the calculation of domain structures during spin-orientation transitions (see the bibliography to Chapter I in the book of Belov *et al.*[1]). The specific characteristic of the present treatment is the introduction of the surface energy, which makes the value of the direction of the magnetic moment at the boundary (the angle θ_S) an additional parameter characterizing the state of the magnet. Apparently there are not at present sufficiently definite experimental data with which to compare the results obtained here. The author hopes that the present publication will stimulate, one the one hand, investigations of reorientational transitions on the basis of surface characteristics (for example, on the basis of reflection of light); and on the other, the development of methods of study of the surface that use the state diagram of magnets near reorientational transitions.

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[1] It has been assumed (and this is important!) that the magnetic moment both on the surface and in the interior is parallel to the plane of the specimen surface. This means that the total anisotropy energy has the following structure:

$$f_V(\vartheta,\ \chi)=k_1^{*}\sin^2\vartheta\cos^2\chi-k_1^{**}\sin^2\vartheta-k_2\sin^4\vartheta,\quad k_1^{*}>0, \tag{1'}$$

χ is the angle between the magnetic moment and the direction of the normal to the surface. If the surface energy also attains a minimum for $\chi=0$, then $\chi(x)\equiv 0$ and $f_V(\theta)=f_V(\theta,\chi=0)$, $k_1=k_1^{**}-k_1^{*}$.

[2] We note a curious detail: if θ_∞ corresponds not to a minimum of the function $f_V(\theta)$ but to its smallest value (this is so in reorientation transitions), then $d\theta/dx$ vanishes at a finite distance x_m from the boundary:

$$\frac{x_m}{C^{1/2}}=\operatorname{sign}(\vartheta_\infty-\vartheta_S)\int\limits_{\vartheta_S}^{\vartheta_\infty}\frac{d\vartheta}{[f_V(\vartheta)-f_V(\vartheta_\infty)]^{1/2}},$$

and this means that the interior of a specimen whose dimension is larger than $2x_m$ does not feel the surface at all. If $f'_V|_{\theta=\theta_\infty}=0$, then $x_m=\infty$.

[3] It is not obligatory to detect narrowing of the hysteresis loop on the basis of the $\theta_S=\theta_S(k_1)$ relation. The surface energy shifts the lability point, i.e. changes the hysteresis loop $\theta_\infty=\theta_\infty(k_1)$.

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Translated by W. F. Brown, Jr.

Giant spin splitting of excitonic states in the hexagonal crystal CdSe:Mn

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(Submitted 25 April 1980)
Zh. Eksp. Teor. Fiz. **79**, 1554–1560 (October 1980)

The effect of giant spin splitting of excitonic states in semiconductors with magnetic impurities was first observed for the hexagonal crystal CdSe:Mn. A theoretical explanation is presented for those peculiarites of the effect which are connected with the anisotropy of the crystal. Comparison with experiment was used to determine the band-structure parameters $\Delta_1=46\pm3$, $\Delta_2=137\pm1$, and $\Delta_3=140.6\pm0.3$ meV. It is shown that in CdSe:Mn, just as in cubic semiconductors, the exchange interaction with the magnetic impurities is ferromagnetic for the electrons of the conduction band and antiferromagnetic for the electrons of the valence band. The exchange constants are of the same order of magnitude as for the crystals CdTe:Mn, ZnTe:Mn, and ZnSe:Mn.

PACS numbers: 71.35. + z, 71.70. − d

INTRODUCTION

The effects of giant spin splittings of electronic (and correspondingly excitonic) states in II–VI semiconductors with magnetic impurities or else in the solid solutions $A^{\mathrm{II}}_{1-x}M_xB^{\mathrm{VI}}$ (here M is a 3*d* ion) were recently observed and investigated in the cubic crystals CdTe:Mn,[1-3] ZnTe:Mn,[4] ZnSe:Mn, and ZnSe:Fe.[5] A phenomenological theory of the effect was considered in Refs. 1, 2, and 4 on the basis of the concept of carrier-impurity exchange interaction. Values were obtained for the energies and for the probabilities of the transitions into the excitonic states. A microscopic theory of the carrier–impurity exchange interaction was previously proposed[6] for semiconductors with cubic symmetry, and the experimentally observed[1-5] difference between the signs of the constants of the exchange interaction of the conduction electrons I_{cM} and the valence electrons I_{vM} with the 3*d* ions was explained. No such effects in noncubic crystals were previously investigated either experimentally or theoretically.

