# On a class of electromagnetic waves

E. G. Bessonov

*P. N. Lebedev Physics Institute, Academy of Sciences of the USSR, Moscow*
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Some general characteristics of electromagnetic waves for which $\int \mathbf{E}(t)dt \neq 0$ are investigated. Consideration is given to methods of generation of such waves, physical processes in which they can be manifested, and conditions of coherence of the radiation. Methods are indicated for increasing the intensity of the long-wave radiation emitted by particles in synchrotron bending magnets and storage rings in the direction of their straight sections. Like synchrotron and undulatory radiation, this radiation may find wide application in various fields in science and technology.

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## INTRODUCTION

To classify electromagnetic waves and describe some of their properties, the present author introduced in Refs. 1 and 2 the parameter

$$\mathbf{I} = \int_{-\infty}^{\infty} \mathbf{E}(t)\,dt, \tag{1}$$

where $\mathbf{E}(t)$ is the electric field vector of the wave. Waves for which $\mathbf{I} \neq 0$ were called strange electromagnetic waves in Ref. 2, since they transfer to a massive charged particle at rest a momentum linear in $\mathbf{E}$ and directed perpendicular to the direction of their propagation. In particular, waves for which the components $\mathbf{E}_t$ have the form of unipolar pulses are strange.

In the present paper, we discuss methods of generation of strange electromagnetic waves and physical processes in which they can be manifested. The force, spectral, angular, and polarization characteristics of such waves are investigated. Conditions for coherence of the radiation are found.

## 1. EMISSION OF STRANGE ELECTROMAGNETIC WAVES

We find conditions for the emission of strange electromagnetic waves. We use the expressions[3,4] for the electric and magnetic fields of a charged particle moving nonuniformly[3,4]:

$$\mathbf{E}(t) = \frac{e(1-\beta^2)(\mathbf{n}-\boldsymbol{\beta})}{R^2(1-\mathbf{n}\boldsymbol{\beta})^3} + \frac{e[\mathbf{n}\times[(\mathbf{n}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}]]}{cR(1-\mathbf{n}\boldsymbol{\beta})^3}, \quad \mathbf{H} = [\mathbf{n}\times\mathbf{E}], \tag{1.1}$$

where $e$, $c\boldsymbol{\beta}$, and $c\dot{\boldsymbol{\beta}}$ are the charge, velocity, and acceleration of the particle, $\mathbf{n}$ is the unit vector directed from the particle to the point of observation, $R$ is the distance from the particle to the point of observation, and $t$ is the time of observation. On the right-hand side of (1.1), $\boldsymbol{\beta}$, $\dot{\boldsymbol{\beta}}$, $\mathbf{n}$, and $R$ must be taken at the earlier time

$$t' = t - R(t')/c. \tag{1.2}$$

The first term in (1.1) describes the rapidly decreasing Coulomb field of the particle, while the second describes the electromagnetic field radiated by the particle. Substituting (1.1) in (1) and using the relation $dt = (1-\mathbf{n}\cdot\boldsymbol{\beta})dt''$, we obtain

$$\mathbf{I} = \mathbf{I}^a + \mathbf{I}^b, \tag{1.3}$$

where

$$\mathbf{I}^a = \int_{-\infty}^{+\infty} \frac{e(1-\beta^2)(\mathbf{n}-\boldsymbol{\beta})}{R^2(1-\mathbf{n}\boldsymbol{\beta})^2}\,dt', \tag{1.4}$$

$$\mathbf{I}^b = \int_{-\infty}^{+\infty} \frac{e[\mathbf{n}[(\mathbf{n}-\boldsymbol{\beta})\dot{\boldsymbol{\beta}}]]}{cR(1-\mathbf{n}\boldsymbol{\beta})^2}\,dt'. \tag{1.5}$$

