Light μ^- atoms in liquid and gaseous hydrogen and deuterium

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De-excitation and Stark mixing in $\mu^- p$ and $\mu^- d$ atoms are considered. The populations of the atomic levels, the intensities of x-ray transitions, and the cascade times are calculated.

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§1. INTRODUCTION

The aim of the present paper is to give a detailed description of cascade processes in $\mu^- p$, $\mu^- d$, and $\mu^- t$, which are the simplest of the exotic atoms consisting of two particles with opposite electric charges. In contrast to heavy exotic atoms with charge $Z>1$ of the nucleus, these atoms do not have their own electrons. Because of their small size and electrical neutrality, they can readily pass through ordinary atoms, when they are subject to the strong intraatomic electric fields, which induce intense transitions between different sublevels nl. Thus, besides the de-excitation processes in the light exotic atoms there is strong Stark mixing, and this makes the cascade process in them extremely complicated and sensitive to the surrounding medium.

The effects of Stark mixing were studied earlier in the mesic atoms $\pi^- p$ and $K^- p$ (Refs. 1 and 2) and antiproton atoms $\bar{p}p$ and $\bar{p}d$,[3,4] where they lead to strong absorption of hadrons in high orbits (the Day–Snow–Sucher[1] mechanism). In contrast to hadronic atoms, there is no nuclear absorption in μ^- atoms, and the muon therefore reaches the ground state with a probability near unity; the atoms $\mu^- p$, $\mu^- d$, and $\mu^- t$ are therefore "pure" objects for studying the influence of Stark mixing on the de-excitation processes. The experimental characteristics of these atoms can serve as a test of the existing models of atomic cascades in light exotic atoms.

Hitherto, cascade processes in light μ^- atoms of the type $\mu^- p$ have been little studied. The available experimental data are limited to measurements of the ratio of the yield of the x-ray K_α line of the atom $\mu^- p$ to the total intensity of the lines of the K series in gaseous and liquid hydrogen.[5,6] Theoretical calculations have been made on the basis of the effective-rate method of the intensities of x-ray transitions for the $\mu^- p$ atom in liquid and gaseous hydrogen with density $N \geqslant 10^{-3} N_0^{\mathrm{H}}$ (N_0^{H} is the density of the liquid hydrogen) (Ref. 7) and of the rates of Stark collisions (in particular,[8] the rate of Stark transition from the metastable $2S$ state).

However, the renewed interest in μ^- catalysis of the nuclear synthesis $d+t \to {}^4\mathrm{He}+n$ reaction[9] requires more detailed information about the properties of the mesic atoms $\mu^- d$ and $\mu^- t$. According to Ref. 10, the mesic molecule $dt\mu^-$ has a weakly bound level whose existence leads to a very high rate of formation of this mesic molecule: In a mixture of deuterium and tritium at liquid hydrogen density, it exceeds the decay rate of the free μ^- meson by two orders of magnitude. This gives grounds for believing that a single μ^- meson is capable of catalyzing $\sim 10^2$ nuclear synthesis reactions in accordance with the scheme

$$\mu^- d+t \to \mu^- t+d \to dt\mu^- \to {}^4\mathrm{He}+n+17.6\ \mathrm{MeV}.$$

To get a more detailed picture of the cycle of μ^- catalysis, it is necessary, in particular, to know the cascade times in the atoms $\mu^- d$ and $\mu^- t$. Since the atoms $\mu^- p$, $\mu^- d$, and $\mu^- t$ have nearly equal reduced masses and their properties are similar to a large degree, we can restrict the investigation to one of them, and to be specific we consider the $\mu^- d$ atom in deuterium.[1)]

The paper is arranged as follows. The processes of de-excitation and Stark mixing are considered in §2. Section 3 gives a method for calculating an atomic cascade valid for any ratio of the de-excitation and Stark mixing rates and permitting exact calculation of all stages of the cascade process in the case of both liquid and gaseous hydrogen (deuterium). The results of the cascade calculations for the atom $\mu^- d$ are given in §4. The available experimental data on the $\mu^- p$ atom are discussed in §5, in which we also make some concluding remarks.

§2. DE-EXCITATION AND STARK MIXING

In the ground state, the mesic atom $\mu^- d$ is characterized by binding energy $\varepsilon_{1S} = 2.66$ keV and Bohr radius $a_1 = 0.27 \times 10^{-10}$ cm. When the μ^- meson is captured in deuterium, the atom $\mu^- d$ is formed in a highly excited state n, whose Bohr radius a_n is close to the Bohr radius of the deuterium atom: $a_B = 0.53 \times 10^{-8}$ cm, i.e., $n \sim 14$.[11,12] The mesic atom goes over to lower levels as a result of the chemical reaction $\mu^- d + D_2 \to \mu^- d + D + D$, the external Auger effect $\mu^- d + D_2 \to \mu^- d + D_2^+ + e^-$ (electron ejected from neighboring atom), and through radiative transitions $\mu^- d \to \mu^- d + \gamma$.[2] The initial stage of the cascade, when chemical de-excitation is important, has been little studied. It is expected that as a result of the dissociation of the D_2 molecules the mesic atom acquires a kinetic energy of order 1 eV (velocity $v \sim 10^6$ cm/sec; see Ref. 2). When $n \lesssim 10$, the chemical de-excitation is replaced by de-excitation through the external Auger effect. The Auger transitions have a dipole nature ($\Delta l = l_i - l_f = \pm 1$), and transitions with the minimal change in the principal quantum number n permitted for ionization of the molecule D_2 are predominant. Figure 1 gives the rates of Auger transitions for circular orbits

