# Electron-hole liquid in thin semiconductor films

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(Submitted 2 April 1980)
Zh. Eksp. Teor. Fiz. **79**, 1509–1517 (October 1980)

The dependence of the ground-state energy of an electron-hole liquid on the carrier density is calculated using an expression obtained earlier for the Coulomb interaction of charges in a thin high-permittivity semiconductor film surrounded by two insulator layers. The possibility of experimental detection of the investigated effects and the nature of the one-particle spectrum of excitations in the liquid are discussed.

PACS numbers: 71.35. + z

Calculations of the properties and behavior of an electron–hole liquid (EHL) in a two-dimensional system have been reported in several earlier papers.[1,2] These calculations are made on the assumption that electrons and holes bound to form an EHL move along an ideal plane and that the Coulomb interaction between them is governed by just one permittivity $\varepsilon$. This model is best approximated experimentally by thin semiconductor films.

However, the permittivity of semiconductors is fairly high, $\varepsilon \sim 10-100$, whereas the permittivity of the media surrounding such a film (this is usually an insulating substrate in vacuum) is of the order of unity. When the distances between the charges in the film is $\gtrsim d$, where $d$ is the film thickness, the field created in the surrounding medium by the charges begins to play a significant role in the interaction between them. Therefore, in the case of films of small thickness $d$ the interaction between charges in a film becomes considerably stronger than in a homogeneous medium with the same permittivity $\varepsilon$. This alters the dependence of the interaction energy on the distance.[3]

We shall assume that a film occupies the space $-d/2 \leqslant z \leqslant d/2$. The substrate occupying the half-space $z < -d/2$ is a homogeneous medium of permittivity $\varepsilon_1$, whereas the half-space $z > d/2$ is a medium of permittivity $\varepsilon_2$. Let $\rho$ be the distance between point charges $e'$ and $e''$ in the film plane $(x, y)$ and $\delta = (\varepsilon_1 + \varepsilon_2)/2\varepsilon$ be a small parameter

$$\delta \ll 1. \tag{1}$$

Then, as shown in Refs. 3–5, if $\rho \gg d$, the interaction energy $V(\rho)$ is described by

$$V(\rho) = \frac{2e'e''}{\varepsilon d}\left(\ln\frac{d}{\rho\delta} - C\right), \quad d \ll \rho \ll \frac{d}{\delta}, \tag{2a}$$

$$V(\rho) = \frac{2e'e''}{(\varepsilon_1 + \varepsilon_2)\rho}, \quad \frac{d}{\delta} \ll \rho, \tag{2b}$$

where $C = 0.577$ is the Euler constant. In the momentum space $(q^2 = q_x^2 + q_y^2)$ the Fourier transform of Eq. (2) is

$$V(q) = \frac{2\pi e'e''}{\varepsilon q}\left(\delta + \frac{dq}{2}\right)^{-1}. \tag{3}$$

We shall consider the case of sufficiently small thicknesses $d$, namely

$$d \ll a_0, \tag{4}$$

where

$$a_0 = \varepsilon\hbar^2/\mu e^2 \tag{5}$$

is the radius of an exciton in the bulk of the film. Here, $e$ and $\hbar$ are, respectively, the electron charge and the Planck constant, and $\mu$ is the reduced effective mass of an electron and a hole. It follows from Eq. (4) that the separation between the size-quantized energy levels is proportional to $\hbar^2/\mu d^2$, which is considerably greater than the interaction energy (2). Therefore, we shall assume that the motion of carriers in such a film is two-dimensional. The interaction energy (2) is also independent of the position of the charges along the $z$ axis.

Formation of a bound state of an electron and a hole (i.e., the formation of an exciton) in such a film is considered in Refs. 4 and 5 and it is shown that if the inequality (4) and the condition

$$a_0\delta^2 \ll d \tag{6}$$

are satisfied, the radii of the ground and first excited states of an exciton lie in the intermediate range of distances $\rho$, where the approximation (2a) for $V$ is valid. Characteristic scales of the distances and energies in this problem are the quantities

$$a_x = {}^1/_2 (a_0 d)^{1/2}, \tag{7}$$

$$E_x = e^2/\varepsilon d. \tag{8}$$

The binding energy of an exciton is given by the formula

$$E_{exc} = -E_x\left\{\ln\frac{4d}{\delta^2 a_0} - \omega_{exc}\right\}, \quad \omega_{exc} = 2.20. \tag{9}$$