We consider further cases in which the acceleration of the particle is nonzero in a restricted region of space whose dimension is appreciably less than the distance to the point of observation. In such cases, $R(t')$ in (1.5) can be taken in front of the integral sign, and $\mathbf{n}$ can be assumed to be constant. Using the relation

$$\frac{[\mathbf{n}[(\mathbf{n}-\boldsymbol{\beta})\dot{\boldsymbol{\beta}}]]}{(1-\mathbf{n}\boldsymbol{\beta})^2} = \frac{d}{dt}\frac{[\mathbf{n}[\mathbf{n}\boldsymbol{\beta}]]}{(1-\mathbf{n}\boldsymbol{\beta})}, \tag{1.6}$$

we transform (1.5) to the form

$$\mathbf{I}^b = \frac{e}{cR}\left[\mathbf{n}\left[\mathbf{n}\left(\frac{\boldsymbol{\beta}_f}{1-\mathbf{n}\boldsymbol{\beta}_f} - \frac{\boldsymbol{\beta}_i}{1-\mathbf{n}\boldsymbol{\beta}_i}\right)\right]\right], \tag{1.7}$$

where $c\boldsymbol{\beta}_f$ and $c\boldsymbol{\beta}_i$ are the final and initial velocities of the particle.

It follows from the definition and (1.7) that the condition $\boldsymbol{\beta}_f \neq \boldsymbol{\beta}_i$ must be satisfied if strange electromagnetic waves are to be radiated. This condition can be satisfied in the cases when at least one of the velocities, $c\boldsymbol{\beta}_f$ or $c\boldsymbol{\beta}_i$, is nonzero, i.e., in the case of infinite motion corresponding to deflection of a particle in external fields, absorption, and emission of the particle by nuclei, etc. We shall consider a number of such cases.[1,2]

1. $\boldsymbol{\beta}_f \neq \boldsymbol{\beta}_i$, $|\boldsymbol{\beta}_f| = |\boldsymbol{\beta}_i| = \boldsymbol{\beta}$. In this case, $I^b = |\mathbf{I}^b|$ takes the maximal value

$$I_m^b = 2e\beta\gamma/cR \tag{1.8}$$

  

in the direction

$$\mathbf{n}_m=(\beta_f+\beta_i)/|\beta_f+\beta_i|, \tag{1.9}$$

when the angle between the vectors $\boldsymbol{\beta}_f$ and $\boldsymbol{\beta}_i$ is

$$\theta_{fi}=2\cos^{-1}\beta. \tag{1.10}$$

In (1.8), we have introduced the relativistic factor $\gamma=(1-\beta^2)^{-1/2}$ of the particle.

A flux of particles characterized by current $i$ in the direction $\mathbf{n}_m$ generates electromagnetic fields with mean intensity[1]

$$\overline{E_m^b}=\overline{H_m^b}=iI_m^b/e=2\beta\gamma i/cR. \tag{1.11}$$

In magnitude and direction, the radiation fields (1.11) are equal in the relativistic case to the static Coulomb fields produced by the space charge and current of a thin cylindrical beam of charged particles at points of space separated from the beam axis by the distance $R/\gamma$ ($\mathbf{I}^a(\mathbf{n}_m)=0$).

2. $\boldsymbol{\beta}_f=-\boldsymbol{\beta}_i$. In this case

$$I^b=\frac{2e}{cR}\frac{\beta\sin\theta}{1-\beta^2\cos^2\theta}, \tag{1.12}$$

where $\theta$ is the angle between the direction of the radiation $\mathbf{n}$ and the direction of the initial velocity $\boldsymbol{\beta}_i$. The quantity $I^b$ takes maximal value

$$I_m^b=\begin{cases}2e\beta/cR, & \beta\leqslant 2^{-1/2}\\ e\gamma/cR, & \beta>2^{-1/2}\end{cases} \tag{1.13}$$

at the angle $\theta=\theta_m$:

$$\theta_m=\arcsin(1/\beta\gamma). \tag{1.14}$$

The expressions (1.12)–(1.14) are also valid in the case $\theta_{fi}\gg 1/\gamma$, $\theta\ll|\theta-\theta_{fi}|$, $\theta\ll 1$ ($\gamma\gg 1$).

