Classical Yang-Mills mechanics. Nonlinear color oscillations

S. G. Matinyan, G. K. Savvidi, and N. G. Ter-Arutyunyan-Savvidi

Erevan Physics Institute
(Submitted 10 July 1980)
Zh. Eksp. Teor. Fiz. **80**, 830–838 (March 1981)

A new class of solutions to the classical Yang-Mills equations in Minkowski space that leads to nonlinear color oscillations is studied. The system describing these oscillations appears to be stochastic. Periodic trajectories corresponding to these solutions are found and studied and it is shown that they form at least a countable set.

PACS numbers: 11.10.Lm

1. INTRODUCTION. THE SIMPLEST NONLINEAR COLOR OSCILLATIONS

There has recently been much discussion of solution of the classical Yang-Mills equations and the possibility of obtaining on the basis of them (without the use of perturbation theory) effects associated with the ground state of quantum chromodynamics.[1,2] In this connection, interest attaches to analysis of the classical Yang-Mills equations without external sources; this may be helpful for the construction and investigation of the structure of the vacuum and the problem of asymptotic states of the theory.

In the earlier Ref. 3, a study was made of periodic solutions of the classical Yang-Mills equations in Minkowski space without external sources describing very simple nonlinear oscillations of the color degrees of freedom and the analog of plane waves in classical electrodynamics. In the present paper, we make a further investigation of nonlinear oscillations in classical Yang-Mills theory. We begin with some results of the previous Ref. 3 needed in what follows.

We consider the Yang-Mills equations without external sources corresponding to the group $SU(2)$:

$$\begin{gathered}\partial_\mu G_{\mu\nu}{}^a+g\varepsilon^{abc}A_\mu{}^b G_{\mu\nu}{}^c=0,\\ G_{\mu\nu}{}^a=\partial_\mu A_\nu{}^a-\partial_\nu A_\mu{}^a+g\varepsilon^{abc}A_\mu{}^b A_\nu{}^c\end{gathered}\tag{1}$$

($a, b, c=1, 2, 3$; $\mu, \nu=0, 1, 2, 3$; $i, j, k=1, 2, 3$) in a coordinate system in which the Poynting vector $G_{0i}^a G_{ji}^a$ ($G_{i,j}\equiv\partial_j G_i$) vanishes. For this, it is sufficient (in the gauge $\partial_i A_i^a=0$, $A_0^a=0$) that one of the following relations be satisfied: a) $A_{i,j}^a=0$), b) $\dot{A}_i^a=0$, c) $A_{i,j}^a-A_{j,i}^a=0$ (the dot denotes differentiation with respect to the time).

In case a), when the potential in a chosen system depends only on the time, the Yang-Mills equations take the form

$$\ddot{A}_i^a+g^2(A_j^b A_j^b A_i^a-A_j^a A_j^b A_i^b)=0.\tag{2}$$

The system described by Eqs. (2) is actually equivalent to a discrete nonlinear mechanical system with Hamiltonian

$$H=\frac{1}{2}(\dot{A}_i^a)^2+\frac{g^2}{4}((A_i^a A_i^a)^2-(A_i^a A_j^a)^2),\tag{3}$$

which is equal to the energy density of the Yang-Mills field system (1).

On the basis of this, it is natural to speak of the classical (nonlinear) Yang-Mills mechanics described by Eqs. (2). We note that the corresponding classical Maxwellian mechanics is trivial and described by the equation $\ddot{A}_i=0$, which corresponds to constant electric field $A_i=-E_i t$; as we shall see below, it is only because of the nonlinearity of the Yang-Mills equations that complicated nonlinear color oscillations arise.

In the simplest case admitting analytic solution, we obtain a nonlinear plane wave that varies in time in accordance with the Jacobian elliptic cosine. Indeed, if we seek a solution to the system (2) in the nine-parameter form

$$A_i^a=O_i^a f^{(a)}(t)/g$$

(no summation over a), where O_i^a is a time-independent orthogonal matrix,

$$O_i^a O_i^b=\delta^{ab},$$

then from (2) we obtain the system of equations

$$\ddot{f}^{(a)}+f^{(a)}(\mathbf{f}^2-f^{(a)2})=0,\tag{4}$$

where

$$\mathbf{f}^2=f^{(1)2}+f^{(2)2}+f^{(3)2}.$$

In the earlier Ref. 3, a study was made of the simple case, admitting analytic solution, when all three colors vary in time in the same way:

$$f^{(1)}(t)=f^{(2)}(t)=f^{(3)}(t)\equiv f(t).$$

Then the system (4) reduces to a nonlinear equation with one degree of freedom whose solution has the form[3]

$$f(t)=(2g^2/3)^{1/4}\mu\,\mathrm{cn}\,[(8g^2/3)^{1/4}\mu(t+t_0);1/\sqrt{2}],\tag{5}$$

where $\mathrm{cn}(x;k)$ is the Jacobian elliptic cosine of argument x and modulus k, t_0 is the arbitrary origin of the time, and μ^4 is the energy density in the considered coordinate system.

