Magnetic susceptibility of narrow-gap semiconductors

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(Submitted 26 June 1980)
Zh. Eksp. Teor. Fiz. **80**, 334–348 (January 1981)

A theory of the magnetic susceptibility of narrow-gap semiconductors of the $Pb_{1-x}Sn_xTe$ and $Hg_{1-x}Cd_xTe$ type is developed in Part I of the paper. A singular contribution of states in the vicinity of the extremal points of the bands separated by a narrow forbidden gap is isolated. It depends significantly on the gap width—i.e., on the alloy composition and also on the temperature and magnetic field. The susceptibility of the crystalline solid solutions $Pb_{1-x}Sn_xTe$ $(0.18 < x < 1)$ is investigated experimentally in Part II over a wide range of carrier densities (from $2 \cdot 10^{16}$ to $6 \cdot 10^{20}$ cm^{-3}). The results are compared with theory. It is shown that a theory that takes into account within the Dimmock model the contribution of the two closely spaced bands to the susceptibility agrees well with the experimental results. The so-called lattice susceptibility has a pronounced temperature dependence due to the appreciable change in the width of the forbidden gap.

PACS numbers: 75.30.Cr

I. THEORY

1. INTRODUCTION

The magnetic susceptibility of electrons in a solid had been examined by many authors. Within the framework of band scheme, we may obtain a general formula (see, for example, Ref. 1) which describes the susceptibility in an extremely weak magnetic field. It resembles the expression for the effective mass in the **k·p** method—it contains a series of terms, each of which represents a sum over the occupied states. This result is not suitable for practical calculations. On the other hand, the electron spectrum in the neighborhood of several selected points of the Brillouin zone in the presence of a magnetic field was used explicitly in an actual calculation of the susceptibility of graphite[2] and also of the contribution of the carriers in indium antimonide.[3] Of course, it is not possible to calculate the contribution of deeper-lying (core) states by this means. In addition, the question arises as to what degree we may neglect the so-called interband interactions which are involved in the **k·p** method and reflected in the general formula.[1]

The answer to this question is most simply understood using as an example of the $Bi_{1-x}Sb_x$ alloys, for which at the L point the conduction band is separated from the valence band by a narrow forbidden gap ε_g. By considering the two-band effective Hamiltonian in a magnetic field we can, taking into account the interaction of the two close bands, determine the Landau level and calculate the thermodynamic potential and the susceptibility.[4] In a weak field, the susceptibility is the sum over the valence and conduction bands and depends on the cyclotron mass and some g factor. As long as we are interested in the contribution of the states from the valence band with energies on the order of ε_g, it is most essential to accurately take into account the effect of the close conduction band on the mass and on the g factor, and the more distant bands introduce small corrections. The susceptibility of core states depends more substantially on the effect of the distant bands. However, it does not depend on ε_g. In the $Bi_{1-x}Sb_x$ alloys, ε_g is small and rapidly changes with a change in x; consequently, the singular contribution of the shallow states χ_s is a rapidly changing function of x, temperature T, carrier density n, and magnetic field. The singularity of this function depends substantially on the shape of the spectrum in the neighborhood of the L point. The contribution of the core states χ_{reg} to a first approximation may be considered independent of the parameters indicated above. A detailed comparison with experiment showed[5] that the theory allows us to explain the high diamagnetism of the $Bi_{1-x}Sb_x$ alloys and its dependence on the experimental variables x, T, n, and H.

Our work is devoted to the susceptibility of the currently popular narrow-gap alloys $Pb_{1-x}Sn_xTe$. The expression obtained here may be also used to describe the susceptibility of the compounds of the type $Hg_{1-x}Cd_xTe$.

2. SPECTRUM IN A MAGNETIC FIELD

The compounds $Pb_{1-x}Sn_xTe$ have a lattice of the rock salt type. The effective Hamiltonian for the point $L = \langle 1, 1, 1\rangle/2$ in a magnetic field parallel to the corresponding $\langle 111\rangle$ axis is given by Dimmock[6]:

$$\hat{H}(k) = \left\| \begin{array}{cc|cc} \varepsilon_- + q_l^- & 0 & v_l k_z & v_t k_- \\ 0 & \varepsilon_- - q_l^- & v_t k_+ & -v_l k_z \\ \hline v_l k_z & v_t k_- & \varepsilon_+ - q_l^+ & 0 \\ v_t k_+ & -v_l k_z & 0 & \varepsilon_+ + q_l^+ \end{array} \right\| \tag{1}$$

$$\varepsilon_\pm = \mp \frac{1}{2}\left(\varepsilon_g + \frac{k_x^2 + k_y^2}{m_t^\pm} + \frac{k_z^2}{m_l^\pm}\right),$$
$$k_\pm = k_x \pm i k_y, \quad q_l^\pm = {}^1/{}_2 g_l^\pm \beta H, \tag{2}$$

where β is the Bohr magneton; v, m, g are constants.

The Hamiltonian is diagonalized if the column of eigenfunctions is expressed in terms of Hermite functions:

$$\psi_{1,2,3,4} \sim \psi_n, \quad \psi_{n+1}, \quad \psi_n, \quad \psi_{n+1}.$$

The spectrum is found from the equation

$$|H(k) - \varepsilon| = 0, \tag{3}$$

and $H(k)$ is obtained from $\hat{H}(k)$ in Eq. (1) by substitution (in the intermediate formulas, $\hbar = 1$)

$$k_\pm, \ (k_x^2 + k_y^2)^{1/2} \to k_\perp = [2eH(n+1)/c]^{1/2}; \quad n = -1, 0, \ldots, \tag{4}$$

$$q_l^\pm \to g_l^\pm \beta H/2 - eH/2cm_t^\pm. \tag{5}$$

Using the matrix equality

$$\begin{vmatrix} A & B \\ C & D \end{vmatrix} = |AD - ACA^{-1}B|,$$

0038-5646/81/010170-08$02.40

where A, B, C, D are the 2×2 matrices marked off by the dotted lines in Eq. (1), we write Eq. (3) in the explicit form

$$\left[(\varepsilon_-+q_l^--\varepsilon)(\varepsilon_+-q_l^+-\varepsilon)-v_l^2k_z^2-v_t^2k_\perp^2\frac{\varepsilon_-+q_l^--\varepsilon}{\varepsilon_--q_l^--\varepsilon}\right]$$
$$\times\left[(\varepsilon_--q_l^--\varepsilon)(\varepsilon_++q_l^+-\varepsilon)-v_l^2k_z^2-v_t^2k_\perp^2\frac{\varepsilon_--q_l^--\varepsilon}{\varepsilon_-+q_l^--\varepsilon}\right]$$
$$-\frac{4v_l^2v_t^2k_z^2k_\perp^2(q_l^-)^2}{(\varepsilon_--q_l^--\varepsilon)(\varepsilon_-+q_l^--\varepsilon)}=0. \tag{6}$$

