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Translated by J. G. Adashko

Possibility of mutual enhancement of electromagnetic waves in semiconductors with narrow conduction band

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(Submitted 22 April 1980)
Zh. Eksp. Teor. Fiz. **79**, 2189–2195 (December 1980)

A quantum kinetic equation for the conduction electrons with arbitrary dispersion law, in the presence of two homogeneous alternating electric fields, is obtained in the approximation of weak electron-phonon coupling. In the case of electromagnetic fields with quantum energy exceeding the width of the conduction band, expressions for the coefficients of the intraband absorption of each of the fields are obtained on the basis of a solution of the kinetic equation. In the considered situation, one of the coefficients turns out to be negative, thus indicating the possibility of mutual enhancement of electromagnetic waves in semiconductors with narrow conduction band.

PACS numbers: 71.25.Cx

Malevich and Épshtein[1] have investigated, on the basis of a quantum kinetic equation, the behavior of the electrons of a semiconductor in the presence of two strong homogeneous alternating electric fields: $\mathbf{E}(t)=\mathbf{E}_1\sin\omega_1 t+\mathbf{E}_2\sin\omega_2 t$. They used a quadratic isotropic electron dispersion law, therefore the results obtained by them are no longer valid at high frequencies, where the energy of the electromagnetic-field quanta is comparable with or larger than the width Δ of the conduction band. In a semiconductor with a narrow conduction band, Δ can be quite small ($\leq 10^{-2}$ eV in semiconductors with superlattice[2] and in organic[3-5] or polaron semiconductors), i.e., the results of Malevich and Épshtein cease to be valid already at $\omega_{1,2}\gtrsim 10^{13}$ sec^{-1}.

No assumptions whatever are made in this paper concerning the electron dispersion law in the conduction band. This makes it possible to investigate the situation[1)] $\omega_{1,2}\gtrsim\Delta$ (a system of units with $\hbar=1$ is used). The frequencies $\omega_{1,2}$ are bounded from above only by the assumption that there are no interband transitions.

The Hamiltonian that describes the interaction of the electrons with the phonons takes in the presence of an electromagnetic field the form

$$\hat{H}=\sum_{\mathbf{p}}\varepsilon\left(\mathbf{p}-\frac{e}{c}\mathbf{A}(t)\right)a_{\mathbf{p}}^{+}a_{\mathbf{p}}+\sum_{\mathbf{q}}\omega_{\mathbf{q}}b_{\mathbf{q}}^{+}b_{\mathbf{q}}+\sum_{\mathbf{p},\mathbf{q}}C_{\mathbf{q}}a_{\mathbf{p}-\mathbf{q}}^{+}a_{\mathbf{p}}(b_{\mathbf{q}}+b_{-\mathbf{q}}^{+}).$$

Here $\varepsilon(\mathbf{p})$ is the electron dispersion law in the conduction band, $\mathbf{A}(t)$ is a certain potential connected with the electromagnetic-wave field

$$\mathbf{E}(t)=\mathbf{E}_1\sin\omega_1 t+\mathbf{E}_2\sin\omega_2 t$$

by the relation

$$\mathbf{E}(t)=-c^{-1}d\mathbf{A}(t)/dt,$$

$a_{\mathbf{p}}^{+}$ and $a_{\mathbf{p}}$ ($b_{\mathbf{q}}^{+}$, $b_{\mathbf{q}}$) are the operators of creation and annihilation of an electron (phonon), $\omega_{\mathbf{q}}$ is the phonon frequency, and $C_{\mathbf{q}}$ is the electron-phonon coupling constant.

