1) There are arguments supporting the view that the real physical space msut be orientable.[6, 7]

2) In the Lemaître model with cosmological constant $\Lambda > 0$ and $\mathscr{K} > 0$, the observation of the same object in two (opposite)directions is possible in the late stages of expansion; see, for example, Refs. 2 and 8.

3) Below, it is assumed that the cosmological constant vanishes, $\Lambda = 0$, and, in addition, the equation of state of the matter is $p = 0$, i.e., we ignore radiation pressure. We recall that in the standard model of a hot Universe the total density of the background radiation at the contemporary epoch is $\rho_{m=0} < 2\times10^{-33}$ g/cm^3,[17] i.e., it is much smaller than the critical density $\rho_{cr} \approx 2\times10^{-29}$ g/cm^3. Here, $\rho_{m=0}$ includes the density of photons, gravitons, all types of neutrinos, and also all as yet unkown particles with zero rest mass that have survived from the superdense phase. If, however, $m_{0\nu} \approx 30$ eV, then today $p_\nu \ll \varepsilon_\nu/3$, i.e., we still have $p_\Sigma \approx 0$.

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Hyperfine structure of the energy levels of μ-mesic molecules of the hydrogen isotopes

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In the first order of perturbation theory in α^2, a calculation accurate to $\sim10^{-3}$ eV is made of the hyperfine structure of the energy levels of all stationary states with quantum numbers $J \leq 1$ and $v \leq 1$ of the total orbital angular momentum and of the vibrational motion, respectively, for mesic molecules of the hydrogen isotopes. The solutions to the nonrelativistic problem of the bound states of a system of three particles with Coulomb interaction found in the adiabatic representation are chosen as the zeroth approximation. Expressions are given for the probability amplitudes of the different values of the total spin of the nuclei and the total spin of the μ-mesic molecules in the stationary states of the hyperfine structure. Calculations are made of the populations of the stationary states of the hyperfine structure of the μ-mesic molecules formed in collisions of the mesic atoms $p\mu$, $d\mu$, and $t\mu$ in the parastate or orthostate with the nuclei p, d, and t.

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1. INTRODUCTION

Recent experiments on the resonance formation of $dd\mu$ and $dt\mu$ mesic molecules[1] have confirmed the theoretical predictions in Ref. 2 that these mesic molecules should have excited weakly bound states with quantum number $J = 1$ for the total orbital angular momentum and quantum number $v = 1$ for the vibrational motion and binding energy $-\varepsilon_{Jv}(dd\mu)=1.91$ eV and $-\varepsilon_{Jv}(dt\mu)=0.64$ eV. The hyperfine structure of the energy levels of these stationary states was not taken into account in the evaluation of the experiment in Ref. 1 or in the theoretical calculations of Ref. 2. So far, the hyperfine structure is known only for the mesic molecules $pp\mu$ and $pd\mu$ in the states with $(J=1, v=0)$ and $(J=0, v=0)$, respectively, for which the hyperfine splitting of the energy levels is ~0.1 eV.[3,4] The hyperfine splitting of the energy levels of μ-mesic atoms of the hydrogen isotopes is also ~0.1 eV, which is comparable in magnitude with the binding energy of the $(J=1, v=1)$ states of the $dd\mu$ and $dt\mu$ mesic molecules and the kinetic energy of the thermal motion at normal temperatures (~0.04 eV). Thus, calculation of the hyperfine structure of the mesic molecules $dd\mu$ and $dt\mu$ is of great interest and, as is noted in Ref. 4, is necessary for calculations of the kinetics of processes taking place in a mixture of hydrogen isotopes.[5]

The stationary states $|(Jv)\mathcal{J}\mathcal{J}_3N\rangle$ of a mesic molecule, i.e., a system of three spin particles $a, b, c=\mu^-$ (with spins $s_i\leqslant 1, i=a, b, c$) with electromagnetic interaction, are characterized by the value $\mathcal{J}$ of the total angular momentum $\vec{\mathcal{J}}=\mathbf{J}+\mathbf{S}$ and its projection $\mathcal{J}_3$ onto the $\hat{Z}$ axis of the laboratory coordinate system and in the zeroth order of perturbation theory are represented by a linear combination of the states $|JSI\mathcal{J}\mathcal{J}_3\rangle$:

$$|(Jv)\mathcal{J}\mathcal{J}_3N\rangle=\sum_{S,I}|JSI\mathcal{J}\mathcal{J}_3\rangle\langle JSI\mathcal{J}|(Jv)\mathcal{J}N\rangle \tag{1}$$

with definite values of the total spin $\mathbf{S}=\mathbf{I}+\mathbf{s}_c$ of the mesic molecule ($s_c=\frac{1}{2}$ is the spin of the μ^- meson) and the total spin $\mathbf{I}=\mathbf{s}_a+\mathbf{s}_b$ of the nuclei a and b; N is the number of the state, $1\leqslant N\leqslant N_{\max}$, and $N_{\max}$ is the degeneracy of the level with the given values of $(Jv\mathcal{J}\mathcal{J}_3)$.

The μ-mesic molecules are formed in collisions of μ-mesic atoms (a, c) in the ground state and characterized by spin $\mathbf{F}=\mathbf{s}_a+\mathbf{s}_b$ with nuclei of the hydrogen isotopes b with spin $\mathbf{s}_b$. The mass of nucleus a is greater than or equal to the mass of the nucleus b; for example,

$$\begin{aligned}&t\mu(F)+d(s_b)\to dt\mu((Jv)\mathcal{J}N),\\&d\mu(F)+d(s_b)\to dd\mu((Jv)\mathcal{J}N).\end{aligned} \tag{2}$$

To calculate the rates of nuclear synthesis reactions in the μ-mesic molecules that are formed, it is necessary to know[4,6] the populations $W_{Jv}^{\mathcal{J}N}(F)$ of the different stationary states $|(Jv)\mathcal{J}\mathcal{J}_2N\rangle$ of the mesic molecule; these depend on the values of F and s_b.

In the present paper, we calculate, in the first order of perturbation theory in α^2, the energy levels $\mathcal{E}_{Jv}^{\mathcal{J}N}$ of the hyperfine structure, the probability amplitudes $\beta_{Jv}^{\mathcal{J}N}\equiv\langle JSI\mathcal{J}|(Jv)\mathcal{J}N\rangle$ of states with definite S and I, and the populations $W_{Jv}^{\mathcal{J}N}(F)$ of the stationary states $|(Jv)\mathcal{J}\mathcal{J}_3N\rangle$ with $J\leqslant 1$ and $v\leqslant 1$ for all μ-mesic molecules of the hydrogen isotopes.

