[1)]The following units are used: $\hbar/(2mgn_0)^{1/2} \equiv \xi$ for distances, $\hbar/2gn_0$ for time, and $n_0^{1/2}$ for the wave function, where m is the mass of the atom and n_0 is the equilibrium density of the condensate. The constant g characterizes the interaction between the particles and is assumed to be small enough to make the coherence length ξ larger than the interatomic distances.

[2)]The function ψ_ν is not a solution of Eq. (1), except at $\nu = 0$, inasmuch as in a medium that is immobile at infinity a straight vortex, being the only perturbation of its ground state, is immobile (see below).

[3)]It is defined as the line the circuit around which (around each of its elements) changes the phase by $2\pi n (n = \pm 1, \pm 2, \ldots)$. In the immediate vicinity of such a line we have $\psi \propto \psi_\nu$.

[4)]It is possible to separate in the gradient of the phase Φ the various contributions (of the flux, of another vortex, of a reflected vortex, of other elements of the considered vortex, etc.) and resolve the Magnus force into several forces (interaction with external flux, interaction with another vortex, interaction with the wall-image force, interaction with other elements of the same vortex-rectifying force acting on a bent vortex).

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Role of spatial dispersion in light absorption by excitons

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A phenomenological analysis is presented of resonant absorption of light by excitons with spatial dispersion taken into account. It is shown that the form of the absorption line contour and its characteristics change drastically when the damping constant goes through a certain critical value. Analytic formulas are obtained for the absorption coefficient, for the integral absorption, for the maximum value of the absorption coefficient, and for the equivalent line width at damping-constant values larger and smaller than critical.

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1. INTRODUCTION

Light absorption by excitons has been the subject of a considerable number of experimental[1-5] (see also the references in these papers) and theoretical[6-10] studies. It has been established that a change in the sample temperature is accompanied by a change in the absorption-line shape and the line parameters such as the half-width, maximum of the line, and integral absorption. One of the distinguishing features of the observed spectra is the decrease of the integral absorption with temperature. There is still no complete theoretical explanation of all the features of the absorption spectra, although it is clear that the experimentally observed changes are connected with the dependence of the phenomenological damping constant ν on the temperature and on the frequency.[1] The role played by spatial dispersion in the decrease of the integral absorption with decreasing temperature was recently considered in Refs. 9 and 10.

We report here a detailed quantitative investigation of the dependence of the absorption line shape and of the line parameters on ν in a crystal model in which spatial dispersion is taken into account. We confine ourselves here to a macroscopic solution of the problem and assume ν to be an independent variable; this assumption is justified because ν has a monotonic dependence on T in a wide temperature interval. In addition, to reveal the role played in absorption by spatial dispersion "in pure form," we assume ν to be constant over the entire range of frequencies to interest to us. Effects connected with the frequency dependence of ν are discussed briefly at the end of the article.

2. ABSORPTION COEFFICIENT

We consider an isotropic nongyrotropic crystal whose dielectric constant in the vicinity of an isolated exciton absorption line can be represented in the form

$$\varepsilon(\omega, \mathbf{K}) = \varepsilon_0 + \frac{p\omega_0^2}{\omega_0^2 - \omega^2 - i\omega\nu + \beta c^2 \mathbf{K}^2}, \tag{1}$$

where ε_0 is the contribution made to the dielectric constant by other resonances, p is the oscillator strength, ω_0 is the natural frequency of the mechanical exciton at $\mathbf{K} = 0$, $\beta = \hbar\omega_0/m_e^* c^2$, and m_e^* is the effective mass of the exciton. Let the crystal be a plane-parallel plate of thickness d. At normal incidence of the light on the face of the plate there will propagate inside the plate, at a given frequency ω, two waves with refractive indices $\tilde{n}_1 = n_1 + i\varkappa_1$ and $\tilde{n}_2 = n_2 + i\varkappa_2$. The complete formula for the amplitude transmission coefficient of the plate, with allowance for the multiple re-