We shall consider the formation of an EHL in a film satisfying the conditions (1), (4), and (6). The Schrödinger equation for the many-body problem is

$$-\frac{\hbar^2}{2m_e}\sum_{i=1}^{N}\nabla_i^2\psi - \frac{\hbar^2}{2m_h}\sum_{j=1}^{N}\nabla_j^2\psi + {}^1/_2\sum_{i_1,i_2=1;\,i_1\neq i_2}^{N} V_{i_1i_2}\psi$$
$$+ {}^1/_2\sum_{j_1,j_2=1;\,j_1\neq j_2}^{N} V_{j_1j_2}\psi + \sum_{i,j=1}^{N} V_{ij}\psi = E_N\psi. \tag{10}$$

We shall adopt a model in which the electron spectrum is static and isotropic. Here, $m_e$ and $m_h$ are the effective masses of electrons and holes assumed (for simplicity) to be equal $m_e = m_h = 2\mu$; $N$ is the number of electron–hole pairs; $i$ and $j$ are the indices labeling electrons and holes, respectively; $\nabla_{i,j}$ is a two-dimensional gradient along the coordinates of the particle under consideration; $\psi$ is the complete wave function of the system which depends on the coordinates of all

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the particles; $V_{e_1e_2}$ is the Coulomb interaction described by Eq. (2), where $\rho$ is the relative distance in the $(x, y)$ plane between the particles $e_1$ and $e_2$. Employing the units $a_x$ and $E_x$ of Eqs. (7) and (8), we obtain the dimensionless Schrödinger equation

$$-\sum_{i=1}^{N}\nabla_{\xi_i}^2\psi-\sum_{j=1}^{N}\nabla_{\xi_j}^2\psi-{}^1/_2\sum_{i_1,i_2=1,i_1\neq i_2}^{N}\ln\xi_{i_1i_2}\psi$$

$$-{}^1/_2\sum_{j_1,j_2=1,j_1=j_2}^{N}\ln\xi_{j_1j_2}\psi+\sum_{i,j=1}^{N}\ln\xi_{ij}\psi=\gamma_N\psi, \tag{11}$$

$$E_N=-E_x\left(N\ln\frac{4d}{\delta^2a_0}-2NC-\gamma_N\right). \tag{12}$$

Here, $\xi_e$ are the dimensionless radius vectors to the particles and $\xi_{e1e2}$ are the dimensionless relative distances between them.

Equation (11) is free of the parameters of the problem, so that the main contribution to the binding energy of an EHL and of any finite many-particle complex is the large, because of Eq. (6), quantity $\ln(4d/\delta^2a_0)$, which also follows for an exciton from Eq. (9). In comparing the binding energies of an exciton and of an EHL we must carry out calculations in the following order in respect of the parameters. This applies only if the average distance between particles in an EHL (or in other many-particle complexes) satisfies the condition

$$d\ll\rho_{av}\ll d/\delta. \tag{13}$$

In calculating the characteristics of an EHL in the isotropic model it is not clear *a priori* whether the spectrum of one-particle excitations is metallic or insulating.

It is known[6-8] that at absolute zero a system of this kind is unstable if a gap forms on the Fermi surface. However, the magnitude of this gap decreases exponentially on increase in the carrier density. We shall assume that at densities

$$n\propto a_x^{-2}, \tag{14}$$

where $n$ is the surface density of carriers in the film, we can ignore the influence of the gap and assume that the initial unperturbed state is a Fermi gas of free electrons and holes. The momentum of the Fermi system is

$$p_F=(2\pi h)^{1/2}. \tag{15}$$

We shall introduce the dimensionless average distance between the particles

$$r_s=(\pi na_x^2)^{-1/2}. \tag{16}$$

The condition (13) can be rewritten as the condition for $r_s$:

$$(d/a_0)^{1/2}\ll r_s\ll(d/a_0\delta^2)^{1/2}, \tag{17}$$

which ensures a fairly wide range of validity of our calculations [this follows from Eqs. (4) and (6)]. The ground-state energy then usually splits into three contributions (for details of the methods of calculation of the EHL energy see Refs. 9 and 10). The kinetic energy is

$$E_{\text{kin}}=2E_x/r_s^2. \tag{18}$$

The exchange energy is

$$E_{\text{exch}}=-2E_x\left(\ln\frac{dp_F}{2\delta}-\frac{1}{2}\right)=-E_x\left(\ln\frac{4d}{\delta^2a_0}-\ln 2r_s^2-1\right). \tag{19}$$