To calculate the hyperfine structure of the mesic molecules, we use the Hamiltonian $H=H^{\text{nrel}}+V^{\text{spin}}$ of a system of three spin particles $(s_i\leqslant 1, i=a, b, c)$ with electromagnetic interaction,[7] where H^{nrel} is the nonrelativistic part of the Hamiltonian, and V^{spin} takes into account spin effects to accuracy $O(\alpha^2)$. In our approach, the spin operator V^{spin} is constructed in accordance with the Foldy–Krajcik procedure.[8] In this case, three-particle forces are absent in the first order in α^2. The operator V^{spin} of the spin interaction consists of two parts: One part represents two-body spin interactions in the center-of-mass system of a pair of particles and is constructed in the framework of the quasipotential approach[9] with allowance for the effects of the internal electromagnetic structure of the particles,[7] and the other part represents the spin effects that depend on the momentum of the center of mass of the pair.[8]

As zeroth approximation in the calculations, we have used the solutions of the nonrelativistic problem with the Hamiltonian H^{nrel} found in the adiabatic representation of the three-body problem with Coulomb interaction,[10] which makes it possible to treat in a unified manner the ground $(J=0, v=0)$, excited $(J, v\neq 0)$, and, particularly important, weakly bound stationary states of the μ-mesic molecules.[2]

2. HYPERFINE STRUCTURE OF THE ENERGY LEVELS OF THE STATIONARY STATES OF μ-MESIC MOLECULES

In the nonrelativistic approximation, the stationary states $|n\tau\rangle=|nvJm_J\lambda\rangle$ of the μ-mesic molecules are characterized by the quantum numbers of the motion of the μ^- meson (n), the vibrational motion of the nuclei (v), the total orbital angular momentum (J), its projection (m_J) onto the $\hat{Z}$ axis of the laboratory coordinate system, and the total parity $[\lambda=+(-)^J]$. The corresponding wave function $\Psi_{n\tau}(\mathbf{R}, \mathbf{r})=\langle\mathbf{Rr}|n\tau\rangle$ is the solution of the nonrelativistic Schrödinger equation[10]

$$(H^{\text{nrel}}(\mathbf{R}, \mathbf{r})-E_{n\tau}^{\text{nrel}})\Psi_{n\tau}(\mathbf{R}, \mathbf{r})=0, \tag{3}$$

where $E_{n\tau}^{\text{nrel}}$ is the total energy of the μ-mesic molecule in the center-of-mass system, $H^{\text{nrel}}(\mathbf{R}, \mathbf{r})$ is the Hamiltonian of the three particles (a, b, c) with Coulomb interaction, $\mathbf{R}$ is the vector joining the nuclei a and b, and $\mathbf{r}$ is the vector joining the center of R and the μ^- meson.

In the adiabatic representation, the wave function $\Psi_{n\tau}(\mathbf{R}, \mathbf{r})$ can be written in the form of the expansion[10]

$$\begin{aligned}\Psi_{n\tau}(\mathbf{R}, \mathbf{r})&=R^{-1}\sum_{m=0}^{J}\sum_{j,p}\{\varphi_{jmp}(\xi, \eta; R)\mathcal{D}_{mm_J}^{J\lambda}(\Phi, \Theta, \varphi)\}\chi_{jmp}^{nJv}(R)\\&=R^{-1}\sum_{m=0}^{J}\sum_{j,p}\langle\xi\eta\Phi\Theta\varphi|jmpRJm_J\lambda\rangle\langle jmpR|nJv\rangle.\end{aligned} \tag{4}$$

Here,

$$\xi=(r_a+r_b)/R, \quad \eta=(r_a-r_b)/R,$$

φ are the spheroidal coordinates of the vector $\mathbf{r}$, r_a and r_b are the distances between the nuclei a and b and the μ^- meson, respectively, $\varphi_{jmp}(\xi, \eta; R)$ is the complete set of solutions to the problem of two fixed Coulomb centers,[11] the dependence of these solutions on the angle φ (around the axis $\hat{z}=\mathbf{R}/|R|$) being included in the symmetrized functions corresponding to total parity $\lambda=+(-)^J$:

$$\mathscr{D}^{J\lambda}_{mm_J}(\Phi,\Theta,\varphi)=[4\pi(1+\delta_{m0})]^{-1/2}\{(-1)^m e^{im\varphi}\mathscr{D}^J_{mm_J}(\Phi,\Theta,0)$$

$$+\lambda(-)^J e^{-im\varphi}\mathscr{D}^J_{-mm_J}(\Phi,\Theta,0)\}, \qquad (5)$$

where $\mathscr{D}^J_{mmJ}(\Phi,\Theta,0)$ are normalized D functions of Wigner, Φ and Θ are the spherical angles of the vector **R**, and m is the projection of the orbital angular momentum **J** onto the $\hat{z}$ axis. The symbol $\sum_{j,p}$ denotes summation over the discrete spectrum and integration over the continuum of the two-center problem,[12] the limits in the present calculations being taken as follows:

$$\sum_{j,p}=\sum_{p}\sum_{n_2=0}^{3}\Big\{\sum_{n_1=0}^{3}+\int_0^{10}dk\Big\}. \qquad (6)$$

Here, $p=g,u$ are the eigenvalues of the operator $\hat{P}_\mu$ which inverts the coordinates $(\xi\to\xi,\eta\to-\eta,\varphi\to\pi-\varphi)$ of the μ^- meson; n_1 and n_2 are the parabolic quantum numbers of the "separated" atom a,c; and k is the momentum of the μ^- meson for motion in the field of the two fixed nuclei a and b with positive energy.[11]