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flections, is of the form[11]

$$\mathcal{T}=\frac{1}{D}\{T_{01}T_1{}^be_1+T_{02}T_2{}^be_2+(T_1{}^bR_{12}{}^b-T_2{}^bR_{11}{}^b)(T_{02}R_{11}{}^a-T_{01}R_{21}{}^a)e_1{}^2e_2 +(T_1{}^bR_{22}{}^b-T_2{}^bR_{21}{}^b)(T_{02}R_{12}{}^a-T_{01}R_{22}{}^a)e_1e_2{}^2\}, \quad (2)$$

where $e_1=\exp(i\tilde{n}_1k_0d)$, $e_2=\exp(i\tilde{n}_2k_0d)$, $k_0=\omega/c$, R_{ik} and T_{ik} are the elements of the reflection and transmission matrices at each of the faces of the plate, while D is given by

$$D=\det\begin{pmatrix} 1-(R_{11}{}^2e_1{}^2-R_{12}R_{21}e_1e_2) & -R_{12}e_1(R_{11}e_1+R_{22}e_2) \\ -R_{21}e_2(R_{11}e_1+R_{22}e_2) & 1-(R_{21}R_{12}e_1e_2-R_{22}{}^2e_2{}^2)\end{pmatrix}.$$

The indices a and b pertain to the entrance and exit faces of the plate. The plate thickness d is assumed to be large enough to satisfy in the resonant-frequency region of interest to us the condition $k_0d\gg\varkappa_{min}^{-1}$, where $\varkappa_{min}$ is the smaller of the absorption coefficients $\varkappa_1$ or $\varkappa_2$. In this case the order of smallness of the terms in (2) is determined by the factors e_1 and e_2, which satisfy the inequalities $|e_1|\ll 1$ and $|e_2|\ll 1$. We need therefore retain in the numerator of (2) only the first two terms, and all the terms proportional to the products e_1 and e_2 can be discarded, while in the denominator we can discard all terms proportional to the first powers of e_1 and e_2. Then $D=1$ and formula (2) takes the form

$$\mathcal{T}=T_{01}T_1{}^be_1+T_{02}T_2{}^be_2. \quad (3)$$

Formula (3) corresponds to the usual approximation of single passage of waves through a crystal (large absorption) and to neglect of the interference phenomena. The transmission coefficients T_{ik} in (3) can be calculated with the aid of additional boundary conditions, as was done in Ref. 11, but we shall not need the explicit forms of these coefficients.

In practice we are interested in the absorption coefficient, which is expressed in terms of the logarithm of the transmission coefficient

$$k(\omega)=(\ln|\mathcal{T}|^{-2})/k_0d \quad (4)$$

and whose frequency dependence corresponds to the contour of the absorption line. The absorption coefficient (4) differs from the usually defined one[12] by a factor $k_0=\omega/c$ in the denominator, and is a dimensionless quantity in our case. When the absolute value of the logarithm of (3) is taken in the frequency regions where e_1 and e_2 are of different order of magnitude, we can discard the smaller of the terms, e.g., $T_{02}T_2{}^be_2$, and we obtain then

$$k(\omega)=2\varkappa_1+(\ln|T_{01}T_1{}^b|^{-2})/k_0d. \quad (5)$$

The transmission coefficients T_{ik} at individual boundaries of the sample are of the order of unity, therefore the logarithmic term in (5) can be neglected compared with $2\varkappa_1$. If $\varkappa_2>\varkappa_1$, we must replace the subscript 1 in (5) by 2. On the other hand if $\varkappa_2\sim\varkappa_1$, then a quantity $\varkappa(\ln 2)/k_0d$ is added to (5), but it can also be neglected compared with $2\varkappa_1$.

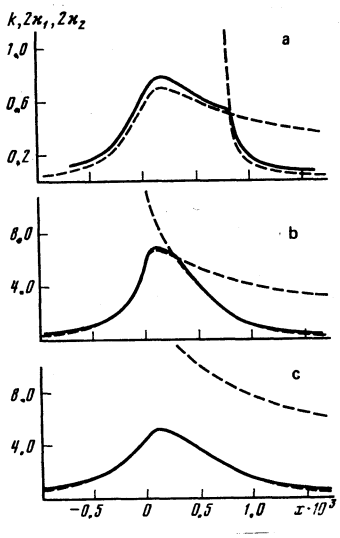

FIG. 1. The dependence of the absorption coefficient $(\ln|\mathcal{T}|^{-2})/k_0d$ (solid curve) and of the doubled damping indices of the two waves in the medium (dashed) on the frequency $[x=(\omega-\omega_0)/\omega_0]$ for three values of the damping constant $(\gamma=\nu/\omega_0)$: a) $\gamma=0.5\cdot10^{-4}$, b) $\gamma=4.5\cdot10^{-4}$, c) $\gamma=8.5\cdot10^{-4}$. We used for the calculations the following parameters: $k_0d=20$, $\varepsilon_0=8.3$, $p=0.013$, $m_e^*=0.9m_e$, $\hbar\omega_0=2.5524$ eV. At these values of the parameters, the critical damping is $\gamma_0=0.548\cdot10^{-3}$.

