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Birefringence and gyrotropy due to nearly Bragglike processes in the x-ray region

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Propagation of electromagnetic waves is considered in scalar spatially periodic media at angles and frequencies that almost satisfy the Bragg conditions. The process of virtual rescattering into other waves and back leads to corrections to the phase velocity of the initial wave. The dependence of the amplitude of the scalar scattering on the polarization causes these corrections to produce birefringence in the region of the two-wave Bragg resonance. Near three-wave and multiwave resonances, subject to definite conditions on the symmetry of the medium, these corrections can lead also to gyrotropy, i.e., to rotation of the plane of polarization of the wave. The possibility of observing the effects in crystal at x-ray frequencies is considered.

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1. INTRODUCTION

It is known that the dielectric constant of a condensed medium differs from unity in the x-ray band ($\lambda \sim 1$ Å) only in the fourth or even fifth decimal point, and is furthermore a pure scalar:

$$\varepsilon_{ik}(\mathbf{r})=\delta_{ik}\left[1-\frac{4\pi N(\mathbf{r})e^2}{m\omega^2}\right],$$

where m and e are the mass and charge of the electron, $N(\mathbf{r})$ is their density, and $\omega=2\pi c/\lambda$ is the radiation frequency. The propagation of x-ray photons through a medium is therefore not accompanied as a rule by a change in their polarization state.

Exceptions are cases when the conditions of Bragg rescattering from a given wave into another at the corresponding Fourier component $e_{\mathbf{q}}\exp(i\mathbf{q}\cdot\mathbf{r})$ of the dielectric constant are satisfied in the crystal. Since the amplitude of the scattering from a wave A into a wave B on scalar perturbations is proportional to $f_{BA} \propto e_{\mathbf{q}}(\mathbf{e}_B^*\cdot\mathbf{e}_A)$, where $\mathbf{e}_A$ and $\mathbf{e}_B$ are the unit vectors of the polarization, it follows, as is well known, that the Bragg interaction is different for the s- and p-polarizations in both the kinematic and the dynamic theory (see, e.g., Refs. 1–3). More complicated are the polarization effects in the case of multiwave refraction (see Refs. 1–4). In all these cases, however, apart from the change in the polarization state of the incident wave itself, diffracted waves are excited in fact.

We wish to discuss in this paper the possibility of observing birefringence and gyrotropy in pure form, i.e., without real excitation of other waves. As the mechanism for producing these effect we propose the process of virtual rescattering into other waves and back. The smallness of the amplitude of the elementary rescattering act can be offset to a considerable degree by the proximity to the Bragg resonances.

2. BIREFRINGENCE IN PROPAGATION NEAR A SOLITARY BRAGG RESONANCE

We consider wave propagation in a direction close to the satisfaction of the Bragg condition for the Fourier component of the dielectric constant

$$\delta\varepsilon(\mathbf{r})=2|\delta\varepsilon_{\mathbf{q}}|\cos(\mathbf{q}\mathbf{r}+\varphi)=\delta\varepsilon_{\mathbf{q}}e^{i\mathbf{q}\mathbf{r}}+\delta\varepsilon_{-\mathbf{q}}e^{-i\mathbf{q}\mathbf{r}}. \quad (1)$$

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In the absence of absorption, $\delta\varepsilon(\mathbf{r})$ is here real, corresponding in terms of (1) to the relation $\delta\varepsilon_{-\mathbf{q}}=\delta\varepsilon_{\mathbf{q}}^*$. We denote by $\mathbf{n}_A$ and $\mathbf{n}_B$ the propagation directions of waves that satisfy exactly the Bragg condition at a certain frequency ω_0:

$$|\mathbf{n}_A|=|\mathbf{n}_B|=1,\quad k_0(\mathbf{n}_A-\mathbf{n}_B)=\mathbf{q}_{AB}=\mathbf{q}, \tag{2}$$

where k_0 is the wave number with allowance for the average refractive index.