The binding energy $-\varepsilon_{Jv}$ of the μ-mesic molecule,

$$\varepsilon_{Jv}\equiv\varepsilon_{n\tau}=E^{nrel}_{n\tau}-E_{na} \qquad (7)$$

(in the ground state of the motion of the μ^- meson), is measured from the energy E_{na} of the ground state $|n\rangle\equiv|n_1n_2m\rangle=|000\rangle$ of the isolated atom a,c, whose nucleus has mass greater than (or equal to) the mass of nucleus b. In what follows, we shall omit some indices, for example, $n=[0,0,0]$ and $\lambda=+(-)^J$, writing $|nvJm_J\rangle\equiv|vJm_J\rangle$. The binding energy $-\varepsilon_{Jv}$ and the wave functions

$$\chi^{Jv}_{jmp}(R)=\langle jmpR|Jv\rangle$$

were found by means of the algorithms[13] for numerical solution of the Sturm–Liouville problem for the system of ordinary integro–differential equations[10] obtained by averaging (3) with respect to the functions $|\xi\eta\Phi\Theta\varphi|jmpRJm_J\lambda\rangle$:

$$\sum_{m'=0}^{J}\sum_{j,p'}\{\langle impRJm_J\lambda|H^{nrel}|jm'p'RJm_J\lambda\rangle$$

$$-\varepsilon_{Jv}\delta_{mm'}\delta_{ij}\delta_{pp'}\langle jm'p'R|Jv\rangle\}=0, \qquad (8)$$

$$\langle jmp0|Jv\rangle=0,\quad \lim_{R\to\infty}\langle jmpR|Jv\rangle=0.$$

The values $-\varepsilon_{Jv}$ of the binding energy ($2J+1$-fold degenerate with respect to m_J) of all the stationary states $|Jv\rangle\equiv|vJm_J\rangle$ with $v,J\leqslant 1$ of the mesic molecules of the hydrogen isotopes were taken from our other paper[12] and are given below in the third columns of Table II.

The addition to the Hamiltonian H^{nrel} of the operator V^{spin} of the spin interaction[7] leads to a hyperfine splitting of the energy level ε_{Jv} of the stationary state $|vJm_J\rangle$. The correct functions of the zeroth approximation corresponding to the stationary state $|(Jv)\mathscr{J}\mathscr{J}_3N\rangle$ have the form[14]

$$\Phi^{\mathscr{J}N}_{(Jv)\mathscr{J}_3}(\mathbf{R},\mathbf{r})=\sum_{S,I}\beta^{\mathscr{J}N}_{SI}\Phi^{SI\mathscr{J}}_{(Jv)\mathscr{J}_3}(\mathbf{R},\mathbf{r})$$

$$=\langle\mathbf{Rr}|(Jv)\mathscr{J}\mathscr{J}_3N\rangle=\sum_{S,I}\langle\mathbf{R},\mathbf{r}|(J,v)SI\mathscr{J}\mathscr{J}_3\rangle\langle JSI\mathscr{J}|(Jv)\mathscr{J}N\rangle, \qquad (9)$$

$$\Phi^{SI\mathscr{J}}_{(Jv)\mathscr{J}_3}(\mathbf{R},\mathbf{r})=\sum_{m_J\zeta}\sum_{I_3\zeta_c}\sum_{\zeta_a\zeta_b}\langle Jm_JS\zeta|JS\mathscr{J}\mathscr{J}_3\rangle$$

$$\times\langle II_3s_c\zeta_c|Is_cS\zeta\rangle\langle s_a\zeta_as_b\zeta_b|s_as_bII_3\rangle\Psi_{vJm_J}(\mathbf{R},\mathbf{r})\chi^{s_c}_{\zeta_c}\chi^{s_a}_{\zeta_a}\chi^{s_b}_{\zeta_b}. \qquad (10)$$

Here

$$\langle Jm_JS\zeta|JS\mathscr{J}\mathscr{J}_3\rangle,\quad \langle II_3s_c\zeta_c|Is_cS\zeta\rangle,\quad \langle s_a\zeta_as_b\zeta_b|s_as_bII_3\rangle$$

are Clebsch–Gordan coefficients[14]; $\chi^{s_c}_{\zeta_c},\chi^{s_a}_{\zeta_a},\chi^{s_b}_{\zeta_b}$ are constant spinors; S,I,s_c,s_a,s_b and $\zeta,I_3,\zeta_c,\zeta_a,\zeta_b$ are the values of the spins $\mathbf{S},\mathbf{I},\mathbf{s}_c,\mathbf{s}_a,\mathbf{s}_b$ and their third projections onto the $\hat{Z}$ axis of the laboratory coordinate system.

For the given values of J and v, the coefficients

$$\beta_{SI}{}^{\mathscr{J}N}\equiv\langle SI|\mathscr{J}N\rangle=\langle JSI\mathscr{J}|(Jv)\mathscr{J}N\rangle,$$

i.e., the probability amplitudes of states with definite S and I in the stationary state $|(Jv)\mathscr{J}\mathscr{J}_3N\rangle$, are solutions of the system of linear algebraic equations

$$\sum_{S',I'}\{\langle(Jv)SI\mathscr{J}\mathscr{J}_3|V^{spin}|(Jv)S'I'\mathscr{J}\mathscr{J}_3\rangle-\mathscr{E}^{\mathscr{J}N}_{Jv}\delta_{SS'}\delta_{II'}\}\beta^{\mathscr{J}N}_{S'I'}=0, \qquad (11)$$

in which $\mathscr{E}^{\mathscr{J}N}_{Jv}$ are found from the secular equation

$$\det|\langle(Jv)SI\mathscr{J}\mathscr{J}_3|V^{spin}|(Jv)S'I'\mathscr{J}\mathscr{J}_3\rangle-\mathscr{E}_{Jv}{}^{\mathscr{J}N}\delta_{SS'}\delta_{II'}|=0 \qquad (12)$$

and represent the required hyperfine splitting of the nonrelativistic energy level ε_{Jv} for fixed $\mathscr{J}$ (the degeneracy with respect to $\mathscr{J}_3$ remains).

In the case of identical nuclei, the only roots of the secular equation among all the N_{max} roots that have physical meaning are those that correspond to states $|(Jv)\mathscr{J}\mathscr{J}_3N\sigma\rangle$ with definite symmetry $\sigma=p(-1)^{J+1}=+1$ under permutation of the nuclei,[14] i.e., $P_n=p\lambda=1$ for even and $P_n=p\lambda=-1$ for odd I.[10]

3. EFFECTIVE SPIN HAMILTONIAN AND HYPERFINE STRUCTURE OF THE STATIONARY STATES OF μ-MESIC MOLECULES

The spin operator V^{spin}, whose explicit form is described in the previous papers of Ref. 7, was averaged

TABLE I*. Coefficients $E_2, \ldots, E_{11}$ (μeV $=10^{-6}$eV) of the effective spin Hamiltonia (13) for the mesic molecules $pp\mu$, $dd\mu$, $tt\mu$, $pd\mu$, $pt\mu$, $dt\mu$ in the stationary states $|Jv\rangle$ with total orbital angular momentum $J\leqslant 1$ and vibrational quantum number $v\leqslant 1$.