Equation (6) connects $k_\perp^2$, k_z^2, ε, and the field H, contained as a factor in $q^\pm$ in Eq. (5). The second term in Eq. (6) is quadratic in H. If this term is neglected, Eq. (6) is broken down into two equations, differing in the sign of $q_l^\pm$:

$$(\varepsilon_-+sq_l^--\varepsilon)(\varepsilon_+-sq_l^+-\varepsilon)-v_l^2k_z^2-v_t^2k_\perp^2\frac{\varepsilon_-+sq_l^--\varepsilon}{\varepsilon_--sq_l^--\varepsilon}=0,$$
$$s=\pm1,\quad \varepsilon_\pm=\mp\frac{1}{2}\left(\varepsilon_g+\frac{k_\perp^2}{m_t^\pm}+\frac{k_z^2}{m_l^\pm}\right). \tag{7}$$

In the absence of a magnetic field, $q_l^\pm=0$ and Eq. (7) describes the valence band and conduction band, which are twofold spin degenerate. For states with $n=-1$, there is only one Eq. (7) with $s=-1$, as can be immediately seen by using Eq. (1) and recognizing that $\psi_n=0$ at $n=-1$. Equation (7) in this case is accurate, since $k_\perp=0$ [see Eqs. (4) and (6)]. The extra solution in Eqs. (6) and (7) may be excluded automatically if instead of n we introduce the Landau number $N=n+\frac{1}{2}-s/2$ and consider that N takes on the values 0, 1,... independently of s. Thus, having determined $k_\perp^2(\varepsilon, k_z, sH)$ from Eq. (6) [or Eq. (7) in a weak field], we find the Landau level from the condition in Eq. (4):

$$k_\perp^2(\varepsilon,k_z,sH)=2\frac{eH}{c}\left(N+\frac{1}{2}+\frac{s}{2}\right),\quad N=0,1,\ldots;\ s=\pm1. \tag{4'}$$

We note that since for $q=0$, Eq. (7) describes the spectrum in the absence of a field, the $k_\perp^2(\varepsilon, k_z, 0)$ determined from it agrees within a factor of π with the area of the intersection of the equal energy surface $\varepsilon(k)=\text{const}$ with the plane $k_z=\text{const}$. Therefore Eq. (4′) differs from the usual quasiclassical quantization condition only by the presence of the spin number s.

Using Eq. (4), we find the spin splitting of the levels in a magnetic field:

$$\omega_s=\varepsilon(N,k_z,s=1)-\varepsilon(N,k_z,s=-1).$$

For the weak-field case this difference may be replaced by the derivative

$$\omega_s=2\frac{\partial\varepsilon}{\partial s}=2\frac{eH}{c}\left(1-\frac{c}{e}\frac{\partial k_\perp^2}{\partial h}\right)\Big/\frac{\partial k_\perp^2}{\partial\varepsilon}\quad\text{at}\quad h=sH\to0. \tag{8}$$

In the same approximation, the cyclotron frequency is

$$\omega_c=\varepsilon(N+1,k_z,s)-\varepsilon(N,k_z,s)=2\frac{eH}{c}\Big/\frac{\partial k_\perp^2}{\partial\varepsilon},\quad h\to0. \tag{9}$$

We call the ratio $g=2\omega_s/\omega_c$ the g factor. Using Eqs. (8) and (9), we find

$$\frac{g}{2}=1-\frac{c}{e}\frac{\partial k_\perp^2}{\partial h}(\varepsilon,k_z,h=0). \tag{10}$$

Experimentalists usually call the parameter connecting ω_s with the Bohr magneton the effective g^* factor:

$$\omega_s=g^*\beta H. \tag{11}$$

Comparing Eq. (10) with Eq. (11), we see that

$$g^*=g\frac{m_0}{m},\quad m=\frac{1}{2}\frac{\partial k_\perp^2}{\partial\varepsilon}$$

where m is the cyclotron mass and m_0 is the free electron mass. The large values of g^* observed in the experiment are explained by the low effective mass in semiconductors with narrow forbidden gaps.

The g factor may be found using Eqs. (10) and (7):

$$g(\varepsilon,k_z)=2[v_t^2-(\varepsilon_+-\varepsilon)g_l^-/4m_0+(\varepsilon_--\varepsilon)g_l^+/4m_0$$
$$+v_t^2k_\perp^2(g_l^-/2m_0-1/m_t^-)/(\varepsilon_--\varepsilon)]$$
$$/[v_t^2-(\varepsilon_+-\varepsilon)/2m_t^-+(\varepsilon_--\varepsilon)/2m_t^+]. \tag{12}$$

At the bottom of the conduction band, i.e., for $k_\perp=k_z=0$ we have $\varepsilon=|\varepsilon_g|/2$ and

$$g_c=2\frac{v_t^2+[\theta(\varepsilon_g)g_l^--\theta(-\varepsilon_g)g_l^+]|\varepsilon_g|/4m_0}{v_t^2+[\theta(\varepsilon_g)/m_t^--\theta(-\varepsilon_g)/m_t^+]|\varepsilon_g|/2}, \tag{13}$$

where $\theta(x)$ is the unit step function. At the top of the valence band, $\varepsilon=-|\varepsilon_g|/2$ and

$$g_v=2\frac{v_t^2-[\theta(-\varepsilon_g)g_l^--\theta(\varepsilon_g)g_l^+]|\varepsilon_g|/4m_0}{v_t^2-[\theta(-\varepsilon_g)/m_t^--\theta(\varepsilon_g)/m_t^+]|\varepsilon_g|/2}. \tag{14}$$

From Eqs. (13) and (14) we see that for the gapless state with $\varepsilon_g=0$, we have $g_{c,v}=2$. The small deviation of the g factor from 2 (on the order of 20%) in the narrow-gap semiconductors under consideration is basically due to corrections to the denominator in Eqs. (13) and (14), since $m_t^\pm\approx0.1m_0$, $g_l^-=-3$, $g_l^+=-1$ (Ref. 7). The same terms determine also the small correction to the transverse effective mass at the bottoms of the conduction and valence bands:

$$(1/m_t)_{c,v}=2v_t^2/\varepsilon_g+1/m_t^\mp.$$

The description in Eq. (4) of the spectrum in a magnetic field is suitable both for analysis of the spin splitting as well as for calculation of the susceptibility. To this end, we also introduce the corresponding expression describing the neighborhood of the Γ point in semiconductors with zincblende structure in the so-called three-band approximation. In this case, the $k_\perp^2(\varepsilon, k_z, sH)$ dependence is determined from the equation[3]

$$\varepsilon(\varepsilon+\varepsilon_g)(\varepsilon+\varepsilon_g+\Delta)-p^2(k_z^2+k_\perp^2)\left(\varepsilon+\varepsilon_g+\frac{2}{3}\Delta\right)+s\frac{eH}{c}p^2(\varepsilon+\varepsilon_g+\Delta)=0, \tag{15}$$

which may be solved with respect to $k_\perp^2$:

$$k_\perp^2(\varepsilon,k_z,sH)=-k_z^2+[\varepsilon(\varepsilon+\varepsilon_g)/p^2+seH/c](\varepsilon+\varepsilon_g+\Delta)(\varepsilon+\varepsilon_g+{}^2/_3\Delta)^{-1}. \tag{16}$$

Here p is a constant which is proportional to the momentum matrix element. Using Eq. (10) we find the g factor

$$g=-2\Delta/3(\varepsilon+\varepsilon_g+{}^2/_3\Delta). \tag{17}$$

The values $\varepsilon=0$, $-\varepsilon_g$, and $-\varepsilon_g-\Delta$ correspond to the extrema of the bands. The corresponding values of the g factor are $-2\Delta/(2\Delta+3\varepsilon_g)$, -1, and 2.