We present here the results of an experimental investigation of giant spin splittings of excitonic states in hexagonal CdSe:Mn, as well as a theory that explains the observed effects.

THEORY

The exchange interaction between an electron with spin $\mathbf{s}_e$ and N localized magnetic moments with spins $\mathbf{S}$ in a crystal (with the orbital angular momentum well quenched) has a Hamiltonian with a component that depends on the mutual orientation of the spins. This component can be represented in the form

$$\mathscr{H}=-2\frac{I_{eM}}{N_0}\sum_{i=1}^{N}\mathbf{s}_e\mathbf{S}^i, \tag{1}$$

where N_0 is the number of cation sites that can be replaced by $3d$ ions, I_{eM} is equal to I_{cM} or I_{vM} for conduction- and valence-band electrons, respectively. For II–VI crystals, in which the valence band is made up mainly by binding p^+ orbitals, the value of I_{vM} is for $3d$ impurities is determined by the d-d interaction ($I_{vM}\sim I_{dd}<0$), whereas I_{cM} is connected with the s-d interaction ($I_{cM}\sim I_{sd}>0$).[6]

In an external magnetic field $\mathbf{H}$ that polarizes the magnetic impurities in spin, the interaction (1) becomes equivalent to the action exerted on the carrier spins by the effective magnetic fields (exchange field)

$$\mathbf{G}_{c(v)}=-2I_{eM}\frac{N}{N_0}\langle\mathbf{S}\rangle, \tag{2}$$

where I_{eM} is equal to I_{cM} or to I_{vM} for $\mathbf{G}_c(\equiv\mathbf{G}_e)$ or $\mathbf{G}_v(\equiv 3\mathbf{G}_h)$ (the symbols $\mathbf{G}_e$ and $\mathbf{G}_h$ were used in Refs. 1, 4, and 5). The mean values $\langle S_\alpha\rangle$, over the crystal, with $\alpha=X$, Y, and Z, are determined by the field $\mathbf{H}$, by the temperature, and by the concrete form of the spin-Hamiltonian of the impurity ions. We shall assume below that the Zeeman interaction of the impurity spins with the field $\mathbf{H}$ exceeds their interaction with the crystal field, and that the quantization axis $-\langle\mathbf{S}\rangle/|\langle\mathbf{S}\rangle|$ is well defined.

It follows from Refs. 1–5 that at magnetic-impurity concentrations larger than 0.1% the interaction with the exchange fields greatly exceeds the direct action of $\mathbf{H}$ on the carrier spins. It will therefore be neglected. The state energies for the conduction-band electrons, just as for a cubic crystal, then take the form

$$E_c^{\pm}=E_c^0\pm{}^1/_2G_e, \tag{3}$$

where E_c^0 is the energy of the conduction electron in the absence of the interaction (1), and $G_{e(v)}$ is the projection of $\mathbf{G}_{e(v)}$ on the magnetic-impurity-spin quantization axis.

The sixfold degeneracy of the states of the valence electrons (spin s_v) at the point Γ of the Brillouin zone is partially lifted by the spin-orbit interaction and by the crystal field[7]

$$\mathscr{H}_0=\Delta_1L_Z^2+2\Delta_2L_Zs_{vZ}+2\Delta_3(L_Xs_{vX}+L_Ys_{vY}), \tag{4}$$

where Δ_1, Δ_2, and Δ_3 are the parameters of the crystal, $L=1$, $s_v=\frac{1}{2}$, and Z is the crystal axis. As a result, the valence band splits into three doubly degenerate bands with energies

$$E_A^0=\Delta_1+\Delta_2, \tag{5}$$

$$E_{B,C}^0={}^1/_2(\Delta_1-\Delta_2)\pm{}^1/_2[(\Delta_1-\Delta_2)^2+8\Delta_3^2]^{1/2}. \tag{6}$$

The interaction (1) in the basis of the eigenvectors of the Hamiltonian (4) constitutes now a 6×6 matrix that describes the splitting and mixing of the subbands (5) and (6) by the exchange field $\mathbf{G}_v$ ($\mathbf{G}_h$). The explicit form the matrix is not presented here for the sake of brevity.