We seek the field in the crystal, with allowance for two-wave refraction at the frequency $\omega=\omega_0+\Delta\omega$, in the form

$$\mathbf{E}(\mathbf{r})=\mathbf{A}(\mathbf{r})\exp(i\mathbf{k}_A\mathbf{r})+\mathbf{B}(\mathbf{r})\exp(i\mathbf{k}_B\mathbf{r}), \tag{3}$$

where $\mathbf{k}_{A,B}=k_0\mathbf{n}_{A,B}$, while $\mathbf{A}(\mathbf{r})$ and $\mathbf{B}(\mathbf{r})$ are slowly varying amplitudes. Maxwell's equations then lead to the system

$$(\mathbf{n}_A\nabla)\mathbf{A}(\mathbf{r})-i\frac{\Delta\omega}{c}\mathbf{A}(\mathbf{r})=i\frac{\omega}{2c}\varepsilon_{\mathbf{q}}\hat{P}_A\hat{P}_B\mathbf{B}(\mathbf{r}), \tag{4a}$$

$$(\mathbf{n}_B\nabla)\mathbf{B}(\mathbf{r})-i\frac{\Delta\omega}{c}\mathbf{B}(\mathbf{r})=i\frac{\omega}{2c}\varepsilon_{-\mathbf{q}}\hat{P}_B\hat{P}_A\mathbf{A}(\mathbf{r}). \tag{4b}$$

Here $\hat{P}_A$ is the operator of projection on a plane perpendicular to the propagation direction of the wave $\mathbf{A}$, i.e.,

$$(\hat{P}_A)_{ik}=\delta_{ik}-(\mathbf{n}_A)_i(\mathbf{n}_A)_k, \tag{5}$$

and analogously for $\hat{P}_B$.

In the right-hand side of Eq. (4a) we have retained, besides the operator $\hat{P}_A$ that is obtained directly from Maxwell's equations and that ensures the transversality condition $\mathbf{n}_A\cdot\mathbf{A}(\mathbf{r})=0$, also the operator $\hat{P}_B$. Its action on the vector $\mathbf{B}$, by virtue of the same transversality condition $\mathbf{n}_B\cdot\mathbf{B}(\mathbf{r})=0$, is equivalent to the action of a unit operator. Introduction of the operator $\hat{P}_B$ into (4a) facilitates the calculations that follow. A similar remark holds for Eq. (4b).

It is easy to verify that in the absence of rescattering ($\delta\varepsilon_{\mathbf{q}}=\delta\varepsilon_{-\mathbf{q}}=0$), Eq. (4a) is satisfied by a plane wave in the form

$$\exp(ik_0\mathbf{n}_A\mathbf{r})\mathbf{A}(\mathbf{r})=\mathbf{A}_0\exp\{i(k_0\mathbf{n}_A+\Delta\omega\mathbf{n}_A/c+k_0\boldsymbol{\psi}_A)\mathbf{r}\},$$

where $\boldsymbol{\psi}_A\cdot\mathbf{n}_A=0$.

Assume that a wave $\mathbf{A}$ that does not satisfy exactly the Bragg condition is incident on the crystal. We write down the solutions for $\mathbf{A}(\mathbf{r})$ and $\mathbf{B}(\mathbf{r})$ in the form

$$\begin{aligned}\mathbf{A}(\mathbf{r})&=\exp\{i(\Delta\omega\mathbf{n}_A/c+k_0\boldsymbol{\psi}_A)\mathbf{r}\}\mathbf{a}(\mathbf{r}),\\ \mathbf{B}(\mathbf{r})&=\exp\{i(\Delta\omega\mathbf{n}_A/c+k_0\boldsymbol{\psi}_A)\mathbf{r}\}\mathbf{b}(\mathbf{r}),\end{aligned} \tag{6}$$

where the angle ψ_A characterizes the deviation from the central directions $\mathbf{n}_A$, (i.e., $\psi_A\cdot\mathbf{n}_A=0$); $\mathbf{a}(\mathbf{r})$ and $\mathbf{b}(\mathbf{r})$ are amplitudes that are changed only by the Bragg interaction. We assume here that, owing to the inexact satisfaction of the Bragg condition, the wave $\mathbf{B}$ is weakly excited. In the equation for $\mathbf{b}(\mathbf{r})$

$$\begin{aligned}(\mathbf{n}_B\nabla)\mathbf{b}(\mathbf{r})+i\Lambda_B(\boldsymbol{\psi}_A,\Delta\omega)\mathbf{b}(\mathbf{r})&=i\frac{\omega}{2c}\varepsilon_{-\mathbf{q}}\hat{P}_B\hat{P}_A\mathbf{a}(\mathbf{r}),\\ \Lambda_B(\boldsymbol{\psi}_A,\Delta\omega)&=k\mathbf{n}_B\boldsymbol{\psi}_A+\frac{\Delta\omega}{c}(\mathbf{n}_B\mathbf{n}_A-1)\end{aligned} \tag{7}$$

we can then neglect the gradient compared with the Bragg detuning Λ_B (which has the dimension of reciprocal centimeters).