In the derivation of the quantum kinetic equation for the description of the processes in the high-frequency fields ($\omega_{1,2}\gg\tau^{-1}$, where τ is the relaxation time), we follow the procedure developed by Épshtein.[7] Assuming the phonons to be in equilibrium and the electron gas to be nondegenerate, we obtain in the lowest order in the coupling constant $C_{\mathbf{q}}$ an equation for the electron distribution function

$$\partial\varphi(\mathbf{p},t)/\partial t=\left\{\sum_{\mathbf{q}}f(\mathbf{q})\,\mathrm{Re}\int_{-\infty}^{t}dt'\exp\left[-i\int_{t'}^{t}d\tau\left(\varepsilon\left[\mathbf{p}+e\int^{\tau}\mathbf{E}(\tau')d\tau'-\mathbf{q}\right]-\varepsilon\left[\mathbf{p}+e\int^{\tau}\mathbf{E}(\tau')d\tau'\right]\right)\right][\varphi(\mathbf{p}-\mathbf{q},t')-\varphi(\mathbf{p},t')]\right\}-\{\mathbf{p}\to\mathbf{p}+\mathbf{q}\},\quad(1)$$

where $\{\mathbf{p}\to\mathbf{p}+\mathbf{q}\}$ stands for the expression written out explicitly in the curly brackets, with the corresponding change in the arguments

and T is the temperature in energy units. In the derivation of (1), the phonon energy $\omega_{\mathbf{q}}$ was assumed to be small compared with the characteristic electron energy $\bar{\varepsilon}$ (quasielastic scattering).

We consider now in greater detail the terms in the argument of the exponential in (1). We have

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$$\varepsilon\left(\mathbf{p}+e\int^{\tau}\mathbf{E}(\tau')d\tau'\right)=\exp\left[e\int^{\tau}\mathbf{E}(\tau')d\tau'\frac{\partial}{\partial\mathbf{p}}\right]\varepsilon(\mathbf{p}). \quad (2)$$

Substituting $\mathbf{E}(t)$ in (2) and expanding the exponential in Bessel functions,[8] we obtain

$$\varepsilon\left(\mathbf{p}+e\int^{\tau}\mathbf{E}(\tau')d\tau'\right)=\sum_{k,l}\tilde{a}_{kl}(\mathbf{p})\cos(k\omega_1+l\omega_2)\tau, \quad (3)$$

where

$$\tilde{a}_{kl}(\mathbf{p})=(-1)^{k+l}I_k(\hat{\xi}_1)I_l(\hat{\xi}_2)\varepsilon(\mathbf{p}), \quad \hat{\xi}_{1,2}=\frac{e\mathbf{E}_{1,2}}{\omega_{1,2}}\frac{\partial}{\partial\mathbf{p}}$$

$I_k(x)$ is a modified Bessel function of integer argument. Substituting (3) in (1) and expanding the exponential in Bessel functions, we obtain at $|m_1\omega_1-m_2\omega_2|\gg\tau^{-1}$ ($m_{1,2}$ are small integers) an equation for the high-frequency (compared with τ^{-1}) part of the distribution function

$$\frac{\partial\tilde{\varphi}(\mathbf{p},t)}{\partial t}=\Big\{\sum_{\mathbf{q}}f(\mathbf{q})\,\mathrm{Re}\int_{-\infty}dt'\sum_{\substack{\{m_{kl}\}\\ \bar{m}_1\omega_1+\bar{m}_2\omega_2\neq 0}}\sum_{\{s_{kl}\}}\prod_{\substack{k,l\\ k^2+l^2\neq 0}}J_{m_{kl}+s_{kl}}(Z_{kl})$$

$$\times J_{s_{kl}}(Z_{kl})\exp[ia_{00}t'+i(\bar{s}_1\omega_1+\bar{s}_2\omega_2)t'-i(\bar{m}_1\omega_1+\bar{m}_2\omega_2)t]$$

$$\times[\tilde{\varphi}(\mathbf{p}-\mathbf{q})-\tilde{\varphi}(\mathbf{p})]\Big\}-\{\mathbf{p}\to\mathbf{p}+\mathbf{q}\}. \quad (4)$$

Here $J_k(x)$ is a Bessel function of real argument,

$$Z_{kl}=a_{kl}/(k\omega_1+l\omega_2), \quad a_{kl}=\tilde{a}_{kl}(\mathbf{p}-\mathbf{q})-\tilde{a}_{kl}(\mathbf{p}),$$