The chromoelectric field

$$E_i^a=O_i^a\dot{f}/g$$

and the chromomagnetic field

$$H_i^a=\varepsilon_{ijk}\varepsilon^{abc}O_j^b O_k^c f^2/g^2$$

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corresponding to (5) are mutually parallel in the "rest" frame, and the three vectors $\mathbf{E}^1, \mathbf{E}^2, \mathbf{E}^3$ are mutually orthogonal. Such a picture differs from the one realized in a plane wave in classical electrodynamics, for which $\mathbf{E} \perp \mathbf{H}$.

The intensities $\mathbf{E}^a$ and $\mathbf{H}^a$ vary in time with period

$$T=\left(\frac{3}{8g^2}\right)^{1/4}\frac{4}{\mu}K\left(\frac{1}{\sqrt{2}}\right),$$

where $K(x)$ is a complete elliptic integral of the first kind. The nonlinear wave with argument $kx=k_\mu x_\mu$, where $k_0=\mu\gamma$, $k_i=\mu\nu_i\gamma$ $(\gamma=(1-\nu^2)^{-1/2})$, $k^2=\mu^2$ (Ref. 3), that arises from (5) under Lorentz transformations differs from Coleman's[4] non-Abelian linear plane wave ($k^2=0$) in that the magnitude of the Poynting vector in Ref. 3 is not equal to the energy density ($k^2=\mu^2$).

Thus, we see that if we investigate the mechanical system (4) we can then make a Lorentz transformation to obtain a solution of the system (1) in an arbitrary coordinate system.[1)] Such a solution will always be a plane wave. Therefore, our problem actually reduces to studying the classical Yang-Mills mechanics (2). In connection with what we have said above, it is interesting to make a further study of the nonlinear color oscillations in the classical Yang-Mills theory. The solution to this problem with more than one degree of freedom reduces to investigation of the system (2) [or—in the simpler variant—the system (4)].

It is also interesting to study the system (2) [or (4)] from another point of view. It is well known that the conservation laws in nonlinear field theory models make it possible to advance far in their study. However, all attempts to find a set of conserved integrals in Yang-Mills theory have hitherto been unsuccessful.[2)] The system (2) [(4)] is much simpler than (1), and the situation with regard to the general system (2) could be clarified by establishing whether the simpler system is completely integrable or not.

In the present paper, we give the results of qualitative analysis and numerical integration of the system of equations (4), and we also study some general properties of the system (2).

2. QUALITATIVE INVESTIGATION OF THE NONLINEAR SYSTEM (4). THE TWO-DIMENSIONAL CASE

We begin our study of the system (4) with the case when $f^{(3)}=0$, so that, introducing the notation $x\equiv f^{(1)}$, $y=f^{(2)}$, we arrive at a nonlinear mechanical system on a plane with Hamiltonian

$$H=\dot{x}^2/2+\dot{y}^2/2+x^2y^2/2 \tag{6}$$

and corresponding equations of motion that we shall investigate here:

$$\ddot{x}+xy^2=0,\quad \ddot{y}+yx^2=0. \tag{7}$$

It follows from the form of H that any conserved integral $F(x,y,p_x,p_y)$ of the system (7) must satisfy the partial differential equation

$$p_x\frac{\partial F}{\partial x}+p_y\frac{\partial F}{\partial y}=xy\left[y\frac{\partial F}{\partial p_x}+x\frac{\partial F}{\partial p_y}\right]$$

from which it can be seen that F cannot depend on only two of variables x, y, p_x, p_y or be a polynomial of finite degree in these variables.

We now analyze the two-dimensional nonlinear mechanical system described by Eqs. (7).