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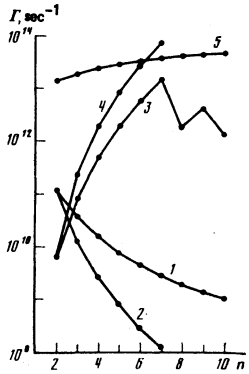

FIG. 1. Rates of radiative transitions Γ^{γ} and Auger transitions Γ^{e} from the states nP (Γ^{γ}_{nP} curve 1, Γ^{e}_{nP} curve 3) and from the states with $l = n-1$ ($\Gamma^{\gamma}_{n,n-1}$ curve 2, $\Gamma^{e}_{n,n-1}$ curve 4) and the Stark mixing rates Γ^{St}_{n} (curve 5) for the $\mu^- d$ atom in liquid deuterium (velocity $v = 10^6$ cm/sec of the mesic atom). The allowed range of the principal quantum number in the Auger transitions is $\Delta n \geq 1$ for $n \leq 7$, $\Delta n \geq 2$ for $n = 8$ and 9, and $\Delta n \geq 3$ for $n = 10$.

($l = n-1$) and P levels, summed over the final states, for the $\mu^- d$ atom in liquid deuterium (calculation based on the expressions of Ref. 2). The Auger transition rate is maximal at $n = 7$, transitions with $\Delta n = 1$ becoming possible from this level. With decreasing n, the Auger transition rate falls rapidly, so that in the final stage of the cascade radiative $E1$ transitions become important, their rate being determined by the well-known expression

$$\Gamma_{if}^{\gamma} = {}^4/_3 \alpha |R_{if}|^2 \omega_{if}^3.$$

The rates of radiative transitions for P levels and levels with $l = n - l$ are given in Fig. 1. Since the rate of the Auger transitions is proportional to the density of the medium, the importance of the radiative transitions is greater when liquid deuterium is replaced by gaseous deuterium (Fig. 2), and in the latter with density $N = 10^{-3} N_0$ (pressure ~1 atm, $T = 300$°K) radiative transitions become dominant for $n_f \leq 3$.

As we have already said, the initial stage of the cascade has been little studied. However, in what follows

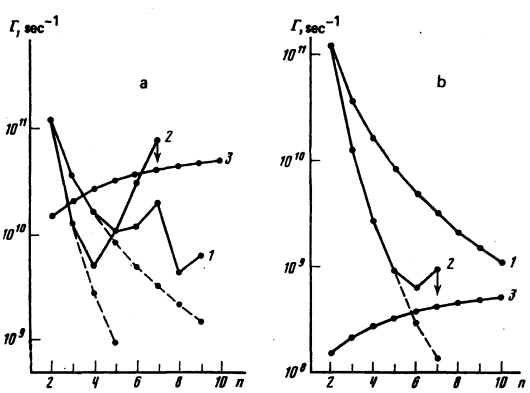

FIG. 2. Total ($\Gamma^{\gamma} + \Gamma^{e}$) rates of de-excitation from the states nP (Γ_{nP} curve 1) and $l = n-1$ ($\Gamma_{n,n-1}$ curve 2) and Stark mixing rates (Γ^{St}_{n} curve 3) for the $\mu^- d$ atom in gaseous deuterium [a) 1 atm, 300°K; b) 0.01 atm, 300°K]. The rates of radiative transitions are indicated by the broken curves.

we shall need the time required for capture of the meson by an atomic level and the cascade between the highly excited states. To estimate this time, we use the fact that these processes for μ^- and π^- mesons must take place with comparable rates ($m_\pi/m_\mu = 1.3$). The lifetime of a π^- meson with initial energy $E = 2$ keV in liquid hydrogen has been measured experimentally:

$$\tau_0 = (2.3 \pm 0.6) \cdot 10^{-12} \text{ sec},^{13} \qquad \tau_0 = (2.0 \pm 0.6) \cdot 10^{-12} \text{ sec}.^{14}$$

Bearing in mind that the π^- meson is absorbed basically from atomic states with $n \leq 4$ and noting that the time τ_0 of the cascade to the lower levels ($n < 7$) is $\sim 10^{-12}$ sec,[2] we conclude that for a meson with initial energy ~2 keV the time of capture and de-excitation to the level $n = 7$ in liquid hydrogen, $\tau_0(E = 2 \text{ keV} \to n = 7)$, is about 10^{-12} sec. Ignoring the mass difference between μ^- and π^- (in reality, the μ^- meson, the lighter particle, is captured more rapidly than the π^- meson[11]), we take for the μ^- meson in liquid hydrogen (deuterium) $\tau_0(E \approx 2 \text{ keV} \to n = 7) = 10^{-12}$ sec. This value agrees with the theoretical estimates[11] of the muon capture time, $\tau_0(E = 2 \text{ keV} \to n = 14) = 0.8 \times 10^{-12}$ sec, and the estimates[2] of the rates of de-excitation on high levels: $\tau_0(n = 14 \to n = 7) = 0.4 \times 10^{-12}$ sec.[2)]

For gaseous hydrogen with density N, the time of muon capture by an atomic level is determined by the expression

$$\tau(E = 2 \text{ keV} \to n = 14) = \tau_0(E = 2 \text{ keV} \to n = 14) N_0/N$$

At densities $N > 10^{-4} N_0$, the initial stage of the cascade is governed by de-excitation processes whose rate is proportional to the density of the medium (chemical dissociation and external Auger effect) and, therefore, $\tau(n = 14 \to n = 7) = \tau_0(n = 14 \to n = 7) N_0/N$. Thus, we finally obtain $\tau(E = 2 \text{ keV} \to n = 7) = (N_0/N) 10^{-12}$ sec.