3. $\boldsymbol{\beta}_i=0$, $\boldsymbol{\beta}_f\neq 0$ ($\boldsymbol{\beta}_f=0$, $\boldsymbol{\beta}_i\neq 0$). In this case

$$I^b=\frac{2e}{cR}\frac{\beta\sin\theta}{1-\beta\cos\theta}, \tag{1.15}$$

where $\theta$ is the angle between the direction of $\mathbf{n}$ and the direction of the velocity $\boldsymbol{\beta}_f(-\boldsymbol{\beta}_i)$. The quantity $I^b$ takes the maximal value

$$I_m^b=e\beta\gamma/cR \tag{1.16}$$

at the angle

$$\theta_m=\arccos\beta. \tag{1.17}$$

A stream of relativistic charged particles produces electromagnetic fields at points of observation situated at the angles (1.14) and (1.17) in cases 2 and 3, respectively, that have half the magnitude corresponding to (1.11). The fields of a wave emitted near the direction $\boldsymbol{\beta}_f$ will be observed together with Coulomb fields of approximately the same magnitude.

The following physical processes correspond to the considered special cases of the emission of strange electromagnetic waves: 1) $e^-$ bremsstrahlung in the field of a nucleus, 2) the emission accompanying $\beta$ decay of nuclei, 3) the radiation of $e^-$ in the case of Compton scattering of $\gamma$ rays on them, 4) the radiation of particles in bending magnets, 5) the radiation of cosmic rays in the magnetic field of the Earth, 6) radiation by $e^+$ when they are reflected from the surface of crystals, etc.

When a particle passes successively through $K$ regions of space in which it changes the magnitude and direction of its velocity, the expression (1.5) reduces to the form

$$\mathbf{I}^t=\frac{e}{c}\sum_{i=1}^{K}\frac{1}{R_i}\left[\mathbf{n}_i\times\left[\mathbf{n}_i\times\left(\frac{\boldsymbol{\beta}_{if}}{1-\mathbf{n}_i\boldsymbol{\beta}_{if}}-\frac{\boldsymbol{\beta}_{ii}}{1-\mathbf{n}_i\boldsymbol{\beta}_{ii}}\right)\right]\right], \tag{1.18}$$

where $\mathbf{n}_i$ is the unit vector directed from the region $i$ to the point of observation; $c\boldsymbol{\beta}_{if}$ and $c\boldsymbol{\beta}_{ii}$ are the final and initial velocity of the particle corresponding to region $i$; and $\boldsymbol{\beta}_{if}=\boldsymbol{\beta}_{i+1i}$. The expression (1.18) is valid when the dimensions of the regions, multiplied by $1-\mathbf{n}_i\cdot\boldsymbol{\beta}_{if(i)}$, are appreciably less than the distances $R_i$ from each region to the point of observation. In the relativistic case, $I^b$ is maximal near the directions $\boldsymbol{\beta}_{if}$ and $\boldsymbol{\beta}_{ii}$.

If the particle moves between the regions along closed trajectories, then $I^b$ near all directions $\boldsymbol{\beta}_{if}$ and $\boldsymbol{\beta}_{i+1i}$ has a distance dependence characteristic for the given case: $I^b\sim 1/R_{i+1}$ at short distances $R_{i+1}\ll R_j$ ($j\neq i+1$) and $I^b\sim(R_i-R_{i+1})/R_{i+1}^2$ at large distances $R_{i+1}\gg R_i-R_{i+1}$. Thus, in the case of finite motion of particles, the value of $I^b$ at short distances falls off in the same way as for infinite motion ($\sim 1/R$), but at large distances faster ($\sim 1/R^2$). The wave emitted by a particle in the direction close to the direction of its motion in a rectilinear section of a closed trajectory consists of two successive strange waves, which arrive from different regions of space. The parameters $\mathbf{I}^b$ of these waves as $R\to\infty$ are equal in magnitude but opposite in direction.

## 2. PROPERTIES OF STRANGE ELECTROMAGNETIC WAVES

1. *Force characteristics*. Strange electromagnetic waves transfer to a massive particle at rest with charge $q$ the momentum

$$\mathbf{P}=q\mathbf{I}, \tag{2.1}$$

which is linear in $\mathbf{E}$ and directed at right angles to the direction of propagation of the wave.[1,2]

If the point of observation is situated between two regions such that the direction $\mathbf{n}_i$ is close to the direction $\boldsymbol{\beta}_i$ and far from the remaining directions $\boldsymbol{\beta}_j$, and $\mathbf{n}_{i+1}=-\mathbf{n}_i$, then $I^b$ is in accordance with (1.18) close to (1.15). In practice, such a case can be realized in a straight section of a synchrotron or a storage ring. In such cases, the strange electromagnetic waves move together with the beams of particles emitting them. If the beams are narrow and have a small angular spread, then the vectors of the mean intensity of the wave fields

$$\overline{\mathbf{E}}^b=\overline{\mathbf{H}}^b=i\mathbf{I}^b/e \tag{2.2}$$

and of the Coulomb fields of the beams have the same direction and are comparable in magnitude.