Thus, the absorption coefficient is determined mainly by that term of (3) which describes the passage of the wave with the smaller of the quantities $\varkappa_1$ or $\varkappa_2$, and we can put

$$k(\omega)=2\varkappa_{min}(\omega). \quad (6)$$

The foregoing is illustrated in Fig. 1, which shows the $2\varkappa_1(\omega)$ and $2\varkappa_2(\omega)$ spectra obtained from the dispersion relation with account taken of formula (1) and of the absorption coefficient (4) calculated with a computer on the basis of the exact formula (2) at $k_0d=20$.

In the plot we used the dimensionless frequency $x=(\omega-\omega_0)/\omega_0$ and the damping constant $\gamma=\nu/\omega_0$. It is seen from the figure that the relation (6) is satisfied with high accuracy in the vicinity of the resonant frequency even at $\gamma=0.5\times10^{-4}$. At larger γ or with increasing k_0d, the formula (6) becomes more accurate. It follows therefore that in the large-thickness approximation the investigation of the absorption in crystals reduced to an investigation of the frequency behavior of the functions $\varkappa_1(x)$ and $\varkappa_2(x)$ in the vicinity of the exciton resonance line. There have been many such investigations (see, e.g., Refs. 1 and 7), and in some of the results we must repeat ourselves here.

3. REFRACTIVE INDICES WITH ALLOWANCE FOR DAMPING, AND THE ABSORPTION-LINE CONTOUR.

The refractive indices of the normal waves in the considered crystal are determined from the dispersion relation $\tilde{n}^2=\varepsilon(\omega,\omega\tilde{n}/c)$, the solutions of which are of the form

$$\tilde{n}_{1,2}^2=\{\beta\varepsilon_0-\Delta\pm[(\varepsilon_0\beta+\Delta)^2+4p\beta]^{1/2}\}/2\beta, \quad (7)$$

where $\Delta=(\omega_0^2-\omega^2-i\omega\nu)/\omega_0^2$. We are interested in the behavior of the functions $\tilde{n}_{1,2}$ in the vicinity of the resonant frequency, and we can therefore put $\Delta=-2x-i\gamma$. Changing over to the complex argument $z=2x/\beta-\varepsilon_0+i\gamma/\beta$, we obtain for the refractive indices the complex-variable function

$$\tilde{n}_{1,2}=\{\varepsilon_0+[z\pm(z^2+q^2)^{1/2}]/2\}^{1/2}, \tag{8}$$

where $q^2=4p/\beta$. The two branches of this function determine the two refractive indices. When extracting the square root in (8) we consider, as usual, only that branch of the function $\tilde{n}(z)$ whose imaginary part is positive. The behavior of the functions $\tilde{n}_1(z)$ and $\tilde{n}_2(z)$ must be investigated on the straight-line contour C shown by the solid line in Fig. 2. The contour C is parallel to the real z axis, with the distance between the contour C and the z axis equal to γ/β. An interesting feature of the function (8) is the presence of a branch point $z=iq$ in the upper half-plane of the variable z. The contour encounters the branch point if $\gamma=\gamma_0=2(p\beta)^{1/2}$. We call this quantity, as in Ref. 7, the critical damping. The behavior of the functions $n_1(z)$ and $n_2(z)$ changes radically when γ goes through the critical value. This singularity can be described in the following manner.