Mesic molecule	J	v	E_2	E_3	E_4	E_5	E_6	E_7	E_8	E_9	E_{10}	E_{11}
$pp\mu$	0	0	103 855	103 855	0	0	0	0	0	0	0	0
	1	0	91 694	91 694	−383	−383	2438	426	−976	−976	0	0
$dd\mu$	0	0	19 066	19 066	0	0	0	0	0	0	0	0
	0	1	16 416	16 416	0	0	0	0	0	0	0	0
	1	0	17 551	17 551	−52	−52	2031	17	−241	−241	−100	−100
	1	1	16 053	16 053	−11	−11	374	3	−44	−44	−23	−23
$tt\mu$	0	0	142 258	142 258	0	0	0	0	0	0	0	0
	0	1	123 384	123 384	0	0	0	0	0	0	0	0
	1	0	133 766	133 766	−299	−299	1638	1020	−2001	−2001	0	0
	1	1	119 534	119 534	−153	−153	723	489	−878	−878	0	0
$pd\mu$	0	0	20 862	101 794	0	0	0	0	0	0	0	0
	1	0	19 974	84 305	−70	−275	2291	83	−178	−1260	−72	0
$pt\mu$	0	0	158 420	101 200	0	0	0	0	0	0	0	0
	1	0	153 347	83 458	−507	−227	2193	620	−1312	−1365	0	0
$dt\mu$	0	0	145 808	19 153	0	0	0	0	0	0	0	0
	0	1	148 869	18 303	0	0	0	0	0	0	0	0
	1	0	136 915	17 642	−377	−41	1860	132	−1814	−267	0	−109
	1	1	189 608	6632	−90	−9	386	30	−364	−57	0	−27

*Mesic-atom energy unit $\varepsilon_\mu=5626.506$ eV.[7]

TABLE II. Hyperfine structure of μ-mesic molecules of the hydrogen isotopes.

A. Hyperfine structure of the levels of the mesic molecule $pp\mu$.

J	v	ε_{Jv}, eV	$\mathcal{J}$	N	$\varepsilon^{\mathcal{J}N}_{Jv}$, eV	$W^{\mathcal{J}N}_{Jv}(\uparrow\downarrow)$	$W^{\mathcal{J}N}_{Jv}(\uparrow\uparrow)$	$\langle 1/2,0\|\mathcal{J}N\rangle$	$\langle 1/2,1\|\mathcal{J}N\rangle$	$\langle 3/2,1\|\mathcal{J}N\rangle$
0	0	−253.0	1/2	1	0	1	1	1	0	0
1	0	−107.0	1/2	1	−0.0905	0.3330	0.0371	0	0.9996	−0.0286
				2	0.0427	0.0002	0.1481	0	0.0286	0.9996
			3/2	1	−0.0924	0.6667	0.0741	0	0.9999	−0.0095
				2	0.0468	0.0001	0.2963	0	0.0095	0.9999
			5/2	1	0.0463	0	0.4444	0	0	1

B. Hyperfine structure of the levels of the mesic molecule $dd\mu$.

J	v	ε_{Jv}, eV	$\mathcal{J}$	N	$\varepsilon^{\mathcal{J}N}_{Jv}$, eV	$W^{\mathcal{J}N}_{Jv}(\uparrow\downarrow)$	$W^{\mathcal{J}N}_{Jv}(\uparrow\uparrow)$	$\langle 1/2,0\|\mathcal{J}N\rangle$	$\langle 1/2,1\|\mathcal{J}N\rangle$	$\langle 3/2,1\|\mathcal{J}N\rangle$	$\langle 3/2,2\|\mathcal{J}N\rangle$	$\langle 5/2,2\|\mathcal{J}N\rangle$
0	0	−325.0	1/2	1	0	0,1667	0,1667	1	0	0	0	0
			3/2	1	−0,0286	0,8333	0,0833	0	0	0	1	0
			5/2	1	0,0191	0	0.7500	0	0	0	0	1
0	1	−35.8	1/2	1	0	0.1667	0.1667	1	0	0	0	0
			3/2	1	−0.0246	0.8333	0.0833	0	0	0	1	0
			5/2	1	0,0164	0	0.7500	0	0	0	0	1
1	0	−226.6	1/2	1	−0,0169	0.2213	0,0560	0	0,9971	−0,0764	0	0
				2	0.0070	0,0565	0,1384	0	0,0764	0.9971	0	0
			3/2	1	−0,0180	0,4436	0.1115	0	0,9988	−0,0487	0	0
				2	0,0084	0,1119	0.2774	0	0.0487	0,9988	0	0
			5/2	1	0,0097	0.1667	0,4167	0	0	1	0	0
1	1	−1.91	1/2	1	−0.0159	0.2222	0.0555	0	1.0000	−0,0149	0	0
				2	0,0077	0,0555	0,1389	0	0.0149	1,0000	0	0
			3/2	1	−0.0161	0.4444	0,1111	0	1,0000	−0,0097	0	0
				2	0,0079	0.1112	0,2778	0	0.0097	1.0000	0	0
			5/2	1	0,0082	0.1667	0,4167	0	0	1	0	0

C. Hyperfine structure of the levels of the mesic molecule $tt\mu$.

J	v	ε_{Jv}, eV	$\mathcal{J}$	N	$\varepsilon^{\mathcal{J}N}_{Jv}$, eV	$W^{\mathcal{J}N}_{Jv}(\uparrow\downarrow)$	$W^{\mathcal{J}N}_{Jv}(\uparrow\uparrow)$	$\langle 1/2,0\|\mathcal{J}N\rangle$	$\langle 1/2,1\|\mathcal{J}N\rangle$	$\langle 3/2,1\|\mathcal{J}N\rangle$
0	0	−363,0	1/2	1	0	1	1	1	0	0
0	1	−83,9	1/2	1	0	1	1	1	0	0
1	0	−289,2	1/2	1	−0,1330	0,3330	0.0372	0	−0,0320	−0.0320
				2	0,0625	0.0003	0,1480	0	0.0320	0,9995
			3/2	1	−0,1342	0,6667	0,0741	0	1,0000	0,0012
				2	0,0695	0,0000	0,2963	0	−0,0012	1,0000
			5/2	1	0.0667	0	0.4444	0	0	1
1	1	−45,2	1/2	1	−0,1191	0,3332	0.0371	0	0,9999	−0,0160
				2	0.0579	0.0001	0,1481	0	0,0160	0.9999
			3/2	1	−0,1198	0.6667	0.0741	0	1,0000	0,0006
				2	0.0609	0.0000	0,2963	0	−0,0006	1,0000
			5/2	1	0.0597	0	0.4444	0	0	1

