Phonon kinetics in nonequilibrium superconductors located in an external electromagnetic field

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A kinetic equation is derived in the quasiclassical approximation for phonons in nonequilibrium superconductors on the basis of the Keldysh formalism. The collision integral of this equation is expressed in terms of polarization operators for which equations are obtained with the aid of the Keldysh technique and on the basis of the analytic-continuation procedure proposed by Éliashberg in an analysis of the electron component of nonequilibrium superconductors. The corresponding expressions, as expected, coincide. It turns out to be possible to express the polarization operators in terms of Green's functions integrated with respect to the energies; these functions were introduced in the equilibrium case by Eilenberger and used subsequently by Éliashberg in an analysis of a nonequilibrium electron subsystem. It is shown how to use the resultant equation to investigate the emission of nonequilibrium phonons from this superconducting film. The problem of the boundary conditions for the phonons is briefly discussed for this case. The phonon flux out of a thin superconducting film into an external medium is calculated analytically for the case when the electromagnetic-field frequency is much less than the gap. It is shown that in a narrow spectral interval (of the order of the external-field frequency) of phonon frequencies close to double the gap, the phonon flux becomes negative in magnitude, i.e., in this spectral interval the film absorbs the phonons. It is noted that this result is closely related to the effect of superconductivity stimulation by a microwave field and is due to the "supercooling" of the electron subsystem under these conditions.

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INTRODUCTION

A number of papers published during the last decade (see, in particular, the articles[1-19] and the bibliographies in them) laid the groundwork for a theoretical description of nonequilibrium superconductors located in external electromagnetic fields. The principal object of study, however, was the electron subsystem. In particular, Eliashberg[6] obtained for the description of this subsystem kinetic equations in terms of energy-integrated Green-Gor'kov functions. These equations made it possible to go outside the framework of the ordinary equations of the Boltzmann type for superconductors, as well as to determine the limits of applicability of these equations. At the same time, in most cases the phonon field was regarded as given, and moreover as being in equilibrium. This approach is justified in those cases when the deviation of the phonons from equilibrium has little effect on the kinetics of the nonequilibrium electrons. Inasmuch as in experiment one usually measures those superconductor electromagnetic characteristics which are directly connected with the behavior of only the electron component, a detailed study of the phonon subsystem turns out to be unnecessary in most cases.

It is possible, however, to formulate the problem differently, and investigate in fact the phonons that result from the action of an external perturbation on the electron subsystem. Problems of this type arise, e.g., in the design of sound generators based on superconducting systems.[11] The basis for their solution should be a kinetic equation for the phonons. There are a number of papers (see, in particular, Refs. 9, 12, and 14) in which a nonequilibrium electron subsystem is considered jointly with a nonequilibrium phonon subsystem. We note in this connection that the question of phonon kinetics is far from simple, since a description of the electron subsystem itself in terms of distribution functions is possible in far from all nonequilibrium situations. In addition, the situation is made complicated by the fact that an external perturbation (e.g., an electromagnetic field) influences the density of the electron levels in the superconductor. This limits the applicability of the ordinary phonon kinetic equations used in the literature.

The main purpose of the present paper is a derivation of a more general kinetic equation for phonons in the case when the electron-phonon system of a superconductor is subjected to the action of an external electromagnetic field. The collision integral of this equation is then expressed not in terms of the electron distribution functions, but in terms of energy-integrated Green-Gor'kov functions (in analogy with the procedure used by Eliashberg[6] in the derivation of the kinetic equations for the electrons). We present below a brief content of the paper.

Using the Keldysh formalism, we derive in the quasiclassical approximation a kinetic equation for phonons in nonequilibrium superconductors that are in a spatially inhomogeneous nonstationary state (Sec. 1). In the right-hand side of this equation, the decisive role is played by polarization operators, expressions for which are obtained both with the aid of the Keldysh technique (Sec. 2) and on the basis of the analytic-continuation procedure proposed by Eliashberg in the derivation of the kinetic equations in nonlinear electrodynamics of superconductors. The corresponding expressions, as expected, coincide, and the final values are expressed in terms of the energy-integrated Green's functions of the electrons (Sec. 3).

We note that the obtained equation can be used jointly with the kinetic equations for the nonequilibrium electron component in those cases when the disequilibrium

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of the phonon field is of importance for the electron component. The most interesting, however, is the use of the equation precisely when the nonequilibrium phonons manage to leave the system without significantly influencing the behavior of the electrons. In this situation, the equations for the nonequilibrium electron subsystem can be solved independently (using in them the equilibrium phonon distribution functions). The corresponding solutions, when substituted in the phonon equation, determine the source of the phonons that are emitted from the nonequilibrium superconductor into the external medium. This circumstance is of interest, in particular because the solutions of the kinetic equations for the electron component are already known for a large number of problems. In Sec. 4 we touch upon the problem of boundary conditions for the phonons, and also discuss briefly the kinetics of the electron subsystem in films.

The approach developed in Sec. 5 can be used to describe phonon emission from a superconducting film situated in a microwave field. The spectrum of the emitted phonons is calculated analytically in the case when the frequency of the external field (ω_0) is much lower than the nonequilibrium value of the gap (Δ). It is shown that the phonon flux is negative in a narrow spectral interval of the phonon frequencies $2\Delta \leqslant \omega_{ph} \leqslant 2\Delta + \omega_0$. Thus, the action of an external RF electromagnetic field can lead not only to phonon emission by a film (the so-called phonon fluorescence, e.g., Ref. 16), but the nonequilibrium film should also absorb phonons in the indicted spectral region. This effect is connected with the decrease of the number of such phonons in a film acted upon by the microwave radiation, and with the onset of a corresponding flux of phonons from the outside, which compensates for the deviation of phonon distribution function from equilibrium. This effect, which we call the "phonon deficit," is essentially of the same nature as the well known phenomenon of superconductivity stimulation by a microwave field.[19] In the concluding Sec. 5 we discuss in greater detail the physical nature of the phonon deficit, and compare the results with those of the numerical calculations of Chang and Scalapino,[14] who investigated the kinetics of nonequilibrium phonons under conditions close to those considered by us.

1. THE KINETIC EQUATION

1. Preliminary definitions

We introduce the Green-Keldysh phonon function in the usual manner[1)]:

$$D_\lambda^{ik}(1,2) = -i\langle T\hat{\varphi}_\lambda(1i)\hat{\varphi}_\lambda(2k)\rangle. \tag{1.1}$$

The Keldysh indices i and k assume values $-$ or $+$, depending on which of two time axes $(-\infty, +\infty)$ or $(+\infty, -\infty)$ the time of each of the phonon-field operators $\hat{\varphi}$ is located.[1] We note that the time on the second axes $(+)$ is assumed to be longer than any time on the first axis $(-)$, and on the second axis the T-ordering of the operators is reversed. The phonon-field operators (since this field is real, $\hat{\varphi}^+ \equiv \varphi$) are expressed in the absence of interaction in the form[2]

$$\hat{\varphi}_0(x) = \frac{1}{V^{1/2}} \sum_{\mathbf{k}} \left(\frac{\omega_0(\mathbf{k})}{2}\right)^{1/2} \{\hat{b}_{\mathbf{k}} \exp\{i[\mathbf{kr} - \omega_0(\mathbf{k})t]\} + \hat{b}_{\mathbf{k}}^+ \exp\{-i[\mathbf{kr} - \omega_0(\mathbf{k})t]\}\}, \tag{1.2}$$

where $\omega_0(\mathbf{k})$ is the phonon dispersion law in the normal metal.

The "bare" phonon Green-Keldysh functions determined on the basis of (1.1) and (1.2) can be easily obtained in the homogeneous and stationary case. We present an expression for D_0^{-+} (cf. Ref. 3):

$$D_0^{-+}(t,\mathbf{r}) = -\frac{i}{2}\int \frac{d^3\mathbf{k}}{(2\pi)^3}\{\omega_0(\mathbf{k}) e^{i\mathbf{kr}}[N_{\mathbf{k}} \exp\{-i\omega_0(\mathbf{k})t\} + (1+N_{-\mathbf{k}})\exp\{i\omega_0(\mathbf{k})t\}]\}, \tag{1.3}$$

in which $N_{\mathbf{k}} = \langle \hat{b}_{\mathbf{k}}^+ \hat{b}_{\mathbf{k}}\rangle$ is the nonequilibrium distribution function of the phonons.

We introduce also the operator $D_{01(2)}^{-1}$, which acts on the first (second) argument of the phonon propagator ($\mathbf{u}$ is the phonon propagation velocity):

$$D_{01(2)}^{-1} = \frac{\partial^2}{\partial t_{1(2)}^2} - u^2\nabla_{1(2)}^2, \tag{1.4}$$

then

$$D_{01(2)}^{-1} D_0^{ik}(12) = u^2\sigma_3^{ik}\delta(t_1 - t_2)\nabla_{1(2)}^2\delta(\mathbf{r}_1 - \mathbf{r}_2), \tag{1.5}$$

where $\hat{\sigma}_3$ is a Pauli matrix.

In the general case the phonon function satisfies the Dyson equation

$$\hat{D}(12) = \hat{D}_0(12) + \int \hat{D}_0(14)\hat{\Pi}(43)\hat{D}(32)\,d^4x_3\,d^4x_4, \tag{1.6}$$

or in an alternate form

$$\hat{D}(12) = \hat{D}_0(12) + \int \hat{D}(13)\hat{\Pi}(34)\hat{D}_0(42)\,d^4x_3\,d^4x_4, \tag{1.7}$$

where the functions are matrices in the Keldysh indices. We recall that by virtue of the very definition (1.1) the Green-Keldysh functions are linearly dependent ($D^{--} + D^{++} - D^{+-} - D^{-+} = 0$), as a result of which the polarization operators are likewise linearly dependent ($\Pi^{--} + \Pi^{++} + \Pi^{-+} + \Pi^{+-} = 0$).

The electron Green-Keldysh-Nambu function is defined similarly (we follow the notation of Ref. 4):

$$G_{\alpha\beta}^{ik}(12) = -i\langle T\psi_\alpha(1i)\psi_\beta^+(2k)\rangle, \tag{1.8}$$

where α and β are the Nambu indices of the field operators

$$\psi_1(1i) = \psi_i(1i), \quad \psi_2(1i) = \psi_i^+(1i). \tag{1.9}$$

The symmetry properties of the function (1.8) are given in Ref. 4.