$\{m_{kl}\}$ and $\{s_{kl}\}$ mean that the summation is carried out over all the variables m_{kl} and s_{kl}; $k,l=0,\pm1,\pm2,\ldots$,

$$\bar{s}_1=\sum_{k,l}s_{kl}k, \quad \bar{s}_2=\sum_{k,l}s_{kl}l, \quad \bar{m}_1=\sum_{k,l}m_{kl}k, \quad \bar{m}_2=\sum_{k,l}m_{kl}l,$$

$\bar{\varphi}(\mathbf{p})$ is the low-frequency (compared with τ^{-1}) part of the electron distribution function, and is determined from the equation

$$0=\Big\{\sum_{\mathbf{q}}f(\mathbf{q})\sum_{\substack{\{m_{kl}\}\\ \bar{m}_1=\bar{m}_2=0}}\sum_{\{s_{kl}\}}\prod_{\substack{k,l\\ k^2+l^2\neq 0}}J_{m_{kl}+s_{kl}}(Z_{kl})$$

$$\times J_{s_{kl}}(Z_{kl})\delta[a_{00}+\bar{s}_1\omega_1+\bar{s}_2\omega_2][\tilde{\varphi}(\mathbf{p}-\mathbf{q})-\tilde{\varphi}(\mathbf{p})]\Big\}-\{\mathbf{p}\to\mathbf{p}+\mathbf{q}\}. \quad (5)$$

The sought coefficients of the intraband absorption of the electromagnetic waves are of the form

$$\alpha_{1,2}=\frac{8\pi e}{c\chi^{1/2}E_{1,2}^2}\sum_{\mathbf{p}}\left\langle\mathbf{v}\left(\mathbf{p}+e\int^{t}\mathbf{E}(t')dt'\right)\mathbf{E}_{1,2}\sin\omega_{1,2}t\tilde{\varphi}(\mathbf{p},t)\right\rangle, \quad (6)$$

where $\mathbf{v}(\mathbf{p})=\partial\varepsilon(\mathbf{p})/\partial\mathbf{p}$ is the electron velocity, χ is the optical dielectric constant,

$$\langle\ldots\rangle=\lim_{T\to\infty}\frac{1}{T}\int_0^T\ldots dt.$$

Expanding

$$\mathbf{v}\left(\mathbf{p}+e\int^{t}\mathbf{E}(t')dt'\right)$$

in analogy with (3) and substituting $\bar{\varphi}(\mathbf{p},t)$ from (4) in (6), we obtain

$$\alpha_1=\frac{8\pi^2\omega_1}{c\chi^{1/2}E_1^2}\sum_{\mathbf{p},\mathbf{q}}f(\mathbf{q})[\tilde{\varphi}(\mathbf{p}-\mathbf{q})-\tilde{\varphi}(\mathbf{p})]$$

$$\times\sum_{\substack{\{m_{kl}\}\\ \bar{m}_1^2+\bar{m}_2^2\neq 0}}\sum_{\{s_{kl}\}}\prod_{\substack{k,l\\ k^2+l^2\neq 0}}\bar{m}_1Z_{\bar{m}_1\bar{m}_2}J_{m_{kl}+s_{kl}}(Z_{kl})J_{s_{kl}}(Z_{kl})\delta[a_{00}+\bar{s}_1\omega_1+\bar{s}_2\omega_2]. \quad (7)$$

The expression for α_2 differs from (7) in the permutation of the indices $1\rightleftharpoons 2$. We note that (7) is the general expression for the intraband absorption coefficient of one of the two strong electromagnetic waves by a semiconductor with arbitrary dispersion law.