Stark mixing

It is easily seen that Stark mixing of the different sublevels of a mesic atom when it moves through the electric field of a deuterium atom plays an important part if the mesic atom is in liquid deuterium. To see this, we compare the frequency ω_{St} of Stark transitions in a homogeneous electric field $\mathscr{E} = \alpha^{1/2} a_B^{-2}$,

$$\omega_{St} = \langle nl | \alpha^{1/2} r \mathscr{E} | n, l-1 \rangle = \alpha a_B^{-2} \langle nl | r | n, l-1 \rangle = n (n^2 - l^2)^{1/2} \cdot 2 \cdot 10^{14} \text{ sec}^{-1},$$

with the characteristic time required for the mesic atom to pass through the deuterium atom:

$$\tau_{St} = a_B/v = 0.5 \cdot 10^{-14} \text{ sec}.$$

Since $\omega_{St} \tau_{St} > 1$, the mesic atom $\mu^- d$ makes many transitions between different states nl with given n. At the same time, the cross section of the Stark mixing process must be in order of magnitude comparable with the geometrical size of the deuterium atom, i.e., the rate of Stark mixing has the scale $Nv\pi a_B^2$, which for liquid deuterium is 5×10^{12} sec^{-1}. Comparing this value with the rates of the de-excitation processes (Fig. 1), we see that Stark mixing has a strong influence on the cascade process.[1,2]

For detailed calculation of the Stark mixing effects, we use the impact-parameter method,[2] in which the mesic atom is assumed to move along a straight path

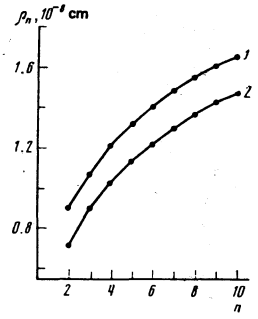

FIG. 3. Dependence of the critical impact parameter ρ_n on the prinicpal quantum number n for $\mu^- d$ atom with velocity $v = 10^6$ cm/sec (curve 1) and $v = 2\times10^6$ cm/sec (curve 2).

through the electric field of the deuterium atom, and the probability of the transition $nl \to n'l'$ is found for given values of the impact parameter ρ and the velocity v:

$$T_{nln'l'} = T(n, l, n', l', \rho, v).$$

The reader can find the details in the original papers Refs. 2, 8, and 15; here, we give the main results.

1. Only transitions between states with the same quantum number n are important.

2. There is a critical value ρ_n of the impact parameter such that for $\rho > \rho_n$ the probabilities of Stark transitions are negligibly small (Fig. 3). Thus, one can introduce the concept of the rate of Stark mixing in collisions,

$$\Gamma_n^{St} = Nv\pi\rho_n^2, \tag{1}$$

and probability of the transition $nl \to nl'$ in a Stark collision:

$$T_n(l, l') = \rho_n^{-2} \int_0^{\rho_n^2} d\rho^2\, T(n, l, n, l', \rho, v). \tag{2}$$

3. After a Stark collision, the mesic atom "forgets" its initial state, since the intensity of mixing is very high when $\rho < \rho_n$ ($\omega_{St}\tau_{St} \gg 1$). This means that $T_n(l, l')$ does not depend on l and is determined by the statistical weight of the final state l':

$$T_n(l, l') = (2l'+1)/n^2. \tag{3}$$

The results of numerical calculation of the Stark mixing rates for the mesic atom $\mu^- d$ are given in Figs. 1 and 2.

§3. METHOD OF CALCULATING THE ATOMIC CASCADE

This section is devoted to the description of a universal method for calculating cascades in light exotic atoms formed in liquid and gaseous hydrogen and deuterium. It was used earlier by the present author to study cascade processes in the antiproton atoms $\bar{p}p$ and $\bar{p}d$.[4] In the limiting case of strong Stark mixing, which occurs in liquid hydrogen and deuterium, the method is equivalent to the effective-rate method of Leon and Bethe.[2]

In contrast to an isolated atom, for which the cascade processes are completely ordered (the atom goes over each time to a lower level and, therefore, any state occurs not more than once), a light exotic atom in matter is characterized by numerous Stark transitions between the energy-degenerate sublevels nl with given principal quantum number n. If Γ_{nl} is the total width of the state nl, which is made up of the rates of de-excitation and absorption (in our case, the only absorption process is the decay of the μ^- meson: $\Gamma_\mu = 4.5 \times 10^5$ sec^{-1}), and Γ_n^{St} is the rate of Stark mixing in collisions, then the probability that a mesic atom does not change its initial state during the time between two successive Stark collisions is determined by the formula

$$s_{nl} = \exp(-\Gamma_{nl}/\Gamma_n^{St}), \tag{4}$$

while the probability of its leaving the state nl due to de-excitation or absorption processes is

$$r_{nl} = 1 - s_{nl}. \tag{5}$$

After the Stark collision, the mesic atom forgets its initial state and goes over to the state nl with probability

$$q_{nl} = (2l+1)/n^2. \tag{6}$$

The connection between the different cascade processes becomes very clear if it is represented in the form of a directed graph whose vertices correspond to the atomic states and the lines to the transitions between them. With the vertices we associate the populations of the atomic levels and with the lines the conditional transition probabilities. In our case, we have the following vertices:

a) p_{nl}, which represent the atomic states nl between the Stark collisions;

b) Z_n, which correspond to states with principal quantum number n during the time of a Stark collision;

c) $\tilde{p}_{nl}$, which denote the set of states to which the atom makes a transition from the state nl during the time between two successive Stark collisions. The population associated with the vertex $\tilde{p}_{nl}$ is defined as the sum of the probabilities of transition from the state nl to states with $n' < n$ and the probability of absorption in the state nl. We call it the *exit population* of the state nl;

d) A_{nl}, which correspond to absorption from the state nl. The graph is shown in Fig. 4. The lines of the graph are designated by the corresponding conditional transition probabilities. These last satisfy a normalization condition: The sum of the conditional transition probabilities over all transitions from a given vertex is equal to unity. Essentially, the constructed graph