The cubic case corresponds to $\Delta_1=0, \Delta_2=\Delta_3=\frac{1}{3}\Delta_{c0}$. The operator (1) retains here the Heisenberg form in the basis of each of the subbands Γ_7 or Γ_8:

$$\mathscr{H}^J_{\text{exch.}}=-2I_J\frac{1}{N_0}\sum_{i=1}^{N}\mathbf{J}\mathbf{S}^i, \tag{7}$$

where J is the total angular momentum, $\frac{3}{2}$ or $\frac{1}{2}$ for the bands Γ_8 and Γ_7, respectively. The values of I_J are connected with I_{vM} by the relations

$$I_{3/2}={}^1/_3I_{vM},\quad I_{1/2}=-{}^1/_3I_{vM}. \tag{8}$$

In the axial case, the band states A, B, and C cannot be described by the total angular momentum J (although J_z is conserved at the point Γ of the Brillouin zone), and the interaction (1) no longer reduces to a Heisenberg form in the representation of the Hamiltonian (4).

In the case $|G_h|\ll|E_A^0-E_C^0|$, $|E_B^0-E_C^0|$, which is the one considered hereafter, the matrix of the operator (1) on the states of the Hamiltonian (4) breaks up approximately into 2×2 and 4×4 matrices for the band C and for the bands A and B, respectively. For the band C we obtain directly

$$E_C^{\pm}=E_C^0\mp\frac{1}{2}\frac{G_v}{|G_v|}[(2f_3^2-g_3^2)^2G_{hZ}^2+g_3^4(G_{hX}^2+G_{hY}^2)]^{1/2}, \tag{9}$$

where $f_3=\Delta_3\sqrt{3}[(E_B^0)^2+2\Delta_3^2]^{-1/2}, g_3=f_3E_B^0/\Delta_3$. The ± signs of $E_C^{\pm}$ [and later of $E_A^{\pm}$ and $E_B^{\pm}$ in (10) and (11)] are chosen in accord with the sign of J_z for the given state at $\mathbf{H}\parallel Z$.

The expressions for the energies of bands A and B are described by a secular 4×4 matrix and take the simplest form for two actual cases:

1. $G_{hZ}\neq0, G_{hX}=G_{hY}=0$ (this corresponds to an external magnetic field parallel to the crystal axis Z). In this case

$$E_A^{\pm}=E_A^0\pm{}^3/_2G_{hZ}, \tag{10}$$

$$E_B^{\pm}=E_B^0\pm{}^1/_2(2g_2^2-f_2^2)G_{hZ} \tag{11}$$

Here

$$f_2=\Delta_3\sqrt{6}/[(E_C^0)^2+2\Delta_3^2]^{1/2},\quad g_2=-f_2E_C^0/2\Delta_3.$$

We note that the ratio of the spin splittings of bands A and B is determined only by the crystal parameters Δ_1, Δ_2 and Δ_3:

$$\frac{E_A^+-E_A^-}{E_B^+-E_B^-}=\frac{\Delta E_A}{\Delta E_B}=\left[1+\frac{8\Delta_3^2}{(\Delta_1-\Delta_2)^2}\right]^{1/2}. \tag{12}$$

This ratio can be used as a necessary additional equation, which makes it possible, jointly with the equations for the energies of the transitions between bands A, B, and C and the conduction band, to determine Δ_1, Δ_2, and Δ_3 independently from the experimental data.

2. $G_{hZ}=G_{hY}=0, G_{hX}\neq0$ (this situation corresponds to an external magnetic field directed along the X axis perpendicular to the crystal axis). In this case

$$E_A^{\pm}={}^1/_2\{(E_A^0+E_B^0\pm g_2{}^2G_{hx})+[(E_A^0+E_B^0\mp g_2{}^2G_{hx})^2+3f_2{}^2G_{hx}{}^2]^{1/2}\},$$
$$E_B^{\pm}={}^1/_2\{(E_A^0+E_B^0\pm g_2{}^2G_{hx})-[(E_A^0+E_B^0\mp g_2{}^2G_{hx})^2+3f_2{}^2G_{hx}{}^2]^{1/2}\}. \quad (13)$$

Both in situation 1 and in situation 2, corrections to the energy arise in the second-order approximation in $|G_h|/|E^0_{A(B)}-E^0_C|$ and are due to the mixing of bands A and B with band C by the interaction (1).