3. SUSCEPTIBILITY OF SEMICONDUCTORS OF THE TYPE $Pb_{1-x}Sn_xTe$ IN A WEAK FIELD

Knowing the levels in the magnetic field (3), we may calculate the susceptibility $\chi=-\partial^2\Omega/\partial H^2$ from the dependence of the potential Ω on the field H:

$$\Omega=-\frac{eHT}{4\pi^2c}\sum_{N,s}\int dk_z\ln\left[1+\exp\left(\frac{\mu-\varepsilon(N,k_z,s)}{T}\right)\right]. \tag{18}$$

In a weak field, the sum over N is calculated using the Euler-Maclaurin formula:

$$\sum_{N=0} F(N)=\int_0 F(N)\,dN+\frac{1}{2}F(0)-\frac{1}{12}F'(0). \tag{19}$$

We integrate by parts the integral term and transform to the variable ε, using Eq. (4):

$$\int_0 F(N)\,dN=-\frac{1}{2}F(0)-\int\left(N+\frac{1}{2}\right)\frac{dF}{d\varepsilon}d\varepsilon.$$

We see that the problem was reduced to calculation of the derivatives with respect to H of the expression

$$J(H)=\int_{\varepsilon_0} d\varepsilon\left[k_\perp^2(\varepsilon,k_z,sH)-\frac{eHs}{c}\right]f(\varepsilon),$$

where $f(\varepsilon)$ is the Fermi function of the distribution, and $\varepsilon_0=\varepsilon(N=0, k_z, sH)$. We find

$$J'(H)=-\frac{\partial\varepsilon_0}{\partial H}\frac{eH}{c}f(\varepsilon_0)+s\int_{\varepsilon_0} d\varepsilon\left(\frac{\partial k_\perp^2}{\partial h}-\frac{e}{c}\right)f(\varepsilon),$$

$$J''(0)=-\frac{\partial\varepsilon_0}{\partial H}f(\varepsilon_0)\left[\frac{e}{c}(1-s)+s\frac{\partial k_\perp^2}{\partial h}\right]_{N=0,h\to0}+\int d\varepsilon f(\varepsilon)\frac{\partial^2k_\perp^2}{\partial h^2}. \tag{20}$$

The difference

$$1-\frac{c}{e}\frac{\partial k_\perp^2}{\partial h}$$

is reduced to the g factor (10), and the derivative $\partial\varepsilon_0/\partial H$ is calculated using Eq. (4):

$$\left.\frac{\partial\varepsilon_0}{\partial H}\right|_{H\to0}=\left[\frac{e}{c}(1+s)-s\frac{\partial k_\perp^2}{\partial h}\right]\Big/2m(k_z), \tag{21}$$

where $m(k_z)=(\frac{1}{2})\partial k_\perp^2/\partial\varepsilon$ is the cyclotron mass in zero field at $N=0$.

Summing over the spin s, we obtain

$$\chi=\frac{1}{8}\left(\frac{e}{\pi c}\right)^2\sum_{L,v,c}\int dk_z\frac{f(\varepsilon_0)}{m(k_z)}\left[\frac{g^2(k_z)}{4}-\frac{1}{3}\right]+\frac{1}{4\pi^2}\int d\varepsilon\,dk_z f(\varepsilon)\frac{\partial^2k_\perp^2}{\partial h^2}. \tag{22}$$

The remaining summation in Eq. (22) is carried out over the bands and different L points. In the first term of Eq. (22), all the values are chosen at $N=0$ $H\to0$, i.e., at

$$k_\perp(\varepsilon, k_z, h\to0)=0. \tag{23}$$

The corresponding value of the g factor is found using Eq. (12):

$$g(k_z)=2\frac{v_t^2-(\varepsilon_+-\varepsilon_0)g_l^-/4m_0+(\varepsilon_--\varepsilon_0)g_l^+/4m_0}{v_t^2-(\varepsilon_+-\varepsilon_0)/2m_t^-+(\varepsilon_--\varepsilon_0)/2m_t^+}, \tag{24}$$
$$\varepsilon_\pm=\mp(\varepsilon_g+k_z^2/m_l^\pm)/2.$$

The cyclotron mass at the limiting point of Eq. (23), and also the dependence of ε_0 on k_z, are determined by the relation (7):

$$\left(\frac{1}{m(k_z)}\right)_{v,c}=\frac{1}{2}\left(\frac{1}{m_t^-}-\frac{1}{m_t^+}\right)\mp\left[2v_t^2+\frac{1}{2}\left(\frac{1}{m_t^-}+\frac{1}{m_t^+}\right)\left(\varepsilon_g+\frac{k_z^2}{M}\right)\right]\Big/F(k_z), \tag{25}$$

$$\varepsilon_0\equiv\varepsilon_{v,c}=\frac{1}{4}k_z^2\left(\frac{1}{m_l^-}-\frac{1}{m_l^+}\right)\mp\frac{1}{2}F(k_z), \tag{26}$$

$$F(k_z)=\left[\left(\varepsilon_g+\frac{k_z^2}{M}\right)^2+4v_l^2k_z^2\right]^{1/2},\quad \frac{1}{M}=\frac{1}{2}\left(\frac{1}{m_l^+}+\frac{1}{m_l^-}\right). \tag{27}$$

For $T=0$, the first integral in Eq. (22) as a function of ε_g is singular—it diverges logarithmically for small k_z as $|\varepsilon_g|\to0$, since in this case $m(k_z)\to0$. The singular contribution χ_s of the small neighborhood about the L point is easily separated if we note that here

$$(1/m(k_z))_{v,c}=\mp2v_t^2/F(k_z). \tag{28}$$

We must also take into account the fact that there are four L points. Expression (22) is correct only for those L points for which the direction of the field coincides with the corresponding ⟨111⟩ axis. However the equal-energy surfaces in $Pb_{1-x}Sn_xTe$ are strongly prolate along the ⟨111⟩ axis; accordingly, the cyclotron mass for $H\parallel\langle111\rangle$ is substantially smaller than for $H\perp\langle111\rangle$. Therefore the contribution from any of the L points is a tensor, for which one principal value [namely, that for which Eq. (22) holds] is substantially greater than the other two—whose magnitudes may be estimates using Eq. (22). Neglecting the latter and going over to the rest of the L points by rotation of coordinates, we see that the summation over the four L points is reduced to multiplication by the coefficient $1+3\cos^2\alpha=2$, where α is the angle between ⟨111⟩ and ⟨$\bar{1}$11⟩.