Putting $\mathbf{E}_2=0$ in (7), we obtain the corresponding expression for the coefficient of the intraband absorption of an electromagnetic wave by a semiconductor with arbitrary dispersion law. In the first nonvanishing approximation in the field $\mathbf{E}_1$, it takes the form

$$\alpha_1=\frac{4\pi^2e^3}{c\chi^{1/2}E_1^2\omega_1^2}\left[\exp\left(\frac{\omega_1}{T}\right)-1\right]\sum_{\mathbf{p},\mathbf{q}}f(\mathbf{q})\Phi(\mathbf{p})$$

$$\times\{\mathbf{E}_1[\mathbf{v}(\mathbf{p}-\mathbf{q})-\mathbf{v}(\mathbf{p})]\}^2\delta[\varepsilon(\mathbf{p}-\mathbf{q})-\varepsilon(\mathbf{p})+\omega_1],$$

$$\Phi(\mathbf{p})=C\exp[-\varepsilon(\mathbf{p})/T]. \quad (8)$$

Next, starting from (7) and (5), we consider separately the case of weak and strong external field.

1. *Weak electromagnetic fields.* Since $\varepsilon(\mathbf{p})$ is a periodic function of p with the period of the reciprocal lattice, we have

$$(\hat{\xi}_{1,2})^n\varepsilon(\mathbf{p})\sim(2\pi E_{1,2}e/\omega_{1,2}G_{1,2})^n\varepsilon(\mathbf{p}),$$

where $\mathbf{G}_1$ and $\mathbf{G}_2$ are the smallest of the reciprocal-lattice vectors and coincide respectively with the directions of $\mathbf{E}_1$ and $\mathbf{E}_2$. We assume below that the external electromagnetic field is weak, so that $\xi_{1,2}\ll 1$, and consider the frequency region in which $\omega_{1,2}>\Delta$ and $|\omega_1-\omega_2|\gg\tau^{-1}$. In this case $Z_{kl}<\xi_1^{|k|}\xi_2^{|l|}\ll 1$, with $Z_{k+1,l}/Z_{k,l}\sim\xi_1\ll 1$ and $Z_{k,l+1}/Z_{k,l}\sim\xi_2\ll 1$. In the highest order in the indicated small parameters we obtain from (7) and (5)

$$\alpha_1=\frac{8\pi^2\omega_1}{c\chi^{1/2}E_1^2}\left[\exp\left(\frac{\omega_1-\omega_2}{T}\right)-1\right]\sum_{\mathbf{p},\mathbf{q}}f(\mathbf{q})\Phi(\mathbf{p})\left[\frac{2a_{11}^2}{(\omega_2-\omega_1)^2}\right.$$

$$\left.+\frac{2a_{11}a_{10}a_{01}}{(\omega_2-\omega_1)\omega_1\omega_2}+\frac{a_{10}^2a_{01}^2}{\omega_1^2\omega_2^2}\right]\delta[\varepsilon(\mathbf{p}-\mathbf{q})-\varepsilon(\mathbf{p})+\omega_1-\omega_2], \quad (9)$$

where

$$a_{10}=-\frac{e\mathbf{E}_1}{2\omega_1}\frac{\partial}{\partial\mathbf{p}}[\varepsilon(\mathbf{p}-\mathbf{q})-\varepsilon(\mathbf{p})],$$

$$a_{01}=-\frac{e\mathbf{E}_2}{2\omega_2}\frac{\partial}{\partial\mathbf{p}}[\varepsilon(\mathbf{p}-\mathbf{q})-\varepsilon(\mathbf{p})],$$

$$a_{11}=\left(\frac{e\mathbf{E}_1}{2\omega_1}\frac{\partial}{\partial\mathbf{p}}\right)\left(\frac{e\mathbf{E}_2}{2\omega_2}\frac{\partial}{\partial\mathbf{p}}\right)[\varepsilon(\mathbf{p}-\mathbf{q})-\varepsilon(\mathbf{p})].$$

The expression for α_2 differs from (9) in the permutation of the indices $1\rightleftharpoons 2$.