We shall designate as classical the refractive index whole limit as $m_e^*\to\infty$ is the quantity $[\varepsilon_0-p/(2x+i\gamma)]^{1/2}$. The second branch of the function (8) will be assumed to be nonclassical. Then at a damping larger than critical the refractive index on one of the sheets of the Riemann surface of the double-valued function $\tilde{n}(z)$ is classical on the entire contour C, and the refractive index on the other sheet of the Riemann surface is nonclassical on the entire contour C. The classical refractive index corresponds to the smaller of the absorption coefficients $\varkappa_{min}(x)$ (Fig. 1c). The $\varkappa_{min}(x)$ curve is close to a Lorentz contour and coincides with the latter at $\gamma\gg\gamma_0$. Thus, at $\gamma>\gamma_0$ the absorption line takes the same form as without allowance for spatial dispersion.

If the inverse inequality holds, $\gamma<\gamma_0$, the contour C passes below the branch point, the sheets of the Riemann surface intersect in this case, and the function (8) has a different behavior on the different ends of the contour C. On one sheet of the Riemann surface, the function $\tilde{n}(z)$ is classical on the left part of the contour C and nonclassical on the right; the situation is reversed on the second sheet. In this case the smaller of the absorption coefficient $\varkappa_{min}(x)$ also corresponds to the classical branch of the function $\tilde{n}(x)$, and $\varkappa_{min}(x)$ on the opposite ends of the contour C lies on different sheets of the Riemann surface. To obtain the absorption line in this case we must draw a cut through the Riemann surface along the line drawn from the branch point on which the absorption coefficients are equal, $\varkappa_1(x)=\varkappa_2(x)$, and to determine $k(x)$ at the point of the intersection of the contour C and the branch cut it is necessary to go over from one sheet of the Riemann surface to the other. The absorption line to the left and to the right of the branch cut will then be determined by the different branches of the function $\tilde{n}(z)$ (see Fig. 1a, 1b). It is shown in Appendix A that the branch cut is determined by the formula

$$z=\frac{\gamma_0^2-\gamma^2}{4\varepsilon_0\beta^2}+i\frac{\gamma}{\beta}.$$

It takes the form of a parabola (shown dashed in Fig. 2). The absorption-line contour has in this case an asymmetrical "trapezoidal" form (Fig. 1a), where the right-hand vertex of the "trapezoid" coincides in position with the point of intersection of the contour C and the branch cut. The left-hand vertex of the "trapezoid" corresponds approximately to the frequency $x=\beta\varepsilon_0/2$. At $\gamma=0$ the distance, along the frequency axis, between the vertices of the trapezoid coincides with the longitudinal-transverse splitting. Such an absorption-line contour was observed in a number of studies.[2-5] Deviation from this contour at low temperatures (when $\gamma<\gamma_0$) occurs when thin crystals were used in the experiment. The wave-interference effects are then significant and the influence of the boundary condition should manifest itself.

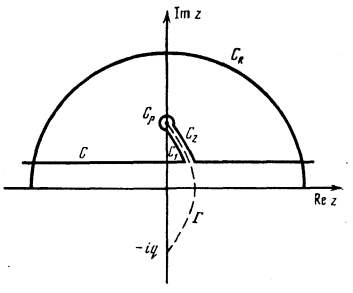

FIG. 2. Integration contour in the plane of the variable z. The dashed line shows the branch cut.

4. INTEGRAL ABSORPTION COEFFICIENT

To find the integral absorption, it is necessary to integrate the absorption coefficient $k(x)$ with respect to the frequencies. We shall integrate with respect to the dimensionless frequency x. The resultant value of the integral absorption coefficient $\mathcal{K}$ differs from the ordinary one[12] by a factor ω_0^2/c. By definition we have

$$\mathcal{K}=2\int_{-\infty}^{\infty}\varkappa_{min}(x)\,dx=2\,\mathrm{Im}\int_{-\infty}^{\infty}\tilde{n}_{кл}(x)\,dx$$
$$=\beta\,\mathrm{Im}\int_C\{\varepsilon_0+[z-(z^2+q^2)^{1/2}/2]\}^{1/2}\,dz, \tag{9}$$

where $n_{cl}(x)$ is the classical refractive index.