D. Hyperfine structure of the levels of the mesic molecule $pd\mu$.

J	v	ε_{Jv}, eV	$\mathcal{J}$	N	$\varepsilon^{\mathcal{J}N}_{Jv}$, eV	$W^{\mathcal{J}N}_{Jv}(\uparrow\downarrow)$	$W^{\mathcal{J}N}_{Jv}(\uparrow\uparrow)$	$\langle 0,1/2\|\mathcal{J}N\rangle$	$\langle 1,1/2\|\mathcal{J}N\rangle$	$\langle 1,3/2\|\mathcal{J}N\rangle$	$\langle 2,3/2\|\mathcal{J}N\rangle$
0	0	−221.5	0	1	0.0046	0.2500	0	1	0	0	0
			1	1	0.0173	0.3853	0.1824	0	0,8964	0,4433	0
				2	−0.0787	0,3647	0.1926	0	−0.4433	0,8964	0
			2	1	0.0359	0	0.6250	0	0	0	1
1	0	−97.4	0	1	0.0135	0,0431	0,0201	0	0,8949	0.4464	0
				2	−0.0653	0,0402	0.0216	0	−0.4464	0,8949	0
			1	1	0,0008	0,2471	0.0015	0.9891	−0.1159	−0.0656	0.0631
				2	0.0122	0,1232	0.0634	0,1390	0.8930	0.4141	−0,1084
				3	−0.0654	0.1268	0,0616	0.0117	−0,4243	0.9052	−0,0221
				4	0.0286	0.0029	0.1235	−0,0475	0.0955	0,0696	0,9919
			2	1	0,0146	0.2000	0,1083	0	0,9108	0.4128	0,0027
				2	−0.0662	0.2166	0,1000	0	−0.4127	0.9107	−0.0137
				3	0.0320	0,0001	0.2083	0	−0,0081	0.0114	0.9999
			3	1	0,0317	0	0.2917	0	0	0	1

E. Hyperfine structure of the levels of the mesic molecule $pt\mu$.

J	v	ε_{Jv}, eV	$\mathcal{J}$	N	$\varepsilon^{\mathcal{J}N}_{Jv}$, eV	$W^{\mathcal{J}N}_{Jv}(\uparrow\downarrow)$	$W^{\mathcal{J}N}_{Jv}(\uparrow\uparrow)$	$\langle 1/2,0\|\mathcal{J}N\rangle$	$\langle 1/2,1\|\mathcal{J}N\rangle$	$\langle 3/2,1\|\mathcal{J}N\rangle$
0	0	−214.0	1/2	1	0.0046	0.1120	0.2960	0,9834	−0.1813	0
				2	−0.1344	0.8880	0,0373	0.1813	0.9834	0
			3/2	1	0.0649	0	0.6667	0	0	1
1	0	−99.0	1/2	1	0.0053	0.0256	0.1026	0.9705	−0.2403	0.0186
				2	−0.1249	0.3076	0.0086	0.2407	0.9703	−0.0254
				3	0.0555	0.0001	0.1111	−0.0120	0.0291	0.9995
			3/2	1	0.0083	0.0548	0.2039	0.9730	−0.2308	0.0007
				2	−0.1262	0.6119	0.0183	0.2308	0.9730	−0.0044
				3	0.0608	0.0000	0.2222	0.0003	0.0044	1.0000
			5/2	1	0,0594	0.0000	0.3333	0	0	1

TABLE II (continued).

F. Hyperfine structure of the levels of the mesic molecule $dt\mu$.

J	v	ε_{Jv}, eV	$\mathcal{J}$	N	$\mathcal{E}_{Jv}^{\mathcal{J}N}$, eV	$W_{Jv}^{\mathcal{J}N}(\uparrow\downarrow)$	$W_{Jv}^{\mathcal{J}N}(\uparrow\uparrow)$	$\langle 0, {}^1/_2 \mid \mathcal{J}N\rangle$	$\langle 1, {}^1/_2 \mid \mathcal{J}N\rangle$	$\langle 1, {}^3/_2 \mid \mathcal{J}N\rangle$	$\langle 2, {}^3/_2 \mid \mathcal{J}N\rangle$
0	0	−319.2	0	1	0.0173	0	0.1111	1	0	0	0
			1	1	0.0282	0.0096	0.3301	0	0.8691	−0.4946	0
				2	−0.1107	0.9904	0.0032	0	0.4946	0.8691	0
			2	1	0.0463	0	0.5556	0	0	0	1
0	1	−34.9	0	1	0.0239	0	0.1111	1	0	0	0
			1	1	0.0312	0.0043	0.3319	0	0.8527	−0.5225	0
				2	−0.1123	0.9957	0.0014	0	0.5225	0.8527	0
			2	1	0.0439	0	0.5556	0	0	0	1
1	0	−232.4	0	1	0.0277	0.0007	0.0368	0	0.8588	−0.5122	0
				2	−0.1039	0.1104	0.0002	0	0.5122	0.8588	0
			1	1	0.0162	0.0000	0.1111	0.9846	−0.1193	0.0742	−0.1039
				2	0.0249	0.0031	0.1101	0.1549	0.8487	−0.4835	0.1481
				3	−0.1035	0.3303	0.0010	−0.0066	0.4978	0.8672	−0.0145
				4	0.0406	0.0000	0.1111	0.0806	−0.1331	0.0935	0.9834
			2	1	0.0273	0.0056	0.1833	0	0.8702	−0.4920	−0.0263
				2	−0.1041	0.5499	0.0019	0	0.4920	0.8706	−0.0065
				3	0.0447	0.0000	0.1852	0	0.0261	−0.0073	0.9996
			3	1	0.0433	0	0.2593	0	0	0	1
1	1	−0.64	0	1	0.0445	0.0001	0.0370	0	0.8293	−0.5589	0
				2	−0.1424	0.1110	0.0000	0	0.5589	0.8293	0
			1	1	0.0407	0.0000	0.1111	0.9967	−0.0531	0.0367	−0.0491
				2	0.0439	0.0002	0.1110	0.0680	0.8271	−0.5545	0.0617
				3	−0.1422	0.3331	0.0001	−0.0010	0.5570	0.8305	−0.0027
				4	0.0501	0.0000	0.1111	0.0453	−0.0481	0.0356	0.9972
			2	1	0.0443	0.0004	0.1851	0	0.8312	−0.5559	−0.0105
				2	−0.1424	0.5552	0.0001	0	0.5560	0.8312	−0.0011
				3	0.0511	0.0000	0.1852	0	0.0093	−0.0049	0.9999
			3	1	0.0508	0	0.2593	0	0	0	1