2. Derivation of kinetic equation

The Green-Keldysh functions depend in the homogeneous and stationary case on differences between space-time coordinates. If the evolution of the phonon subsystem is gradual enough, we can assume that the quantities characterizing this system depend weakly on the sum variable (1 + 2) and are functions mainly of the difference variable (1 − 2). (This is essentially the condition for quasiclassical behavior.) Separating these

variables [we use below a notation of the type $1 \equiv x_1 \equiv (\mathbf{x}, t)$]:

$$\hat{D}(x_1x_2) = \hat{D}\left(\frac{x_1+x_2+(x_1-x_2)}{2}; \frac{x_1+x_2-(x_1-x_2)}{2}\right), \quad (1.10)$$

we take Fourier transforms with respect to the difference variables $\mathbf{R} = \mathbf{r}_1 - \mathbf{r}_2$ and $\Theta = t_1 - t_2$

$$D^{ik}(\mathbf{q}, \omega, \mathbf{r}, t) = \int D^{ik}(\mathbf{r}, t; \mathbf{R}, \Theta) e^{i\mathbf{q}\mathbf{R}+i\omega\Theta} d\mathbf{R}\, d\Theta, \quad (1.11)$$

where obviously $\mathbf{r} = (\mathbf{r}_1 + \mathbf{r}_2)/2$ and $t = (t_1 + t_2)/2$. Acting with the operator D_{01}^{-1} on Eq. (1.6) and with D_{02}^{-1} on (1.7) and subtracting the second from the first, we write the results for the (−+) component:

$$(D_{02}^{-1} - D_{01}^{-1})D^{-+}(x_1x_2) = -\int d^4x_3\, d^4x_4 \{[D^{--}(13)\Pi^{-+}(34) + D^{-+}(13)\Pi^{++}(34)]\delta(t_2-t_4)\nabla_2^2\delta(\mathbf{r}_4-\mathbf{r}_2) + [\Pi^{--}(43)D^{-+}(32) + \Pi^{-+}(43)D^{++}(32)]\delta(t_1-t_4)\nabla_1^2\delta(\mathbf{r}_1-\mathbf{r}_4)\}. \quad (1.12)$$

We turn first to the right-hand side of (1.12) and transform it with allowance for the quasiclassical conditions. For the phonon subsystem the latter mean that the quantities characterizing its variation in time (Δt) and in space (Δr) should be large compared with reciprocal frequencies $(\omega(\mathbf{q}))^{-1}$ and wave vectors q^{-1} which are typical of phonons, i.e.,

$$\omega(\mathbf{q})\Delta t \gg 1, \quad q\Delta r \gg 1 \quad (\hbar = 1), \quad (1.13)$$

which is a good approximation when the perturbation of the phonon subsystem is due to the superconducting electron subsystem. The condition (1.13) makes it possible to simplify in the usual manner (see, e.g., Ref. 3) the right-hand side of (1.12). Recognizing that the operator $(D_{02}^{-1} - D_{01}^{-1})$ in the left-hand side of (1.12) can be represented in the form

$$D_{02}^{-1} - D_{01}^{-1} = -\frac{\partial^2}{\partial t\, \partial\Theta} + u^2\frac{\partial^2}{\partial \mathbf{r}\, \partial \mathbf{R}}, \quad (1.14)$$

we take the Fourier transform of Eq. (1.12) and obtain as a result the expression [the arguments of all the functions in the curly brackets are $(\mathbf{q}, \omega; \mathbf{r}, t)$]

$$2\left\{i\omega\frac{\partial D^{-+}}{\partial t} + iu^2\mathbf{q}\frac{\partial}{\partial \mathbf{r}}D^{-+}\right\} = -(uq)^2\{\Pi^{-+}D^{+-} - D^{-+}\Pi^{+-}\} \quad (1.15)$$

(we have used here the linear dependence of the components of D^{ik} as well as of Π^{ik}).

To change over to the kinetic equation in terms of the phonon distribution functions $N(\mathbf{q}, \mathbf{r}, t)$ we must establish a connection between N and $\hat{D}$. Unfortunately, the Green-Keldysh function for phonons, defined in accordance with (1.1), while convenient from the point of view of the dynamics of electron-phonon interaction, is not as directly connected with the density matrix which as the electron function G (Refs. 2 and 3).[2)] Nevertheless, in the quasiclassical case of interest to us, such a connection can be established. We note for this purpose that the superconducting transition has hardly any effect ($\Delta\omega_0(\mathbf{q})/\omega_0(\mathbf{q}) \sim 10^{-4}$) on the base phonon spectrum, $\omega_0(\mathbf{q}) \approx \omega(\mathbf{q})$. Using the quasiclassical condition for the phonons, we assume that $N(\mathbf{q}, \mathbf{r}, t)$ and $\hat{D}(\omega, \mathbf{q}; t, \mathbf{r})$ are related as

$$D^{-+}(\mathbf{q}, \omega; t, \mathbf{r}) = -\frac{i}{2}\omega(\mathbf{q})[(1+N(-\mathbf{q}, \mathbf{r}, t))\delta(\omega+\omega(\mathbf{q})) + N(\mathbf{q}, \mathbf{r}, t)\delta(\omega-\omega(\mathbf{q}))]. \quad (1.16)$$

This relation is similar to that which follows from (1.3) for the quantities $D_0^{-+}(\mathbf{q}, \omega)$ and $N_{\pm q}$. (Concerning the subsitutions

$$D^{-+}(\mathbf{q}, \omega) \to D^{-+}(\mathbf{q}, \omega; \mathbf{r}, t), \quad N_q \to N(\mathbf{q}, \mathbf{r}, t),$$

where t and $\mathbf{r}$ serve as parameters, see, e.g., Ref. 3.) Next, for acoustic phonons (the only ones that appear in the phenomenon of interest to us), the following relation holds for momenta that are small compared with the limiting values:

$$\mathbf{q} = \frac{\partial\omega(\mathbf{q})}{\partial \mathbf{u}} \approx \frac{\omega(\mathbf{q})}{u^2}\mathbf{u}. \quad (1.17)$$

Using also the property

$$D^{+-}(\mathbf{q}, \omega; \mathbf{r}, t) = D^{-+}(-\mathbf{q}, -\omega; \mathbf{r}, t), \quad (1.18)$$

which follows from (1.1) and (1.11), and substituting (1.16)–(1.18) in (1.15), we obtain, after integrating with respect to ω within the limits $(0, \infty)$, the kinetic equation

$$\frac{\partial N(\mathbf{q}, \mathbf{r}, t)}{\partial t} + \mathbf{u}\frac{\partial N(\mathbf{q}, \mathbf{r}, t)}{\partial \mathbf{r}} = I(N), \quad (1.19)$$

the source in which is the quantity (cf. Ref. 13)

$$I(N) = \frac{i\omega(\mathbf{q})}{2}\{\Pi^{-+}(\mathbf{q}, \omega(\mathbf{q}); \mathbf{r}, t)[1+N(\mathbf{q}, \mathbf{r}, t)] - \Pi^{+-}(\mathbf{q}, \omega(\mathbf{q}); \mathbf{r}, t)N(\mathbf{q}, \mathbf{r}, t)\}. \quad (1.20)$$

We note that this expression can be represented also in a somewhat different form, if we change over in usual fashion (see, e.g., Ref. 3) from the matrix Π^{ik} to the linearly independent functions $\Pi = \Pi^{--} + \Pi^{++}$, $\Pi^R = \Pi^{--} + \Pi^{-+}$, and $\Pi^A = \Pi^{--} + \Pi^{+-}$

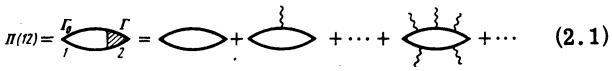
$$I(N) = \frac{i\omega(\mathbf{q})}{2}\left\{(\Pi^R - \Pi^A)N - \frac{1}{2}[\Pi - (\Pi^R - \Pi^A)]\right\}. \quad (1.21)$$

This representation turns out to be more convenient.

2. THE COLLISION INTEGRAL

1. The polarization operation in the Keldysh technique

In the Keldysh-Nambu technique, the polarization operator is given by a diagram expansion similar to the usual one:



(2.1)

[The electron propagators in the left-hand side of (2.1) should be represented by thick lines.] The diagrams are set in correspondence with analytic expressions by the usual Feynman rules,[2] the only difference being that all the quantities, including the vertices, are matrices in the indices of Keldysh and Nambu. Since the superconducting transition, just as an interaction with an external electromagnetic field, affects in the polarization operator only a small smeared region ($\sim T/\varepsilon_F$, Δ/ε_F) near the Fermi surface, it follows that, just as in the case of a normal metal,[5] we can set, with adiabatic accuracy ($\sim u/v_F$), the total vertex Γ equal to $\Gamma_0 \sim g$. On the other hand, the electron function G^{ik} [the thick lines in (2.1)] will be assumed accurate in the sense that they contain the interaction of the electrons with the external field, with phonons, with impurities, and with one another. Recognizing that in the technique em-

ployed by us[4] the vertices Γ_0 have a matrix structure

$$\Gamma_{\alpha\beta}^{ij,k} \sim \sigma_3^{ij}\delta_{jk}\sigma_3^{\alpha\beta}, \tag{2.2}$$

we obtain (g is the electron-phonon interaction constant)

$$\Pi^{kk'}(12) = -ig^2(-1)^{k+k'}\{G_{11}^{kk'}G_{11}^{k'k} - G_{12}^{kk'}G_{21}^{k'k} - G_{21}^{kk'}G_{12}^{k'k} + G_{22}^{kk'}G_{22}^{k'k}\}. \tag{2.3}$$