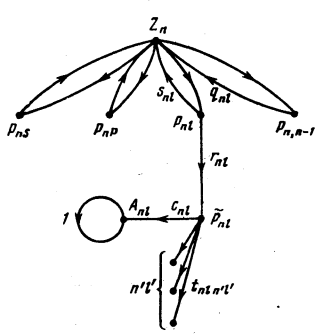

FIG. 4.

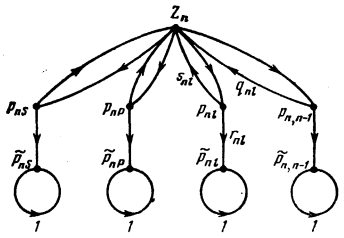

FIG. 5.

determines a Markov chain,[16] which enables us to use the standard methods of the theory of Markov processes.

Suppose the initial distribution has the form

$$p_{nl}=0,\ n>N;\quad p_{nl}=p_{nl}^{(N)},\ n\leqslant N. \tag{7}$$

We shall call the populations $p_{nl}^{(N)}$ for $n=N$ the *entry populations* of the states nl. We consider the subgraph consisting of the vertices with given $n=N$ and the corresponding lines (Fig. 5), and find for the given entry populations p_{nl} the resulting exit populations. The populations of the vertices Z_n and A_{nl} at the initial time are set equal to zero. It is here convenient to introduce a simpler notation for the vertices: $p_i, i=1,2n+1$. Vertices of the subset $\{p_i, i=1,n\}=\{\tilde{p}_{nl}\}$ corresponds to so-called absorbing states—transitions from them to other vertices of the subgraph are impossible. The remaining states $\{p_i, i=n+1,2n+1\}=\{p_{nl}, Z_n\}$ are called transition states. The conditional probabilities of the transitions $p_j\to p_i$ form a transition matrix (P_{ij}) of the form

$$(P_{ij})=\begin{pmatrix}\mathbf{I} & \mathbf{R}\\ \mathbf{0} & \mathbf{Q}\end{pmatrix}. \tag{8}$$

Here, I is the $n\times n$ unit matrix, and 0 is the $(n+1)\times n$ null matrix, which corresponds to an absorbing nature of the states $i=1,n$. The $n\times(n+1)$ matrix R describes the transitions from the transition states to the absorbing states:

$$\mathbf{R}=\left(\begin{array}{ccc|c} r_{nS} & & & \\ & r_{nP}\ \ \ 0 & & \\ & \ddots & & 0 \\ & r_{nl} & & \\ & \ddots & & \\ 0 & & r_{n,n-1} & \end{array}\right). \tag{9}$$

The $(n+1)\times(n+1)$ matrix Q describe transitions between the transition states:

$$\mathbf{Q}=\begin{pmatrix} & & q_{nS}\\ & 0 & \vdots\\ & & q_{nl}\\ & & \vdots\\ & & q_{n,n-1}\\ s_{nS}s_{nP}\ldots s_{nl}\ldots s_{n,n-1} & & 0\end{pmatrix}. \tag{10}$$

Let B be a $n\times(n+1)$ matrix whose elements B_{ij} are, by definition, equal to the probability that a process which begins in the transition state p_{n+j} ends in the absorbing state p_i. The matrix B satisfies the equation[16]

$$\mathbf{B}=\mathbf{R}+\mathbf{BQ}, \tag{11}$$

whose solution has the form

$$\mathbf{B}=\mathbf{RN}, \tag{12}$$

where $\mathbf{N}=(\mathbf{I}-\mathbf{Q})^{-1}$ is the fundamental matrix.

Now, when the matrix B has been found, we can calculate the resulting exit populations $\tilde{p}_{nl}$:

$$p_k=\sum_{i=1}^{n}B_{ki}p_i,\quad \{p_i\}=\{p_{nl}\},\quad \{p_k\}=\{\tilde{p}_{nl}\}.$$

The probabilities of the transitions $nl\to n'l'$ and of absorption in the state nl are determined by

$$y_{nl\to n'l'}=\tilde{p}_{nl}t_{nln'l'},\qquad a_{nl}=\tilde{p}_{nl}c_{nl}, \tag{13}$$

where $t_{nln'l'}$ and c_{nl} are the conditional probabilities of transitions and absorption:

$$t_{nln'l'}=\Gamma_{nl\to n'l'}/\Gamma_{nl},\qquad c_{nl}=\Gamma_{\mu}/\Gamma_{nl}. \tag{14}$$

As a result of the considered transitions, a new distribution is formed,

$$p_{Nl}^{(N)}\to p_{Nl}^{(N-1)}=0,\quad p_{nl}^{(N)}\to p_{nl}^{(N-1)}=p_{nl}^{(N)}+\sum_{l'}y_{Nl'nl},\quad n<N,$$

and we can treat it in exactly the same way as the original one ($N\to N-1$).

