complicated and multiply connected Fermi surface.

We are grateful to A. A. Bulgakov for the numerical calculations of the angular dependences of the speed and of the absorption.

[1)]Formulas (2.1)–(2.6) which follow were written in 1958 by L. D. Landau but remained unpublished.

[2)]We do not consider the question of resonant coupling of the sound with weakly damped electromagnetic waves, when the field E′ must be determined from the exact equation (2.9).

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