It is more convenient, however, to operate with the linearly-independent quantities G, G^R, and G^A.[3] Changing simultaneously over also to the quantities Π, Π^R, and Π^A and leaving out terms of the type $G_{11}^A(12)$ $G_{11}^A(21)$, which vanish indentically, we obtain

$$\Pi^{A(R)}(12) = -{}^1/_2\, ig^2\{G_{11}^{A(R)}(12)G_{11}(21) + G_{11}G_{11}^{R(A)} + G_{22}^{A(R)}G_{22} + G_{22}G_{22}^{R(A)} - G_{12}^{A(R)}G_{21} - G_{12}G_{21}^{(R)A} - G_{21}^{A(R)}G_{12} - G_{21}G_{12}^{R(A)}\} \tag{2.4}$$

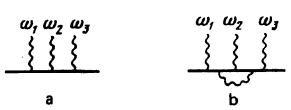
and correspondingly

$$\Pi(12) = -{}^1/_2 ig^2\{G_{11}(12)G_{11}(21) + G_{22}G_{22} - G_{21}G_{12} - G_{12}G_{21} + G_{11}{}^AG_{11}{}^R + G_{11}{}^RG_{11}{}^A + G_{22}{}^AG_{22}{}^R + G_{22}{}^RG_{22}{}^A - G_{12}{}^AG_{21}{}^R - G_{12}{}^RG_{21}{}^A - G_{21}{}^AG_{12}{}^R - G_{21}{}^RG_{12}{}^A\}. \tag{2.5}$$

It remains to go over to the (x,p) representation, as is required for a kinetic equation written in the form (1.19). It becomes clear then that the polarization operators can be expressed in terms of the so-called "Green's functions integrated over the energies." They were introduced for the equilibrium case by Eilenberger[8] and generalized by Eliashberg[6] for the description of a nonequilibrium electron subsystem of a superconductor in the model with phonon thermostat. We shall dwell in somewhat greater detail on the physical and formal aspects of this model, for the purpose of clarifying certain features of the behavior of the electron-phonon system, which are of importance for the subsequent application of Eqs. (1.19), (2.4), and (2.5).

2. Model with phonon thermostat

To use the analytic-continuation method for the derivation of the kinetic equations for a nonequilibrium electron in a model with a phonon thermostat, the temperature electron Green's functions were expanded in powers of the external field, and this gave rise[6] to diagrams of the type



(the wavy lines correspond to interaction with Bose fields, and $\omega_k = 2i\pi k$ are discrete imaginary frequencies of the Bose field).

The physical quantities were determined by analytic continuation of the quantities involved in the phonon model of the superconductor into the upper half-plane with respect to each of the discrete imaginary field frequencies. It is significant here that the phonons are in equilibrium. From the formal point of view this means that in the exact phonon Green's function

$$D(\mathbf{p}'-\mathbf{p}) = g^2\frac{2\omega(\mathbf{p}'-\mathbf{p})}{\omega^2(\mathbf{p}'-\mathbf{p}) - (\varepsilon'-\varepsilon)^2} \tag{2.6}$$

the initial distribution of the discrete imaginary Bose frequencies ($\varepsilon - \varepsilon' = 2i\pi Tn$, with n an integer,[3]) is fixed. As a result the lines of the cuts of a diagram of arbitrary type (e.g., case b in the diagram) coincide with those for diagrams that do not contain self-energy inserts (but are of the same order in the field, see case a), and it is this which determines the equality of the analytic structures of diagrams of a given order with respect to the external field. In the general case, when the phonons are not in equilibrium, the phonon Green's function can be repeesented in the form[4)]

$$D_\omega(\mathbf{q}) = \frac{1}{[D_{0\omega}(\mathbf{q})]^{-1} - \Pi_\omega(\mathbf{q})}. \tag{2.7}$$

The real part $\mathrm{Re}\,\Pi_\omega(\mathbf{q})$, with which the renormalization of the sound velocity is connected, is determined by the entire mass of the electrons, and the region of the temperature smearing near the Fermi surface yields a correction of the order of T/ε_F. As already stipulated, the renormalization of the sound velocity in the transition to the superconducting state has an analogous smallness (Δ/ε_F), and the influence of the electromagnetic field is also small and affects mainly only the smearing region. It is therefore possible to disregard the small corrections and assume that these renormalizations have already been included in (2.6). (We note that this circumstance has allowed us in Sec. 1 above, when working with Dyson matrix equations, to retain only the components that determine the distribution function.) On the other hand, the imaginary part $\mathrm{Im}\,\Pi_\omega(\mathbf{q})$ is determined entirely by the vicinity of the Fermi surface, and is therefore sensitive to the distribution of the excitations. The neglect of this quantity in (2.7) and the transition to the initial representation (2.6) correspond to the physical assumption that a relaxation source stronger than the one connected with the interaction of the phonons and electrons is active in the phonon system. Such a source can be a connection of the phonon system with the external medium (thermostat), which causes the phonons themselves to assume the role of the thermostat for the electron subsystem. If this connection were to be included explicitly in the analysis, then the right-hand side of the kinetic equation obtained above would become equal to zero in the cases corresponding to an equilibrium phonon distribution. Another equivalent procedure was used in the derivation of the kinetic equations in nonlinear electrodynamics of superconductors,[7] wherein the connection of the phonon system with the thermostat was obtained by simply equating the polarization operator to zero. In both cases the resultant phonon system is in equilibrium.

The foregoing arguments clarify the physical meaning of the kinetics equation (1.19) obtained by us. If it is assumed that the phonon functions in its right-hand side, as well as in the complete electron Green's functions, are in equilibrium, then the left-hand side of this equation corresponds to departure of phonons to the external thermostat, i.e., it determines the emission of phonons by the sytem. In situations when this assumption is valid, the kinetic equations of Eliashberg for the electron component are likewise applicable. We shall show below that the right-hand side of the phonon kinetic equation is expressed in terms of the solution of the Eliashberg equation, which in many cases of practical interest (e.g., for thin films in an

external electromagnetic field or for weak-coupling bridges) have already been well investigated.

3. COLLISION INTEGRAL AS A PHONON SOURCE

It is indicated in the preceding section, the imaginary part of the polarization operator, which is of interest to us, is entirely determined by the vicinity of the Fermi surface. This circumstance is quite important, since it enables us to describe the behavior of the electrons in the collision integral (1.21), (2.4), (2.5) on the basis of Green's functions integrated with respect to the energies. Before we do that, it is advisable to obtain expressions for the polarization operators by the Eliashberg method.

1. Polarization operators in the Éliashberg method

Thus, we obtain the quantities Π, Π^R, and Π^A by the analytic-continuation method generalized to include the nonlinear case.[6] In the representation of discrete imaginary frequencies ($\varepsilon_n = i\pi(2n+1)T$, $\omega_m = 2i\pi Tm$) we have for polarization operator the expression

$$\Pi_{\omega\omega-\omega'}(\mathbf{p},\mathbf{p}-\mathbf{p}') = g^2 T \sum_{\varepsilon_1} \int \frac{d^3\mathbf{p}_1}{(2\pi)^3} \{G_{\varepsilon_1}\bar{G}_{\omega-\varepsilon_1} + \bar{G}_{\varepsilon_1}G_{\omega-\varepsilon_1} - F_{\varepsilon_1}F^+_{\omega-\varepsilon_1} - F_{\varepsilon_1}{}^+F_{\omega-\varepsilon_1}\}. \quad (3.1)$$

For the sake of brevity, we omit the second arguments of the Green's functions ($G_{\varepsilon_1\varepsilon_2}$ etc.), which can be reconstructed from the "decay" conservation law for the internal variable:

$$\varepsilon_1+\varepsilon_2=\omega, \quad \omega_1+\omega_2=\omega'. \quad (3.2)$$

Connected with the use of the rules (3.2) is the appearance of the functions G in (3.1), which differ in G that the directions of the arrows on the diagrams are reversed. In addition, the presence of the pair FF^+ in (3.1) is connected, as usual, with the change of the sign of the diagram. Proceeding to the analytic continuation of the polarization operator, we shall regard each of the terms in (3.1) as a sum of diagrams of different order in the external field [cf. (2.1)]. The entire procedure is perfectly analogous to the one used by Eliashberg in Ref. 6 when finding the analytic continuation of the self-energy parts of the electron-electron collisions, the only difference being that now we have Bose external frequencies (and, naturally, the number of electron lines is equal to two). Since the direction of the arrows on the lines of the diagrams does not influence essentially the procedure of the analytic continuation, we consider for the sake of argument the expression

$$\pi_{\omega\omega-\omega'} = T \sum_{\varepsilon_1} G_{\varepsilon_1\varepsilon_1-\omega} G_{\omega-\varepsilon_1,\omega-\varepsilon_1+\omega_1-\omega'}. \quad (3.3)$$

Diagrams of a definite order in the external field have, as functions of the complex variable ω at fixed imaginary frequencies of the field vertices, cuts on the lines $\mathrm{Im}(\omega-\Omega_m)=0$, located between the outermost upper and lower cuts

$$0 \leqslant \mathrm{Im}\,\omega \leqslant \mathrm{Im}\,\omega'. \quad (3.4)$$

Just as in the case of the electron-electron self-energy parts,[6] the quantities Ω_m constitute certain combinations

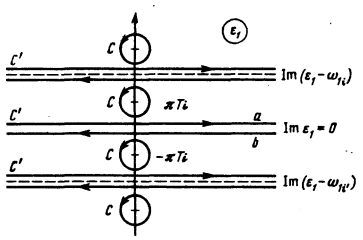


FIG. 1. Transformation of the integration contour. The dashed lines show the positions of the cuts on the ε_1 plane. When there is no cut (e. g. at $\mathrm{Im}\,\varepsilon_1=0$) the sum of the contribution along lines a and b is zero.