In the limiting case when the Stark mixing rate is large compared with the de-excitation and absorption rates, the states nl with given n are populated in accordance with their statistical weight. It is therefore sufficient to determine the total populations $p_n=\sum p_{nl}$, which can be done by means of the effective-rate method.[2] The de-excitation is described by an effective rate determined in accordance with the graph in Fig. 4 by the formula

$$\Gamma_{n\to n'}=\Gamma_n^{St}\sum_{l}\sum_{l'}\frac{(2l+1)}{n^2}\left(1-\exp\left(-\frac{\Gamma_{nl}}{\Gamma_n^{St}}\right)\right)\frac{\Gamma_{nl\to n'l'}}{\Gamma_{nl}}\Bigg|_{(\Gamma_{nl}/\Gamma_n^{St})\to 0}$$
$$=\sum_{l}\sum_{l'}\frac{(2l+1)}{n^2}\Gamma_{nl\to n'l'}. \tag{15}$$

The effective absorption rate for the mesic atom μ^-d is equal to the decay rate Γ_μ of the free μ^- meson. The conditional probabilities of the transitions $n\to n'$ and of absorption in the state with principal quantum number n are determined by the effective rates:

$$t_{n\to n'}=\Gamma_{n\to n'}/\Gamma_n^{tot},\quad c_n=\Gamma_\mu/\Gamma_n^{tot},$$
$$\Gamma_n^{tot}=\Gamma_\mu+\sum_{n'}\Gamma_{n\to n'}.$$

The effective-rate method can be used in the case of liquid deuterium; in gaseous deuterium, as we have shown above (see Fig. 2), the de-excitation in the final stage of the cascade occurs more rapidly than the Stark mixing, and it is necessary to use the universal method of calculation described above.

Cascade time

An atomic cascade consists of a set of processes, each characterized by a time. To introduce the concept of the cascade time, we consider the problem of calculating the probability with which a μ^-d atom reaches the ground state.

The required probability has the form

$$p_{1S}=1-\sum_{nl\neq 1S}a_{nl}, \tag{16}$$

where a_{nl} is the probability of absorption from the state nl:

$$a_{nl}=\tilde{p}_{nl}\Gamma_\mu/\Gamma_{nl}.$$

If it is borne in mind that the decay rate of the μ^- meson is small compared with the de-excitation rates ($\Gamma_\mu \ll \Gamma_{nl}$ for $nl\neq 1S, 2S$), then the expression (16) can be written in the form[3)]

$$p_{1s}=1-a_{2s}-\Gamma_\mu T, \qquad (17)$$

where T, the cascade time, is defined by[4)]

$$T=\sum_{nl\neq 1s,\,2s}\tilde{p}_{nl}\Gamma_{nl}^{-1}. \qquad (18)$$

§4. RESULTS OF CASCADE CALCULATIONS FOR THE MESIC ATOM $\mu^- d$

The level populations p_{nl}, the intensities of x-ray transitions, and the cascade time were calculated for liquid and gaseous (1 atm and 10^{-2} atm, 300°K) deuterium by the method described in §3. The calculation commenced with the level $n=10$. The initial stage of the cascade ($n>10$) is of no great interest (the time of muon capture by an atomic orbit and the cascade time between the highly excited states were estimated in §2). The initial distribution at level $n=10$ was taken to be statistical ($p_{nl}=(2nl+1)/n^2$), since the Stark mixing rate in the high levels $n\geq 10$ exceeds the de-excitation rate in deuterium with density $N>10^{-5}N_0$. The velocity $v=10^6$ cm/sec of the mesic atom corresponded to a kinetic energy of 1 eV.[5)]

The results of the calculations are shown in Tables I, II, and III. The total populations of the atomic levels with given principal quantum number n (p_n) and the intensities of the radiative transitions between levels n and n_f ($Y_{n\to nf}$) are normalized to one stopped μ^- meson.

In liquid deuterium, the majority of atoms pass through the states $n\leq 7$, since here Auger transitions with $\Delta n=1$ are predominant. The importance of the radiative transitions is characterized by the quantities

$$S_n=\sum_{n_f}Y_{n\to n_f}p_n^{-1}, \quad S_{n_f}'=\sum_{n}Y_{n\to n_f}p_{n_f}^{-1}.$$

The first of these determines the contribution of the radiative processes to the de-excitation of an atom in state n; the second, the contribution of the radiative processes to the population of the state n_f. In liquid deuterium, radiative processes are responsible for 95% of the population of the state 1S, 5% of the population of the states with $n=2$, and 0.1% of the population of the states with $n=3$. The rate of the de-excitation processes in the different stages of the cascade is characterized by the times of de-excitation from states with given n:

$$T_n=\sum_l \tilde{p}_{nl}\Gamma_{nl}^{-1}.$$

In liquid deuterium, the slowest stage of the cascade corresponds to the transitions from the levels with $n=3$ and 2: $T_2=7.2\times10^{-12}$ sec and $T_3=5.4\times10^{-12}$ sec. The total cascade time (with allowance for capture of a muon with initial energy 2 keV by an atomic level) is $T=1.5\times10^{-11}$ sec. The probability of absorption of the μ^- meson during the deceleration time and the cascade to the levels $n>1$ is 7.6×10^{-6}, and the probability of absorption from the state 2S is 7.6×10^{-7}. Note that the rate of formation of the $\mu^- d$ atom in the ground state in the case of liquid hydrogen, $\lambda=1/T=7\times10^{10}$ sec^{-1}, appreciably exceeds the rate of resonance formation of $dt\mu^-$ molecules: $\lambda_{dt\mu}\sim10^8$ sec^{-1}.