The energies of the twelve possible spin excitons (for each type of relative motion of the electron and hole) can be obtained from the relation

$$E_{ex}=E_c^{\pm}-E_n^{\pm}-E_n^{cв},\quad n=A,\ B,\ C. \quad (14)$$

Here $E_c^{\pm}$ and $E_n^{\pm}$ are described by formulas (2), (10), (11), (9), and (13), respectively, and E_n^b is the binding energy for the A, B, and C excitons.

In the general case it is necessary to take into account the intra-exciton electron-hole exchange[7] and the dimagnetic contribution to the energy of an exciton in an external magnetic field **H**. In the real experimental situation, however, the contributions of these interactions are small for an 1s exciton in a II–VI compound.

To obtain the selection rules for the optical transitions, we used the wave functions of the states with energies $E_c^{\pm}, E_A^{\pm}, E_B^{\pm}, E_C^{\pm}$. For situation 1, the probabilities of the ten allowed transitions are independent of $G_{e(v)}$ and are proportional to the following quantities:

$$\begin{gathered}
|\langle+|\sigma^-|A^+\rangle|^2=|\langle-|\sigma^+|A^-\rangle|^2=3,\\
|\langle+|\sigma^+|B^-\rangle|^2=|\langle-|\sigma^-|B^+\rangle|^2=(\delta_1+2^{1/2}\delta_2)^2,\\
|\langle+|\pi|B^+\rangle|^2=|\langle-|\pi|B^-\rangle|^2=(2^{1/2}\delta_1-\delta_2)^2,\\
|\langle+|\sigma^+|C^-\rangle|^2=|\langle-|\sigma^-|C^+\rangle|^2=(2^{1/2}\delta_4+\delta_3)^2,\\
|\langle+|\pi|C^+\rangle|^2=|\langle-|\pi|C^-\rangle|^2=(\delta_4-2^{1/2}\delta_3)^2.
\end{gathered} \quad (15)$$

Here $\langle+|, \langle-|, |A^{\pm}\rangle, |B^{\pm}\rangle, |C^{\pm}\rangle$ denote states corresponding to the energies (2), (10), (11), (9); σ^+, σ^-, and π indicate the polarization of the light relative to the direction of the external field **H**,

$$\delta_1=\frac{1}{3}(2g_2+f_2),\quad \delta_2=\frac{\sqrt{2}}{3}(f_2-g_2),\quad \delta_3=\frac{\sqrt{2}}{3}(f_3-g_3),$$
$$\delta_4=\frac{1}{3}(g_3+2f_3).$$

In a cubic crystal g_2, f_2, g_3, f_3 become equal to unity and expressions (15) become the expressions known for the cubic case.

For situation 2, practically all the possible transitions are allowed and their probabilities depend on G_h. The matrix elements for the transitions between the states $\langle\pm|$ and $|A^{\pm}\rangle, |B^{\pm}\rangle, |C^{\pm}\rangle$ [with energies (3), (13), and (9)] under in influence of light that is linearly polarized in the X, Y, and Z directions are proportional to the quantities listed in the table, where

$$\begin{gathered}
\varepsilon_\mu^\lambda={}^1/_2(1+\lambda\gamma_\mu)^{1/2},\\
\gamma_\mu=(E_A^0-E_B^0+\mu g_2{}^2G_{hx})/[3f_2{}^2G_{hx}{}^2+(E_A^0-E_B^0+\mu g_2{}^2G_{hx})^2]^{1/2},\\
\delta_{\pm}=\delta_1\pm2^{\pm1/2}\delta_2,\quad \delta_{\pm}'=\delta_4\pm2^{\mp1/2}\delta_3,\quad \lambda,\ \mu=+,\ -.
\end{gathered}$$

The transition probabilities are proportional to the squares of the corresponding matrix elements. For arbitrary orientations of the magnetic field relative to the crystal axis, the corresponding energy and transition probabilities can be obtained on the basis of the solution of the secular matrix of the operator (1) in the representation of the Hamiltonian (4).

TABLE I.