of field-vertex frequencies, and in this case, too, the set of these combinations and their total number depend on the distribution of the vertices over the electron lines. We assume that the respective functions correspond to cuts with $\mathrm{Im}(\varepsilon_1-\omega_{1i})=0$ and $\mathrm{Im}(\varepsilon_2-\omega_{2k})=0$ and transform the sum over the frequencies in (3.3) into a contour integral

$$\pi_{\omega\omega-\omega'} = \oint_C \frac{dz}{4\pi} \mathrm{th}\left(\frac{z}{2T}\right) G_z G_{\omega-z}, \quad (3.5)$$

where C is the contour shown in Fig. 1. Unwinding the contour C and C' along the edges of the cuts and recognizing that for diagrams of arbitrary order the integrals along the arcs of large circles vanish when the corresponding radii go to infinity, we obtain after simple transformations

$$\pi_{\omega\omega-\omega'} = \sum_{ik} \int \frac{dz}{4\pi i} \left\{\delta_i(G_{z+\omega_{1i}})G_{\omega-z-\omega_{1i}} \mathrm{th}\frac{z}{2T} - G_{z+\omega-\omega_{2k}}\delta_k(G_{-z+\omega_{2k}})\mathrm{th}\frac{z}{2T}\right\}, \quad (3.6)$$

where $\delta_{i,k}(G)$ is the jump of the Green's functions on the corresponding cut. The external variable ω and the field frequencies remained imaginary. Their combinations determine the set of cuts of a given diagram. Putting in all the diagrams $\omega > \omega'$ (the upper edge of the upper cut), shifting in them the integration variables, and summing the perturbation-theory series in all orders, we obtain the value of π^R:

$$\pi^R_{\omega\omega-\omega'} = \int_{-\infty}^{\infty} \frac{dz}{4\pi i} \{G_z G^R_{\omega-z} + G_z{}^R G_{\omega-z}\}, \quad (3.7)$$

where the function G is defined by

$$G_{\varepsilon\varepsilon-\omega} = \sum_{N=0}^{\infty}\sum_{i=0}^{N} \delta_i(G^{(N)})\mathrm{th}\frac{\varepsilon-\omega_i}{2T}, \quad (3.8)$$

and the functions $G^{R(A)}$ are determined directly from the diagram expansion (or from an equation of the Dyson type, where all the electron Green's functions are retarded (advanced), and the entire set $\{G, G^R, G^A\}$ coincides directly with the functions that enter in Ref. 6. In the same manner, an expression for π^A follows from (3.6) at $\omega<0$ (lower edge of the lower cut):

$$\pi^A_{\omega\omega-\omega'} = \int_{-\infty}^{\infty} \frac{dz}{4\pi i} \{G_z G^A_{\omega-z} + G_z{}^A G_{\omega-z}\}. \quad (3.9)$$

Using for $\Pi_{\omega\,\omega-\omega'}$ a representation similar to (3.8) but

modified in connection with the Bose character of the external frequencies:

$$\pi_{\omega\omega-\omega'}=\sum_{N=0}^{\infty}\sum_{k}\delta_k(\pi^{(N)})\operatorname{cth}\frac{\omega-\Omega_k}{2T}, \tag{3.10}$$

we obtain after substituting (3.6) in (3.10) and using the identity

$$\operatorname{cth}(x-x')(\operatorname{th}x-\operatorname{th}x')=1-\operatorname{th}x\operatorname{th}x'$$

the value

$$\pi_{\omega\omega-\omega'}=\int_{-\infty}^{\infty}\frac{dz}{4\pi i}\{G_zG_{\omega-z}+(G_z^R-G_z^A)(G^R_{\omega-z}-G^A_{\omega-z})\}. \tag{3.11}$$

By the same token, we have found for the polarization operator (3.1) the entire set of functions Π, Π^R, and Π^A which arise in the nonequilibrium case. It can be shown that they are equivalent to those obtained earlier on the basis of the Keldysh formalism. This agreement is natural, inasmuch, as shown by Volkov and Kogan,[4] the functions $G^{(R,A)}$, $\bar{G}^{(R,A)}$, $F^{(R,A)}$, and $F^{+(R,A)}$ coincide, apart from the sign, with the functions $G_{11}^{(R,A)}$, $G_{22}^{(R,A)}$, $G_{12}^{(R,A)}$, and $G_{21}^{(R,A)}$, and therefore the polarization operators quadratic in the Green's functions should be identically equal.[5)] We note also that even though some of the terms in (3.11) make a zero contribution, we shall use precisely this form of the equation, which will be convenient subsequently.

2. Integration over the energies

We now express the source of the obtained kinetic equations in terms of the energy-integrated Green's functions.[6,8] To this end we consider some term, say the first, in the expression for $\Pi_{\omega\,\omega-\omega'}(\mathbf{p},\mathbf{p}-\mathbf{p}')$, which follows from (3.1) when (3.11) is taken into account. In this term we can change from integration with respect to $d^3\mathbf{p}_1$ to integration over the angles and energies, on the basis of the relation

$$\frac{d^3\mathbf{p}_1}{(2\pi)^3}\approx\frac{mp_F}{2\pi^2}\frac{d\Omega_{p_1}}{4\pi}d\xi_1. \tag{3.12}$$

Using the auxiliary δ-function $\delta(\xi_{\mathbf{f}}-\xi_1+\mathbf{q}\cdot\mathbf{p}_1/m)$, we can introduce also integration with respect to the variable ξ_2, so that we can express the quantity

$$\frac{mp_F}{2\pi^2}\int\frac{d\Omega_{p_1}}{4\pi}\int\int d\xi_1\,d\xi_2\delta(\xi_2-\xi_1+\mathbf{q}\mathbf{p}_1/m)G_1\bar{G}_2 \tag{3.13}$$

in terms of the energy-integrated functions defined by relations of the type

$$g_{\varepsilon\varepsilon-\omega}(\mathbf{p},\mathbf{k})=\int_{-\infty}^{\infty}d\xi G_{\varepsilon\varepsilon-\omega}(\mathbf{p},\mathbf{p}-\mathbf{k}), \tag{3.14}$$

since the δ function in (3.13) limits mainly the integration with respect to angle (cf. Ref. 6) and can therefore be moved outside the integrals with respect to ξ. We have thus

$$\int\frac{d^3\mathbf{p}_1}{(2\pi)^3}G_1\bar{G}_2=\frac{mp_F}{2\pi^2}\frac{1}{2qv_F}\int\frac{d\Omega_{p_1}}{4\pi}\delta\left(\frac{\mathbf{qp}_1}{qp_1}\right)g_1\bar{g}_2. \tag{3.15}$$

Introducing, to abbreviate the subsequent notation, the operator M:

$$M=\frac{q^2}{2i}\frac{mp_F}{2\pi^2}\frac{1}{2qv_F}\int\frac{d\varepsilon_1d\omega_1d^3\mathbf{k}}{(2\pi)^5}\frac{d\Omega_{p_1}}{4\pi}\delta\left(\frac{\mathbf{qp}_1}{qp_1}\right), \tag{3.16}$$

as well as the symbol

$$[A,B]_+=A_{\varepsilon_1\varepsilon_2-\omega_1}(\mathbf{p}_1,\mathbf{k})B_{\omega-\varepsilon_1,\,\omega-\varepsilon_1+\omega_1-\omega'}(\mathbf{p}-\mathbf{p}_1,\mathbf{p}'-\mathbf{k})+B\cdot A,$$

we obtain the final expressions for the Fourier components of the quantities that enter in the source of the phonon kinetic equation:

$$\begin{aligned}\Pi_{\omega\omega-\omega'}&=M\{[g,\bar{g}]_+-[f,f^+]_++[g^R-g^A,\bar{g}^R-\bar{g}^A]_+\\&\quad-[f^R-f^A,f^{+R}-f^{+A}]_+\},\\(\Pi^R-\Pi^A)_{\omega\omega-\omega'}&=M\{[g,\bar{g}^R-\bar{g}^A]_++[\bar{g},\bar{g}^R-g^A]_+\\&\quad-[f,f^{+R}-f^{+A}]_+-[f^+,f^R-f^A]_+\}.\end{aligned} \tag{3.17}$$

We note that the collision integral (1.21) turned out to be expressed in terms of functions that do not depend on ξ. This enables us to use Eq. (1.19) even in those cases when the concept of the excitation spectrum becomes meaningless for the electron subsystem (ξ becomes a poor quantum number).

The electron functions contained in (3.17) (g, f, etc.) coincide, as is evident from the foregoing analysis, with those used in Ref. 6, which is devoted to a description of the electron subsystem of a superconductor in the presence of an electromagnetic field. Therefore our method leads to a closed system for describing the phonon kinetics.

Using the expressions given in Ref. 6 for the $\hat{g}$-functions, we can show that the kinetic equation (1.19), (1.21), (3.17) describes the relaxation of the phonon system to equilibrium as a result of inelastic collisions with electrons.

We consider next in this paper a spatially homogeneous case and use expressions for the functions $g^{R(A)}$ in an approximation diagonal in the energies

$$\begin{aligned}\hat{g}_\varepsilon&=2\pi i\begin{pmatrix}\varepsilon&\Delta\\-\Delta&-\varepsilon\end{pmatrix}\frac{\theta(\varepsilon^2-\Delta^2)}{(\varepsilon^2-\Delta^2)^{1/2}}(1-2f_\varepsilon)\operatorname{sign}\varepsilon,\\(\hat{g}^R-\hat{g}^A)_\varepsilon&=2\pi i\begin{pmatrix}\varepsilon&\Delta\\-\Delta&-\varepsilon\end{pmatrix}\frac{\theta(\varepsilon^2-\Delta^2)}{(\varepsilon^2-\Delta^2)^{1/2}}\operatorname{sign}\varepsilon\end{aligned} \tag{3.18}$$

[we have changed here from the excitation distribution function n_ε that is even in ε (Ref. 6) to the quasiparticle distribution function f (see, e.g., Ref. 10)]. It can be shown that in this case the collision integral takes the canonical form

$$\begin{aligned}I=\int_{-\infty}^{\infty}\int d\varepsilon_1\,d\varepsilon_2L(\varepsilon_1,\varepsilon_2)\theta(\varepsilon_1\varepsilon_2)\{&[(1+N_{\varepsilon_1})f_{\varepsilon_1}f_{\varepsilon_2}\\&-(1-f_{\varepsilon_1})(1-f_{\varepsilon_2})N_{\varepsilon_1}](1+\Delta^2/\varepsilon_1\varepsilon_2)\delta(\varepsilon_1+\varepsilon_2-\omega)\\+[(1+N_{\varepsilon_1})f_{\varepsilon_1}(1-f_{\varepsilon_2})&-N_{\varepsilon_1}(1-f_{\varepsilon_1})f_{\varepsilon_2}](1-\Delta^2/\varepsilon_1\varepsilon_2)\delta(\varepsilon_2-\varepsilon_1+\omega)\},\end{aligned} \tag{3.19}$$

$$L(\varepsilon_1,\varepsilon_2)=\frac{\pi\lambda}{2}\frac{\omega_D}{\varepsilon_F}\frac{\varepsilon_1\theta(\varepsilon_1^2-\Delta^2)}{(\varepsilon_1^2-\Delta^2)^{1/2}}\frac{\varepsilon_2\theta(\varepsilon_2^2-\Delta^2)}{(\varepsilon_2^2-\Delta^2)^{1/2}}$$

(here $\lambda=g^2mp_F/2\pi^2$ is a dimensionless constant of the electron-phonon interaction, $\lambda\sim1$; ω_D is the phonon Debye frequency), equivalent to that obtained by Bar'yakhtar *et al.*[9] by another method. Some seeming difficulty is that the terms of the type $\xi_1\xi_2/\varepsilon_1\varepsilon_2$ that are used in Ref. 9 are absent from the coherence factors in (3.19). Actually the contribution of these terms is identically equal to zero if it is recognized that $\varepsilon=(\Delta^2+\xi^2)^{1/2}$ (cf. Ref. 12).