In gaseous deuterium, radiative transitions become the domanant process for $n_f\leq 3$ at density $N=10^{-3}N_0$ (1 atm, 300°K) and $n_f\leq 6$ at density $10^{-5}N_0$ (see Tables II and III and Fig. 2). The slowest part in the final stage of the cascade ($n\leq 7$) corresponds to levels to which de-excitation by the Auger effect gives way to radiative

TABLE I.

n	p_n		$Y_{n\to n_f}$ $n_f=1$	2	3	$\sum_{n_f} Y_{n\to n_f}$	S_n
7	0.849	0.021					
6	0.947	0.055					
5	0.972	0.17					
4	0.983	0.75	0.0019	0.0012	0.0013	0.0045	0.0046
3	0.988	5.4	0.06	0.047	–	0.11	0.11
2	0.937	7,2	0.89	–	–	0.89	0.95
1	1.000	–	–	–	–	–	–
		$\sum_n Y_{n\to n_f}$	0.953	0.049	0.0014		
		S'_{n_f}	0.953	0.052	0.0014		

Note. The $\mu^- d$ atom in liquid deuterium: p_n are the populations of the atomic levels, T_n are the times of de-excitation from states with principal quantum number n, $Y_{n\to nf}$ are the intensities of the radiative transitions $n\to n_f$. Decay time $\tau_0(n=7\to n=1)=13.6$ psec. The probability of absorption in the levels $n=2$–7: $a(7-2)=6.9\times10^{-6}$, including the probability of absorption in the state 2S: $a_{2S}=7.2\times10^{-7}$(velocity of the mesic atom). $v=10^6$ cm/sec).

TABLE II.

n	p_n	T_n, psec	$Y_{n\to n_f}$ $n_f=1$	2	3	4	$\sum_{n_f} Y_{n\to n_f}$	S_n
7	0.808	18	0.0036	0.0021	0,0016	0.0014	0,011	0,014
6	0.899	45	0.016	0.0096	0.0078	0.0074	0,049	0.055
5	0.889	105	0,077	0,051	0.046	0.056	0.23	0.26
4	0.743	108	0.21	0.17	0.20	–	0.58	0.78
3	0.434	26	0.17	0.26	–	–	0.43	0.99
2	0.503	4.1	0.503	–	–	–	0.503	1.00
1	1.000	–	–	–	–	–	–	–
		$\sum_n Y_{n\to n_f}$	1.000	0.496	0.263	0.070		
		S'_{n_f}	1.000	0.994	0.61	0,095		

Note. The $\mu^- d$ atom in gaseous deuterium (1 atm, 300°K). The cascade time is $\tau(n=7\to n=2)=3\times10^{-10}$ sec. The probability of absorption in the levels $n=2$–7: $a(7-2)=1.4\times10^{-4}$, including absorption in the state 2S: $a_{2S}=3.5\times10^{-6}$ ($v=10^6$ cm/sec).

TABLE III.

n	p_n	T_n, psec	$Y_{n\to n_f}$ $n_f=1$	2	3	4	$\sum_{n_f} Y_{n\to n_f}$	S_n
7	0.303	345	0.011	0,016	0,019	0.022	0.12	0.41
6	0.344	416	0.014	0.028	0,044	0.065	0.25	0.72
5	0.319	244	0.015	0.038	0.081	0.17	0.30	0.94
4	0.370	108	0.027	0,087	0.25	–	0.37	0.994
3	0.505	42	0,085	0.42	–	–	0.50	1.000
2	0.711	5.7	0.71	–	–	–	0.71	1.000
1	1.00	–	–	–	–	–	–	–
		$\sum_n Y_{n\to n_f}$	1.00	0.71	0.50	0.35		
		S'_{n_f}	1.000	1.000	0.995	0.94		

Note. The $\mu^- d$ atom in gaseous deterium (0.01 atm, 300°K). The decay time is $\tau(n=7\to n=2)=1.2\times10^{-9}$ sec. The probability of absorption in the levels $n=2$–7: $a(7-2)=7.4\times10^{-4}$, including absorption in the state 2S: $a_{2S}=2.1\times10^{-4}$ ($v=10^6$ cm/sec).

de-excitation: $n=4$ and 5 for $N=10^{-3}N_0$ and $n=6$ for $N=10^{-5}N_0$; it is characterized by the times $T_4=10^{-6}$ sec and $T_6=4\times10^{-10}$ sec, respectively. In contrast to the case of liquid deuterium, transitions between the low levels of the μ^-d atom ($n\leq7$) in gaseous deuterium with density $N<10^{-2}N_0$ take place more rapidly than the processes of deceleration and capture of the muon by an atomic level and the transitions between the highly excited states ($n>7$), whose characteristic time is

$$\tau(E=2\,\text{keV}\to n=7)\approx(N_0/N)\cdot10^{-12}\,\text{sec}.$$

In gaseous deuterium, a μ^- meson with initial energy 2 keV is captured by an atomic level and reaches the ground state in a time $T=1.3\times10^{-9}$ sec at density $N=10^{-3}N_0$ and $T=10^{-7}$ sec at $N=10^{-5}N_0$.