	$A+$	$A-$	$B+$	$B-$	$C+$	$C-$
$\langle+\|X$	0	$-3^{1/2}\varepsilon_+^+-\varepsilon_+^-\delta_+$	0	$3^{1/2}\varepsilon_+^--\varepsilon_+^+\delta_+$	0	$-\delta_+'$
$\langle+\|Y$	$3^{1/2}\varepsilon_-^++\varepsilon_-^-\delta_+$	0	$3^{1/2}\varepsilon_-^--\varepsilon_-^+\delta_+$	0	$-\delta_+'$	0
$\langle+\|Z$	$2\varepsilon_-^-\delta_-$	0	$-2\varepsilon_-^+\delta_-$	0	δ_-'	0
$\langle-\|X$	$-3^{1/2}\varepsilon_-^++\varepsilon_-^-\delta_+$	0	$-3^{1/2}\varepsilon_-^--\varepsilon_-^+\delta_+$	0	$-\delta_+'$	0
$\langle-\|Y$	0	$3^{1/2}\varepsilon_+^+-\varepsilon_+^-\delta_+$	0	$-3^{1/2}\varepsilon_+^--\varepsilon_+^+\delta_+$	0	$-\delta_+'$
$\langle-\|Z$	0	$2\varepsilon_+^-\delta_-$	0	$2\varepsilon_+^+\delta_-$	0	$-\delta_-'$

The derived expressions provide a complete description of the energies and of the optical-transition probabilities in giant spin splitting of excitonic states in uniaxial crystals.

EXPERIMENT AND DISCUSSION

The measurements were made at $T=1.94$ K in magnetic fields up to 30 kOe on hexagonal CdSe:Mn crystals, in which 1.3% Mol. (crystal 1) and 10% Mol. (crystal 2) of Mn were introduced during the growth. The Mn concentration was not verified again after the crystals were grown. We investigated the spectra of near-normal excitonic reflection of light in regions A and B of the excitonic transitions at various orientations of the external magnetic field relative to the hexagonal Z axis of the crystal, and at various polarizations of the light. The family of the reflection spectra for **H**$\|Z$ and **E**$\perp Z$ (crystal 2) is shown in Fig. 1. Figure 2 shows the positions of the minima of the excitonic reflection as a function of H for different polarizations of the light for the same crystal (points). The results for crystal 1 were qualitatively similar to those presented, but the splittings were approximately one-third as large. The energy distance (for both crystals) between excitons A and B remained in the range 25.4 ± 0.2 meV, in good agreement with the known value[8] for pure CdSe, and their spectra were shifted with increasing concentration towards higher energies: by 15.8 ± 0.5 meV and by 140.0 ± 0.5 meV for crystals 1 and 2, respectively.

The signs of the circular polarizations of the spectrum components for **H**$\|Z$ and the general picture of

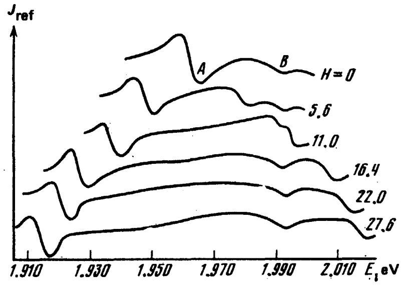

FIG. 1. Plots of the reflection spectra of light with polarization **E** ⊥ Z (σ polarization) in the region of the A and B excitonic bands of the crystal $Cd_{0.90}Mn_{0.10}Se$ at $T=1.94$ K and at different values of the magnetic field **H**$\|Z$. The numbers at the curves indicate the field strengths H (kOe).

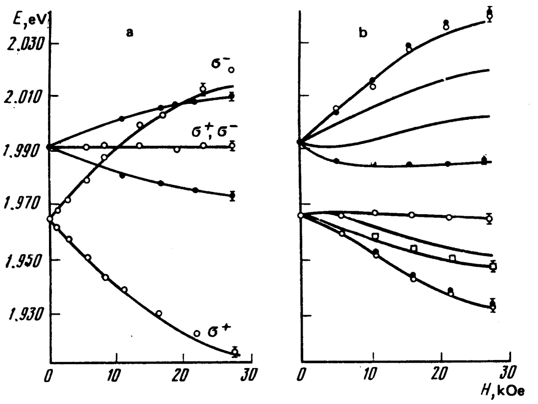

FIG. 2. Positions of the maxima of the A and B excitonic-reflection bands vs the magnetic field $\mathbf{H}\parallel Z$ (a) and $\mathbf{H}\perp Z$ (b). The experimental points (○) and (●) in Fig. a correspond to measurements in polarizations $\mathbf{E}\perp\mathbf{H}$ (σ light) and $\mathbf{E}\parallel\mathbf{H}$ (π light). In Fig. b, the experimental points correspond to the experiment configurations: $\mathbf{E}\perp Z$, $\mathbf{E}\perp\mathbf{H}$ (○); $\mathbf{E}\parallel Z$, $\mathbf{E}\perp\mathbf{H}$ (●); $\mathbf{E}\perp Z$, $\mathbf{E}\parallel\mathbf{H}$ (□). The calculation results are the solid curves.