4. NONEQUILIBRIUM PHONONS AND ELECTRONS IN THIN FILMS

1. Boundary conditions

The kinetic equation obtained above for phonons, together with the corresponding kinetic equations for the electron system, must generally speaking be supplemented also by boundary conditions for the phonon system. (In the electron system, the question of the boundary conditions is usually circumvented by assuming a large number of nonmagnetic impurities, so that the reflection of the electrons from the boundaries can be neglected.) We consider hereafter the process of phonon emission from superconducting films situated in a high-frequency electromagnetic field, and advance here arguments that permit simplification of the boundary-condition problem.

The propagation of phonons with frequency considerably higher than u/d (where d is the film thickness) can be considered in the "geometric-optics" approximation, when their behavior on the boundary is determined by the ratio of the "optical" densities $\rho_1 u_1/\rho_2 u_2$ of the media ($\rho_{1,2}$ are the densities of media 1 and 2). These phonons give up their entire energy in each collision with the wall (the latter is assumed to be smooth enough), i.e., they will be freely radiated to the outside if the phonon "optical density" of the outer medium ρu (ρ is the mechanical density) coincides with the "optical density" of the film metal. Imposing this lower limit on the frequencies of the considered phonons, we choose the superconducting-film thickness such that the phonon time of flight is less that the travel time connected with the interaction with the electrons, the order of magnitude of which in a bulky sample is[6] $v_F/u\omega(\mathbf{q})$.[5] Under this condition the model with the phonon thermostat becomes applicable for the electron subsystem (since the reaction of the phonons on the electrons can be neglected). Let, e.g., the film have a thickness d comparable with the correlation radius $\xi_0 \sim v_F/T_c$. For films of this thickness, the foregoing means that in the frequency region

$$T_c u/v_F \lesssim \omega(\mathbf{q}) \lesssim T_c \qquad (4.1)$$

it is possible to use directly the scheme developed for the phonons.

We note that we have neglected above everywhere the damping of the phonons, i.e., the phonon lifetime was assumed to be long enough. Assuming that the phonon is absorbed in each collision with the wall, this time can be estimated at $\tau \sim d/u$, and in order for the scheme developed by us to be applicable [see, in particular, formula (1.16)], we must have $\tau \gg 1/\omega(\mathbf{q})$. If we put $d \sim \xi_0$, then this condition takes the form $\xi_0/u \sim v_F/T_c u \gg 1/\omega(\mathbf{q})$, which coincides in fact with the condition for the applicability of the "geometric-optics" approximation [see the left-hand inequality of (4.1)]. Thus, the damping of phonons with frequencies in the interval at (4.1) can indeed be neglected.

2. Kinetics of electron subsystem in films

Bearing in mind that for sufficiently thin films the coupled system of kinetic equations becomes uncoupled, we assume that a film of an isotropic superconductor of thickness d comparable with the correlation radius ξ_0, irradiated by an external electromagnetic field, is in a thermostat with temperature T. The corresponding Eliashberg kinetic equations in matrix form now become ($\mathbf{v} = \mathbf{v}_F = \mathbf{p}_F/m$):

$$\begin{pmatrix} (\omega - \mathbf{vk})g & (2\varepsilon - \omega - \mathbf{vk})f \\ (2\varepsilon - \omega + \mathbf{vk})f^+ & -(\omega + \mathbf{vk})\bar{g} \end{pmatrix} = \begin{pmatrix} H_1 & 0 \\ 0 & \bar{H}_1 \end{pmatrix} \hat{g} - \hat{g} \begin{pmatrix} H_1 & 0 \\ 0 & \bar{H}_1 \end{pmatrix} + \hat{g}\hat{\Sigma}^A - \hat{\Sigma}^R \hat{g} + \hat{g}^R \hat{\Sigma} - \hat{\Sigma} \hat{g}^A, \qquad (4.2)$$

where

$$\hat{g}^{(R,A)} = \begin{pmatrix} g & f \\ -f^+ & \bar{g} \end{pmatrix}^{(R,A)}, \quad \hat{\Sigma}^{(R,A)} = \begin{pmatrix} \Sigma_1 & \Sigma_2 \\ -\Sigma_2^+ & \bar{\Sigma} \end{pmatrix}^{(R,A)},$$

$$H_1 = -\frac{e}{c}\mathbf{vA} + e\varphi, \quad \bar{H}_1 = \frac{e}{c}\mathbf{vA} + e\varphi, \qquad (4.3)$$

and the matrix product must be understood also as a convolution in the internal variables. The matrices $\hat{\Sigma}^{R,A}$, which are the self-energy parts of the electron interactions with phonons, impurities, and with one another, are in turn expressed through $\hat{g}$-function. We shall not present the corresponding expressions for them, nor for $\hat{g}^{R,A}$, so as not to clutter up the exposition (for details see Ref. 6). The solutions of (4.2), when substituted in the right-hand side of the kinetic equation (1.21) for the phonons, determine the phonon flux from the film to the outside, and an equilibrium phonon distribution function should be assumed in (1.21).

Let us recall briefly the assumptions under which the solutions of Eq. (4.2) are obtained.[7,15] It is assumed that the external electromagnetic field of frequency ω_0 is perpendicularly incident on the film and is described in a gauge with a vector potential $\mathbf{A}$ lying in the plane of the film. It is assumed for simplicity that the vector potential is constant over the film cross section. This means that the film is thin, (less than the depth of penetration of the field). We note, however, that the condition that $\mathbf{A}$ be constant over the cross section is not critical (from the point of view of homogeneity of the picture over the film thickness), owing to the large diffusion length of the nonequilibrium electronic excitations. It is assumed also that the density of the nonmagnetic impurities is high enough. The dynamics of the electron system then simplifies, and in particular, the electron mean free path l becomes small ($l = v_F\tau \ll d$) and it is possible to disregard the reflection of the electrons from the walls. Even when all these simplifying circumstances are taken into account, the kinetic equations (4.2) are quite complicated. They can be further simplified if $\omega_0\tau_0 \gg 1$, where τ_0 is a certain effective excitation-energy relaxation time. In conjunction with the locality conditions $\omega_0\tau \ll 1$ and $\tau\Delta \ll 1$, it enables us to show that the decisive role for the electron subsystem is played by the functions $\hat{g}_\varepsilon = \hat{g}_{\varepsilon\varepsilon} = 2\pi\delta(\omega)\hat{g}_{\varepsilon\varepsilon-\omega}$, which are diagonal in the energies and the equations for which take into account the successive one-photon transitions [the probability, e.g., of two-photon transitions has a smallness $(\omega_0\tau_0)^{-2}$]. We use next the solutions given in Refs. 7 and 15 for these equations to calculate the phonon emission from the films.

5. EFFECT OF "PHONON DEFICIT" INDUCED IN SUPERCONDUCTING FILMS BY A HIGH-FREQUENCY FIELD

Remaining within the framework of the approximations made above, we impose on the electromagnetic-field frequency the limitation $\omega_0 < 2\Delta$. It will be formally assumed hereafter that the parameter ω_0/Δ is small, but actually the results, as will be made clear later, remain meaningful also at $\omega_0 > 2\Delta$.

1. Nonequilibrium electron distribution

We shall use below for the nonequilibrium electronic component of the superconductor in the stationary state the expression obtained in Refs. 7 and 15. We repeat here some essential factors connected with this solution. Using the definition (3.18) we can obtain on the basis of (4.2), taking into account the conditions listed in Sec. 4.2, a kinetic equation for n_ε ($n_\varepsilon \equiv f$ at $\varepsilon > \Delta$) in the form

$$0=2(e/c)^2 D\mathbf{A}_{\omega_0}\mathbf{A}_{-\omega_0}[U_{\varepsilon\varepsilon-\omega_0}(n_{\varepsilon-\omega_0}-n_\varepsilon)-U_{\varepsilon+\omega_0\varepsilon}(n_\varepsilon-n_{\varepsilon+\omega_0})]+I_\varepsilon, \quad (5.1)$$

where $\varepsilon > \Delta$ and we have designated by $D = v_F l/3$ the diffusion coefficient, and

$$U_{\varepsilon\varepsilon-\omega_0} = \frac{\varepsilon(\varepsilon-\omega_0)+\Delta^2}{[(\varepsilon-\omega_0)^2-\Delta^2]^{1/2}(\varepsilon^2-\Delta^2)^{1/2}}\theta(\varepsilon-\omega_0-\Delta). \quad (5.2)$$

The collision integral I has a canonical form[6,7] and contains collisions of the electrons with one another and with phonons, while the collisions with the impurities drop out in the derivation of (5.1). (The latter is the result of the assumption that these collisions are elastic, an important factor when averaging over the angles.) We do not present here the expressions for I_ε in the general case, since we shall use hereafter only the solutions obtained in Ref. 7 for (5.1) in the relaxation-time approximation.