Since the Hamiltonian (6) is positive-definite ($H\geqslant 0$), the "material point" described by (7) cannot leave the region bounded by the equipotential curves $xy=\pm\sqrt{2}\,\mu^2$ (Fig. 1), where μ^4 is the "total energy of the point." The equipotential lines are symmetric with respect to the origin and the coordinate axes and the straight lines $x=\pm y$. It is obvious that if the "point" with "total energy" μ^4 described by (7) and (6) is at some instant on the equipotential curve $xy=\pm\sqrt{2}\,\mu^2$, then it will leave this curve along the normal into the region.

It follows from the symmetry of the problem that the trajectory will be periodic if any of the events listed below occur twice (in any combination): a) the trajectory passes through the origin; b) the trajectory is perpendicular to one of the symmetry axes; c) the trajectory reaches an equipotential curve.

These sufficient conditions of periodicity are helpful for the classification and description of the trajectories given below (see Sec. 3), but we do not rule out the possibility that one could find other weaker sufficient criteria of periodicity of the trajectories of the system (7).

Along the symmetry axes $x=\pm y$, the system executes, of course, the periodic oscillations (5) [events a) and c)]. Along the axes $x=0$ and $y=0$ the point, as in electrodynamics, goes away to infinity ($\ddot{x}=\ddot{y}=0$, $\dot{x},\dot{y}\neq 0$). But if at some instant the velocity of the system is not directed along the x or y axis, then it will not go to infinity, though in some cases it may travel an arbitrarily large distance from the center and return in a finite time to the region $x\sim y$, as is readily seen from the negativity of $\ddot{x}/x$ and $\ddot{y}/y$.[3)] We describe this qualitatively by considering the motion of the point when it moves away from the center in one of the narrow "channels" bounded by the equipotential lines.

In polar coordinates ($x=\rho\cos\varphi$, $y=\rho\sin\varphi$), Eqs. (7) have the form

$$\ddot{\varphi}+\frac{2\dot{\rho}}{\rho}\dot{\varphi}+\frac{\rho^2}{4}\sin 4\varphi=0, \tag{8'}$$

$$\ddot{\rho}-\rho\dot{\varphi}^2+\frac{\rho^3}{2}\sin^2 2\varphi=0. \tag{8''}$$

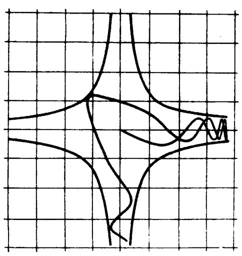

FIG. 1.

In the case of motion away from the center ($\rho \gg \mu$; note that in our problem x, y, ρ have dimensions of mass and not length!), for example, along the channel $\varphi \ll \pi/4$, $\sin 4\varphi \approx 4\varphi$, $\dot{\rho} > 0$, it can be seen from Eq. (8′) that the "frequency" of the oscillations with respect to the coordinate φ increases with increasing distance from the center, while the amplitude decreases until $\dot{\rho} = 0$ ("turning point") [this last occurs in a finite interval of time, since $\ddot{\rho} \approx -a(t)\rho^3$, $a > 0$], after which the "damping" regime is replaced by an "excitation" regime. Figure 1 shows a characteristic example of such behavior obtained on a computer.

The motion with respect to ρ, averaged over the rapid oscillations of φ, consists of a random walk with large amplitudes ($\ddot{\rho} + a\rho^3 \approx 0$) from channel to channel with complicated motion in the region $x \sim y$ [which can be followed in a numerical integration of the system (7) on a computer]. In the language of the variation in time of the color amplitudes $f^{(1)}$ and $f^{(2)}$, this picture corresponds alternately to rapid oscillations and decrease of one color amplitude and growth of the other.

As will be shown in Sec. 3, the system (7) also has at least a countable set of periodic trajectories satisfying the sufficient conditions listed above [events a)–c)].

With regard to the behavior of the system with three degrees of freedom (4), it will follow from the general analysis of the system (2) in Sec. 4 that it reproduces qualitatively the main features in the behavior of the system (7), whose periodic trajectories we shall now investigate.

3. PERIODIC TRAJECTORIES OF THE SYSTEM (7)

In Fig. 2, we show examples of some periodic trajectories photographed on the display of the computer used to integrate the system (7). In Figs. 2(a)–2(f), we show trajectories that pass through the coordinate origin and are perpendicular to either an equipotential line [Figs. 2(a), 2(b), 2(d), and 2(f)] or the symmetry axis $y = 0$ [Figs. 2(c) and 2(e)]. The trajectories are arranged in the order of decreasing slope relative to the x axis at the origin. The trajectory in Fig. 2(a) corresponds to the oscillations in accordance with the elliptic cosine law (5) studied earlier in Ref. 3.