Finally, we obtain the singular contribution to the susceptibility (it is independent of the field direction, as it should be for a cubic crystal):

$$\chi_s=\left(\frac{e}{2\pi c}\right)^2\sum_{v,c}\int dk_z\frac{f(\varepsilon_0)}{m(k_z)}\left[\frac{g^2(k_z)}{4}-\frac{1}{3}\right], \tag{29}$$

where g, m, and ε_0 are given by Eqs. (24), (26)–(29), and integration over k_z may be carried out with infinite limits since the integral in Eq. (29) converges.

The contribution of the deeper-lying states χ_{reg} must be calculated within the framework of the **k** · **p** method. It is important that it does not depend on ε_g and other experimental variables. Combining it with the term regular in ε_g in Eq. (22), we write the complete susceptibility in the form

$$\chi=\chi_s+\chi_{reg}.$$

The susceptibility of a semiconductor without carriers is often called the lattice susceptibility. In the absence of carriers, $f=0$ for the conduction band and $f=1$ for the valence band. The singular contribution to the lattice susceptibility is equal to

$$\chi_{s,lat}=\left(\frac{e}{2\pi c}\right)^2\int_v\frac{dk_z}{m(k_z)}\left[\frac{g^2(k_z)}{4}-\frac{1}{3}\right], \tag{30}$$

where the integration is carried out over the valence band. Close to the extremum of the valence band in semiconductors of the type PbTe with narrow band gaps, the mass in Eq. (28) is negative, and the g factor (14) is close to 2. Such a semiconductor is diamagnetic, and its susceptibility depends on the temperature, basically as a consequence of the change in $\varepsilon_g(T)$. The parameter v_l, determining the mass in Eqs. (28), (27), is actually small. Therefore the evaluation of the integral in Eq. (30) depends on the relation between $|\varepsilon_g|$ and $4Mv_l^2$. If $|\varepsilon_g|\ll4Mv_l^2$, then the major logarithmic contribution arises from the region $|\varepsilon_g|\ll2v_lk_z\ll4Mv_l^2$. In this region, we may neglect the dependence of the g factor on k_z under the condition

$$M(1/m_t^+-1/m_t^-)v_l^2v_t^{-2}\ll1.$$

Evaluating the integral in Eq. (30), we find in this case

$$\chi_{s,\mathrm{lat}}=-\frac{1}{3\hbar v_l}\left(\frac{ev_t}{\pi c}\right)^2\ln\left|\frac{16Mv_l^2}{\varepsilon_g}\right|.$$

In the other limit, $|\varepsilon_g|\gg 4Mv_l^2$, the singular contribution to the lattice susceptibility has the form

$$\chi_{s,\mathrm{lat}}=-\frac{1}{2\pi\hbar}\left(\frac{ev_t}{c}\right)^2\left|\frac{M}{\varepsilon_g}\right|^{1/2}\left(\frac{g_v^2}{4}-\frac{1}{3}\right)\varphi(\varepsilon_g),$$

where $\varphi(\varepsilon_g)=1$ for $\varepsilon_g>0$, $\varphi(\varepsilon_g)=\ln|4\varepsilon_g/Mv_l^2|$ for $\varepsilon_g<0$, and g_v is determined by Eq. (14).

From Eq. (24) it is seen that the g factor decreases with an increase in k_z and ε_g. Therefore the contribution of sufficiently deep-lying states in Eq. (30) may be paramagnetic; and for a large forbidden gap with, the contribution of the entire filled valence band may become paramagnetic.

The difference between Eqs. (29) and (30) gives the carrier susceptibility. Since $1-f$ for the valence band is the hole distribution function, the carrier susceptibility is equal to

$$\chi_{s,\mathrm{car}}=\left(\frac{e}{2\pi c}\right)^2\sum_{c,v}\int dk_z\frac{f(\varepsilon_0)}{|m(k_z)|}\left[\frac{g^2(k_z)}{4}-\frac{1}{3}\right],\tag{31}$$

where for the valence band

$$f=(\exp(\mu-\varepsilon_v/T)+1)^{-1},$$

and ε_v is given by Eq. (26); for the conduction band, f is the usual Fermi function. The expression (31) is a generalization of the Pauli-Landau formula to the case of several anisotropic bands. The essential difference between Eq. (31) and the Peierls-Landau formula is the presence of the paramagnetic term, which must be taken into account simultaneously with the diamagnetic term if the spin-orbit interaction is not small compared with the forbidden gap width.

At $g=2$, the relation between the Pauli paramagnetism and the Landau diamagnetism is the same as for free electrons. However, the carrier susceptibility in the semiconductor may have any sign, depending on the ratio of $g^2/4$ to $\frac{1}{3}$.

The measurement of the susceptibility of a degenerate semiconductor allows us to determine the g factor using Eq. (31). For example, for an n-type semiconductor with a low carrier density and $\varepsilon_g>0$, Eq. (31) gives

$$\chi_{s,\mathrm{car}}=\left(\frac{e}{\pi c}\right)^2\left(\frac{g_c^2}{4}-\frac{1}{3}\right)\left[\frac{1}{2}m_l\left(\mu-\frac{\varepsilon_g}{2}\right)\right]^{1/2}\Big/ m_l,\tag{31a}$$

where μ is the Fermi energy, and the g factor and the longitudinal mass m_l are taken at the bottom of the band. Noting that the carrier density is

$$n=\frac{8}{3\pi^2}m_t(2m_l)^{1/2}\left(\mu-\frac{\varepsilon_g}{2}\right)^{3/2},$$

we obtain

$$\chi_{s,\mathrm{car}}=\left(\frac{3m_ln}{2^5\pi^4m_t^4}\right)^{1/3}\frac{e^2}{c^2}\left(\frac{g_c^2}{4}-\frac{1}{3}\right).\tag{31b}$$

We recall that the value of the g factor is given by Eq. (13), and the effective masses at the bottom of the conduction band are equal to

$$\frac{1}{m_{l,t}}=\frac{2v_{l,t}^2}{\varepsilon_g}+\frac{1}{m_{l,t}^-}.$$

The carrier susceptibility in Eq. (31) depends on the temperature both as a consequence of the explicit change of the distribution function, and because of the temperature dependences of the gap ε_g and the chemical potential: the temperature dependence of the latter is determined from the electroneutrality condition.