The field term of the kinetic equation (5.1) can be written in the form

$$\alpha\left\{-\omega\frac{dn^{(0)}}{d\varepsilon}(U_{\varepsilon\varepsilon-\omega_0}-U_{\varepsilon+\omega_0\varepsilon})+\frac{\omega_0^2}{2}\frac{d^2n^{(0)}}{d\varepsilon^2}(U_{\varepsilon\varepsilon-\omega_0}+U_{\varepsilon+\omega_0\varepsilon})\right\}. \quad (5.3)$$

We have introduced here the symbol $\alpha = D(e/c)^2\mathbf{A}_{\omega_0}\mathbf{A}_{-\omega_0}$, while the derivatives $dn^{(0)}/d\varepsilon$ and $d^2n_\varepsilon^{(0)}/d\varepsilon^2$ are obtained by expanding the quantities in the parentheses in (5.1) (this procedure is generally speaking meaningful at $\omega_0 \ll \omega$), and then using an approximation by the equilibrium value $n_\varepsilon^{(0)} = [1+\exp(|\varepsilon|/T)]^{-1}$. Using (5.2), we easily see that the first of the field terms in (5.3) differs from zero in the region $\varepsilon \sim \Delta$, while the second varies in the region $\varepsilon \sim T$. If we are interested only in the linearized equation, then we can assume that the change of the distribution function also breaks up into corresponding parts, $n^{(1)} = n_1^{(1)} + n_2^{(1)}$. In the relaxation-time approximation we can write for the collision integral in (5.1) (Refs. 7, 15)

$$I_\varepsilon \approx -\frac{1}{\tau_0}n^{(1)}\frac{\varepsilon}{(\varepsilon^2-\Delta^2)^{1/2}}, \quad (5.4)$$

which leads to the following solution for $n^{(1)}$:

$$n_1^{(1)} = \alpha\tau_0\left\{\frac{\omega_0}{2T}\left(\mathrm{ch}\frac{\varepsilon}{2T}\right)^{-2}(U_{\varepsilon\varepsilon-\omega_0}-U_{\varepsilon+\omega_0\varepsilon})\right\}\frac{(\varepsilon^2-\Delta^2)^{1/2}}{\varepsilon}, \quad \varepsilon > \Delta. \quad (5.5)$$

It can be shown by iteration that the corrections to $n_1^{(1)}$ are of the order of Δ/T at $\varepsilon \sim \Delta$, but in contrast to (5.5) they are different from zero in the interval $\varepsilon \sim T$. The quantity $n_2^{(1)}$ is of the same order of smallness. As a result it can be assumed that the change of the distribution function $n^{(1)} = n_\varepsilon - n_\varepsilon^{(0)}$ consists of the quantity $n_1^{(1)}$ (5.5) and a small increment ("tail," which drops off at $\varepsilon \sim T$). The relaxation-time approximation is insufficient for the calculation of this small increment. In our case, however, this increment is immaterial, since we shall operate hereafter with expressions in which the decisive role is played by the region $\varepsilon \sim \Delta \ll T$ (at $T \sim T_c$), where this correction can be neglected compared with $n_1^{(1)}$ (5.5). It is therefore legitimate to use the τ-approximation in the vicinity of the transition temperature. Analysis[7] shows that the linear approximation (5.5) is valid up to an external-field intensity α given by

$$\alpha \leqslant \gamma, \quad (5.6)$$

where γ coincides with double the excitation damping determined by the single-particle Green's function [it has the same value as in a normal metal accurate to $(\Delta/T)^2$]. It is precisely this case, when condition (5.6) is satisfied, which we shall have in mind in the present study.

2. Classification of phonon sources

Within the framework of the approximations made for the electron subsystem and in the approximation linear in the external-field intensity, expression (1.19) is written, with allowance for (3.19), in the form

$$\frac{\partial N(\omega_q,t)}{\partial t} = 2\int_\Delta^\infty\int L(\varepsilon_1,\varepsilon_2)\{(n_{\varepsilon_1}^{(1)}-n_{\varepsilon_2}^{(1)})N_{\omega_q}^{(0)}-n_{\varepsilon_1}n_{\varepsilon_2}^{(1)} - n_{\varepsilon_2}n_{\varepsilon_1}^{(1)}+n_{\varepsilon_1}^{(1)}\}\left(1-\frac{\Delta^2}{\varepsilon_1\varepsilon_2}\right)\delta(\varepsilon_1-\varepsilon_2-\omega_q)d\varepsilon_1 d\varepsilon_2$$
$$+\int_\Delta^\infty\int L(\varepsilon_1,\varepsilon_2)\{(n_{\varepsilon_1}^{(1)}+n_{\varepsilon_2}^{(1)})N_{\omega_q}^{(0)}+n_{\varepsilon_1}n_{\varepsilon_2}^{(1)}+n_{\varepsilon_2}n_{\varepsilon_1}^{(1)}\} \times\left(1+\frac{\Delta^2}{\varepsilon_1\varepsilon_2}\right)\delta(\varepsilon_1+\varepsilon_2-\omega)d\varepsilon_1 d\varepsilon_2, \quad (5.7)$$

where

$$\omega_q = u|\mathbf{q}|, \quad N_\omega^{(0)} = (e^{\omega/T}-1)^{-1}. \quad (5.8)$$

As follows from the foregoing, expression (5.7) determines directly the phonon flux in the frequency region (4.1) if the parameters (ρu) of the outer medium and of the film coincide. Generally speaking, the problem of the boundary condition is not so simple (on this subject, see, e.g., the bibliography of the review[16]). In this respect we adhere to the approach developed in Sec. 4 and assume that expression (5.7) determines the phonon fluxes directly. In the more general case, when explicit account of the processes on the boundary is necessary, one can treat this expression as representing an internal source of nonequilibrium phonons.

The source (right-hand side) in expression (5.7), which determines the kinetics of the phonons, can be classified in the following manner. The first integral in (5.7), which is connected with the relaxation processes in the electron subsystem, will be called the relaxation source (I_{rel}). The second integral is connected

with the processes of excitation recombination and the "pair breaking" of the condensate. It can be called the recombination source (I^{rec}). Each of these sources consists of an aggregate of terms proportional to the phonon occupation numbers, and terms of another type which do not contain explicitly this proportionality. The aggregate of the first type can be called the induced source (I_{ind}) and that of the second type the spontaneous phonon source (I_{sp}). By the same token, the entire source can be represented in the form

$$I=I_{ind}^{rel}+I_{sp}^{rel}+I_{ind}^{rec}+I_{sp}^{rec}, \tag{5.9}$$

which is convenient for analysis.

We note first that at frequencies $\omega_q < \Delta$ the following relation holds in the vicinity of the transition temperature

$$N_{\omega_q}^{(0)}/n_s \gg 1 \tag{5.10}$$

and consequently the spontaneous sources in (5.9) are small in this frequency region and can be left out. We are interested in the spectral dependence $I=I(\omega_q)$ for different values of the frequency ω_0 of the external electromagnetic field. It turns out that this dependence can be established analytically in the most interesting region of the spectrum ω_q comparable with Δ if $\omega_0 \ll \Delta$. Further study of the behavior of the relaxation and recombination sources is best carried out separately. A general analysis of the results and a discussion of their physical meaning is given in subsection 5 of the present section.

3. Recombination source

We consider first the recombination term I_{ind}^{rec}. It can be represented in the form

$$I_{ind}^{rec}=2N_{\omega_q}^{(0)}\int_{\Delta}^{\infty}\int L(\varepsilon_1,\varepsilon_2)n_{\varepsilon_1}^{(1)}\left(1+\frac{\Delta^2}{\varepsilon_1\varepsilon_2}\right)\delta(\varepsilon_1+\varepsilon_2-\omega_q)\,d\varepsilon_1 d\varepsilon_2. \tag{5.11}$$

Using (5.5), we can write

$$I_{ind}^{rec}=\frac{\alpha\tau_0\omega_0}{T}N_{\omega_q}^{(0)}\int_{\Delta}^{\infty}L(\varepsilon_1,\omega_q-\varepsilon_1)\left(\mathrm{ch}\frac{\varepsilon_1}{2T}\right)^{-2}(U_{\varepsilon_1\varepsilon_1-\omega_0}$$
$$-U_{\varepsilon_1+\omega_0\varepsilon_1})\left(1+\frac{\Delta^2}{\varepsilon_1\varepsilon_2}\right)\frac{(\varepsilon_1^2-\Delta^2)^{1/2}}{\varepsilon_1}d\varepsilon_1, \tag{5.12}$$

which reduces, after simple transformations with allowance for (5.2) and (5.8), to the form

$$I_{ind}^{rec}=\frac{\pi\lambda\tau_0\omega_D}{2\varepsilon_F\omega_q}\alpha\omega_0[B_1(\omega_q)-B_2(\omega_q)], \tag{5.13}$$

where

$$B_1(\omega_q)=\int_{\Delta+\omega_0}^{\omega_q-\Delta}\frac{A(\varepsilon,\omega_0)}{(\omega_q-\varepsilon-\Delta)^{1/2}(\varepsilon-\omega_0-\Delta)^{1/2}(\varepsilon-\Delta)^{1/2}}\theta(\omega_q-2\Delta-\omega_0)\,d\varepsilon,$$
$$B_2(\omega_q)=\int_{\Delta}^{\omega_q-\Delta}\frac{A(\varepsilon,-\omega_0)}{(\omega_q-\varepsilon-\Delta)^{1/2}(\varepsilon+\omega_0-\Delta)^{1/2}(\varepsilon-\Delta)^{1/2}}\theta(\omega_q-2\Delta)\,d\varepsilon, \tag{5.14}$$

while the function $A(\varepsilon,\omega_0)$ is defined by the relation

$$A(\varepsilon,\omega_0)=\frac{[\varepsilon(\omega_q-\varepsilon)+\Delta^2][\varepsilon(\varepsilon-\omega_0)+\Delta^2]}{\varepsilon(\omega_q-\varepsilon+\Delta)^{1/2}(\varepsilon-\omega_0+\Delta)^{1/2}(\varepsilon+\Delta)^{1/2}}. \tag{5.15}$$

Results in this form already reveals two important spectral features of the source I_{ind}^{rec}. First, there exists a minimum threshold frequency of nonequilibrium phonons (equal to double the gap), above which the source (5.13) becomes different from zero. Second, inasmuch as $B_2(\omega_q)$ [which is positive, as is seen from (5.14) and (5.15)] enters in the source (5.13) with a minus sign, and since it differs from zero in a certain frequency interval, where $B_1(\omega_q)$ is equal to zero, we can conclude that there exists a region of frequencies ω_q in which I_{ind}^{rec} is negative. This result is at first glance somewhat unexpected. It is therefore of interest to study in greater detail this "anomalous" behavior of the recombination source.