Note. Here, J and v are the orbital and vibrational quantum numbers, respectively; $-\varepsilon_{Jv}$ is the binding energy of the stationary state $|Jv\rangle$ of the mesic ,olecule in the nonrelativistic approximation; $\mathcal{J}$ is the total angular momentum of the μ-mesic molecule; $\mathcal{E}_{Jv}^{\mathcal{J}N}$ is the energy of the stationary state $|(Jv)\mathcal{J}\mathcal{J}_3N\rangle$, measured from ε_{Jv}; $W_{Jv}^{\mathcal{J}N}(\uparrow\downarrow)$ and $W_{Jv}^{\mathcal{J}N}(\uparrow\uparrow)$ are the populations of the level $(J, v, \mathcal{J}, N)$ of the mesic molecule formed as a result of collision of the mesic atom $p\mu(F)$, $d\mu(F)$, or $t\mu(F)$ in the parastate ($F=\uparrow\downarrow$) or orthostate ($F=\uparrow\uparrow$), respectively, with the p, d, or t nucleus (of the same kind or lighter); $\langle SI|\mathcal{J}N\rangle = \beta_{SI}^{\mathcal{J}N}$ is the probability amplitude of the state $|SI\rangle = |JSI\mathcal{J}\mathcal{J}_3\rangle$ with definite values of the total spin S and the total spin I of the nuclei of the mesic molecule in the stationary state $|\mathcal{J}N\rangle = |(\mathcal{J}\mathcal{J}_3N\rangle$:

$$|\mathcal{J}N\rangle = \sum_{SI} |SI\rangle\langle SI|\mathcal{J}N\rangle.$$

N is the number of the state $1 \leqslant N \leqslant N_{\max}$: where $N_{\max}$ is the degeneracy of the level with the given values of $(J, v, \mathcal{J}, \mathcal{J}_3)$.

in two stages. After the first averaging of V^{spin} with respect to the variables $\mathbf{R}$ and $\mathbf{r}$ with the function $\Psi_{vJ\,mJ}(\mathbf{R},\mathbf{r})$ of the stationary state $|n\tau\rangle$ we have the effective spin Hamiltonian H^{eff}, which is defined on the direct product $J\otimes s_c\otimes s_a\otimes s_b$ of the spaces of representations of ranks J, s_c, s_a, s_b of the group SU_2 (Ref. 7):

$$\begin{aligned}H^{\text{eff}}=&E_1\mathbf{s}_a\mathbf{s}_b+E_2\mathbf{s}_a\mathbf{s}_c+E_3\mathbf{s}_b\mathbf{s}_c+E_4\mathbf{s}_a\vec{\mathscr{L}}^{(J)}+E_5\mathbf{s}_b\vec{\mathscr{L}}^{(J)}+E_6\mathbf{s}_c\vec{\mathscr{L}}^{(J)}\\&+E_7(\mathbf{s}_a\mathscr{T}^{(J)}\mathbf{s}_b)+E_8(\mathbf{s}_a\mathscr{T}^{(J)}\mathbf{s}_c)+E_9(\mathbf{s}_b\mathscr{T}^{(J)}\mathbf{s}_c)\\&+E_{10}(\mathbf{s}_a\mathscr{T}^{(J)}\mathbf{s}_a)+E_{11}(\mathbf{s}_b\mathscr{T}^{(J)}\mathbf{s}_b).\end{aligned}\tag{13}$$

Here, $\vec{\mathscr{L}}^{(J)}$ are the generators of the representation of rank J of the group SU_2 acting on the constant spinors χ_{mJ}^J, and the symbol $(\mathbf{x}\mathscr{T}^{(J)}\mathbf{y})$ is defined by the equation $(\mathbf{x},\mathbf{y}\equiv\mathbf{s}_a,\mathbf{s}_b,\mathbf{s}_c)$

$$(\mathbf{x}\mathscr{T}^{(J)}\mathbf{y})={}^3/_2\{(\mathbf{x}\vec{\mathscr{L}}^{J})(\mathbf{y}\vec{\mathscr{L}}^{(J)})+(\mathbf{y}\vec{\mathscr{L}}^{(J)})(\mathbf{x}\vec{\mathscr{L}}^{(J)})\}-J(J+1)\mathbf{x}\mathbf{y}.$$

The values of the coefficients $E_2, \ldots, E_{11}$ in μ eV $=10^{-6}$ eV ($E_1=0$ to within the adopted relative accuracy $\sim 5\times 10^{-5}$ for E_2 and E_3 and $\sim 5\times 10^{-4}$ for $E_4, \ldots, E_{11}$) for the stationary states $|n\tau\rangle$ with $J, v \leqslant 1$ are given in Table I for all the μ-mesic molecules of the hydrogen isotopes. For the scheme for calculating the coefficients E_i, and also the values of all the employed constants, masses and magnetic moments of the particles, the parametrizations of the p, d, and t form factors, and so forth, see the previous papers of Ref. 7.

Averaging of the effective spin Hamiltonian (13) with the wave function $\chi_{\mathcal{J}_3}^{JSI\mathcal{J}}$ of the eigenstate $|JSI\mathcal{J}\mathcal{J}_3\rangle$ of the total angular momentum $\vec{\mathcal{J}}$, which is obtained from (10) by the formal substitution $\Psi_{vJ\,m}(\mathbf{R},\mathbf{r})\to\chi_{mJ}^J$,

$$\begin{aligned}\chi_{\mathcal{J}_3}^{JSI\mathcal{J}}=&\sum_{\zeta I_3\zeta_a}\langle J\mathcal{J}_3-\zeta S\zeta|J\mathcal{J}\mathcal{J}_3\rangle\langle II_3s_c\zeta-I_3|Is_cS\zeta\rangle\\&\times\langle s_a\zeta_a s_b I_3-\zeta_a|s_a s_b I I_3\rangle\chi_{\mathcal{J}_3-\zeta}^{J}\chi_{\zeta-I_3}^{s_c}\chi_{\zeta_a}^{s_a}\chi_{I_3-\zeta_a}^{s_b},\end{aligned}\tag{14}$$

leads for fixed J and v to the system of linear algebraic Eq. (11) for $\beta_{Jv}^{\mathcal{J}N}$ and to the secular Eq. (12) for $\mathcal{E}_{Jv}^{\mathcal{J}N}$.