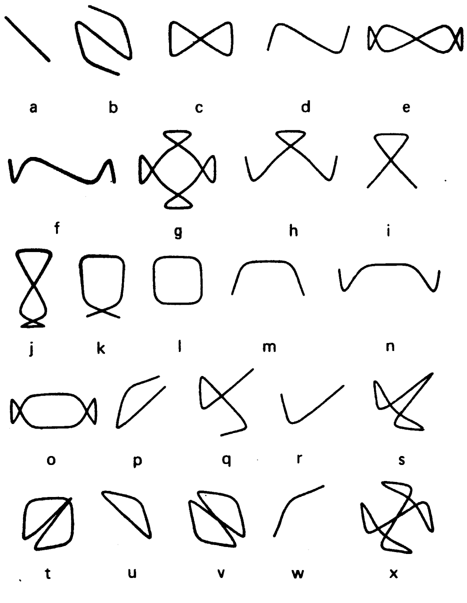

FIG. 2.

A further decrease in the slope leads to an increase in the number of intersections with the x axis as in the trajectories in Figs. 2(c) and 2(e) and 2(d) and 2(f). We denote these angles for trajectories of the type in Figs. 2(c) and 2(e) by α_n^0 and for trajectories of the type in Figs. 2(d) and 2(f) by β_n^0, where n is the number of intersections of the trajectories with the x axis. In the limit $n \to \infty$, the angles α_n^0 and β_n^0 tend to zero. It is obvious that trajectories with slope α^0, where $\alpha_n^0 \geqslant \alpha^0 \geqslant \beta_n^0$, do not go deeper into the channel than the trajectory with slope β_n^0.

These figures clearly reveal the tendency to an increase in the frequency and decrease in the amplitude of the oscillations as the "particle" moves further into the channel along the x axis the smaller is the angle between the x axis and the trajectory at the origin—in agreement with the qualitative analysis made above (Sec. 2) for large $\rho (\gg \mu)$.

In Figs. 2(g)–2(m) we show examples of trajectories that pass perpendicular to the y axis at different distances from the center and perpendicular to either the coordinate axes [Figs. 2(g), 2(j), 2(l), and 2(m)], or the equipotential lines. With decreasing distance of these trajectories along the y axis from the center, they all then enter the channel, and the picture considered in Sec. 2 for large ρ is again reproduced qualitatively. Figures 2(p), 2(q), and 2(r) show trajectories that are twice perpendicular to the equipotential lines. Finally, Figs. 2(s)–2(x) represent trajectories perpendicular to the symmetry axes $x = \pm y$.

On the basis of the above analysis of the trajectories in Fig. 2 it can be seen that the number of periodic trajectories of the type of Figs. 2(c)–2(f), and also of the type in Figs. 2(n) and 2(m) is countable, so that we can assert that the set of periodic solutions of the system (7) with fixed energy density μ^4 is at least countable.

We now make a remark concerning the initial conditions.

Since no trajectory of the system (7) can lie entirely in a signal quadrant of Fig. 1 (which is obvious from the nature of the "force field" of the problem), it follows from this and the symmetry of the problem that all possible trajectories of the system (7) can be obtained by specifying initial conditions in the form

$$y=0, \quad x=x_0>0, \quad \dot{x}=\sqrt{2}\mu^2 \cos\alpha, \quad \dot{y}=\sqrt{2}\mu^2 \sin\alpha \quad (0 \leqslant \alpha \leqslant \pi).$$

It is also helpful to compare the system (7) with the system in which the potential energy $U(x, y)$ has the form $(x^2y^2)^n$. In the limiting case $n \to \infty$ in the plane xy, we are concerned in this case with the motion of a material point which undergoes perfectly elastic collisions with an infinite barrier in the form of the hyperbolic cylinder $x^2y^2 = 1$. Thus, the trajectory at the point in the xy plane is a sequence of rectilinear segments, and it can be constructed by means of the laws of geometrical optics.

The periodic ("broken") trajectories of this system correspond completely to the smooth trajectories of the system (7), examples of which are given in Fig. 2. Note that for the system with $n=\infty$ one can in principle calculate analytically, for example, the characteristic angles α_n^0 and β_n^0 discussed above.

To conclude this section, we give an example of a periodic trajectory of the system (7) found by numerical integration on the computer (Fig. 3) that does not satisfy any of the sufficient criteria of periodicity [events a)–c)] listed in Sec. 2.