The exchange energy includes the same large logarithm as the binding energy of an exciton given by Eq. (9) and, because of Eq. (12), it represent the main part of the interaction energy. The third term in the total energy is the correlation energy $E_{\text{corr}}$, which is small compared with the exchange energy in the range $r_s\sim1$; since in this case the Coulomb interaction (3) is effectively screened, the main contribution to $E_{\text{corr}}$ corresponds to the transferred momenta $q\sim p_F\sim a_x^{-1}\gg d^{-1}\delta$. Therefore, in calculating $E_{\text{corr}}$ we can replace (3) with the following expressions:

$$V(q)=4\pi e'e''/\varepsilon dq^2, \tag{20}$$

and $|E_{\text{corr}}|\sim E_x\ll|E_{\text{exch}}|$. The binding energy of an EHL, $|E_{\text{min}}|$, represents the minimum of $E=E_{\text{kin}}+E_{\text{exch}}+E_{\text{corr}}$ considered as a function of $n$ or $r_s$ and differs only by a numerical factor (in units of $E_x$) from the binding energy of an exciton given by Eq. (9).

We shall calculate the correlation energy employing the Nozières–Pines method (see Refs. 9 and 10), in which $E_{\text{corr}}$ is represented by an integral of the transferred momentum

$$E_{\text{corr}}=\int_0^\infty dqI(q). \tag{21}$$

The function $I(q)$ is calculated within the limits of large and small $q$ and the results are joined in the range of intermediate values of $q$:

$$I(q)=I_1(q)=-a+bq-cq^2,\quad q\ll1, \tag{22a}$$

$$I(q)=I_2(q)=-d_1/q^5,\quad q\gg1, \tag{22b}$$

$$a=\frac{4}{\pi},\quad b=\frac{\sqrt{2}}{r_s},\quad c=\frac{2}{r_s^2}\left(\frac{1}{2}+\frac{1}{\pi}\right)-\frac{1}{6\pi},\quad d_1=3r_s^2,$$

where $I$ is measured in units of $E_x$ and $q$ is measured in units of $p_F$. The expansion of $I_1$ is obtained in the random phase approximation, whereas $I_2$ is the main contribution to the sum of the direct and exchange vacuum diagrams of the second order. The form of the function appropriate to our problem is shown in Fig. 1. We shall joint the results using two variantes. In the first variant, the function $I(q)$ is split into a sum of two functions:

$$I=I'(q)+I''(q), \tag{23}$$

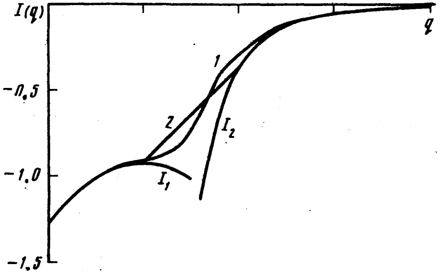

FIG. 1. Expansion of the function $I(q)$ for high and low values of the momenta in the case when $r_s=2$. Curves 1 and 2 correspond to the first and second joining variants. Here, $I(q)$ is in units of $E_x$ and $q$ is in units of $p_F$.

where

$$I'(q)=\begin{cases}-a+bq, & q\leqslant a/b\\ 0, & q>a/b\end{cases};\quad I''(q)=-\frac{cq^2}{1+cq^7/d_1}.$$