If a particle is deflected in an external field through angle $\Delta\theta \gtrsim 1/\gamma$ and then passes near some system, then the system is subject to not only the field of the strange electromagnetic wave radiated by the deflected particle but also the Coulomb field of the particle, distorted in the deflection process. This circumstance must affect, for example, the bremsstrahlung radiated by $e^+e^-$ beams colliding in the straight sections of storage rings and the ionizing capacity of ultrarelativistic electrons that undergo appreciable deflections in a nuclear field.[2]

2. *Spectral and angular characteristics*. The spectral density of the energy distribution in an electromagnetic wave is determined by the expression

$$\frac{d\varepsilon}{d\omega\, ds}=c|\mathbf{E}_\omega|^2, \tag{2.3}$$

where

$$\mathbf{E}_\omega=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\mathbf{E}(t)e^{i\omega t}dt$$

is the Fourier transform of the electric field of the wave, $\omega$ is the frequency, and $ds$ is the element of surface normal to $\mathbf{n}$. Since $\mathbf{E}_\omega \to \mathbf{I}/2\pi$ as $\omega \to 0$, it follows from (1.7) and (2.3) that in the case of strange electromagnetic waves

$$\frac{d\varepsilon}{d\omega\, ds}=\frac{e^2}{4\pi^2cR^2}\left|\left[\mathbf{n}\times\left(\frac{\boldsymbol{\beta}_f}{1-\mathbf{n}\boldsymbol{\beta}_f}-\frac{\boldsymbol{\beta}_i}{1-\mathbf{n}\boldsymbol{\beta}_i}\right)\right]\right|^2. \tag{2.4}$$

The expression (2.4) is well known in the theory of bremsstrahlung of colliding particles.[3-5] The range of frequencies for which the expression (2.4) is valid depends on the energy of the particle, the nature of the external deflecting fields, and so forth. Thus, for deflection of a particle through angle $\Delta\theta \gg 1/\gamma$ in the field of a magnet, the expression (2.4) is valid up to the frequencies characteristic for synchrotron radiation:

$$\omega_s=3\Omega\gamma^3, \tag{2.5}$$

where $\Omega=eH/mc^2\gamma$ is the cyclotron frequency, and $H$ is the intensity of the external magnetic field.

In accordance with (2.4) the radiation emitted by the particle in the magnet at low frequencies $\omega \ll \omega_s$ in directions of $\mathbf{n}$ near $\boldsymbol{\beta}_f$ and $\boldsymbol{\beta}_i$ is concentrated mainly in the range of angles

$$\Delta\theta=1/\gamma \tag{2.6}$$

relative to the initial and final velocities of the particle. This range is appreciably less than the range

$$\Delta\theta=\frac{1}{\gamma}\left(\frac{\omega_s}{\omega}\right)^{1/3}$$

within which the synchrotron radiation emitted at the same frequencies by particle moving in circular orbits in a magnetic field of the same intensity is concentrated (Ref. 4).[1)]

In a single passage through a magnet, particles lose a small fraction of their energy through the electromagnetic radiation. Therefore, to create effective sources of strange electromagnetic waves it is desirable to use a scheme of energy recouperation. Such a scheme is realized in synchrotrons and charged-particle storage rings, in which strange electromagnetic waves are emitted in the direction of the straight sections (see the end of Sec. 1). The spectral density of the intensity distribution of this radiation at distances of the order of the section length and frequencies $\omega \ll \omega_s$ is appreciably higher [by $\sim\omega_s/\omega)^{2/3}$ times] than for the synchrotron radiation emitted in the other direactions in the same synchrotron or storage ring (see footnote 1). It is easy to show that at large distances the spectral composition of the radiation emitted from the straight sections is shifted to the high-frequency region, remaining significantly different from the synchrotron radiation spectrum. For example, a rapid decrease in the spectrum intensity of the radiation in the direction of low frequencies will be observed at $\omega < \omega_l$, where

$$\omega_l=\pi c\gamma^2/l, \tag{2.7}$$

in which $l$ is the length of the straight section. In the interval $\omega_l < \omega < \omega_s$, oscillations of the spectral intensity as a function of the frequency will be observed.[6,7]