It is necessary for this purpose to calculate the integrals (5.14). This can be done approximately by taking into account the smoothness of the function $A(\varepsilon,\omega_0)$ in the integration region. This circumstance enables us to apply the mean-value theorem and represent (5.14) in the form

$$B_1(\omega_q)=A(\bar{\varepsilon},\omega_0)\frac{2}{(\omega_q-2\Delta)^{1/2}}K\left(\left[\frac{\omega_q-2\Delta-\omega_0}{\omega_q-2\Delta}\right]^{1/2}\right)\theta(\omega_q-2\Delta-\omega_0),$$
$$B_2(\omega_q)=A(\tilde{\varepsilon},-\omega_0)\frac{2}{(\omega_q-2\Delta+\omega_0)^{1/2}}K\left(\left[\frac{\omega_q-2\Delta}{\omega_q-2\Delta+\omega_0}\right]^{1/2}\right)\theta(\omega_q-2\Delta), \tag{5.16}$$

where $\bar{\varepsilon}$ and $\tilde{\varepsilon}$ lie respectively in the regions $(\Delta+\omega_0, \omega_q-\Delta)$ and $(\Delta, \omega_q-\Delta)$, while $K(k)$ is a complete elliptic integral of the first kind in normal form,[20] and appears on going from (5.14) to (5.16). For this function we have

$$K(k)\approx\frac{\pi}{2}\left(1+\frac{k^2}{4}\right),\quad k^2\ll 1;\quad K(k)\approx\ln\frac{4}{(1-k^2)^{1/2}}\quad k^2\ll 1. \tag{5.17}$$

Of greater interest for the sequel are the frequencies $\omega_q-2\Delta$ of scale ω_0. We note that for these frequencies the calculations obtained by the mean value theorem are asymptotically exact in the case $\omega_0 \ll \Delta$. It is precisely this case which we shall bear in mind. We therefore put $\bar{\varepsilon}=\tilde{\varepsilon}=\Delta$ [in which case $A(\bar{\varepsilon},\omega_0)\approx A(\tilde{\varepsilon}-\omega_0)\approx 2^{1/2}\Delta^{3/2}$] and obtain from (5.16) and (5.15) the threshold value of the function $B_2(\omega_q)$ and $\omega_q=2\Delta$. As a result we get

$$B_2(\omega_q=2\Delta)\approx 2^{1/2}\pi\Delta(\Delta/\omega_0)^{1/2}. \tag{5.18}$$

From this expression, together with (5.13) which determines the depth of the dip on the spectral curve of the recombination source (see Fig. 3), it follows that the dip becomes deeper when the frequency of the external field ω_0 increases. An investigation of expression (5.16) shows that $B_2(\omega_q)$ decreases with increasing frequency ω_q and assumes at $\omega_q-2\Delta=\omega_0$, accurate to ω_0/Δ, the value (we use here the numerical values of the coefficients)

$$B_2(\omega_q=2\Delta+\omega_0)\approx 0{,}8B_2(\omega_q=2\Delta). \tag{5.19}$$

With further increase of ω_0, the term $B_1(\omega_q)$ comes into play, and accurate to ω_0/Δ its threshold value coincides with that of $B_2(\omega_q)$, i.e.,

$$B_1(\omega_q=2\Delta+\omega_0)\approx B_2(\omega_q=2\Delta). \tag{5.20}$$

From this, taking (5.19) into account, it follows directly that the difference $B_1(\omega_q)-B_2(\omega_q)$ is positive at $\omega_q=2\Delta+\omega_0$ and amounts to approximately one-fifth of $B_2(\omega_q)$ at $\omega_q=2\Delta$. Inasmuch as with further increase of ω_q each of the quantities $B_1(\omega_q)$ and $B_2(\omega_q)$, and consequently also their difference, tends to zero, we ar-

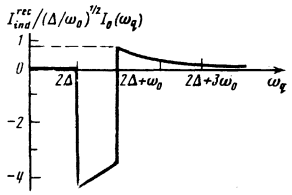


FIG. 2. Spectral dependence of the recombination source of the phonons.

rive at the conclusion that the function I_{ind}^{rec} has a "spike" (maximum). An investigation of the expression

$$B(\omega_q)=B_1(\omega_q)-B_2(\omega_q)$$
$$=2^{1/2}\Delta^{3/2}\left\{\frac{1}{(\omega-2\Delta)^{1/2}}\times K\left(\left[\frac{\omega-2\Delta-\omega_0}{\omega-2\Delta}\right]^{1/2}\right)\right.$$
$$\left.-\frac{1}{(\omega-2\Delta+\omega_0)^{1/2}}K\left(\left[\frac{\omega-2\Delta}{\omega-2\Delta+\omega_0}\right]^{1/2}\right)\right\} \qquad (5.21)$$
$$\omega\geqslant 2\Delta+\omega_0,$$

shows that the half-width of the "spike" is of the order of ω_0. The results of the calculation of the source I_{ind}^{rec} are illustrated in Fig. 2. (These results will be discussed in subsection 5).

4. Relaxation source

We proceed to consider the relaxation source I_{ind}^{rel}, which is represented on the basis of (5.7) in the form

$$I_{ind}^{rel}=2N_\omega\left\{\int_{\Delta+\omega_q}^{\infty}L(\varepsilon_1,\varepsilon_1-\omega_q)n_{\varepsilon_1}^{(1)}\left(1-\frac{\Delta^2}{\varepsilon_1(\varepsilon_1-\omega_q)}\right)d\varepsilon_1\right.$$
$$\left.-\int_{\Delta}^{\infty}L(\varepsilon_2+\omega_q,\varepsilon_2)n_{\varepsilon_2}^{(1)}\left(1-\frac{\Delta^2}{(\varepsilon_2+\omega_q)\varepsilon_2}\right)d\varepsilon_2\right\} \qquad (5.22)$$

or else, taking into account (5.8) and the relation $\omega_q \ll T$, as well as the obvious transformations, in the form

$$I_{ind}^{rel}=\pi\lambda\frac{T\omega_D}{\varepsilon_F\omega_q}\int_{\Delta}^{\infty}\frac{[(\varepsilon_1+\omega)\varepsilon_1-\Delta^2][n_{\varepsilon_1+\omega_q}^{(1)}-n_{\varepsilon_1}^{(1)}]}{(\varepsilon_1^2-\Delta^2)^{1/2}[(\varepsilon_1+\omega_q)^2-\Delta^2]^{1/2}}d\varepsilon_1, \qquad (5.23)$$

where $n_\varepsilon^{(1)}$ is defined in (5.5). The succeeding analysis is perfectly similar to that for the recombination source. Leaving out the details of the calculations, we present the results in the case $\omega_0 \ll \Delta$ for the source (5.23):

$$I_{ind}^{rel}=\pi\lambda\frac{\alpha\omega_D}{2\varepsilon_F\omega_q}\tau_0\omega_0\sum_{i=1}R_i(\omega_q,\omega_0), \qquad (5.24)$$

where

$$R_1(\omega,\omega_0)=\frac{2}{\omega^{1/2}}K\left(\left[\frac{\omega_0}{\omega}\right]^{1/2}\right)\varphi_1(\varepsilon\approx\Delta),\quad \omega>\omega_0,$$
$$R_1(\omega,\omega_0)=\frac{2}{\omega_0^{1/2}}K\left(\left[\frac{\omega}{\omega_0}\right]^{1/2}\right)\varphi_1(\varepsilon\approx\Delta+\omega_0-\omega),\quad \omega<\omega_0,$$
$$R_2(\omega,\omega_0)=-\frac{2}{(\omega+\omega_0)^{1/2}}K\left(\left[\frac{\omega_0}{\omega_0+\omega}\right]^{1/2}\right)\varphi_2(\varepsilon\approx\Delta),$$
$$R_3(\omega,\omega_0)=-\frac{2}{(\omega+\omega_0)^{1/2}}K\left(\left[\frac{\omega}{\omega_0+\omega}\right]^{1/2}\right)\varphi_3(\varepsilon\approx\Delta+\omega_0), \qquad (5.25)$$
$$R_4(\omega,\omega_0)=\frac{2}{\omega^{1/2}}K\left(\left[\frac{\omega-\omega_0}{\omega}\right]^{1/2}\right)\varphi_4(\varepsilon\approx\Delta),\quad \omega>\omega_0,$$
$$R_4(\omega,\omega_0)=\frac{2}{\omega_0^{1/2}}K\left(\left[\frac{\omega_0-\omega}{\omega}\right]^{1/2}\right)\varphi_4(\varepsilon\approx\Delta),\quad \omega<\omega_0,$$

and the quantities $\varphi_i(\varepsilon)\equiv\varphi(\varepsilon;\omega,\omega_0)$, which arise when the mean-value theorem is used, are equal to

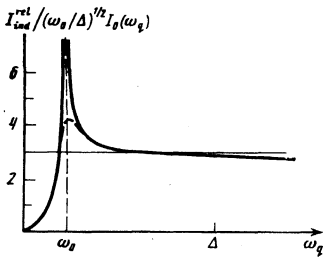


FIG. 3. Spectral dependence of the relaxation source of the phonons.

$$\varphi_1(\varepsilon;\omega,\omega_0)=\frac{[(\varepsilon+\omega)\varepsilon-\Delta^2][(\varepsilon+\omega)(\varepsilon+\omega-\omega_0)+\Delta^2]}{(\varepsilon+\omega)(\varepsilon+\Delta)^{1/2}(\varepsilon+\omega-\omega_0+\Delta)^{1/2}(\varepsilon+\omega+\Delta)^{1/2}}$$
$$\varphi_3(\varepsilon;\omega,\omega_0)=\frac{[(\varepsilon+\omega)\varepsilon-\Delta^2][\varepsilon(\varepsilon-\omega_0)+\Delta^2]}{\varepsilon(\varepsilon+\Delta)^{1/2}(\varepsilon+\omega+\Delta)^{1/2}(\varepsilon-\omega_0+\Delta)^{1/2}} \qquad (5.26)$$
$$\varphi_{2(4)}(\varepsilon,\omega,\omega_0)=\varphi_{1(3)}(\varepsilon;\omega,-\omega_0).$$

As is evident from the presented expressions, the function I_{ind}^{rel} has no threshold singularities. By investigating (5.24)–(5.26) we can verify that the relaxation source vanishes (from the positive side) at the start of the spectrum ($\omega_q \approx 0$). At larger values of the argument, I_{ind}^{rel} increases and is characterized at $\omega_q=\omega_0$ by a logarithmic divergence. Further increase of the argument is accompanied by a decrease, first logarithmic [the term $R_1(\omega,\omega_0)$] and then in power-law fashion and, for example at $\omega_q \approx \Delta$, the source is given by $I_{ind}^{rel}\approx 3(\omega_0/\Delta)^{1/2}$ (in units of $I_0=\pi\lambda\alpha\tau_0\omega_0\omega_D/2\varepsilon_F\omega_q$). The foregoing is illustrated in Fig. 3.