In this case $\mathbf{b}(\mathbf{r})$ is excited only virtually:

$$\mathbf{b}(\mathbf{r})=\Lambda_B^{-1}\frac{\omega}{2c}\varepsilon_{-\mathbf{q}}\hat{P}_B\hat{P}_A\mathbf{a}(\mathbf{r}), \tag{8}$$

and substitution of this expression in (4a) yields an equation for the change of the amplitude $\mathbf{a}(\mathbf{r})$ along the ray $\mathbf{n}_A$:

$$(\mathbf{n}_A\nabla)\mathbf{a}(\mathbf{r})=i\hat{T}\mathbf{a}(\mathbf{r})=i\Lambda_B\left|\frac{\omega\varepsilon_{\mathbf{q}}}{2c}\right|^2\hat{P}_A\hat{P}_B\hat{P}_A\mathbf{a}(\mathbf{r}). \tag{9}$$

It is convenient here to express the symmetrical matrix $\hat{T}$, whose components lie only in a plane perpendicular to $\mathbf{n}_A$, in terms of coordinates such that

$$\mathbf{n}_A=\mathbf{e}_z,\quad [\mathbf{n}\times\mathbf{n}_B]\propto\mathbf{e}_y,\quad \mathbf{n}_B-\mathbf{n}_A(\mathbf{n}_B\mathbf{n}_A)\propto\mathbf{e}_x;$$

so that its components take the form

$$T_{yy}=\Lambda_B^{-1}\left|\frac{\omega\varepsilon_{\mathbf{q}}}{2c}\right|^2,\quad T_{xy}=T_{yx}=0,\quad T_{xx}=T_{yy}(\mathbf{n}_A\mathbf{n}_B)^2. \tag{10}$$

Thus, the wave A is subject under these conditions to birefringence with principal axes $\mathbf{e}_y$ and $\mathbf{e}_x$, the rate of phase advance being

$$\frac{d}{dz}(\varphi_y-\varphi_x)=T_{yy}[1-(\mathbf{n}_A\mathbf{n}_B)^2]. \tag{11}$$

It is convenient to express the value of the Fourier component $|\varepsilon_{\mathbf{q}}|$ in terms of the thickness l_e over which complete transfer from one wave to the other takes place within the framework of Ewald's Pendellosung solution in the dynamic diffraction regime. Namely, for the symmetrical Laue case

$$l_e\left|\frac{\omega\varepsilon_{\mathbf{q}}}{2c}\right|=\frac{\pi}{2}\cos\frac{\theta}{2}, \tag{12}$$

where $\cos\theta\equiv\mathbf{n}_A\cdot\mathbf{n}_B$. In addition, the ratio of the deviation Λ_B from the Bragg condition to the width of the dynamic resonance in terms of the same variables is given by

$$\Lambda_B=\alpha|\varepsilon_{\mathbf{q}}|\frac{\omega}{2c\cos(\theta/2)}, \tag{13}$$

and this ratio is $\alpha\gg 1$.

Of course, the perturbation-theory approach itself is valid only as an expansion in terms of the small parameter $\alpha^{-1}\ll 1$. In the solution (6), (8) the energy fraction contained in the wave $\mathbf{B}$ is $\sim\alpha^{-2}$, and is negligibly small already at $\alpha\sim 10$. The rate of growth of the phase difference $\varphi_y-\varphi_x$ then takes the form

$$\frac{d}{dz}(\varphi_y-\varphi_x)=\left(\frac{2}{\pi}\right)^2\frac{1}{l_e\alpha}[1-(\mathbf{n}_A\mathbf{n}_B)^2]. \tag{14}$$

Estimates of the effect in accord with (14) are given in Sec. 4 below.

3. BIREFRINGENCE AND GYROTROPY NEAR THREE-WAVE BRAGG RESONANCES

We consider the propagation of the wave $\mathbf{A}$ near a direction that satisfies the three-wave diffraction conditions. Assume that the three reciprocal-lattice vectors $\mathbf{q}_{AB}$, $\mathbf{q}_{BC}$, and $\mathbf{q}_{CA}$ form a closed triangle

$$\mathbf{q}_{AB}+\mathbf{q}_{BC}+\mathbf{q}_{CA}=0,\quad \mathbf{q}_{AB}=-\mathbf{q}_{BA},\quad \mathbf{q}_{BC}=-\mathbf{q}_{CB},\quad \mathbf{q}_{CA}=-\mathbf{q}_{AC}. \tag{15}$$