4. SUSCEPTIBILITY OF SEMICONDUCTORS OF THE TYPE $Hg_{1-x}Cd_xTe$

We use Eqs. (16), (17), and (22) to determine the contribution of the Γ point to the susceptibility of narrow-gap semiconductors with a zincblende structure. In Eq. (22) it is convenient to transform from k_z to the variable ε, the connection between which is given by Eq. (15) at $H=k_\perp=0$. Noting that

$$\frac{\partial k_\perp^2}{\partial\varepsilon}\frac{\partial\varepsilon}{\partial k_z}\bigg|_{k_\perp=0}=\frac{\partial k_z^2}{\partial\varepsilon}\frac{\partial\varepsilon}{\partial k_z}\bigg|_{k_\perp=0}=2k_z|_{k_\perp=0}$$

$$=\frac{2}{p}\left[\frac{\varepsilon(\varepsilon+\varepsilon_g)(\varepsilon+\varepsilon_g+\Delta)}{\varepsilon_g+{}^2/_3\Delta+\varepsilon}\right]^{1/2},$$

we rewrite Eq. (22) in the form

$$\chi=\left(\frac{e}{2\pi c}\right)^2p\sum_{v,c}\operatorname{sign}m\int d\varepsilon f(\varepsilon)\left[\frac{\varepsilon+\varepsilon_g+{}^2/_3\Delta}{\varepsilon(\varepsilon+\varepsilon_g)(\varepsilon+\varepsilon_g+\Delta)}\right]^{1/2}\left(\frac{g^2}{4}-\frac{1}{3}\right),\tag{32}$$

where the factor sign m takes into account the sign of the mass

$$m=\frac{(\varepsilon+\varepsilon_g+\Delta)(\varepsilon+\varepsilon_g+{}^2/_3\Delta)(2\varepsilon+\varepsilon_g)-{}^1/_3\varepsilon(\varepsilon+\varepsilon_g)\Delta}{2p^2(\varepsilon+\varepsilon_g+{}^2/_3\Delta)^2},$$

and the g factor is determined by Eq. (17). It is easy to see that for the conduction band the mass is always positive, and for the valence band it is always negative. The second integral in Eq. (22) in this case vanishes identically, since $\partial^2k_\perp^2/\partial h^2=0$, as is evident from Eq. (16).

We estimate the singular contribution to the lattice susceptibility in a semiconductor with small Δ and ε_g compared with the characteristic atomic energies E_a. For this we set $f=1$ in Eq. (32) and integrate over the light [extremum at $\varepsilon=\min(0,-\varepsilon_g)$] and spin-split (extremum at $\varepsilon=-\varepsilon_g-\Delta$) hole bands. The contribution of the band of heavy holes, which are not considered in Eq. (15), is small as a consequence of their large mass.

The integral in Eq. (32) has the following singularities: logarithmic for small $|\varepsilon|\sim|\varepsilon_g|\ll\Delta$ and large $|\varepsilon|\gg\Delta$ square-root for $\varepsilon=-\varepsilon_g-2\Delta/3$. Integration for small $|\varepsilon|$ is actually limited by the region of applicability of the $\mathbf{k}\cdot\mathbf{p}$ method, i.e., by energies which are small compared with the atomic E_a. The contribution of these distant regions is small with respect to the parameter $1/m_0$ and does not depend on ε_g and Δ. The singularity at $\varepsilon=-\varepsilon_g-2\Delta/3$ results from the openness of the light-hole band: in Eq. (15), k^2 increases without limit as $\varepsilon\to-\varepsilon_g-2\Delta/3$ and at $H=0$. In fact, this openness disappears if the terms of second order in k are included in the effective Hamiltonian. This means that we must exclude the region of the point $\varepsilon=-\varepsilon_g-2\Delta/3$ with dimensions on the order of

$$\frac{\Delta}{3p}\left(\frac{2\Delta}{3m_0}\right)^{1/2},$$

at the boundary of which the mass becomes comparable with m_0. The contribution of the excluded region is proportional to $1/m_0$, and it must be included in the regular part of the susceptibility.

Estimating the contribution of the spin-split band to the lattice susceptibility, i.e., integrating over ε from $-\varepsilon_g-\Delta$ to $-E_a$, we see that this contribution is paramagnetic:

$$\chi_{s,\text{lat}}^{(1)}=\frac{p}{3\hbar}\left(\frac{e}{2\pi c}\right)^2\ln\frac{E_a}{\Delta};\quad |\varepsilon_g|\ll\Delta\ll E_a. \tag{33}$$

The contribution of the filled band of light holes $(-\varepsilon_g-2\Delta/3<\varepsilon\lesssim|\varepsilon_g|)$ to the lattice susceptibility is equal to

$$\chi_{s,\text{lat}}^{(2)}=-\frac{p}{\hbar}\left(\frac{e}{2\pi c}\right)^2\left[\left(\frac{3m_0p^2}{2\Delta}\right)^{1/4}-\frac{1}{12}\left(\frac{2}{3}\right)^{1/2}\ln\left|\frac{2\Delta}{\varepsilon_g}\right|\right]. \tag{34}$$

As seen from Eq. (34), the diamagnetism of the semiconductors of the type under consideration, in contrast to the alloys $Pb_{1-x}Sn_xTe$ and $Bi_{1-x}Sb_x$, should decrease with a decrease in the gap width $|\varepsilon_g|$.

The expression for the carrier susceptibility is obtained using Eq. (32) in the manner used above for $Pb_{1-x}Sn_xTe$ [see Eq. (31)]. Taking Eq. (17) into account, we see that the sought expression agrees with the formula given by Bowers and Yafet,[3] who calculated the carrier susceptibility in indium antimonide (see also Ref. 7). As seen from Eq. (32), in a semiconductor of the type under consideration the light carriers located close to the extrema of the bands $\varepsilon=0$, $-\varepsilon_g$ are diamagnetic, since they have $g^2/4<\frac{1}{3}$.

We note that the expression within the brackets in Eq. (32) is small close to the extrema of the light bands—it tends toward $\frac{1}{12}$ as $\varepsilon_g\to 0$. Therefore, in contrast to $Pb_{1-x}Sn_xTe$ and $Bi_{1-x}Sb_x$, where the corresponding factor is close to 2, the carrier susceptibility is small compared with the lattice susceptibility right up to densities on the order of 10^{18} cm^{-3}, and is quite sensitive to small corrections in the spectrum. Gel'mont took into account these corrections close to the extrema.[8] We note that the numerical value of the g factor given by Kim and Narita,[9] taking into account the isotropic corrections, does not change the ratio of $g^2/4$ to $\frac{1}{3}$ and does not explain the observed paramagnetism of the light carriers in n-HgTe. With increase in the number of light holes, they may prove to be paramagnetic, since their g factor increase as $\varepsilon\to-\varepsilon_g-2\Delta/3$. The carriers close to the extremum $\varepsilon=-\varepsilon_g-\Delta$ of the spin-split valence band are paramagnetic, since here $g^2/4\to 1$.

In the gapless state ($\varepsilon_g\to 0$), the increase in the susceptibility (34), or more precisely, the increase in the total susceptibility (32), when the carriers are taken into account, stops when ε_g becomes comparable with the largest of the parameters T, μ, or $\omega_c\sim eHp^2/c|\varepsilon_g|$. In this case, the dependence of χ on T, H, or the composition is described by Eq. (34), in which we must make the substitution

$$\varepsilon_g\to\max\{T,\ \mu(eHp^2/c)^{1/2}\}.$$

For comparison of Eq. (34) with experiment, the alloys $Hg_{1-x}Cd_xTe$ close to the gapless state are apparently the most suitable, since these alloys can be obtained with low carrier density.