the gain spin splitting are equivalent to those previously observed in cubic crystals with manganese impurity[1-5] (accurate to crystal splitting of the states with $J_Z = \pm\frac{3}{2}$ and $J_Z = \pm\frac{1}{2}$). It follows directly therefore that the signs of the constants I_{cM} and $I_{vM}(I_h)$ in CdSe:Mn, as well as in CdTe:Mn, ZnTe:Mn and ZnSe:Mn,[1-5] are opposite, and I_{cM} and I_{vM} are respectively of the ferromagnetic and antiferromagnetic type. The ratio of the splittings of the π components (B exciton) and $\sigma^\pm$ components (A exciton), as well as the fact that the $\sigma^\pm$ components for the B excitons are not split, cannot be understood on the basis of the relations for the cubic crystal,[1,2] but is easily understood when account of expressions (14) is taken in (2), (10), and (11). From a comparison of these formulas with experiment it follows that $G_e = -0.666 G_h$, and the ratio of the spin splittings of the A and B valence bands (12) is $\Delta E_A/\Delta E_B = 4.5 \pm 0.2$. Neglecting the difference between the binding energies of the A, B, and C excitons and assuming (on the basis of the conservation of $E_A^0 - E_B^0$) that $E_{A(B)}^0 - E_C^0$ likewise remain unchanged when Mn is added to the crystal, we obtain from relations (5), (6), and (12) and from the data of Ref. 8 for $E_A^0 - E_B^0$ that $\Delta_1 = 46.6 \pm 3$; $\Delta_2 = 137.3 \pm 1$; $\Delta_3 = 140.6 \pm 0.3$ meV, which is close to the values in Table 40.3 of Ref. 7.

The magnetic-field dependence of the spin splitting of the excitonic states demonstrates the tendency to saturate in strong fields. Nonetheless, the spectra at $\mathbf{H}\parallel Z$ for either spectrum cannot be described under the assumption

$$G_e = G_e(H\to\infty) B_{S=5/2}(g\beta H/kT),$$

but can be described by a Brillouin function $B_{5/2}$ with effective temperatures $T' = T + \Theta$. The spin splittings are described within the limits of errors if $\Theta = 3.3 \pm 0.1$ K and if $G_e(H\to\infty)$ is equal to 2.8 and 22.0 meV for crystals 1 and 2, respectively. This results points to a substantial Mn–Mn antiferromagnetic interaction at the employed Mn concentrations. The equality of Θ for crystals 1 and 2 is from our point of view an accident, and no such dependence will apparently hold in stronger fields for crystal 2. It can be assumed that the value of $G_e(H\to\infty)$ obtained for crystal 1 is close to the true value, whereas that for crystal 2 is significantly undervalued and cannot be used to estimate I_{cM} and I_{vM} in accordance with (3) and (7).

Checks on the theory described above, on the correctness of G_e/G_h, and on the values of Δ_1, Δ_2, and Δ_3 can be provided by experiments at $\mathbf{H}\perp Z$, corresponding to the theoretical situation 2. Inasmuch as in the employed fields the Zeeman splitting of the ground states of the Mn^{2+} ions substantially exceed the classical splitting, the connection of H with G_e and G_h should be the same in case 2 as in case 1.

The solid lines in Figs. 2a and 2b were plotted using the presentated values of Δ_1, Δ_2, and Δ_3 as well as formulas (2), (10), (11), and (13) with the same connection of H with G_e and G_h. It is seen that the agreement with experiment is excellent. The observed relative intensities of the reflection bands, their polarizations, and their dependence on H (on G_e and G_h at $\mathbf{H}\perp Z$) are in full qualitative agreement with the results (15) and with the data in the table. Low intensity transitions did not manifest themselves in the reflection, and no exact qualitative measurements of the intensity were made.

In weak fields (< 5 kOe) and at $\mathbf{H}\perp Z$, we observed for the lines of exciton A in crystal 1 systematic deviations of the positions of the exciton spin components from the theoretical calculations. This may be due to the presence of intra-exciton electron-hole exchange interaction and to effects of dielectric interaction between closely lying exciton resonances having the same polarization.[9] No detailed study of these effects was made.