For a given triad of vectors q_{ik} satisfying the condition

(15), it is convenient to use them as the sides of a triangle ABC and find in its plane the point O of intersection of the perpendicular bisectors (the center of the circumscribed circle of the triangle, Fig. 1). We draw from this point a line OO' perpendicular to the plane of the triangle ABC. It is easy to show that the waves for which the Ewald sphere intersects the ABC plane along the circumscribed circle of the triangle satisfy the Bragg three-wave diffraction condition. Thus, the manifold of waves with exact Bragg directions $\mathbf{n}_A$, $\mathbf{n}_B$, and $\mathbf{n}_C$ depends on a single parameter. This parameter can be, for example, the frequency ω_0, so as to have at $k_0=\omega_0(\varepsilon_0)^{1/2}/c$

$$k_0(\mathbf{n}_A-\mathbf{n}_B)=\mathbf{q}_{AB},\quad k_0(\mathbf{n}_B-\mathbf{n}_C)=\mathbf{q}_{BC},\quad k_0(\mathbf{n}_C-\mathbf{n}_A)=\mathbf{q}_{CA}. \tag{16}$$

In our problem we take into account three Fourier components of the dielectric constant (in the absence of absorption, the zeroth component can be included in $k_0=\omega_0(\varepsilon_0)^{1/2}/c$):

$$\delta\varepsilon(\mathbf{r})=\delta\varepsilon^*(\mathbf{r}) =2\{|\delta\varepsilon_{AB}|\cos(\mathbf{q}_{AB}\mathbf{r}+\varphi_{AB})+|\delta\varepsilon_{BC}|\cos(\mathbf{q}_{BC}\mathbf{r}+\varphi_{BC}) +|\delta\varepsilon_{CA}|\cos(\mathbf{q}_{CA}\mathbf{r}+\varphi_{CA})\}=\delta\varepsilon_{AB}\exp(i\mathbf{q}_{AB}\mathbf{r})+\dots. \tag{17}$$

The phase shifts $\varphi_{ik}=-\varphi_{ki}$ of the Fourier components depend on the choice of the origin, and a shift $\Delta\mathbf{r}$ causes them to change in accord with the law

$$\Delta\varphi_{ik}=\mathbf{q}_{ik}\Delta\mathbf{r}. \tag{18}$$

By the same token, at least two out of three phases in (17) can be made to vanish (the two components Δx and Δy in the ABC plane). By virtue of the condition $\mathbf{q}_{AB}+\mathbf{q}_{BC}+\mathbf{q}_{CA}=0$ the sum φ_{ABC} of these phases, taken in accordance with the rule for going around the triangle in some definite direction (e.g., ABC), turns out to be independent of the choice of the origin

$$\varphi_{ABC}=\varphi_{AB}+\varphi_{BC}+\varphi_{CA}=\mathrm{inv}, \tag{19}$$

with $\varphi_{ABC}=-\varphi_{ABC}$.

In the general case φ_{ABC} is not equal to zero, although in some concrete cases the equality $\varphi_{ABC}=0$ can follow from the symmetry of the crystal (for more details see, e.g., Refs. 1–3).

The gyrotropy effect of interest to us, as will be seen from the sequel, is proportional to $\sin\varphi_{ABC}$. In analogy with the two-wave case, at the frequency $\omega=\omega_0+\Delta\omega$ the field in the crystal, which differs slightly from the exact Bragg frequency, will be represented, given $\mathbf{n}_A$, $\mathbf{n}_B$, and $\mathbf{n}_C$, in the form

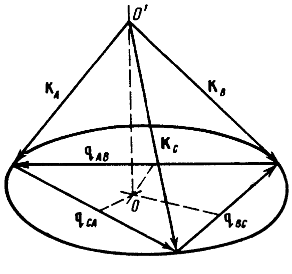

FIG. 1. Orientation of wave vectors $\mathbf{k}^A$, $\mathbf{k}^B$, and $\mathbf{k}^C$ in three-wave Bragg resonance; $\mathbf{q}^{AB}$, $\mathbf{q}^{BC}$, and $\mathbf{q}^{CA}$ are the crystal reciprocal-lattice vectors.