II. SUSCEPTIBILITY OF $Pb_{1-x}Sn_xTe$ SOLID SOLUTIONS. COMPARISON WITH THEORY

Investigations of the narrow-gap semiconductor solid solutions based on IV–VI compounds and characterized by a narrow forbidden gap ε_g, have intensively progressed within the last decade. In these semiconductors, ε_g changes with composition in such a way that the gapless state arises at a definite temperature-dependent value.[6] Infrared detectors and lasers based on these solid solutions have been constructed.[10] Therefore the problem of obtaining the most complete information concerning their band spectrum is timely.

The magnetic susceptibility method occupies a special position among the methods for studying the band structure of semiconductors. It does not depend on the scattering mechanisms and is a universal characteristic of the band spectrum. Experimental investigations of the magnetic susceptibility as a function of temperature, carrier density, and composition makes it possible to determine the magnetic susceptibility of the crystal lattice, of the defects, and of the impurities.

The goal of this work is to study the magnetic susceptibility of $Pb_{1-x}Sn_xTe$ solid solutions over wide ranges of carrier densities ($2\cdot 10^{16}$–$6\cdot 10^{20}$ cm^{-3}), temperatures (4.5–500°K), and compositions ($0.18<x<1$), and to compare the results with the theory. Here we have used results of both the latest investigations as well as our own earlier results.[11,12] The methods for growing and heat-treating the single crystals are presented in Ref. 13. The technique of measuring the magnetic susceptibility is described in Refs. 12 and 14.

Previously[12] we attempted to interpret our results on the basis of Zawadzki's work,[15] who calculated the dimagnetic part of the carrier susceptibility within the framework of the Kane isotropic model, using the Landau-Peierls formula. By extrapolation to zero carrier density, the magnetic susceptibility of the crystal lattice was determined at 77°K. However, the cause of the susceptibility remained unclear, as did the substantial temperature dependence of the ensuing fit parameter.

The theory presented in Part I takes into account the contribution to the magnetic susceptibility of the valence band, separated from the conduction band by a narrow gap, and also the anisotropy of the band spectrum of the $Pb_{1-x}Sn_xTe$ solid solutions. The magnetic susceptibility of the latter is represented as

$$\chi=\chi_s+\chi_{\text{reg}}. \tag{35}$$

The expression for χ_s in Eq. (29) is determined by the small forbidden gap and describes the carrier magnetic susceptibility and the singular (i.e., substantially dependent on composition and temperature) part of the lattice susceptibility. Notwithstanding the anisotropy of the Dimmock band spectrum, the summation over the ellipsoids in the L points leads to the fact that χ_s is an isotropic parameter independent of the direction of the magnetic field. The contribution χ_{reg} of the deeper-lying states does not depend substantially on the temperature, and can therefore be separated from the total susceptibility.

TABLE I. Characteristics of $Pb_{1-x}Sn_xTe$ samples and the regular part of the magnetic susceptibility.

Sample number	s, mole fraction	Hole density, cm^{-3}	$\chi_{reg} \cdot 10^7$ cm^3/g	
			100 K	300 K
a*	0.18	$2.0 \cdot 10^{16}$	−3.56	−3.30
1*	0.18	$1.2 \cdot 10^{17}$	−3.56	−3.30
2	0.18	$3.0 \cdot 10^{19}$	−3.50	−3.48
3	0.18	$6.0 \cdot 10^{20}$	−3.00	−3.10
4	0.35	$3.0 \cdot 10^{18}$	−3.52	−3.35
5	0.35	$2.0 \cdot 10^{19}$	−3.50	−3.55
6	0.35	$1.0 \cdot 10^{20}$	−2.50	−2.87
7	0.59	$5.1 \cdot 10^{19}$	−2.54	−2.46
8	0.59	$4.8 \cdot 10^{20}$	−2.50	−2.84
9	0.87	$1.0 \cdot 10^{20}$	−1.43	−1.70
10	1.00	$1.0 \cdot 10^{21}$	+0.25	−0.70

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*n type samples. The rest are p type.

We previously determined the numerical values of the matrix elements of the momentum operators[16]:

$$v_t = 7.98 \cdot 10^7 \text{ cm/s}; \; v_l = (1.98 + 1.44x) \cdot 10^7 \text{ cm/s}.$$

The parameters m and g are found experimentally[17]:

$$m_t^+ = 0.1m_0, \; m_l^+ = 0.8m_0, \; g_t^+ = -1,$$

$$m_t^- = 0.07m_0, \; m_l^- = 0.413m_0, \; g_t^- = -3.$$

The dependence of ε_g (in eV) on the composition and the temperature[18] has the form

$$\varepsilon_g = 0.18 - 0.52x + 4.1 \cdot 10^{-4}\, T.$$

The carrier density and the compositions of the investigated samples are given in Table I.

For calculation of the magnetic susceptibility from Eq. (29) it is necessary to know the temperature dependence of the chemical potential μ. This dependence is determined from the electroneutrality condition, which for n-type samples is of the form

$$n_l - p_l - p_h = n_0,$$

and for p-type samples

$$p_l + p_h - n_l = p_0.$$

The densities of the electrons n_l and the light holes p_l

$$n_l, p_l = \left| \int N(\varepsilon) \frac{\partial f}{\partial \varepsilon} d\varepsilon \right| \tag{36}$$

are determined by the number of states $N(\varepsilon)$; $\partial f/\partial \varepsilon$ is the derivative of the Fermi function. The integration in Eq. (36) is carried out over positive values of ε for electrons and over negative values for holes, if ε is reckoned from the center of the forbidden gap.

The number of states $N(\varepsilon)$ in the Dimmock model is given by an elliptic integral. This expression is simplified if we neglect the effect of distant bands on the transverse mass, i.e., the term with $1/m_t^\pm$ compared with $2v_t^2/\varepsilon_g$. The correction to the longitudinal mass is more substantial and is due to the low value of v_l. In this approximation, as shown by Fal'kovskiĭ

$$N(\varepsilon) = \frac{8k_z}{5\pi^2\hbar^3 v_t^2} \left[\varepsilon^2 - \frac{\varepsilon_g^2}{4} - \frac{k_z^2}{6} \left(v_l^2 + \frac{\varepsilon_g + 2\varepsilon}{4m_l^-} + \frac{\varepsilon_g - 2\varepsilon}{4m_l^+} \right) \right]_{k_z^{min}}^{k_z^{max}}, \tag{37}$$

$$k_z^2 \big|_{max,\,min} = C \pm (C^2 + D)^{1/2}, \; D = m_l^+ m_l^- (4\varepsilon^2 - \varepsilon_g^2), \tag{38}$$
$$C = -2m_l^- m_l^+ v_l^2 - m_l^+ (\varepsilon + \varepsilon_g/2) - m_l^- (\varepsilon_g/2 - \varepsilon).$$

In Eq. (37), the limiting value k_z^{max} or k_z^{min} is set equal to zero if the corresponding value k_z^2, given by Eq. (38), proves to be negative.