We note in conclusion that although the low accuracy of the measurement of the concentration of the Mn, as well as the effects of the Mn–Mn interaction, does not make it possible to determine accurately in our experiment I_{cM} and I_{vM}, the values of $G_e N_0/N$ and $G_h N_0/N$ are of the same order as in CdTe:Mn, ZnTe:Mn, and ZnSe:Mn.[1-5] The I_{cM} and I_{vN} signs that follow from the character of the polarization of the observed lines are in full agreement with the theory.[6]

The effects of giant spin splitting in cubic and hexagonal II–VI semiconductors are of the same nature.

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Translated by J. G. Adashko

Conductivity of two-dimensional systems with macroscopic inhomogeneities

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(Submitted 29 April 1980)
Zh. Eksp. Teor. Fiz. 79, 1561-1572 (October 1980)

The conductivity of certain two-dimensional systems with periodic arrangement of the inhomogeneities is considered. An exact solution is obtained for models with dielectric as well as superconducting square inclusions arranged in checkerboard fashion. The field distribution in the conducting region, the fluctuations of the current density, and the effective conductivity are obtained. Taken together with the results obtained by Dykhne [Sov. Phys. JETP 32, 63 (1971)] this makes it possible, when the properties of the components differ drastically, to provide a relatively complete description of such a system both in the vicinity of the metal–insulator transition and far from it. The conductivities of similar systems with inclusions of different shape are qualitatively estimated and the limits of the critical exponent are established. A relation that is valid also in the three-dimensional case is obtained between the permittivity of a medium with metallic inclusion and the conductivity having the same structure with superconducting inclusions.

PACS numbers: 72.10.Bg, 71.30. + h, 64.60.Fr

1. In the study of the physical properties of inhomogeneous media, principal attention is usually paid to systems with random distribution of the components (see, e.g., Refs. 1 and 2). This is natural, since real systems of this kind are frequently randomly inhomogeneous. Thus, for example, thin films obtained by sputtering on a substrate, have during the initial sputtering stage a strongly inhomogeneous random structure (island films). A theoretical analysis of such systems is fraught with extreme difficulties, which have not been overcome to this day. In essence, the only study in which some exact analytic results were obtained is that of Dykhne[3] (and its generalizations[4-6]). A number of important results in the vicinity of the metal–insulator (MI) phase transition are obtained by similarity theory.[2] To investigate the properties of randomly inhomogeneous media in the entire range of concentrations it is necessary to resort to computer calculations and to model experiments.

Another important class of inhomogeneous media comprises the so-called systems with topological structure (STS), i.e., systems in which all the inclusions are identical and form a periodic lattice. Among the real systems of this type are, for example, thin films with topological structure, used in semiconductor devices.[7] These films, which are also obtained by sputtering, are on the one hand bulky enough to be practically homogeneous, and on the other thin enough to permit the distributions of the electric field and of the current in them to be regarded in a number of cases as two-dimensional. The topological structure, i.e., the periodic distribution of the elements that cause the inhomogeneity, is obtained, for example, by subsequent selective etching. It is important that the dimensions of the inhomogeneities are large in comparison with the mean free path, so that a macroscopic description with a coordinate-dependent conductivity is applicable.

The periodicity of the STS leads to periodicity in the distribution of the electric field (current) and makes it possible by the same token to restrict oneself in the determination of the potential to a single unit cell of the structure. Clearly, this problem is simpler than that of the randomly inhomogeneous medium. The difference in the complexity of the solution of these problems is similar to a considerable degree to the corresponding difference when it comes to finding the phonon spectra of an ideal and a disordered crystal. In two-dimensional STS, when the properties of the components differ greatly, it is possible in a number of cases to make use of the powerful methods of the theory of functions of complex variable.[8,9,10]

In addition to the obvious need for investigating the properties of the STS for various applications, their study is also of fundamental interest since a metal-insulator phase transition can take place in these systems. From this point of view such systems can serve as a relatively simple object for a theoretical and experimental study of the phase-transition problem. The investigation of the STS properties, which is also of independent interest, permits on the one hand to disclose the universality of the conclusions of the general theory of phase transitions and determine the role played in it by the STS. On the other hand, exactly solvable STS models can serve as the touchstones for a variety of possible approximate methods.

We consider in this paper certain STS models. Principal attention is paid to a system in which the inclusions, quadratic in shape, are arranged in checkerboard fashion. The investigation of such a system is