If a screen of diameter $\sim a/l$ is placed on the continuation of the axis of the straight section of a synchrotron at distance $a \ll l$ from the point of entry of the particle into the nearest magnet, then it will reflect the major part of the energy of the radiation emitted by the particle from this magnet in the direction of the axis of the section. For $a \ll l$, the radiation from a more distant magnet will be reflected to a lesser extent. As a result, at distances $R \gg l$ the spectral composition of the radiation is shifted in the direction of lower frequencies down to the frequency $\omega_a=\gamma c/a$. A similar effect will be observed in the case of diaphragming (using lenses and focusing mirrors) of the radiation emitted in the direction of the straight section.

3. *Polarization characteristics*. The radiation emitted by particles moving nonuniformly is polarized. The nature of the polarization is determined by the ratio of the imaginary and real parts of $\mathbf{E}_\omega$. Since $\mathbf{E}_\omega \to \mathbf{I}/2\pi$ as $\omega \to 0$ in the case of strange electromagnetic waves, and $\mathbf{I}$ is a real quantity, the low-frequency part of the radiation of strange electromagnetic waves has complete linear polarization. The direction of the plane of polarization is determined by the direction of

the vector **I**. The degree of linear polarization of this radiation is less than unity if the radiation is emitted by a beam of particles deflected in different directions (as in the bremsstrahlung of particles in matter) or if the particles in the beam have an initial angular spread $\theta_b \gtrsim 1/\gamma$.

4. *Time, spectral, and coherence characteristics of strange electromagnetic waves emitted by particle beams.* Suppose for simplicity that the radiation is emitted by a beam of charged particles moving along a circular arc subtending the angle $\theta_c \leqslant 2/\gamma$ with radius $\rho$. In this case, the electric field components of the wave emitted in the direction $\mathbf{n}_m$ determined by the expression (1.9) will have the form of unipolar pulses, and the duration of the radiation of each particle will be

$$\Delta t \approx \rho\theta_c/\beta c\gamma^2. \tag{2.8}$$

In this case the main fraction of the radiation will be concentrated in the frequency range $0 \leqslant \omega \leqslant 1/\Delta t$.

In the considered case, there are two possible radiation regimes—the low-current regime, $i < i_c$, and the high-current regime, $i > i_c$, where the characteristic current

$$i_c = e/\Delta t = e\beta c\gamma^2/\rho\theta_c \tag{2.9}$$

corresponds to the current at which the radiation pulses emitted by neighboring particles begin to overlap. With increasing current, the radiation spectrum is also displaced in the direction of lower frequencies since the waves with lower frequencies emitted by the particles are added with equal phases. For $i < i_c$, the total intensity of the radiation is proportional to the current; for $i > i_c$, to the square of the beam current. A quadratic dependence of the radiation intensity on the current is characteristic of the coherent radiation of a beam of particles grouped in short bunches (with an extension of the order of the wavelength of the emitted wave). For $i > i_c$, strange electromagnetic waves are also emitted coherently by an extended beam with uniform density, since in this case (unipolar pulses) the rms value of the electromagnetic field intensity of these waves is close to the mean value.[2]

What we have said above concerning the frequency characteristics of strange electromagnetic waves emitted by particle beams moving in a circular arc subtending angle $\theta_c \leqslant 2/\gamma$ is also true in the general case. For $\theta_c \gg 1/\gamma$, it is necessary to replace $\theta_c$ in (2.9) by $1/\gamma$.

In the field of a nucleus, beam particles are deflected in different directions. In this case, the polarities of the strange electromagnetic waves emitted by different particles will be different, and therefore the radiation spectrum will be suppressed in the low-frequency region $\omega < i/e$. In calculating the effective radiation emitted by a beam of particles scattered by atoms, it is necessary to take into account the interference between the radiation fields of the different particles.

As is well known, the radiation emitted by a single particle moving in matter is also suppressed in the low-frequency region of the spectrum because of the circumstance that the particle is not deflected on the average from its initial direction of motion by the successive collisions with the atoms.[8]

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