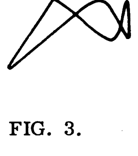

FIG. 3.

4. QUALITATIVE ANALYSIS OF THE GENERAL SYSTEM (2)

The analysis of the two-dimensional system made in Secs. 2 and 3 facilitates the qualitative understanding of a number of characteristic features of the solutions of the classical nonlinear Yang-Mills mechanics described by Eqs. (2).

As follows from the foregoing, an important part is played here by knowledge of the shape of the equipotential surface ($\dot{A}_i^a=0$)

$$U(A_i^a)=\frac{g^2}{4}[(A_i^aA_i^a)^2-(A_i^aA_j^a)^2]=\mu^4 \tag{9}$$

in the nine-dimensional space of the components of the vector potential A_i^a. The functional $U(A_i^a)$ can be represented in the following form, which shows explicitly that it is positive definite:

$$U=\frac{g^2}{2}\sum_{\substack{a<b\\ i<j}}(A_i^aA_j^b-A_j^aA_i^b)^2. \tag{10}$$

It is readily seen that the sum in (10) is a sum of the squares of the cofactors of all the elements A_i^a of a 3×3 matrix; this will be helpful in what follows.

Let us consider different sections of the surface (10), increasing their dimensionality, i.e., the number of nonvanishing elements of the matrix A_i^a, successively.

The two-dimensional sections of the surface (10) are of two types. The first of them corresponds to a matrix A_i^a in which the nonzero elements (two) belong one row or one column. In this case, the equations for these elements of the matrix A_i^a decouple, and the trajectories always go to infinity, corresponding to either a constant electric field of Abelian electrodynamics (for example, when $A_1^1, A_2^1\neq 0$) or two independent "electrodynamics" (when, for example, $A_1^1, A_1^2\neq 0$).

A nontrivial case arises when the two-dimensional sections of (10) correspond to nonvanishing elements of the matrix A_i^a that do not lie on one column or one row. The case $A_1^1, A_2^2\neq 0$, which corresponds to the system (7), was the subject of detailed analysis in Secs. 2 and 3.

For the three-dimensional sections, we begin with the case when the three nonvanishing elements in the matrix A_i^a are each on a different row and a different column. There are six such sections. One of them ($A_1^1, A_2^2, A_3^3\neq 0$) corresponds to our system (4). In these cases, U degenerates into a sum of squares of products of the components A_i^a in pairs. For the system (4), for example (the matrix A_i^a is diagonal) we have[4]

$$U=\frac{g^2}{2}[(A_1^1A_2^2)^2+(A_2^2A_3^3)^2+(A_3^3A_1^1)^2].$$

In all these six cases, the behavior of the three-dimensional system is qualitatively similar to the behavior that we considered in detail in Secs. 2 and 3 for the nontrivial two-dimensional system. In this case, there are six channels along the coordinate axes, and the motion in them is analogous to the motion in the four channels of the two-dimensional system, i.e., with increasing distance from the center, the frequency of the oscillations of the trajectories increases, and the amplitude decreases until it stops, after which the regime of damping with respect to the spherical angle ϑ is replaced by an excitation regime. As in the two-dimensional case, the particle goes to infinity only in the case of motion along the coordinate axes.

The general picture of the variation of the color amplitudes in this three-dimensional case is characterized by alternate rapid oscillations and decrease of two color amplitudes and growth of the third. There is a "transfer" of color between the amplitudes ("beats"). The periodic trajectories of the three-dimensional system have as projections onto the corresponding planes trajectories of the type of those shown in Fig. 2 in the analysis of the two-dimensional case.

We now consider the three-dimensional sections when two and only two of the three nonzero elements of the matrix A_i^a belong to one row or one column. Suppose, for example, only the matrix elements A_1^1, A_2^1, A_3^3 are nonzero:

$$U=\frac{g^2}{2}(A_3^3)^2[(A_1^1)^2+(A_2^1)^2].$$

There are here just two channels along the A_3^3 axis, their shape being specified by a figure of revolution with hyperbolas as generators. The motion in the plane $A_3^3=0$ is here infinite, while in the planes (A_1^1, A_3^3) and (A_2^1, A_3^3) it is analogous to the two-dimensional motion considered in Secs. 2 and 3.