The probability of transition from states with $n>2$ to the state $2S$ (entry population p_{2S}) is of interest for experiments on the detection of the delayed component of the K_α line, i.e., the radiative $2P\to1S$ transition following the Stark transition $2S\to2P$. For liquid deuterium $p_{2S}=0.16$, and for gaseous deuterium $p_{2S}=0.070$ ($N=10^{-3}N_0$) and $p_{2S}=0.41$ ($N=10^{-5}N_0$). Our results agree with the analysis in Ref. 18 of experimental data on x-ray emission of the μ^-p atom in gaseous hydrogen,[5] according to which $p_{2S}=0.08\pm0.02$; for gaseous hydrogen with density $N=5\times10^{-3}N_0^{\text{H}}$ (4 atm, 300°K), we obtained $p_{2S}=0.077$.

A special investigation must be made into $2S-2P$ Stark mixing and the probability of absorption of the μ^- meson from the state $2S$ in connection with the fine structure of the levels with $n=2$ ($\delta E=E_{2S}-E_{2P}=-0.2$ eV). For μ^-d atoms with kinetic energy $E\approx1$ eV ($v\sim10^6$ cm/sec), allowance for the fine structure leads to a slight decrease in the rate of the $2S\to2P$ Stark transitions.[8] At the same time, the time of $2S-2P$ Stark mixing is appreciably shorter than τ ($E=2$ keV $\to n=7$) $=10^{-12}N_0/N$:

$$\tau_{2S-2P}=1/\Gamma_{n=2}^{St}\approx(N_0/N)\cdot10^{-13}\,\text{sec},$$

and absorption in the $2S$ state, where some of the atoms "get stuck" in the case of a low-density gas, is weak compared with the absorption in the initial stage of the cascade.

If the μ^-d atom has kinetic energy $E<0.3$ eV, a $2S-2P$ transition as a result of collision with a D_2 molecule is impossible, but during the collision time intense $2S-2P$ Stark mixing must occur if the μ^-d atom is in a strong electric field $\mathscr{E}$ for which the Stark splitting $\Delta E_n^{\text{St}}=3\alpha^{1/2}a+\mathscr{E}n(n-1)$ of the levels exceeds the fine structure $|\delta E|$. Thus, de-excitation of the state $2S$ can occur through radiative transition to the state $1S$ in a Stark collision; the rate of such a process can be estimated in accordance with the formula[8]

$$\Gamma_{2S\to2P\to1S}=\tfrac{3}{4}\tilde{\Gamma}^{St}(1-\exp(-\bar{\Gamma}_{2P\to1S}\tau_{St}))\approx\tfrac{3}{4}\tilde{\Gamma}^{St}\tau_{St}\bar{\Gamma}_{2P\to1S}^{\gamma}.$$

Here, $\tilde{\Gamma}^{St}=Nv\pi R^2$ is the rate of Stark collisions for which $\Delta E_{n=2}^{St}\geq|\delta E|$ ($R=7\times10^{-9}$ cm; see also Ref. 8), and $\tau_{St}=(4/3)R/v$ is the mean time of a Stark collision. As a result, the rate of de-excitation of the state $2S$ for mesic atoms with energy $E<0.3$ eV is determined by

$$\Gamma_{2S\to2P\to1S}=\pi R^2N\Gamma_{2P\to1S}^{\gamma}=6\cdot10^{-2}\frac{N}{N_0}\Gamma_{2P\to1S}^{\gamma}=6\cdot10^{9}\frac{N}{N_0}\,[\text{sec}^{-1}].$$

The probability of absorption from the state $2S$ is

$$a_{2S}(E<0.3\,\text{eV})=p_{2S}q\Gamma_\mu/\Gamma_{2S\to2P\to1S}=8\cdot10^{-5}\,p_{2S}qN_0/N.$$

Here, p_{2S} is the entry population of the state $2S$, and q is the probability of moderation to the energy $E<0.3$ eV. Note that at liquid deuterium density there is for a_{2S} an upper bound: $a_{2S}\leq1.3\times10^{-5}$ ($p_{2S}=0.16, q\leq1$); this value is comparable with the probability of absorption in the levels $n>2$ ($a(n>2)=7\times10^{-6}$) (it is to be expected that in reality the probability of moderation to the energy $E<0.3$ eV is appreciably smaller than unity[8]). Thus, we conclude that the probability of decay of a μ^- meson with initial energy $E=2$ keV during the time of deceleration, capture, and cascade in the atomic levels $n>1$ in liquid deuterium does not exceed 2×10^{-5}. A more accurate estimate requires calculation of q, which goes beyond the scope of the present paper.

§5. CONCLUDING REMARKS

Experimental data on the properties of light neutral μ^- atoms are very sparse. For the μ^-p atom in gaseous hydrogen (4 atm, 300°K) a measurement has been made[5] of the ratio of the intensity of the K_α line to the total intensity of the lines of the K series:

$$X=Y_{2P\to1S}\Big/\sum_n Y_{nP\to1S}=0.42\pm0.10.$$

A theoretical value of X can be obtained without detailed cascade calculations by exploiting the following circumstances, which hold for gaseous hydrogen under the indicated conditions ($N=5\times10^{-3}N_0^{\text{H}}$).

1) The states with $n=1$ and 2 are populated predominantly by radiative transitions.

2) For states with $n\geq3$ the Stark mixing rate exceeds the rate of the de-excitation processes.