$$\mathbf{E}(\mathbf{r})=\mathbf{A}(\mathbf{r})\exp(i\mathbf{k}_A\mathbf{r})+\mathbf{B}(\mathbf{r})\exp(i\mathbf{k}_B\mathbf{r})+\mathbf{C}(\mathbf{r})\exp(i\mathbf{k}_C\mathbf{r}), \tag{20}$$

where $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ are slowly varying amplitudes that satisfy the equations

$$(\mathbf{n}_A\nabla)\mathbf{A}-i\Delta\omega\mathbf{A}/c=i(\hat{f}_{AB}\mathbf{B}+\hat{f}_{AC}\mathbf{C}), \tag{21a}$$

$$(\mathbf{n}_B\nabla)\mathbf{B}-i\Delta\omega\mathbf{B}/c=i(\hat{f}_{BA}\mathbf{A}+\hat{f}_{BC}\mathbf{C}), \tag{21b}$$

$$(\mathbf{n}_C\nabla)\mathbf{C}-i\Delta\omega\mathbf{C}/c=i(\hat{f}_{CA}\mathbf{A}+\hat{f}_{CB}\mathbf{B}),\quad \hat{f}_{\alpha\beta}=\omega\varepsilon_{\alpha\beta}\hat{P}_\alpha\hat{P}_\beta/2c. \tag{21c}$$

Assume that incidence on the crystal of a wave $\mathbf{A}(\mathbf{r})\exp(i\mathbf{k}_A\cdot\mathbf{r})$ with certain values of the angle ψ_A and of the frequency $\omega_0+\Delta\omega$, which led to a detuning from the exact Bragg condition. It is convenient then to seek the solution of Eqs. (21) in the form

$$\{\mathbf{A}(\mathbf{r}),\mathbf{B}(\mathbf{r}),\mathbf{C}(\mathbf{r})\}=\{\mathbf{a}(\mathbf{r}),\mathbf{b}(\mathbf{r}),\mathbf{c}(\mathbf{r})\}\exp\{i(k_0\psi_A+\Delta\omega\mathbf{n}_A/c)\mathbf{r}\}, \tag{23}$$

where $\psi_A\cdot\mathbf{n}_A=0$, while $\mathbf{a}(\mathbf{r})$, $\mathbf{b}(\mathbf{r})$, and $\mathbf{c}(\mathbf{r})$ are even slower functions of the coordinates. It follows for them from (21) that

$$(\mathbf{n}_A\nabla)\mathbf{a}(\mathbf{r})=if_{AB}\mathbf{b}+if_{AC}\mathbf{c}, \tag{24a}$$

$$(\mathbf{n}_B\nabla)\mathbf{b}(\mathbf{r})+i\Lambda_B\mathbf{b}(\mathbf{r})=if_{BA}\mathbf{a}+if_{BC}\mathbf{c}, \tag{24b}$$

$$(\mathbf{n}_C\nabla)\mathbf{c}(\mathbf{r})+i\Lambda_C\mathbf{c}(\mathbf{r})=if_{CA}\mathbf{a}+if_{CB}\mathbf{b}, \tag{24c}$$

$$\Lambda_C(\Delta\omega,\psi_A)=k_0\psi_A\mathbf{n}_C+\Delta\omega c^{-1}[\mathbf{n}_A\mathbf{n}_c-1]. \tag{25}$$

The expression for Λ_B differs from (25) by the change of subscripts $C\to B$, and is contained in formula (7).

If we stipulate the vanishing of both Bragg detunings

$$\Lambda_B(\Delta\omega,\psi_A)=\Lambda_C(\Delta\omega,\psi_A)=0,$$

then we can obtain the dependence of the admissible slopes $\psi_A=\mathbf{m}\Delta\omega$ that preserve the exact Bragg condition and correspond to a shift of the center of the Ewald sphere along the line OO' in Fig. 1. The vector $\mathbf{m}\perp\mathbf{n}_A$ lies in this case in the plane containing the vector $\mathbf{n}_A$ and the line OO', and its value can be easily determined by equating (25) to zero.

We shall be interested in the situation wherein the Bragg condition is not satisfied exactly for both the pair AB and the pair AC, the waves B and C being admixed only virtually, with a small amplitude of the order of $f_{iA}/\Lambda_i\sim\alpha_{iA}^{-1}$ and of higher powers of this parameter.

We represent the amplitudes of the waves $\mathbf{b}$ and $\mathbf{c}$ in the form of a series in powers of the small parameter f/Λ, i.e., $\mathbf{b}=\mathbf{b}_1+\mathbf{b}_2+\dots$ and $\mathbf{c}=\mathbf{c}_1+\mathbf{c}_2+\dots$; we assume here that $\mathbf{a}(\mathbf{r})$ is of zeroth order of smallness. In addition we make the assumption (whose validity will be confirmed by the calculation results) that differentiation with respect to the coordinates in (24) leads to an additional smallness of order f/Λ or of a higher power of this parameter.