In the second variant we join by drawing a straight line from the maximum of the curve $I_1$ to the point $q=2$ on the curve $I_2$. In both cases the integral (21) can be calculated analytically. We thus obtain the ground-state energy as a function of $r_s$. The minimum of this function determines the binding energy and the equilibrium density of an EHL. The functions $I(q)$ obtained for both joining variants are similar and the results depend weakly on the nature of the joining:

$$E_{min}=-E_x\left\{\ln\frac{4d}{\delta^2 a_0}-\omega_{met}\right\}, \tag{24}$$

where in the first variant we have $\omega_{met}=1.68$ and $r_{s\,min}^{(m)}=1.8$, whereas in the second variant we have $\omega_{met}=1.70$ and $r_{s\,min}^{(m)}=2.3$. The corresponding quantity which occurs in the binding energy of an exciton is $\omega_{exc}=2.20$. Consequently, an EHL is preferred for energy reasons to an exciton gas, and the difference between the binding energies is

$$|E_{min}|-E_{exc}\approx 0.5E_x. \tag{25}$$

For example, in the case of a GaAs film of thickness $d=30$ Å this difference is ≈19.5 meV, i.e., $\approx 40\% E_{exc}$. It should be noted that Eqs. (24) and (25) represent only estimates of the binding energy of an EHL, because they are obtained for an isotropic dispersion law of electrons and holes. Allowance for the anisotropy of the dispersion law reduces the kinetic energy and increases the binding energy of an EHL as well as its equilibrium density.[9]

A shortcoming of the above calculations of the ground-state energy of an EHL is the fact that the energy does not reduce to the binding energy of an exciton when the densities of electrons and holes tend to zero. This shows that the electron–hole correlations of the multiple scattering type are not properly allowed for. In the isotropic case and for low densities a system of electrons and holes is clearly better described if the zeroth approximation is a free gas of correlated electron–hole pairs (excitons).[11-13] In this case we can again obtain the dependence of the ground-state energy on the density $E(n)$. The minimum of this energy corresponds to an EHL whose spectrum of one-particle excitations is of the insulating type. We shall assume that the equilibrium density of an insulating EHL is such that the average distance satisfies the conditions (13) and (17), so that we can use the Coulomb potential in the form given by Eq. (2a).

Canonical transformation of the Hamiltonian (10) allowing for coherent pairing of electrons and holes gives expressions differing from the corresponding results in Refs. 11–13 only by the nature of the Coulomb interaction. The ground-state energy considered in the Hartree–Fock approximation is

$$U=2\int\frac{d^2p}{(2\pi)^2}\frac{\hbar^2p^2}{2\mu}\sin^2\varphi_p-2\int\frac{d^2p\,d^2p'}{(2\pi)^4}V(\mathbf{p}-\mathbf{p}')\times\left({}^1/_4\sin 2\varphi_p\sin 2\varphi_{p'}+\sin^2\varphi_p\sin^2\varphi_{p'}\right). \tag{26}$$

The energy $U$ is a functional of $\varphi_p$. The function $\varphi_p$ is found from the conditions of minimum energy and stability of the ground state. This function $\varphi_p$ satisfies the normalization condition

$$n=2\int\frac{d^2p}{(2\pi)^2}\sin^2\varphi_p \tag{27}$$

and in the limit $n\to 0$ it is related to the wave function of the ground state of an exciton $\psi_0(p)$ in the momentum representation:

$$(n/2)^{1/2}\psi_0(p)=\sin\varphi_p, \tag{28}$$

so that in the limit $n\to 0$ the energy $U/n$ reduces to the energy of the ground state of an exciton. In the simplest case if we select $\psi_0(p)$ to be the Fourier transform of the simplest wave function $\psi_0(\rho)=e^{-\alpha\rho}$,

$$\psi_0(p)=(8\pi)^{1/2}a_x/[1+(pa_x)^2]^{3/2}, \tag{29}$$

we can obtain the ground-state energy of an exciton in the form of Eq. (9) but with a somewhat less satisfactory value $\omega=2.31$, and the slope of the curve $E(n)$ considered in this approximation is

$$\left.\frac{d(U/n)}{dn}\right|_{n=0}^{HF,f}=8\pi({}^2/_5\ln 2-{}^{16}/_{525})E_xa_x^2=6.2\,E_xa_x^2. \tag{30}$$