In accordance with 1),

$$Y_{2P\to1S}=p_{2P}+p_{2S}=\sum_{n\geq3}(Y_{nD\to2P}+Y_{nS\to2P}+Y_{nP\to2S}).$$

In accordance with 2), the different sublevels nl with given principal quantum number n are populated according to their statistical weights, and therefore the following relationships hold between the intensities of the radiative transitions:

$$\frac{Y_{nD\to2P}}{Y_{nP\to1S}}=\frac{5\Gamma_{nD\to2P}^{\gamma}}{3\Gamma_{nP\to1S}^{\gamma}}=\begin{cases}0{,}64 & n=3\\ 0{,}50 & n=4,\\ 0{,}46 & n=5\end{cases}$$

$$\frac{Y_{nP\to2S}}{Y_{nP\to1S}}=\frac{\Gamma_{nP\to2S}^{\gamma}}{\Gamma_{nP\to1S}^{\gamma}}=\begin{cases}0.13 & n=3\\ 0{,}14 & n=4,5\end{cases},$$

$$\frac{Y_{nS\to2P}}{Y_{nP\to1S}}=\frac{\Gamma_{nS\to2P}^{\gamma}}{3\Gamma_{nP\to1S}^{\gamma}}=0.01.$$

As a result, the intensity of the K_α line can be expressed in terms of the intensities of the remaining K lines

$$Y_{2P\to1S}=\sum_{n\geq3}\left(\frac{5\Gamma_{nD\to2P}^{\gamma}}{3\Gamma_{nP\to1S}^{\gamma}}+\frac{\Gamma_{nS\to2P}^{\gamma}}{3\Gamma_{nP\to1S}^{\gamma}}+\frac{\Gamma_{nP\to2S}^{\gamma}}{\Gamma_{nP\to1S}^{\gamma}}\right)Y_{nP\to1S}$$

$$=\left(\frac{5\Gamma_{nD\to2P}^{\gamma}+\Gamma_{nS\to2P}^{\gamma}+3\Gamma_{nP\to2S}^{\gamma}}{3\Gamma_{nP\to1S}^{\gamma}}\right)_{\text{av}}\sum_{n\geq3}Y_{nP\to1S}\simeq0.69\sum_{n\geq3}Y_{nP\to1S}$$

(we have used the arithmetic mean $(\ldots)_{av}$ for $n=3,4,5$). From this we find the theoretical value $X = Y_{2P\to 1S}/\sum_{n\geq 2} Y_{nP\to 1S} = 0.41$, which agrees well with the experimental value of X. The results of the exact calculation by the method described in §3 is $X=0.47$. The effective-rate method gives the similar value $X=0.44$.[7]

In a denser medium, the condition 1) does not hold, while in a less dense medium the condition 2) does not; therefore, X must depend on the density. For the $\mu^- d$ atom in liquid deuterium $X=0.9$ (the states with $n=2$ are populated predominantly by Auger transitions); in gaseous deuterium under normal conditions $X=0.5$ (the Stark mixing rate in levels with $n=3$ is insufficiently high compared with the $3P\to 1S$ transition rate). For the $\mu^- p$ atom in liquid hydrogen, X has been measured[6] with low accuracy: $X=0.7\pm 0.2$. The result of our calculation for this case is $X=0.89$, which is close to the result $X=0.92$ of Ref. 7.

Experimental investigations into the dependence of X on the target density are within the scope of present-day experimental possibilities and would be extremely desirable for testing models of the cascade processes in light exotic atoms. More accurate investigations of the x-ray spectrum, including measurements of the intensities of the individual lines of the K and L series, are of great interest for establishing the details of the cascade in the lower levels. The information obtained in this manner would be very helpful for simulating cascade processes in more "complicated" light exotic atoms of the type $\bar{p}p$ and $\bar{p}d$.

With regard to the problem of μ catalysis of the reaction $d+t\to {}^4\mathrm{He}+n$, our main result is that the rate of capture of the μ^- meson by the K orbit of these mesic atoms appreciably exceeds the expected rate of μ catalysis. At the same time, the probability of decay of the muon in an excited state of the mesic atoms $\mu^- d$ and $\mu^- t$ is negligibly small.

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When the paper had been prepared for press, I became acquainted with the paper Ref. 19 of Borie and Leon, in which they investigate theoretically the dependence of the intensities of the K_α and L_α lines and the population of the state 2S of the atom $\mu^- p$ on the density of the medium and obtain results in good agreement with those of the present paper.

[1)] The case of hydrogen differs by an insignificant change in the density: for liquid hydrogen $N_0^{\mathrm{H}} = 4.25\times 10^{22}$ cm^{-3}, for liquid deuterium $N_0 = 5.27\times 10^{22}$ cm^{-3}.

[2)] We have used the value $\tau_0(n=14\to n=7)$ for the $\pi^- p$ atom converted to the case of the $\mu^- d$ atom in accordance with the expressions of Ref. 2.

[3)] De-excitation of the state $2S$ is possible only through the two-photon radiative transition, whose rate $\Gamma^{2\gamma}_{2S\to 1S} = 1.06\times 10^5$ sec^{-1} (Ref. 17) is comparable with the muon decay rate $\Gamma_\mu = 4.54\times 10^5$ sec^{-1}.

[4)] Here, in finding the exit populations, we can ignore the decay of the muon in levels $nl\neq 1S,\ 2S$.

[5)] The results of the cascade calculations are not very sensitive to the velocity of the mesic atom. If the velocity is doubled ($v = 2\times 10^6$ cm/sec), the Stark mixing rate increases by 1.3–1.6 times, while the cascade time and the intensity of the x-ray transitions change by only a few percent. The only exceptions are the $2S-2P$ Stark mixing rate and the probability of absorption from the state $2S$ (see below).

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