Let $\gamma>\gamma_0$. Then in (9) the integration contour C can be deformed into a semicircle C_R of radius $R\to\infty$, drawn in the upper half-plane of the variable z (Fig. 2). The contributions made to the integral by the arcs located on the ends of the semicircle and contained between the contour C and the real z axis are infinitely small and can be neglected. We make in (9) the change of variable $z=R\exp i\varphi$, and assume that $R\to\infty$, so that in the integrand we can expand in terms of the small parameter $1/R$. We then obtain

$$\mathcal{K}=\lim_{R\to\infty}\operatorname{Im} i\beta R\int_{\pi}^{0}\left(\varepsilon_0-\frac{q^2}{4R}e^{-i\varphi}\right)^{1/2}e^{i\varphi}\,d\varphi$$

$$=-\frac{\beta q^2}{8\varepsilon_0^{1/2}}\int_{\pi}^{0}d\varphi=\frac{\pi p}{2\varepsilon_0^{1/2}}. \tag{10}$$

It is seen therefore that when the damping exceeds the critical value the integral absorption is independent of γ and is proportional to the oscillator strength.

When the inverse inequality holds, $\gamma<\gamma_0$, the integrand in (9) has a discontinuity at the point of intersection of the contour C with the branch cut (the point z_0). To simplify the calculations we make the integrand continuous, closing the integration contour by the lines C_1 and C_2 along the edges of the cut and by the infinitely small circle C_ρ around the branch point. The integral over the closed contour is then equal to the integral over the contour C_R. Recognizing that the integral over the contour C_ρ tends to zero as $\rho\to 0$, the sought integral is equal to

$$\mathcal{K}=\frac{\pi p}{2\varepsilon_0^{1/2}}-\beta\operatorname{Im}\int_{C_1}\left(\varepsilon_0+\frac{z+(z^2+q^2)^{1/2}}{2}\right)^{1/2}dz$$
$$-\beta\operatorname{Im}\int_{C_2}\left(\varepsilon_0+\frac{z-(z^2+q^2)^{1/2}}{2}\right)^{1/2}dz \tag{11}$$

and reduces to calculation of the integrals along the edges of the cut.

We make the changes of variables $t=[z+(z^2+q^2)^{1/2}]/2$ and $t=[z-(z^2+q^2)^{1/2}]/2$ in the first and second integrals of (11), respectively. The two integrals in (11) can then be combined into one:

$$\mathcal{K}=\frac{\pi p}{2\varepsilon_0^{1/2}}-\beta\operatorname{Im}\int_{t_1}^{t_2}(\varepsilon_0+t)^{1/2}\frac{q^2+4t^2}{4t^2}\,dt \tag{12}$$

with integration limits on the lower ends of the contours C_1 and C_2:

$$t_1=[z_0+(z_0^2+q^2)^{1/2}]/2,\quad t_2=[z_0-(z_0^2+q^2)^{1/2}]/2.$$

The integral in (12) can be easily calculated:

$$\mathcal{K}=\frac{\pi p}{2\varepsilon_0^{1/2}}-\beta\operatorname{Im}\left\{\frac{2}{3}(\varepsilon_0+t)^{3/2}-\frac{q^2(\varepsilon_0+t)^{1/2}}{4t}\right.$$
$$\left.-\frac{q^2}{8\varepsilon_0^{1/2}}\ln\frac{\varepsilon_0^{1/2}-(\varepsilon_0+t)^{1/2}}{\varepsilon_0^{1/2}+(\varepsilon_0+t)^{1/2}}\right\}\Bigg|_{t_1}^{t_2}. \tag{13}$$

Substituting the integration limits in (13) and separating the imaginary part (see Appendix B), we obtain

$$\mathcal{K}=\frac{p}{\varepsilon_0^{1/2}}\left(\operatorname{arctg}\frac{\gamma}{(\gamma_0^2-\gamma^2)^{1/2}}\right.$$
$$\left.+\frac{\gamma(\gamma_0^2-\gamma^2)^{1/2}}{\gamma_0^2}\right). \tag{14}$$

At $\gamma\ll\gamma_0$, formula (14) can be written in the form

$$\mathcal{K}\approx\gamma(p/\varepsilon_0\beta)^{1/2}. \tag{15}$$

Thus, at small γ the integral absorption depends linearly on γ and tends to zero at $\gamma\to 0$. The derivative

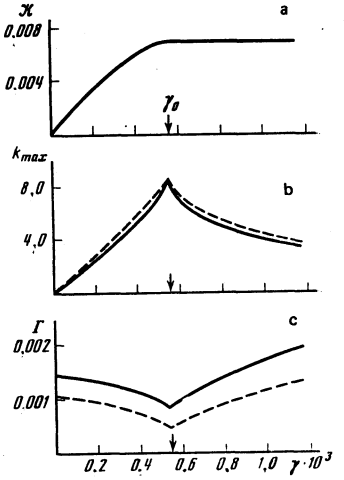

FIG. 3. Plots of the integral absorption coefficient (a), of the maximum value of the absorption coefficient [dashed—exact value, solid—calculated with the aid of (17) and (20)], and c) of the equivalent line width (solid curve) and the line half-width (dashed) against the damping constant.