We are interested in the variation of the amplitude $\mathbf{a}(\mathbf{r})$ along the ray $\mathbf{n}_A$, assuming, for example, that the crystal boundary is perpendicular to the direction $\mathbf{n}_A=\mathbf{e}_z$, and then $\mathbf{a}=\mathbf{a}(z)$, $\mathbf{b}=\mathbf{b}(z)$, and $\mathbf{c}=\mathbf{c}(z)$. We divide the right and left sides of (24b) and (24c) by Λ_B and Λ_C, respectively. Substituting $\mathbf{b}(z)=\mathbf{b}_1(z)+\mathbf{b}_2(z)+\dots$ and a similar expression for $c(z)$, and equating terms of like powers of Λ^{-1}, we obtain

$$\mathbf{b}_1(z)=\Lambda_B^{-1}\hat{f}_{BA}\mathbf{a}(z),\quad \mathbf{b}_2(z)=\Lambda_B^{-1}\Lambda_C^{-1}\hat{f}_{BC}\hat{f}_{CA}\mathbf{a}(z), \tag{26}$$

so that the equation for $d\mathbf{a}/dz$, accurate to terms of second order in Λ^{-1} inclusive, takes the form

$$\frac{d\mathbf{a}}{dz}=i\,[\Lambda_B^{-1}\hat{f}_{AB}\hat{f}_{BA}+\Lambda_C^{-1}\hat{f}_{AC}\hat{f}_{CA}+\Lambda_B^{-1}\Lambda_C^{-1}(\hat{f}_{AB}\hat{f}_{BC}\hat{f}_{CA}+\hat{f}_{AC}\hat{f}_{CB}\hat{f}_{BA})]\,\mathbf{a}(z). \tag{27}$$

The terms proportional to Λ_B^{-1} and Λ_C^{-1} in (27) describe the contributions of the virtual admixture of the waves B and C to the average refractive index and to the birefringence of the wave A; each of these contributions coincides with the one considered in Sec. 2 above. The gyrotropy of interest to us is due to the term $\alpha\Lambda^{-2}f^3$. To be able to observe this small gyration we must stipulate that there be no stronger birefringence of order $\Lambda^{-1}f^2$. In other words, conditions must be found such that the matrix acting in the xy plane

$$\hat{T}_1=\Lambda_B^{-1}\hat{f}_{AB}\hat{f}_{BA}+\Lambda_C^{-1}\hat{f}_{AC}\hat{f}_{CA}$$

is a multiple of the unit matrix, i.e., that it yield only a correction to the average refractive index of the wave A.

It is easy to verify that this is possible only if the following two conditions are simultaneously satisfied:

1) the planes made up by the vector pairs $(\mathbf{n}_A, \mathbf{n}_B)$ and $(\mathbf{n}_A, \mathbf{n}_C)$ are perpendicular to each other;

2) the detunings Λ_B and Λ_C satisfy the relation

$$\frac{|\varepsilon_{BA}|^2}{\Lambda_B}[1-(\mathbf{n}_B\mathbf{n}_A)^2]=\frac{|\varepsilon_{CA}|^2}{\Lambda_C}[1-(\mathbf{n}_A\mathbf{n}_C)^2]. \tag{28}$$

We assume these two conditions to be satisfied. The action of the terms $\propto f^2\Lambda^{-1}$ then yields the relation $\mathbf{a}(z)=\tilde{\mathbf{a}}(z)\exp(i\mu z)$, where

$$\mu=\frac{|\varepsilon_{AB}|^2}{\Lambda_B}\frac{1-(\mathbf{n}_A\mathbf{n}_B)^2(\mathbf{n}_A\mathbf{n}_C)^2}{1-(\mathbf{n}_A\mathbf{n}_C)^2}. \tag{29}$$