One further type of three-dimensional section of the surface (10) is obtained if the nonvanishing elements of A_i^a belong to one row or one column of the matrix A_i^a. Then $U=0$, and the motion along all directions of the corresponding three-dimensional space is infinite. If, for example, the nonvanishing components are A_1^a, A_2^a, A_3^a ($a=1,2,3$), which lie on one row, we arrive at the well-known configuration of the covariantly constant chromoelectric field introduced in the investigation of vacuum polarization in non-Abelian gauge theory.[7,8]

The sections of higher dimensionality can be studied similarly. The general picture obtained is again that of beats of the color like those described in detail above.

The question arises of whether the dynamical system (2) considered here is stochastic. An argument in favor of this is the fact[9] that, as we have seen, the trajectories of the system (2) are unstable with respect to small changes in the initial conditions (x_0, α; see Sec. 3), to which the failure of searches for new integrals of the motion of the general system (1) can be attributed.

All the foregoing analysis was done in the chosen coordinate system, in which the vector potential $A_i^a(t)$ depends only on the time. In an arbitrary coordinate system, as noted in Sec. 1, we shall have solutions in the form of nonlinear plane waves A_μ^a that depend on the argument $\xi = kx = k_\mu x_\mu$:

$$A_\mu^a(x) = \alpha_{\mu\nu}(\mathbf{v}) A_\nu^a(\xi) \qquad (A_0^a(\xi) = 0),$$

where $\alpha_{\mu\nu}(\mathbf{v})$ is the matrix of the Lorentz transformation $x_\mu = \alpha_{\mu\nu} x'_\nu$; $k_0 = \mu\gamma$, $k_i = \mu\nu_i\gamma$, $k^2 = \mu^2$.

Thus, we see that the solutions are analogous to the plane waves of classical electrodynamics. It is of interest to establish whether these solutions could play the same part in quantization of Yang-Mills theory as do plane waves in quantum electrodynamics. A difficulty in the realization of such a program obviously arises because of the absence of a superposition principle for such solutions, so that the ordinary quantization scheme does not apply here.

5. CONCLUSIONS

In this paper, we have investigated in detail a new class of solutions of the classical Yang-Mills equations; it leads to nonlinear plane waves in Minkowski space. The analysis of these solutions has led to a fairly interesting nontrivial picture of the oscillations of the color amplitudes, and this could be helpful in the development of our ideas about the nature of vacuum fluctuations in quantum chromodynamics.

The stimulus for this investigation was the conviction that although the study of the Yang-Mills equations at the classical level has already led to numerous remarkable results (monopoles,[10-13] instantons,[1] merons,[14-16] and so forth) we have still (in contrast to the situation in electrodynamics) not yet studied in adequate detail the numerous manifestations of unusual properties of the Yang-Mills fields associated with their nonlinearity.

At the same time, it must be borne in mind that although classical solutions play an important part in quantum field theory,[1,5,11] it is entirely possible that deeper penetration into the essence of the purely quantum aspects of Yang-Mills fields will radically change our notions about the nature of the vacuum fluctuations of non-Abelian gauge fields based on classical solutions.

It is a pleasant duty to thank V. A. Ambartsumyan, A. Ts. Amatuni, A. A. Ansel'm, G. Z. Baseyan, A. A. Belavin, V. N. Gribov, V. E. Zakharov, O. V. Kancheli, R. M. Muradyan, Ya. A. Smorodinskiĭ, and L. D. Faddeev for their interest in the work and numerous helpful discussions. We are grateful to our colleagues K. V. Grigoryan, Yu. P. Mozharov, and A. S. Nanasyan at the Computational Center of the Erevan Physics Institute for assistance in the computer calculations.

[1)] In linear theories describing a particle of mass $\mu(\Box A + \mu^2 A = 0)$ and in the rest frame, the amplitude of the corresponding wave oscillates in accordance with the law $\cos\mu t$, which corresponds in the case considered here to oscillation in accordance with (5). In a massless linear theory such as electrodynamics ($\Box A = 0$), there does not exist a coordinate system moving with the wave, which, as we see, is not correct in Yang–Mills theory ($\Box A + A^3 = 0$). It should be noted that one could from the start seek a solution of the system (1) in the for $A_\mu^a(x) = A_\mu^a(kx)$, where $k^2 = \mu^2$, without making a special choice of the coordinate system. However, the analogy with classical mechanics given here appears to be helpful.

[2)] See Ref. 5 for conservation laws on the manifold of contours in Yang–Mills theory for the case of three dimensions.

[3)] One can say that such a motion occupies an intermediate position between finite and infinite motions.

[4)] This case corresponds to a configuration of three mutually perpendicular color fields as considered earlier in Ref. 6.

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Translated by Julian B. Barbour