The values of $\mathcal{E}_{Jv}^{\mathcal{J}N}$ of the energy levels of the hyperfine structure measured from the nonrelativistic value ε_{Jv} (7) (see Table II) and the probability amplitudes $\beta_{SI}^{\mathcal{J}N}\equiv\langle SI|\mathcal{J}N\rangle$ of states with definite S and I of the stationary states $|(Jv)\mathcal{J}\mathcal{J}_3N\rangle$ with $J\leqslant 1$ and $v\leqslant 1$ are given in Table II for all the μ-mesic molecules of the hydrogen isotopes.

It is well known[4,6] that μ-mesic molecules are formed in collisions of μ-mesic atoms in the 1s state with spin $\mathbf{F}=\mathbf{s}_a+\mathbf{s}_c$ in the parastate ($F=\uparrow\downarrow$) or orthostate ($F=\uparrow\uparrow$) with nuclei of the hydrogen isotopes with spin $\mathbf{s}_b$, i.e., in accordance with the scheme of type (2). Under the assumption of an arbitrary orientation of the spins $\mathbf{F}$ and $\mathbf{s}_b$ and the total orbital angular momentum $\mathbf{J}$ of the produced μ-mesic molecule, its original spin state is represented by the density matrix[15]

$$\rho=[(2F+1)(2s_b+1)(2J+1)]^{-1}\sum_{F_3\zeta_b m_J}\Pi_{FF_3}\otimes\Pi_{s_b\zeta_b}\otimes\Pi_{Jm_J}, \tag{15}$$

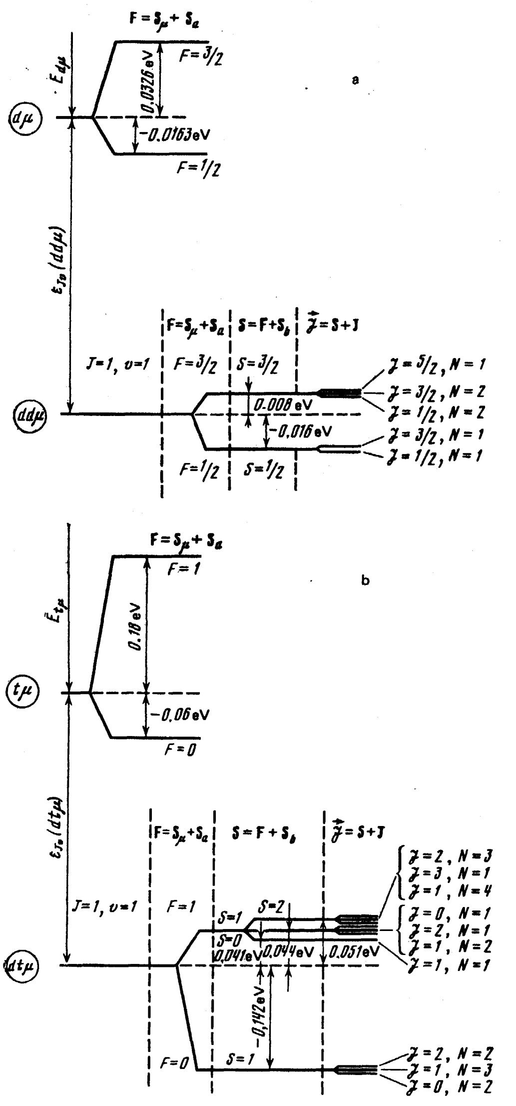

FIG. 1. "Transformation" of the energy levels of the hyperfine structure of the mesic atoms $d\mu$ (*a*) and $t\mu$ (*b*) in the $1s$ state with spins $\mathbf{F}=\mathbf{s}_\mu+\mathbf{s}_a$ in a collision with d nuclei with spins $\mathbf{s}_b$ into the hyperfine-structure energy levels of the stationary states $|\mathcal{J}\mathcal{J}_3N\rangle$ (1) with total orbital angular momentum $J=1$ and vibrational quantum number $v=1$ of the mesic molecules $dd\mu$ and $dt\mu$. Here, $\vec{\mathcal{J}}=\mathbf{S}+\mathbf{J}$ is the total angular momentum, $\mathbf{S}=\mathbf{s}_\mu+\mathbf{s}_a+\mathbf{s}_b$ is the total spin of the μ-mesic molecule. N is the number of the hyperfine-structure state for given $\mathcal{J}$, and $W_{Jv}^{\mathcal{J}N}(\uparrow\downarrow)$ abd $W_{Jv}^{\mathcal{J}N}(\uparrow\uparrow)$ are the para- and ortho-populations (17)–(19) of the stationary states $|(Jv)\mathcal{J}\mathcal{J}_3N\rangle$ of the mesic molecule formed through the spin state characterized by the density matrix (15) from the parastate $F+\uparrow\downarrow$ and orthostate $F=\uparrow\uparrow$, respectively, of the mesic atoms $d\mu$ or $t\mu$; $E_{d\mu}=-2663.226$ eV, $\Delta E_{d\mu}^{\mathrm{hfs}}=0.049$ eV, $\varepsilon_{Jv}(dd\mu)=-1.91$ eV, $\Delta E_{dd\mu}^{\mathrm{hfs}}=0.024$ eV, $E_{t\mu}=-2711.268$ eV, $\Delta E_{t\mu}^{\mathrm{hfs}}=0.240$ eV, $\varepsilon_{Jv}(dt\mu)=-0.64$ eV, $\Delta E_{dt\mu}^{\mathrm{hfs}}=0.193$ eV.

$$\Pi_{FF_3}=|FF_3\rangle\langle FF_3|,\quad \Pi_{s_b\zeta_b}=|s_b\zeta_b\rangle\langle s_b\zeta_b|,\quad \Pi_{Jm_J}=|Jm_J\rangle\langle Jm_J|. \tag{16}$$

Then the probability of finding the μ-mesic molecule in the stationary state $|(Jv)\mathcal{J}\mathcal{J}_3N\rangle$ of the hyperfine structure is determined as a function of the two possible spin states, $F=\uparrow\downarrow$ or $F=\uparrow\uparrow$, of the mesic atom (a,c) by the expression[15]

$$W_{Jv}^{\mathcal{J}N}(F)=\sum_{\mathcal{J}_3=-\mathcal{J}}^{\mathcal{J}}\langle(Jv)\mathcal{J}\mathcal{J}_3N|\rho|(Jv)\mathcal{J}\mathcal{J}_3N\rangle, \tag{17}$$

if, as in Ref. 16, allowance is not made for transitions between states of the hyperfine structure.