Separating this relation, we obtain for $\tilde{\mathbf{a}}(z)$

$$\frac{d\tilde{\mathbf{a}}}{dz}=i\hat{T}_2\tilde{\mathbf{a}}(z). \tag{30}$$

We shall assume that in the xy plane, which is perpendicular to $\mathbf{n}_A=\mathbf{e}_z$, the x axis lies in the plane $(\mathbf{n}_A, \mathbf{n}_B)$, and the y axis in the plane $(\mathbf{n}_A, \mathbf{n}_C)$; we recall that we assume that the last two planes are perpendicular to each other, so as to set equal to zero the birefringence of first order in f/Λ. The matrix $\hat{T}_2$ contains a symmetrical real part that describes the average refractive index $\hat{T}_2^0$ and the birefringence $\hat{T}_2^c$, and an antisymmetrical pure imaginary part that describes the gyration or the optical activity $\hat{T}_2^a$:

$$\hat{T}_2^0=\left(\frac{\omega}{2c}\right)^3|\varepsilon_{AB}\varepsilon_{BC}\varepsilon_{CA}|\frac{\cos\varphi_{ABC}}{\Lambda_B\Lambda_C}[(\mathbf{n}_A\mathbf{n}_B)^2+(\mathbf{n}_A\mathbf{n}_C)^2]\begin{pmatrix}1&0\\0&1\end{pmatrix}, \tag{31a}$$

$$\hat{T}_2^c=\left(\frac{\omega}{2c}\right)^3|\varepsilon_{AB}\varepsilon_{BC}\varepsilon_{CA}|\frac{\cos\varphi_{ABC}}{\Lambda_B\Lambda_C}\times\begin{pmatrix}(\mathbf{n}_A\mathbf{n}_B)^2-(\mathbf{n}_A\mathbf{n}_C)^2 & (\mathbf{n}_B\mathbf{n}_C)\sin AB\sin AC\\(\mathbf{n}_B\mathbf{n}_C)\sin AB\sin AC & (\mathbf{n}_A\mathbf{n}_C)^2-(\mathbf{n}_A\mathbf{n}_B)^2\end{pmatrix}, \tag{31b}$$

$$\hat{T}_2^a=\left(\frac{\omega}{2c}\right)^3|\varepsilon_{AB}\varepsilon_{BC}\varepsilon_{CA}|\frac{\sin\varphi_{ABC}}{\Lambda_B\Lambda_C}(\mathbf{n}_B\mathbf{n}_C)\sin AB\sin AC\begin{pmatrix}0&-i\\i&0\end{pmatrix}. \tag{31c}$$

We have introduced here the notation $AB\equiv(\mathbf{n}_A\cdot\mathbf{n}_B)$.

We consider first the case most favorable for the observation of gyration: $\varphi_{ABC}=\pm\pi/2$. Then there is no birefringence in second order, and the rate $d\beta/dz$ of the polarization plane (β is the angle) is

$$\frac{d\beta}{dz}=i\frac{T_{12}-T_{21}}{2}=\left(\frac{\omega}{2c}\right)^3|\varepsilon_{AB}\varepsilon_{BC}\varepsilon_{CA}|\frac{\sin\varphi_{ABC}}{\Lambda_B\Lambda_C}(\mathbf{n}_B\mathbf{n}_C)\sin AB\sin AC, \tag{32}$$

where we must put $\sin\varphi_{ABC}=\pm1$.

In the general case, however, then φ_{ABC} is not equal to $\pm\pi/2$, birefringence appears in second order in addition to gyration. When a linearly polarized wave enters the crystal, its average polarization rotates in the course of propagation and becomes elliptic as well. The corresponding formulas are straightforward in principle, but are quite unwieldy and will not be presented here.

It is easily understood that when the Bragg interactions involve four and more waves it is likewise possible in principle to obtain birefringence and gyration. A highly important problem is posed by the symmetry requirements that the medium must satisfy in order to obtain $\varphi_{ABC}\neq 0$ in the three-wave problem. This question is not considered in this paper.

4. DISCUSSION OF EXPERIMENTAL POSSIBILITIES

We assume for numerical estimate a tentative value of the order of 10^{-5}–10^{-4} cm for l_e in (12) (see Ref. 2, Chap. 9). A deviation of 10 widths of the dynamic-diffraction curve from the Bragg condition, $\alpha\sim10$ [where α is defined by Eq. (13)] will then yield, in the two-wave case, a birefringence of the order of

$$\frac{d}{dz}(\varphi_y-\varphi_x)\sim\frac{1}{l_e\alpha}\sim10^{+3}-10^{+4}\ \text{cm}^{-1}.$$

On the other hand, the absorption coefficient μ can amount under typical conditions to 10^2–5×10^2 cm^{-1}. Thus, a phase difference $\varphi_y-\varphi_x\sim2$–100 rad can accumulate over an absorption length $l_{\text{abs}}=\mu^{-1}$. This means that the production of strong birefringent elements based on almost-Bragg two-wave resonances is really feasible. We note that it is possible to use nonideal (mosaic) crystals, since we need not satisfy exactly the conditions of dynamic diffraction.