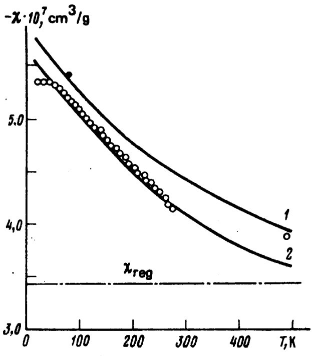

FIG. 1. Temperature dependence of the magnetic susceptibility of $Pb_{0.82}Sn_{0.18}Te$ for (1) $N = 0$ and (2) $N = 2.0 \cdot 10^{16}$ cm^{-3}. (The solid line indicates the theoretical curve in all the figures.) ● - data from Ref. 12, ○- data from Ref. 11.

The density of the heavy holes, the mass of which is on the order of the free electron mass m_0,[19] is reduced to the well-known Fermi integral

$$p_h = \frac{1}{2\pi^2\hbar^3}(2m_0kT)^{3/2} F_{1/2}\left(\frac{\mu - \varepsilon_{lh}}{kT}\right),$$

where ε_{lh} is the energy separation between the extrema of the bands of light and heavy holes.

The lattice magnetic susceptibility, χ_{lat}, may be designated as the value of the sum in Eq. (35) in the state for which the valence band is fully occupied and the conduction band is empty. The theoretical dependence of the specific lattice susceptibility (this differs from the volume susceptibility, considered in Part I, by the factor ρ^{-1}, where ρ is the density) on the temperature is shown on Fig. 1. The figure shows also the calculated and measured magnetic susceptibilities of a sample with a low carrier density.

The values of χ_{reg}, selected so that the theoretical and experimental temperature dependences of the magnetic susceptibility agree, are given in Table I for different samples. It is seen that the magnitude of χ_{reg} indeed depends weakly on both the temperature interval over which the comparison is made, and on the composition of the sample, up to rather high carrier densities

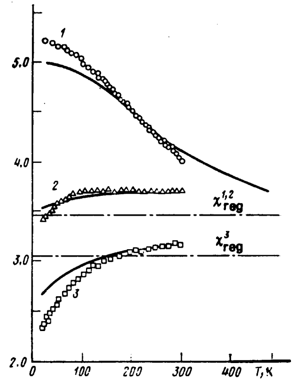

FIG. 2. Temperature dependences of the magnetic susceptibility of $Pb_{0.82}Sn_{0.18}Te$ (the curve numbers correspond to the sample numbers here and on Figs. 3–5).

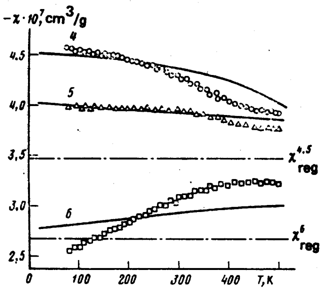

FIG. 3. Temperature dependences of the magnetic susceptibility of $Pb_{0.065}Sn_{0.35}Te$.

on the order of $2\cdot 10^{19}$ cm^{-3}. The poorer agreement between theory and experiment at larger carrier densities is apparently explained both by the deviation of the dispersion law from the Dimmock model for such highly occupied bands, and generally speaking, by the lower quality of the crystals, which contain in particular a large number of vacancies.

In a prior work[12] χ_{lat} was determined experimentally for $Pb_{0.82}Sn_{0.18}Te$ at 80 °K. The value of χ_{lat} at 500 °K may be obtained from our data[11] for the sample with $p_0 = 2.0\cdot 10^{16}$ cm^{-3}, since at this temperature the conduction is intrinsic and the chemical potential is close to the center of the forbidden gap. As seen from Fig. 1, the results of our calculations agree rather well with the data of these papers.

In contrast to the established views of the crystal lattice magnetic susceptibility as a weak function of temperature, in $Pb_{1-x}Sn_xTe$ solid solutions it has a significant temperature dependence due to the substantial change in ε_g. Therefore the separation of the carrier magnetic susceptibility in the crystals studied, assuming constancy of χ_{lat}, cannot be considered correct.

On Fig. 2 we present the experimental and theoretical temperature dependences of the magnetic susceptibility for crystals with $x = 0.18$ and different carrier densities. At low carrier densities (sample 1) the magnetic susceptibility decreases with an increase in temperature. This is connected with the increase of ε_g with temperature and also with the washout of the Fermi function and the increase in the contribution to the magnetic susceptibility from states with higher effective masses. For the case of crystals with high carrier densities (samples 2 and 3), the magnetic susceptibility increases with temperature. This is explained by the fact that for $n_0 > 10^{18}$ and $p_0 > 10^{18}$ cm^{-3} the level of the chemical potential μ is located in an allowed energy band. With an increase in temperature, μ approaches the edge of this band, increasing the contribution to the magnetic susceptibility from the small neighborhoods of the L point and, consequently, the diamagnetism of the crystal.

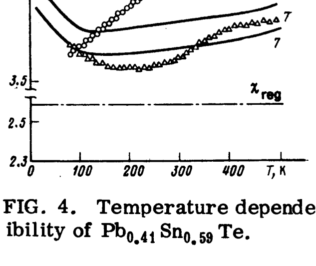

FIG. 4. Temperature dependences of the magnetic susceptibility of $Pb_{0.41}Sn_{0.59}Te$.

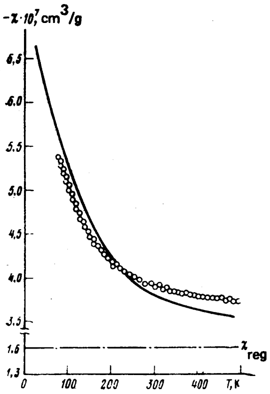

FIG. 5. Temperature dependences of the magnetic susceptibility of $Pb_{0.13}Sn_{0.87}$ TE.

On Fig. 3 we give the temperature dependences of the magnetic susceptibility for $x = 0.35$ and different hole densities. For $x = 0.59$ (Fig. 4) band inversion takes place at 270 °K.[18] If $\varepsilon_g \to 0$, the effective mass $m^*(\varepsilon_F) \sim \varepsilon_F$; ε_F and m^* have a maximum at the inversion temperature, and consequently the magnetic susceptibility has a minimum. However, the minimum in the experimental and theoretical curves (7) is observed at a temperature which is substantially lower (120–170 °K) than the inversion point and it is not pronounced. This is also due to the high level of the chemical potential, which leads to a weak dependence of the magnetic susceptibility on ε_g.

In Fig. 5 we present the temperature dependence of the magnetic susceptibility of sample 9 of the solid solution with $x = 0.87$. In contrast to the case $x = 0.18$, for such a carrier density the magnetic susceptibility decreases with increase in temperature, owing to the reversed band arrangement and to the increase in ε_F and $1/m^*$. We note that, regardless of the lower value of χ_{lat}, the theory describes the temperature dependence of the magnetic susceptibility well.