The probability $W_{Jv}^{\mathcal{J}N}(F)$ is usually called the population of the stationary state $|(Jv)\mathcal{J}\mathcal{J}_3N\rangle$ of the mesic molecule.[4,6] It can be expressed in terms of the probability amplitudes $\beta_{Jv}^{\mathcal{J}N}\equiv\langle JSI\mathcal{J}|(Jv)\mathcal{J}N\rangle$ [see (11) and Table II] of the states $|JSI\mathcal{J}\mathcal{J}_3\rangle$ (1) as follows:

$$W_{Jv}^{\mathcal{J}N}(F)=\frac{2\mathcal{J}+1}{(2F+1)(2s_b+1)(2J+1)}\sum_S\left|\sum_I\langle JSF\mathcal{J}|JSI\mathcal{J}\rangle\beta_{SI}^{\mathcal{J}N}\right|^2, \tag{18}$$

where

$$\langle JSF\mathcal{J}|JSI\mathcal{J}\rangle\equiv\langle JSF\mathcal{J}\mathcal{J}_3|JSI\mathcal{J}\mathcal{J}_3\rangle$$

does not depend on $\mathcal{J}_3$, $|\mathcal{J}_3|\leqslant\mathcal{J}$:

$$\begin{aligned}\langle JSF\mathcal{J}|JSI\mathcal{J}\rangle=&\sum_{\zeta F_3I_3}[\langle J\mathcal{J}_3-\zeta S\zeta|JS\mathcal{J}\mathcal{J}_3\rangle]^2\\ &\times\langle FF_3s_b\zeta-F_3|Fs_bS\zeta\rangle\langle s_aI_3+F_3-\zeta s_c\zeta-I_3|s_as_cFF_3\rangle\\ &\times\langle II_3s_c\zeta-I_3|Is_cS\zeta\rangle\langle s_aI_3+F_3-\zeta s_b\zeta-F_3|s_as_bII_3\rangle.\end{aligned} \tag{19}$$

In Fig. 1, we show schematically the "transformation" of the hyperfine energy levels of the $d\mu$ and $t\mu$ mesic atoms into the hyperfine energy levels of the $dd\mu$ and $dt\mu$ mesic molecules in accordance with the scheme for adding angular momenta in reactions of the type (2), i.e., the spins $\mathbf{F}$ of the mesic atom a,c, the deuteron b ($\mathbf{s}_b$), and the total orbital angular momentum $\mathbf{J}$ of the system a,b,c are added successively to the total spin $\mathbf{S}=\mathbf{F}+\mathbf{s}_b$ and total angular momentum $\mathcal{J}=\mathbf{S}+\mathbf{J}$ of the mesic molecules. Thus, the complete set of energy levels of the states (1) of the hyperfine structure of the mesic molecule, which are expanded in the perturbation-theory scheme with respect to the states $|JSI\mathcal{J}\mathcal{J}_3\rangle$ with coefficients $\beta_{SI}^{\mathcal{J}N}$ (11), decomposes naturally into two groups in accordance with the possible ways in which the μ-mesic molecule can be formed from the parastate ($F=\uparrow\downarrow$) or orthostate ($F=\uparrow\uparrow$) of the μ-mesic atom a,c.

In Table II we give the parastate, $W_{Jv}^{\mathcal{J}N}(\uparrow\uparrow)$, and orthostate, $W_{Jv}^{\mathcal{J}N}(\uparrow\downarrow)$, populations of the stationary states $|(Jv)\mathcal{J}\mathcal{J}_3N\rangle$ with $J,v\leqslant1$ for all the μ-mesic molecules of the hydrogen isotopes calculated in accordance with Eqs. (18) and (19). (We recall that the mass of nucleus a is greater than or equal to the mass of nucleus b.)

4. CONCLUSIONS

In the present paper (see Table II) we have calculated in the first order of perturbation theory in α^2 to accuracy ~10^{-4} eV the hyperfine structure of the energy levels of the stationary states ($J\leqslant1, v\leqslant1$) of the μ-mesic molecules of the hydrogen isotopes. In the calculation of the coefficients of the effective spin Hamiltonian (13) in the expansion (4)–(6) of the wave function of the μ-

mesic molecule we have retained the terms corresponding to the states of the first four shells in accordance with the classification of the "separated" atom[7] of the discrete spectrum of the two-center problem.[11] The terms corresponding to the continuum states of the two-center problem were not taken into account, since their contribution is less than the adopted accuracy ~10^{-5} eV of the calculations.[7]

As can be seen from Table I, the coefficients of the effective spin Hamiltonian (13) that determine the spin-orbit (E_4, E_5, E_6) and spin-tensor $(F_7, \ldots, E_{11})$ interactions are appreciably smaller than the coefficients that determine the spin-spin (E_2, E_3) interaction of the μ^- meson with the nuclei. This has the consequence that some of the energy levels $\mathscr{E}_{Jv}^{\mathscr{J}N}$ are degenerate to accuracy ~10^{-3} eV; for example, see Fig. 1b and Table II E. At the same time, the contribution of the corrections $o(\alpha^2)$ is $<10^{-3}$ eV according to estimates.

It follows from our calculations that the corrections for the internal electromagnetic structure of the nuclei to the coefficients of the effective spin Hamiltonian (13) are ~0.5–1.5% of the corresponding quantities calculated without allowance for them.[7] The Foldy–Krajcik corrections,[8] which represent the motion of the center of mass of a pair of particles, significantly change the coefficients (E_4, E_5, E_6), which determine the spin-orbit interaction in the μ-mesic molecule.[7] Allowance for both these effects changes the hyperfine splitting of the energy levels $\mathscr{E}_{Jv}^{\mathscr{J}N}$ of the μ-mesic molecules by ~0.5% (the corrections for the internal electromagnetic structure of the nuclei) and ~0.05–0.1% (Foldy–Krajcik corrections).[7]

Despite the smallness of the effects due to the internal electromagnetic structure of the nuclei, their inclusion is of fundamental importance, since in this case the operators of the spin interaction does not have ~$\delta(r)$ and $|r^{-3}|$ singularities.[17]

The results of the present paper can be used to make more detailed calculations of the rates of resonance formation of $dd\mu$ and $dt\mu$ mesic molecules[2] and the kinetics of mesic-molecular processes in a mixture of hydrogen isotopes.[18]

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