It follows from (9) that $\alpha_{1,2}$ differ from zero only if $|\omega_1-\omega_2|\leqslant\Delta$. Inasmuch as the sum over p and q in (9) contains a positive definite function, the coefficients α_1 and α_2 reverse sign on going through the point $\omega_1=\omega_2$. Thus, α_1 and α_2 differ from zero noticeably only near the frequencies $\omega_1=\omega_2\pm\bar{\varepsilon}$, with the plus sign corresponding to $\alpha_1>0$ and $\alpha_2<0$, and the minus sign to $\alpha_1<0$ and $\alpha_2>0$.

The physical picture of the phenomenon is the following. Since $\bar{\varphi}(\mathbf{p})$ decreases rapidly (exponentially) with increasing $\varepsilon(\mathbf{p})$, the transitions with increasing electron energy $\varepsilon(\mathbf{p})$ become more favored than the inverse transition with decreasing $\varepsilon(\mathbf{p})$, since the transition frequency is proportional to the number of electrons in the initial state. At $\omega_{1,2}>\Delta$, only processes with simultaneous absorption (emission) of a photon ω_1 and emission

(absorption) of a photon ω_2 satisfy the energy conservation law. If in this case $\omega_1 > \omega_2$, then the absorption of the photon ω_1 is accompanied by emission of the photon ω_2 with increase of the electron energy by an amount $\omega_1 - \omega_2$, whereas emission of the photon ω_1 proceeds with absorption of the photon ω_2 and with a decrease of the electron energy by the same amount. Thus, the processes that occur with increase of electron energy are those with absorption of the photon ω_1 and emission of the photon ω_2, i.e., $\alpha_1 > 0$ and $\alpha_2 < 0$. In the opposite case, $\omega_1 < \omega_2$, the processes that lead to the increase of the electron energy are those with absorption of the photon ω_2 and emission of the photon ω_1, i.e., we have $\alpha_2 > 0$ and $\alpha_1 < 0$. We emphasize that in accordance with (9) the described effect is connected only with the fact that the conduction band has a finite width, and does not depend on the concrete form of the dispersion law.

In the case of a sufficiently narrow band, when $\Delta \ll T$ and $\Delta \ll \omega_{1,2}$, the expression for the absorption coefficient is simpler:

$$\alpha_1 = \frac{2\pi^2 e^2 n}{c\chi^{1/2}\omega_1(\omega_1-\omega_2)V}\left(\frac{eE_2}{\omega_2}\right)^2 \Phi(\omega_1-\omega_2), \tag{10}$$

where

$$\Phi(\omega) = \sum_{p,q}\frac{|C_q|^2}{\omega_q}\left\{\frac{\partial^2}{\partial p_1 \partial p_2}[\varepsilon(\mathbf{p}-\mathbf{q})-\varepsilon(\mathbf{p})]\right\}^2 \delta[\varepsilon(\mathbf{p}-\mathbf{q})-\varepsilon(\mathbf{p})+\omega]$$

is an even function of ω, p_1 and p_2 are the momentum components along $\mathbf{E}_1$ and $\mathbf{E}_2$, V is the volume of the Brillouin zone, and n is the conduction-electron density.

Let us clarify the character of the function $\Phi(\omega)$ using as an example the dispersion law $\varepsilon(\mathbf{p}) = \Delta(1 - \cos \mathbf{p}\cdot\mathbf{a})/2$, used as a model for organic semiconductors.[9] As a result we obtain (acoustic and unpolarized optical phonons)

$$\Phi(\omega) = \frac{(VC_0\omega a_1 a_2)^2}{\Delta}\begin{cases} P_{-1/2}(2\omega^2/\Delta^2-1), & |\omega|\leqslant\Delta \\ 0, & |\omega|>\Delta \end{cases} \tag{11}$$

where $C_0^2 = |C_q|^2/\omega_q$, while a_1 and a_2 are the projections of the vector $\mathbf{a}$ on $\mathbf{E}_1$ and $\mathbf{E}_2$, and $P_\nu(x)$ is a Legendre function.[8] The discontinuities in (11) at $|\omega| = \Delta$ are due to transitions between the singularities in the state density of the electrons and are connected with the specific features (quasi-one-dimensionality) of the chosen dispersion law.