For comparison, we shall give the slope obtained in the same approximation for a purely two-dimensional case,[12] corresponding to the form of the potential (2b)

$$\left.\frac{d(U/n)}{dn}\right|_{h=0}^{HF,2d}=2\pi(1-315\,\pi/2^{12})E^*a^{*2}=4.77\,E^*a^{*2}, \tag{31}$$

and in the case of a three-dimensional system[11,12] with the usual Coulomb potential we find that

$$V(q)=\frac{4\pi e'e''}{\varepsilon q^2},\quad q^2=q_x^2+q_y^2+q_z^2, \tag{32}$$

$$\left.\frac{d(U/n)}{dn}\right|_{n=0}^{HF,3d}=\frac{13\pi}{6}E_0a_0^3=6.8\,E_0a_0^3, \tag{33}$$

where $E_0$, $a_0$ and $E^*$, $a^*$ are the binding energy and radius of three- and two-dimensional excitons, respectively. In all three cases a self-consistent approximation allowing for terms linear in respect of the carrier density gives, for the same values of the density right up to $r_s\sim 2$, much lower energies than $E_{HF}=E_{kin}+E_{exch}$ for an EHL with a metallic spectrum (Fig. 2).

Minimization of the functional $U$ for arbitrary values

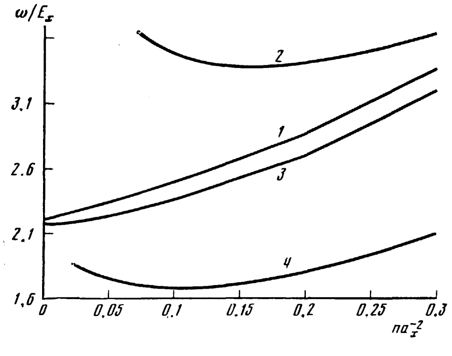

FIG. 2. Dependences of the energy of an EHL on the carrier density. The ordinate gives the energy $\omega$ in units of $E_x$, whereas the abscissa gives the density $n$ in units of $a_x^{-2}$: 1), 2) energy in the Hartree–Fock approximation; 3), 4) total energy; 1), 3) insulating EHL; 2, 4) metallic EHL. The total energy of the insulating EHL (curve 4) is calculated using the parameters of GaAs.

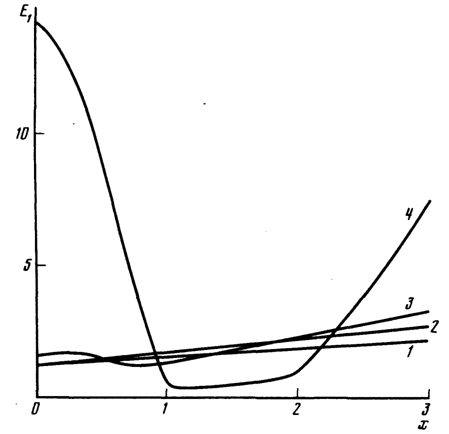

FIG. 3. Spectrum of one-particle excitations in an insulating EHL calculated in the Hartree–Fock approximation for various densities. The abscissa gives the values of $x = kq^2$ proportional to the square of the momentum: 1) $n = 10^{-3} a_x^{-2}$, $k = 4.4$; 2) $n = 7 \times 10^{-3} a_x^{-2}$, $k = 2.75$; 3) $n = 10^{-1} a_x^{-2}$, $k = 1.5$; 4) $n = a_x^{-2}$; $k = 0.22$.

of the density was carried out for the class of functions

$$\sin^2 \varphi_p = \frac{1}{1+\psi_p^2}, \quad \psi_p = a + 2bp^2 + cp^4, \tag{34}$$

where the parameter $c$ is found from the condition (30), whereas the parameters $a$ and $b$ are variational. The resultant dependence $U(n)$ is shown in Fig. 2. For low values of $n$, we have

$$(U/n)_{min} = -E_x \left( \ln \frac{4d}{\delta^2 a_0} - 2.21 - 2.6\, n a_x^2 \right). \tag{35}$$

In the limit $n \to 0$, Eq. (35) gives the ground-state energy of an exciton which corresponds to the value $\omega = 2.21$ in Eq. (9), i.e., our selection of the test function (34) is highly successful.