$\partial\mathcal{K}/\partial\gamma$ is continuous in the entire range of variation of the damping constant, including the point $\gamma=\gamma_0$. Figure 3a shows a plot of $\mathcal{K}(\gamma)$ calculated from formulas (10) and (14). If it is recognized that the parameter γ depends linearly on temperature in a wide range of its values, a similar $\mathcal{K}(\gamma)$ plot was obtained in experiment.[5]

5. MAXIMUM VALUE OF THE ABSORPTION COEFFICIENT

The exact value of the frequency $x=x_{max}$, on which the absorption is maximal, is difficult to calculate analytically. From the numerical calculations (see Fig. 1) it is seen, however, that at any damping parameter the maximum of the absorption line is located on the right of the frequency $x=0$. To simplify the calculations we obtain the value of the absorption coefficient at the point $x=\beta\varepsilon_0/2$ located between the points $x=0$ and $x=x_{max}$. In the case $\gamma>\gamma_0$, the classical refractive index at this frequency is

$$\tilde{n}_{cl}=n_{cl}+i\varkappa_{min}=\{\varepsilon_0+i[\gamma-(\gamma^2-\gamma_0^2)^{1/2}]/2\beta\}^{1/2}, \tag{16}$$

from which we get

$$k_{max}=2^{1/2}\left\{\left[\varepsilon_0^2-\frac{p}{\beta}+\frac{\gamma(\gamma-(\gamma^2-\gamma_0^2)^{1/2})}{2\beta^2}\right]^{1/2}-\varepsilon_0\right\}^{1/2}. \tag{17}$$

In the asymptotic case $\gamma\gg\gamma_0$, the maximum value of the absorption coefficient can be obtained by carrying out the expansion in (16). In this case

$$k_{max}=p/\varepsilon^{1/2}\gamma, \tag{18}$$

i.e., k_{max} is inversely proportional to γ, as in the case of a Lorentzian absorption line.

When the opposite inequality $\gamma<\gamma_0$ is satisfied, the refractive index with the smaller imaginary part is ob-

tained from the formula

$$n_d+i\varkappa_{min}=(\varepsilon_0+(\gamma_0^2-\gamma^2)/2\beta+i\gamma/2\beta)^{1/2}. \quad (19)$$

Hence

$$k_{max}=2^{1/2}\left\{\left[\varepsilon_0^2+\frac{\gamma_0^2+4\beta\varepsilon_0(\gamma_0^2-\gamma^2)^{1/2}}{4\beta^2}\right]^{1/2}-\varepsilon_0-\frac{(\gamma_0^2-\gamma^2)^{1/2}}{2\beta}\right\}^{1/2}. \quad (20)$$

If $\gamma \ll \gamma_0$, we can carry out the expansion in (19) and obtain

$$k_{max}=\gamma/\{2\beta(2\varepsilon_0\beta+(\gamma_0^2-\gamma^2)^{1/2})\}^{1/2}, \quad (21)$$

i.e., in the region of small γ the maximum value of the absorption coefficient depends linearly on the damping constant. At $\gamma=\gamma_0$, the maximum of the absorption coefficient takes on the largest value, equal to

$$k_{max}(\gamma_0)=2^{1/2}[(\varepsilon_0^2+p/\beta)^{1/2}-\varepsilon_0]^{1/2}. \quad (22)$$

This formula describes the absorption coefficient at the branch point and coincides with the exact value of the maximum of the absorption coefficient.

A plot of the maximum absorption against γ, calculated from formulas (17) and (20), is shown in Fig. 3b in the form of a solid curve. The computer-calculated plot for the exact value of the maximum of the absorption coefficient is shown dashed. The error in the determination of the maximum of the absorption coefficient from the above formulas is ~10% and is approximately the same for all γ, with the exception of $\gamma \approx \gamma_0$.