We note that formulas (9)–(11) obtained above are valid only so long as $|\Delta\xi_1\xi_1|/|\omega_1 - \omega_2| \ll 1$. A calculation of the absorption coefficients in the field region in which this inequality is violated will be presented below.

2. *One weak and one strong electromagnetic field.* In the calculation of the absorption coefficient in the region when one of the fields is strong ($\xi_1 \ll 1 \ll \xi_2$), we confine ourselves to the frequency region in which

$$\omega_{1,2} \gg \Delta,\ \omega_{1,2} \gg |\omega_1 - s\omega_2| \gg \tau^{-1},$$

where s is a small natural number. In this case we have

$$Z_{kl}/Z_{\pm 1,\mp s} \sim |\omega_1 - s\omega_2|/\omega_{1,2} \ll 1,$$
$$Z_{kl} \ll 1\ (k \neq \pm 1;\ \mp s \neq l).$$

In the highest order in the indicated small parameters we obtain from (7)

$$\alpha_1 = \frac{8\pi^2\omega_1}{c\chi^{1/2}E_1^2}\sum_{p,q} f(\mathbf{q})[\bar{\varphi}(\mathbf{p}-\mathbf{q}) - \bar{\varphi}(\mathbf{p})]\sum_k kJ_k^2(2Z_{1s})$$
$$\times\sigma[a_{00}+k(\omega_1-s\omega_2)],\ \alpha_2 = -(E_1^2/E_2^2)(s\omega_2/\omega_1)\alpha_1. \tag{12}$$

In this case the equation for $\bar{\varphi}(\mathbf{p})$ takes according to (5) the form

$$0 = \sum_q f(\mathbf{q})[\bar{\varphi}(\mathbf{p}-\mathbf{q}) - \bar{\varphi}(\mathbf{p})]\sum_k J_k^2(2Z_{1s})\delta[a_{00}+k(\omega_1-s\omega_2)]. \tag{13}$$

It follows from (12) that on going through the points $\omega_1 = s\omega_2$, $s = 1, 2, \ldots$, the signs of α_1 and α_2 are reversed, thus pointing to the possibility of mutual enhancement of the weak and strong electromagnetic waves in semiconductors with a narrow conduction band.

Next, we put, for the sake or argument, $s = 1$ and, just as in the case of weak fields, carry out concrete calculations with the dispersion law $\varepsilon(\mathbf{p}) = \Delta(1 - \cos \mathbf{p}\cdot\mathbf{a})/2$ as the example. In this case, taking the δ functions in (12) and (13) into account, we have

$$2Z_{1s} = k\xi_1 J_1(\xi_2)/J_0(\xi_2),\ \xi_{1,2} = e\mathbf{E}_{1,2}\mathbf{a}/\omega_{1,2}.$$

By virtue of $\xi_1 \ll 1$, we have $2Z_{1s} \ll k$, far from the roots of $J_0(\xi_2)$, and we need retain in (13) only the term with $k = 0$, and in (12) the term with $k = \pm 1$. Substituting

$$\bar{\varphi}(\mathbf{p}) = G\exp[-\varepsilon(\mathbf{p})J_0(\xi_2)/T]$$

from (13) in (12), we obtain at $|J_0(\xi_2)|\Delta \ll T$ in the cases of scattering by acoustic and optical unpolarized phonons

$$\alpha_1 = \frac{8\pi^2 e^2 n J_1^2(\xi_2)}{c\chi^{1/2}\omega_1(\omega_1-\omega_2)V|J_0(\xi_2)|a_2^2}\Phi\left(\frac{\omega_1-\omega_2}{J_0(\xi_2)}\right). \tag{14}$$

At $\xi_2 \ll 1$ we easily obtain from (14) the result (10), (11). Near the roots of $J_0(\xi_2)$, when $|J_0(\xi_2)\Delta| < |\omega_1 - \omega_2|$, the δ functions in (12) vanish and there is no electromagnetic-wave absorption under the assumptions made above.