A calculation of the correlation energy of the system with a new initial state is very difficult since in this case there is no specific class of diagrams.[13] The total energy of an insulating EHL can be estimated using again the representation (21) and calculating the function $I(q)$ by interpolation between the limiting cases $q \ll 1$ and $q \gg 1$. For the largest transferred momenta, $q \gg 1$, the nature of the spectrum has little effect on the energy and the function $I(q)$ is described by Eq. (22b) with the same value of the coefficient $d_1$. In the opposite limiting case, the approximation for $I(q)$ is a sum of the diagrams in the random phase approximation which represents—in the present case—some improvement compared with the second order of perturbation theory.[2] If $q \ll 1$, the expansion of $I(q)$ begins from a term which is linear in respect of $q$. The complete expression for $I(q)$ is obtained by harmonic matching of two limiting cases. The correlation energy found by integration of $I(q)$ is added to the energy deduced in the Hartree–Fock approximation $(U/n)_{\min}$. The total energy of an insulating EHL considered as a function of the carrier density is plotted in Fig. 2. It should be pointed out that the correlation energy of an insulating EHL depends, in the approximation employed, on the parameters of the actual material not only via the quantities $E_x$, $a_x$, and $\ln(4d/\delta^2 a_0)$ in a formula analogous to Eq. (24), but also via the modified energy of one-particle excitations. We calculated the correlation energy of an insulating EHL using the parameters $\varepsilon = 12.35$ and $a_0 = 146$ Å, which represent GaAs, and selecting $d = 30$ Å, $\varepsilon_1 = 1$, and $\varepsilon_2 = 3$. We found that $E_x = 39.6$ meV, $a_x = 33$ Å, $\ln(4d/\delta^2 a_0) = 3.44$, and the binding energy of an exciton $E_{exc} = 49.2$ meV. The total energy has a minimum at $n_{\min}^{(d)} = 0.007 a_x^{-2}$ or at $r_{s\min}^{(d)} = 6.7$. This point corresponds to $\omega_{insul} = 2.17$ [see Eq. (24)]. Figure 3 shows the dependence of the energy of one-particle excitations on the momentum in an insulating EHL (modified dispersion law) calculated for various values of the density. The insulating nature of the dispersion law justifies the name of "insulating EHL."

A comparison of similar calculations of the EHL energy for germanium and silicon with the experimental data[9,10] shows that the results obtained approximate well the dependence of the exact energy of the ground state of the investigated Coulomb system on the carrier density in the range $n a_x^2 \gtrsim 0.1$. We are assuming that the exact dependence should have two minima separated by a considerable density interval, as indicated by our calculations. These minima correspond to the formation of an EHL with insulating or metallic spectra of one-particle excitations. The state of an insulating EHL is clearly metastable.

It should be pointed out that insufficient accuracy of the calculations of metallic and insulating EHL's does not allow us to draw the final conclusions on their relative stability. In all likelihood, more accurate information on the nature of the phase diagram of an EHL and on the metal–insulator transition can be made by calculating the energy gap in the spectrum of one-particle excitations, similar to that carried out in Ref. 14 for a three-dimensional EHL.

The results obtained in the present paper demonstrate that it should be possible to detect an EHL in a thin semiconductor film on a insulating substrate. The binding energy of this EHL is considerably greater than the binding energy of an EHL in a three-dimensional material. The upper limit on the thickness of the film is $d_{\max} = a_0$, whereas the lower limit is $d_{\min} = 6 a_0 \delta^2$. For most substances the value of $d_{\min}$ is less than or of the order of interatomic distances and is unattainable experimentally. In the case of practically all the semiconductors there is a range of film thicknesses in which the results are valid. For most semiconductors the appropriate film thickness is ~10–100 Å (GaAs, Ge), whereas for some semiconductors it is sufficient if the film thickness amounts to a few hundreds of angstroms (PbS, InSb).

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Translated by A. Tybulewicz

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# The sign of the thermoelectric power in white tin

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(Submitted 15 April 1980)
Zh. Eksp. Teor. Fiz. 79, 1518–1526 (October 1980)

The diffusion component of the thermoelectric power is investigated in the region of electron scattering by phonons. A value $(\partial \ln S/\partial \ln \varepsilon)_{\varepsilon=\varepsilon_F} = -1.5$ is obtained. It is shown that this value can be attributed to topological singularities of the Fermi surface of tin.