6. LINE WIDTH

It is difficult to calculate analytically the line half-width in this case. We therefore take here the line width to be the width of the equivalent rectangular contour, whose area equals the area of the considered contour and whose maximum absorption is equal to the height of this contour. For example, for a Lorentz contour, the equivalent line width differs from the half-width by a factor $\pi/2$. In each of the considered cases the line width is then equal to the ratio of the integral absorption to the maximum absorption coefficient. We write down below the formulas only for the limiting cases. At $\gamma \gg \gamma_0$ the equivalent line width is

$$\Gamma=\pi\gamma/2, \quad (23)$$

and at $\gamma \ll \gamma_0$ the formula for Γ is of the form

$$\Gamma=\gamma_0[1+(\gamma_0^2-\gamma^2)^{1/2}/2\beta\varepsilon_0]^{1/2}. \quad (24)$$

Figure 3c shows the curves for the equivalent line width, calculated as a ratio of the integral absorption [formulas (10) and (14)] to the maximum absorption [formulas (17) and (20)], and also for the line half-width calculated with a computer using the exact formula (2). It is seen from the figure that for all γ the two quantities differ by approximately the same factor, ~1.5. It is therefore possible in practice to convert from the equivalent width to the half-width by dividing the latter by this factor.

7. DISCUSSION OF RESULTS

We have investigated the shape and parameters of the absorption-line contour in a medium with spatial dispersion. We see that the spatial dispersion exerts a strong influence on the integral absorption, on the line width, and on the line maximum as functions of the damping parameter. These curves change shape radically at the point $\gamma=\gamma_0$, which depends both on the effective mass of the exciton and on the oscillator strength. Analytic formulas for the indicated dependences were obtained also in Ref. 8, but did not contain the dependence on the parameter m_d^* that characterizes the spatial dispersion, and seem therefore incorrect to us. The characteristic value obtained in Ref. 8 for the damping constant, starting with which the integral absorption begins to decrease, differs from γ_0 by two orders of magnitude. A discrepancy of the same order with the results of Ref. 8 was found by numerical calculations in Ref. 10. This is apparently the reason why that theory led previously to a disagreement with the experimental data of Ref. 3.

In the reasoning above we did not take into account the frequency dependence of γ, which, as noted in Ref. 1, can substantially influence both the absorption-line shape and the integral absorption. However, the presence of such a dependence can be easily revealed by comparing the experimental and theoretical absorption curves. For example, in the region $\gamma<\gamma_0$ the absorption coefficient at each frequency depends approximately linearly on γ. Therefore in this case the absorption-line contour is represented in the form of a product of the theoretical contour by the function $\gamma(\omega)$. Such a simple dependence makes it possible to determine directly the $\gamma(\omega)$ dependence from a comparison of the experimental contour with the theoretical one.

APPENDIX A

DETERMINATION, OF THE GEOMETRIC LOCUS OF THE z-PLANE POINTS AT WHICH THE IMAGINARY PARTS OF THE REFRACTIVE INDICES ARE EQUAL

The refractive indices of the two waves in the medium (7) can be expressed in the form

$$n_1+i\varkappa_1=[\varepsilon_0/2+x/\beta+u+i(\gamma/2\beta+v)]^{1/2}, \quad (A.1)$$

$$n_2+i\varkappa_2=[\varepsilon_0/2+x/\beta-u+i(\gamma/2\beta-v)]^{1/2}, \quad (A.2)$$

where

$$u+iv=1/2[(\varepsilon_0-2x/\beta)^2+4p/\beta-\gamma^2/\beta^2+(\varepsilon_0-2x/\beta)i\gamma/\beta]^{1/2}. \quad (A.3)$$

We equate the imaginary parts of the refractive indices, which can be calculated by the general rules from (A.1) and (A.2):

$$\{[(\varepsilon_0/2+x/\beta+u)^2+(\gamma/2\beta+v)^2]^{1/2}-(\varepsilon_0/2+x/\beta+u)\}^{1/2}=\{[(\varepsilon_0/2+x/\beta-u)^2+(\gamma/2\beta-v)^2]^{1/2}-(\varepsilon_0/2+x/\beta-u)\}^{1/2}. \quad (A.4)$$