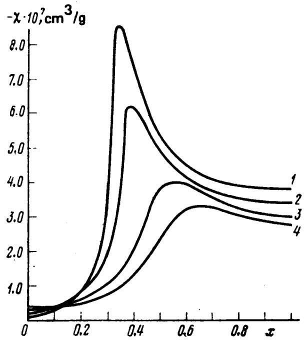

FIG. 6. Dependence of the magnetic susceptibility of $Pb_{1-x}Sn_xTe$ ($N = 0$) on composition for temperatures T = (1) 20 °K; (2) 80 °K; (3) 200 °K; (4) 300 °K.

On Fig. 6 we show the theoretical dependence of the magnetic susceptibility of $Pb_{1-x}Sn_xTe$ solid solutions on composition for zero carrier density at different temperatures. For 20 °K, the value of χ reaches $-8.5 \cdot 10^{-7}$ cm^3/g for $x=0.36$. Such a significant diamagnetism is due to the large contribution to the magnetic susceptibility from integrating over the small neighborhood of the L point as $\varepsilon_g \to 0$. With increase in temperature, the magnetic susceptibility at the maximum decreases and the maximum becomes less pronounced. This is explained by the temperature washout of the Fermi function.

The measurement of the magnetic susceptibility of a degenerate semiconductor makes it possible to determine the g factor [see Eq. (31b)]. For samples with $x=0.18$ and low carrier concentration ($n_0, p_0 < 10^{18}$ cm^{-3}) $g=1.45$ at $T=30$ °K, which is close to the value 1.36 obtained by Melngailis *et al.* in an investigation of the Shubnikov–de Haas effect.[20]

Thus, the theory which takes into account the contribution to the magnetic susceptibility from the two closely spaced bands and is developed within the Dimmock model describes well the experimental results for a large number of $Pb_{1-x}Sn_xTe$ solid solutions over a wide range of carrier densities and temperatures. We must point out that allowance for the transfer of the holes to the heavy valence band is essential for agreement of theory with experiment at high carrier densities and at temperatures in the liquid helium range.

In the calculations we used, the longitudinal momentum operator matrix element, which is substantially dependent on composition; v_l increases[16] with an increase in x, in contrast to the result of Appold *et al.*,[17] where it drops. The calculation of the magnetic susceptibility of SnTe for both versions of the $v_l(x)$ dependence showed that the experimental data is described best by using the $v_l(x)$ corresponding to our results.[16]

The authors express their deep appreciation to E. V. Mozdor for carrying out all the necessary computer calculations.

[1] P. K. Misra and L. M. Roth, Phys. Rev. **177**, 1089 (1969).
[2] M. P. Sharma, L. G. Johnson, and J. W. McClure, Phys. Rev. **B9**, 2467 (1974).
[3] R. Bowers and Y. Yafet, Phys. Rev. **115**, 1165 (1959).
[4] S. D. Beneslavskiĭ and L. A. Fal'kovskiĭ, Zh. Éksp. Teor. Fiz. **69**, 1063 (1975). [Sov. Phys. JETP **42**, 541 (1975)].
[5] N. B. Brandt, M. V. Semenov, and L. A. Fal'kovskiĭ, J. Low Temp. Phys. **27**, 75 (1977).
[6] J. O. Dimmock, J. Phys. Chem. Sol. **32** (Suppl. 1), 319 (1971).
[7] B. L. Gel'mont, V. I. Ivanov-Omskiĭ, B. T. Kolomiets, and V. M. Mel'nik, Fiz. Tekh. Poluprovodn. **4**, 299 (1970) [Sov. Phys. Semiconductors **4**, 244 (1970)].
[8] B. L. Gel'mont, Fiz. Tverd. Tela **11**, 1096 (1969) [Sov. Phys. Solid State **11**, 646 (1969)].
[9] R. S. Kim and S. Narita, Phys. Status Solidi B **73**, 741 (1976).
[10] T. C. Harman and I. Melngailis, Appl. Sol. St. Sci. **4**, 1 (1974).
[11] G. V. Lashkarev, K. D. Tovstyuk, V. B. Orletskii, and A. D. Shevchenko, Ukr. Fiz. Zh. **19**, 864 (1974); G. V. Lashkarev, D. F. Migleĭ, K. D. Tovstyuk, and A. D. Shevchenko, Fiz. Tekh. Poluprovodn. **8**, 1425 (1974) [Sov. Phys. Semiconductors **8**, 929 (1975); K. D. Tovstyuk, G. V. Lashkarev, V. B. Orletskiĭ, and A. D. Shevchenko, Fiz. Tverd. Tela **16**, 221 (1974). [Sov. Phys. Solid State **16**, 140 (1974)]; G. V. Lashkarev, in: Poluprovodniki s uzkoi zapreshchennoĭ zonoĭ i polumentally (Narrow-gap Semiconductors and Semimetals), Reports of the Fourth All-Union Symposium, L'vov, 1975, Part II, p. 64; G. V. Lashkarev, V. B. Orletskiĭ, M. V. Radchenko, K. D. Tovstyuk, and A. D. Shevchenko, *ibid*, p. 71.
[12] G. V. Lashkarev, R. O. Kikodze, and A. V. Brodovoĭ, Fiz. Tekh. Poluprovodn. **12**, 1066 (1978). [Sov. Phys. Semiconductors **12**, 633 (1978)].
[13] V. B. Orletskiĭ, F. F. Sizov, G. V. Lashkarev, and K. D. Tovstyuk, Fiz. Tekh. Poluprovodn. **9**, 269 (1975). [Sov. Phys. Semiconductors **9**, 176 (1975)].
[14] G. V. Lashkarev, D. F. Miglei, A. D. Shevchenko, and K. D. Tovstyuk, Phys. Status Solidi B **63**, 663 (1974).
[15] W. Zawadzki, Phys. Status Solidi **3**, 1421 (1963).
[16] G. V. Lashkarev and A. I. Dmitriev, Visnik Akad. Nauk Ukr. SSR **2**, 12 (1978).
[17] G. Appold, R. Grisar, G. Bauer, H. Burkhard, R. Ebert, H. Pasher, and H. Hasele, Inst. Phys. Conf. of Semicond., Edinburgh, Ser. N43, 1978, p. 29.
[18] J. O. Dimmock, J. Melngailis, and A. J. Strauss, Phys. Rev. Lett. **16**, 1193 (1966).
[19] G. V. Lashkarev, F. F. Sizov, V. B. Orletskii, and K. D. Tostyuk, Poluprovodniki s uzkoĭ zapreshchennoĭ zonoĭ i polumetally (Narrow-gap Semiconductors and Semimetals) Report of the Fourth All-Union Symposium, L'vov, 1975, Part IV, p. 67.
[20] J. Melngailis, T. C. Harman, J. G. Mavroides, and J. O. Dimmock, Phys. Rev. **B3**, 370 (1971).

Translated by Cathy Flick