PACS numbers: 72.15.Jf, 72.10.Di, 71.25.Hc

The diffusion part of the thermoelectric power of a metal is defined by the expressions[1]

$$\alpha_e = \frac{\pi^2 k^2 T}{3e\varepsilon_F}\xi = aT, \tag{1}$$

$$\xi = \left(\frac{\partial \ln \sigma}{\partial \ln \varepsilon}\right)_{\varepsilon=\varepsilon_F} = \left(\frac{\partial \ln \Lambda}{\partial \ln \varepsilon} + \frac{\partial \ln S}{\partial \ln \varepsilon}\right)_{\varepsilon=\varepsilon_F}, \tag{2}$$

where $\sigma$ is the conductivity of the metal, $\Lambda$ is the electron mean free path, and $S$ is the area of the Fermi surface (FS). We have shown in a preceding paper[2] that in the case of electron scattering by impurities the value of $\xi$ is determined mainly by the first term of the right-hand side of (2) and depends on the type of impurity. For a pure metal (when electron-phonon scattering predominates), however, this question has so far remained unanswered. The point is that the experimental $\xi$ for a number of polyvalent metals (gallium,[3] tin[2]) are essentially negative, and this is difficult to reconcile with the prevailing notions. In fact, as shown by Klemens,[4] in the isotropic case we have $\partial \ln \tau/\partial \ln \varepsilon = 0$. The physical reason is that the energy relaxation is much faster than the momentum relaxation. It can therefore be assumed that $\partial \ln\langle \tau \rangle/\partial \ln \varepsilon$ will be small also in a weakly anisotropic case.

On the other hand, the FS of tin, as well as of most polyvalent metals, is in the broadened-band scheme close to a sphere of free electrons, for which $\partial \ln \nu S/\partial \ln \varepsilon = 3/2$. The negative experimental $\xi$ are therefore puzzling [$\xi = -$ (1 to 3) for Sn].

Nielsen and Taylor[5] have shown recently that $\partial \ln \tau/\partial \ln \varepsilon$ can have large negative values on account of scattering processes in which virtual phonons participate. According to Ref. 2, however, the contribution they obtained for the pure metal cannot be significant at low temperatures.

In the present study we have attempted to resolve this contradiction in the case of tin, using the size effect for this purpose. The electron mean free path, which is governed by the finite dimensions of the sample, is independent of energy, and this makes it possible to estimate the contribution of both terms in the right-hand side of (2).

The theory of the size effect for electric conductivity was developed by Dingle.[6] It is known that in this theory a distinction is made between two limiting cases. For cylindrical samples at $d \ll \Lambda_\infty$, where $d$ is the sample diameter and $\Lambda$ is the electron mean free path in the bulk sample,

$$\rho_d/\rho_\infty = \Lambda_\infty/d. \tag{3}$$

Substituting $\rho_d$ from (3) in (2), we readily obtain

$$a_d = \frac{\pi^2 k^2}{3e\varepsilon_F}\frac{\partial \ln S}{\partial \ln \varepsilon} = \mathrm{const}(d). \tag{4}$$

For cylinders with $d \gg \Lambda_\infty$

$$\rho_d/\rho_\infty = 1 + \gamma\Lambda_\infty/d. \tag{5}$$

From (5), (2), and (1) we have

$$a_d\rho_d = -\frac{\pi^2 k^2}{3e\varepsilon_F}\left(\frac{\partial \rho_\infty}{\partial \ln \varepsilon} - \frac{A}{d}\frac{\partial \ln S}{\partial \ln \varepsilon}\right), \tag{6}$$

where

$$A = \gamma\rho_\infty\Lambda_\infty, \quad \frac{\partial \ln S}{\partial \ln \varepsilon} = -\left(\frac{\partial \ln \rho_\infty}{\partial \ln \varepsilon} + \frac{\partial \ln \Lambda_\infty}{\partial \ln \varepsilon}\right).$$

It is seen from the presented relations that by measuring $\rho$ and $a$ as functions of the sample diameters we can determine $\partial \ln \Lambda_\infty/\partial \ln \varepsilon$ and $\partial \ln S/\partial \ln \varepsilon$.

## EXPERIMENT

We investigated single crystals of ultrapure tin with $\rho_{300\,\mathrm{K}}/\rho_0 \geq 6\cdot 10^5$. The sample diameter ranged from 4 to