As for gyrotropy (rotation of the polarization plane), we get for it from (32) the estimate

$$\frac{d\beta}{dz}\sim\sin\varphi_{ABC}\frac{1}{l_e\alpha^2}. \tag{33}$$

Assuming $\sin\varphi_{ABC}\sim1$, $l_e\approx10^{-4}$, and $\alpha^2\approx10^{+2}$, we obtain $d\beta/dz\sim10^{+2}$ cm^{-1}. Over an absorption thickness $l_{\text{abs}}\sim10^2$ cm, the gyration angle can amount to $\beta\sim1$ rad, i.e., a perfectly observable value.

We have thus predicted in this paper strong x-ray birefringence and gyration effects resulting from two- and multiwave almost-Bragg interactions. Observation of these effects uncovers wide possibilities for the application of various methods of polarization optics of the visible band in the x-ray region of the spectrum.

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Kinetic phenomena in the flow of a strongly rarefied molecular gas in an external field

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A kinetic theory is developed for the effects that arise when a free-molecular polyatomic gas flows between two surfaces in an external field. The influence of the field on the transport processes is due to the nonequilibrium polarization of the gas molecules when they are nonspherically scattered from the surface of a solid, and to the destruction of this polarization in the field. The change of the gas flow velocity in a channel in a magnetic field, and the onset of a transverse heat flux between the surfaces (whose temperatures are equal) is examined in detail. In contrast to the previously investigated thermomagnetic phenomena, the considered effects in a gas stream can occur when the molecules are scattered from the surface not only inelastically but also elastically. At the same time, these effects occur only if the interaction with the surface is such that the states of the molecule before and after the collision are correlated.

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1. INTRODUCTION

The influence of a magnetic field on heat flow in a strongly rarefied ($l \gg L$, where l is the molecule mean free path and L is the characteristic dimension) polyatomic gas (the thermomagnetic effect) has already been observed and investigated earlier.[1,2] Other possible effects in an inhomogeneously heating gas in a magnetic field were also analyzed, such as the appearance of transverse heat and mass fluxes in a gas contained between two surfaces having different temperatures, or of thermomagnetic forces acting on the walls.[3] The physical causes common to the changes in the transport processes in a magnetic field are the polarization of the molecules inelastically scattered from the solid surface and the precession of the magnetic moment of the molecule about the field direction. A distinguishing feature of the foregoing effects is the oscillatory character of the dependence of the macroscopic fluxes in the gas on the intensity of the constant external field at a fixed geometry of the problem. The concrete dependence of the macroscopic quantities on the intensity and orientation of the field is determined entirely by the law of nonspherical scattering of molecules by walls. Therefore the kinetic effects in a strongly rarefied gas in an external field serve as a unique source of information on the physical mechanism of the orientation-dependent interaction between molecules and the surface of a solid, and on the properties of the surface itself.[2,4,5]

By virtue of the isotropy of the distribution of the molecules of the equilibrium gas with respect to their orientations and directions of motion, polarization of molecules deflected from the surface can occur only in a nonequilibrium gas. The effects listed above are due to the temperature inhomogeneity of the system. It can be assumed that the molecules reflected (elastically and inelastically) from the walls become polarized also in the case of gas flow. The presence of a predominant direction of the velocity of the molecules incident on the surface and the dependence of the probability of the scattering on the mutual orientation of the velocity $\mathbf{v}$ and of the angular momentum $\mathbf{M}$ of the molecule should make the distribution function dependent also on the orientation of the vector $\mathbf{M}$, i.e., should lead to polarization of the molecules. The molecule precession produced when the external field is turned-on changes this dependence (it destroys partially the polarization). As a result, the kinetic properties of the system are altered in an external field; in particular, the scalar transport coefficients acquire a tensor character.

In this paper we construct a theory of the phenomena connected with the influence of an external field on the transport processes in a stream of strongly rarefied polyatomic gas. We solve the problem of the flow of collisionless gas in a channel made up of two infinite surfaces in a magnetic field. We investigate the change of the channel resistance in the field and the onset of heat flow between the surfaces (which have equal temperatures). These effects are the Knudsen analogs of the known viscomagnetic effect[6] and of the effect of viscomagnetic heat flow,[7] which take place if $l \lesssim L$. They are produced, however, by another physical mechanism, namely polarization of the molecules by nonspherical scattering from the surface.

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