Thus, the behavior of the two contributions I^{rec} and I^{rel} has been established in the entire phonon-frequency spectral range of interest to us.

5. Discussion of the phonon-deficit effect

The results obtained in subsections 3 and 4 are graphically combined in Fig. 4a. We note first that the regions of action of the relaxation and recombination sources in the case considered us, when the external-radiation frequencies are low compared with the gap, do not overlap in practice. For this reason the total phonon flux $I=I^{rec}+I^{rel}$ is negative in the frequency interval $2\Delta \ll \omega_q \leqslant 2\Delta+\omega_0$ ("dip" on Fig. 4a). This means that a superconducting film acted upon by high-frequency electromagnetic radiation should absorb phonons selectively. The origin of this effect is closely connected with the processes that cause superconductivity stimulated by a microwave field.[19] In fact, at

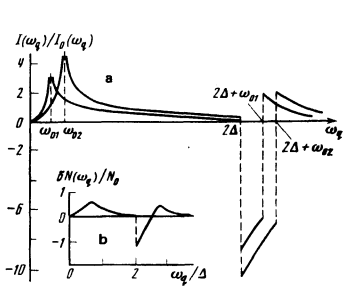


FIG. 4. Spectrum of phonon emission from a thin film. The negative "dips" on the curves correspond to the phonon-deficit effect.

a temperature $T \neq 0$ the superconductor always contains a definite number of excitations above the gap, and these excitations are in thermodynamic equilibrium with the phonons. The phonons, whose energy is approximately double the gap, produce effectively quasiparticles that recombine and emit phonons of the same type. We note that in accordance with the detailed balancing principle the probabilities of the direct and inverse processes are equal. The situation changes when an external high-frequency electromagnetic field is turned on. It is known (see, e.g., Ref. 15) that if the frequency of the latter does not exceed the quasiparticle production threshold, then the action of the microwave field reduces mainly to a change of the "center of gravity" of the distribution function of the quasiparticles, whose number above the gap becomes less than the thermodynamic equilibrium value (in other words, an effective "supercooling" of the quasiparticles takes place). This means violation of the detailed balancing when the excitations interact with phonons in the presence of an external field, as a result of which the probability of absorption of phonons with frequency 2Δ becomes larger than the probability of their emission. This produces in the indicated phonon-frequencies a phonon deficit, which can be replenished from the outside in the presence of a connection with the external medium, and it is this which leads to the negative phonon flux (the entire picture remaining stationary).

We note in this connection that an experimental observation of phonon absorption could be regarded not only as a direct confirmation of the theory developed above, but also as one more proof of the validity of the premises used in the construction of the theory of stimulated superconductivity.

A few words now on the paper of Chang and Scalapino.[14] They have introduced in the kinetic equation for the phonons an additional term (of the type $\delta N/\tau_{es}$, where τ_{es} is the time of escape of the nonequilibrium phonons from the film), which takes into account the coupling of the phonons to the outer medium. Since in free exchange ($\tau_{es} \to 0$) the phonon deficit produced by the external field is completely offset by the influx from the outside ($\delta N \to 0$), all that has been noted in Ref. 14 is that an equilibrium phonon distribution is produced in the interior of the film. This equilibrium, however, is dynamic and is directly related with the presence of phonon fluxes of opposite sign. These phonon fluxes were not calculated in Ref. 14. Figure 4b shows the curve obtained in Ref. 14 by numerical calculation for $\delta N(\omega_q)$ at $\tau_{es} \neq 0$, from which it is seen that negative values $\delta N(\omega_q) < 0$ are present. The shape of this curve points to the presence of a phonon deficit inside the film, and also points indirectly to the presence of negative phonon fluxes (Fig. 4a).

As for the spike on the central curve (Fig. 4), it is due at frequencies $\omega_q \sim 2\Delta + \omega_0$, as indicated in Ref. 14, to the fact that the quasiparticles displaced by the external field from the edge of the gap by an energy ω_0 recombine with the quasiparticles remaining at the edge. In our analysis this is particularly evident. Indeed, as indicated in Sec. 4, the considered situation is characterized by one-quantum transitions. Consequently, the excess of the quasiparticles is produced mainly at energies $\varepsilon \approx \Delta + \omega_0$. As seen from (5.5), the increment to the distribution function of the quasiparticles has a singularity at these energies. However, since we have confined ourselves to an approximation linear in the intensity of the external field, it follows from (5.7) that the principal role is played here by recombination of nonequilibrium quasiparticles of energy $\varepsilon \approx \Delta + \omega_0$ with equilibrium quasiparticles of energy $\varepsilon \approx \Delta$ at the edge of the gap, where the density of the electronic states is large. As a result, phonons with energy $\omega_q \approx 2\Delta + \omega_0$ are obtained.

In conclusion, we discuss the logarithmic divergence of the phonon (relaxation) flux at the frequency $\omega = \omega_0$ (Figs. 3 and 4a). This formal divergence is connected with a singularity in the density of the electronic states [see, e.g., (3.19)], which is typical of superconductors in the absence of external fields. Such a logarithmic divergence vanishes if account if taken of the "smoothing" influence of the electromagnetic field on the state density of the quasielectrons. In other words, the approximation diagonal in the energies for the $\hat{g}$ functions, which caused the transition from the general expression (1.19) to the canonical form (3.19) for the collision integrals, must be supplemented by allowance for definite off-diagonal terms. [We note that allowance of certain similar off-diagonal field terms was made in Refs. 7 and 16 when the nonequilibrium distribution function of the electronic excitations (5.5) was determined on the basis of Eq. (4.2).] There exist also other ways of eliminating divergences of this kind. Thus, for example, in Ref. 14 the divergence was eliminated by artificially smoothing out the singularities in the electronstate density. In Ref. 21, the divergence of the nonequilibrium increment to the distribution function (at energy $\varepsilon = \Delta + \omega_0$) was eliminated on the basis of considerations connected with the Pauli principle. One can indicate one other method of eliminating this divergence. The point is that the frequency of the emitted phonons is determined accurate to their damping, which in order of magnitude is comparable with the damping of phonons in a bulky normal sample,[5] i.e., $\delta\omega_q \sim \omega_q u/v_F$. Substituting this value in the expression for $R_1(\omega, \omega_0)$, which determines the divergence of the relaxation source (5.24), we obtain $I_{ind}(\omega \approx \omega_0) \approx I_0(\omega_0/2\Delta)^{1/2}\ln(v_F/u)$. The dashed lines in Fig. 3 shows the corresponding cutoff.

More accurate estimates can also be made, but they do not influence the results to any appreciable degree.

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[1] We omit hereafter the index λ connected with the phonon polarization. Allowance for the polarization, as will be made clear later, leads to a simple addition of the index λ to the phonon quantities in the final expressions.

[2] We note in this connection the paper by Prange and Kadanoff,[13] who used a different approach to the problem of electron-phonon interaction.

[3] Indeed, by changing from the expression for the Green's function (2.5) to the phonon distribution function (see Ref. 2) we can verify by direct summation over the frequencies that the resultant distribution function coincides in the case of integer n with the equilibrium Bose distribution: $N_\omega^0 = [\exp(\omega/T) - 1]^{-1}$.

[4] Such a schematic representation of the nonequilibrium phonon Green's function was used in Ref. 7 and turns out to be quite useful in the discussion of many physical problems.

[5] We did not dwell in detail on the proof on this statement. A more detailed exposition of this problem as well as of other aspects touched upon in the paper will be given elsewhere.

[6] This estimate is satisfactory in those cases when the frequency of the emitted phonons is less than the threshold of quasiparticle production in the superconductor.

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Nonlocal electron-interaction effects in the spontaneous-current model

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We consider the influence of hybridization of the electron bands of a semimetal on the symmetry of the order parameter $\Delta(\mathbf{k})$ of the exciton phase. It is shown that an arbitrarily small hybridization suppresses the symmetrical component $\Delta_s(\mathbf{k})$ of the order parameter in a one-dimensional system with weakly screened Coulomb interaction of the electrons and holes. In a three-dimensional system one can indicate for the hybridization a limit below which a first-order phase transition is possible with formation of a symmetrical component $\Delta_s(\mathbf{k}) = \Delta_s(-\mathbf{k})$. One of the possibilities of formation of an inhomogeneous state for an excitonic dielectric is noted. The results of the study have a direct bearing on the spontaneous-current model.

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INTRODUCTION

An indispensable part of the model of spontaneous currents in an excitonic dielectric[1] is the presence of interband dipole transitions in the system. If an electron-hole condensate with an imaginary component of the order parameter appears in a semimetal (semiconductor) system, then a macroscopic electron current can flow in the presence of interband dipole transitions in this system.[1] This, however, still leaves open the question of the influence of the interband dipole transitions of the electrons on the phase transition of the semimetal (semiconductor) into the state of an excitonic dielectric with imaginary order parameter. As will be shown in the present paper this influence is particularly important when the electron-hole interaction is nonlocal.

The most widely used and simplest description of electron bands, which takes into account the interband dipole transitions, is the approximation of Luttinger and Kohn.[2] In this approximation these transitions are represented in the form of band hybridization. The hybridization is expressed in the Hamiltonian in the

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