Squaring both halves of (A.4) and performing simple calculations we obtain the relation that must be satisfied by the quantities u and v:

$$\frac{\gamma^2}{(2\beta)^2}(v^2-u^2)=u^2v^2-\frac{\gamma}{\beta}\left(\frac{\varepsilon_0}{2}+\frac{x}{\beta}\right)uv. \tag{A.5}$$

Substituting here the expressions for u and v calculated from (A.3), we get an equation for x:

$$\left(\varepsilon_0-\frac{2x}{\beta}\right)^2+\frac{4p}{\beta}-\frac{\gamma^2}{\beta^2}=\left(\varepsilon_0+\frac{2x}{\beta}\right)\left(\varepsilon_0-\frac{2x}{\beta}\right)-\frac{1}{4}\left(\varepsilon_0-\frac{2x}{\beta}\right)^2, \tag{A.6}$$

whose solution is

$$x=1/2(\varepsilon_0\beta+p/\varepsilon_0-\gamma^2/4\varepsilon_0\beta). \tag{A.7}$$

Changing from x to the variable z, we obtain an equation that specifies parametrically the sought contour Γ:

$$z=(\gamma_0^2-\gamma^2)/4\beta^2\varepsilon_0+i\gamma/\beta. \tag{A.8}$$

The parameter in this equation is the quantity γ. The relation $\varkappa_1=\varkappa_2$ is valid at each point of the contour Γ in the parameter range $0<\gamma<\gamma_0$.

APPENDIX B
CALCULATION OF THE INTEGRAL ABSORPTION COEFFICIENT

The refractive indices $\tilde{n}_1$ and $\tilde{n}_2$ and the variables t_1 and t_2 on the contours C_1 and C_2 are connected by the relations

$$\begin{aligned}&\tilde{n}_1^2=\varepsilon_0+t_1, \quad \tilde{n}_2^2=\varepsilon_0+t_2,\\ &t_1t_2=-q^2/4, \quad t_1+t_2=(\gamma_0^2-\gamma^2)/4\varepsilon_0\beta^2+i\gamma/\beta.\end{aligned} \tag{B.1}$$

With the aid of (B.1) we can obtain the following two identities:

$$\tilde{n}_1\tilde{n}_2=\varepsilon_0+i\frac{\gamma}{2\beta}, \quad \tilde{n}_1-\tilde{n}_2=\frac{1}{2\beta}\left(\frac{\gamma_0^2-\gamma^2}{\varepsilon_0}\right)^{1/2}. \tag{B.2}$$

Substituting in (13) the integration limits and changing from t_1 and t_2 to the variables $\tilde{n}_1$ and $\tilde{n}_2$, we obtain

$$\begin{aligned}\mathcal{K}=\frac{\pi p}{2\varepsilon_0^{1/2}}+\beta\,\mathrm{Im}\Big\{&\frac{2}{3}(\tilde{n}_1-\tilde{n}_2)(\tilde{n}_1^2+\tilde{n}_2^2+\tilde{n}_1\tilde{n}_2)\\ &-(\tilde{n}_1-\tilde{n}_2)(\tilde{n}_1\tilde{n}_2+\varepsilon_0)+\frac{q^2}{8\varepsilon_0^{1/2}}\ln\frac{\varepsilon_0-\tilde{n}_1\tilde{n}_2-\varepsilon_0^{1/2}(\tilde{n}_1-\tilde{n}_2)}{\varepsilon_0-\tilde{n}_1\tilde{n}_2+\varepsilon_0^{1/2}(\tilde{n}_1-\tilde{n}_2)}\Big\}.\end{aligned} \tag{B.3}$$

Substituting in (B.3) the formulas (B.2) and separating the imaginary part, we obtain ultimately

$$\mathcal{K}=\frac{p}{\varepsilon_0^{1/2}}\left(\mathrm{arctg}\frac{\gamma}{(\gamma_0^2-\gamma^2)^{1/2}}+\frac{\gamma(\gamma_0^2-\gamma^2)^{1/2}}{\gamma_0^2}\right). \tag{B.4}$$

The expression for the derivative of $\mathcal{K}$ with respect to γ is

$$d\mathcal{K}/d\gamma=(\gamma_0^2-\gamma^2)^{1/2}/2\beta\varepsilon_0^{1/2}. \tag{B.5}$$

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