We note that the effects should manifest themselves most strongly in crystals with narrow allowed bands $\Delta < \omega_{1,2}$ and with wide forbidden bands $\Delta_g > \omega_1 + \omega_2$, where there is no absorption of any one of the waves separately. This situation is realized in semiconductors with superlattice[2] and in organic semiconductors.[3-6,9] It is with these semiconductors as examples that we shall make numerical estimates of the predicted effect.

It follows from (9) with (8) that, in order of magnitude

$$|\alpha_{1,2}| \sim (\omega_1-\omega_2)\omega_{1,2}^{-1}\xi_{2,1}^2\alpha_0(\omega_1-\omega_2),$$

where $\alpha_0(\omega)$ is the coefficient of intraband absorption of the electromagnetic wave. In polymer semiconductors such as radiation-thermally modified polyethylene, α_0 can reach 10^2 cm^{-1} (Ref. 4,5), which at $\Delta \sim 10^{-2}$ eV ($\Delta g \sim 1$ eV), $d \sim 5\times10^{-8}$ cm,[3-6] $\omega_2 \sim 1.2\times10^{-2}$ eV, $\omega_1 - \omega_2 \sim 0.8\times10^{-2}$ eV, and $E_{1,2} \sim 2.4\times10^4$ V/cm ($\xi_{1,2} \sim 0.1$) yields a value $|\alpha_{1,2}| \sim 1 - 10^{-1}$ cm^{-1}. Which lends itself readily to experimental verification. Analogous values of $|\alpha_{1,2}|$ can be obtained also for semiconductors with superlattices, putting $\alpha_0 \sim 0.5\times10^2$ cm^{-1}. In the last case, however, the condition $\xi_{1,2} \sim 0.1$ corresponds to weaker fields: $E_{1,2} \sim 5\times10^3$ V/cm, inasmuch as $d \sim 2.5\times10^{-7}$ cm.[2]

In the case of a weak field and a strong field, substi-

tuting in (8) $\varepsilon(p)=\Delta(1+\cos \mathbf{p}\cdot\mathbf{a})/2$, and assuming $\Delta \ll T$, we obtain from (14) with (8), in order of magnitude,

$$|\alpha_1| \sim [J_1(\xi_2)/J_0(\xi_2)]^2[(\omega_1-\omega_2)/J_0(\xi_2)]^3\omega_1^{-1}\Delta^{-2}\alpha_0[(\omega_1-\omega_2)/J_0(\xi_2)],$$
$$(|\alpha_2|=(E_1^2/E_2^2)(\omega_2/\omega_1)|\alpha_1|),$$

so that at $\Delta \sim 10^{-2}$ eV ($\Delta_\varepsilon \sim 1$ eV), $d \sim 5\times10^{-8}$ cm, $\omega_2 \sim 10^{-1}$ eV, $|\omega_1-\omega_2| \sim 0.8\times10^{-2}$ eV, $E_1 \sim 2.4\times10^4$ V/cm ($\xi_1 \sim 10^{-2}$), $E_2 \sim 10^6$ V/cm ($\xi_2 \sim 0.5$), and $\alpha_0 \sim 10^2$ cm^{-1} we get $|\alpha_1| \sim 1-10^{-1}$ cm^{-1} and $|\alpha_2| \sim 10^{-3}-10^{-4}$ cm^{-1}. Similar values of $|\alpha_{1,2}|$ are obtained also for semiconductors with superlattices in fields $E_1 \sim 5\times10^2$ V/cm and $E_2 \sim 2\times10^5$ V/cm.

The author thanks E. M. Épshtein for a discussion of the results and for helpful remarks.

[1)] A similar situation in the presence of one electromagnetic wave was considered by Strekalov.[6]

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Translated by J. G. Adashko