A feature of the modulation of the space-energy distribution of the particles in this case is that despite the preserved form of the energy spectrum, continuous energy transfer takes place from the moving inhomogeneities to the charged particles. The similarity, characteristic of this case ($\nabla\cdot\mathbf{u}=0$), of the form of the energy spectrum of the particles in space is the consequence of the balance of the particle flux **J** in the space and the "transfer" of the particles over the J_p system.

6. CONCLUSION

Our analysis shows that the energy exchange between charged particles and moving magnetic-field inhomogeneities, at a given law of variation of the velocity of the medium in space, is determined by the concrete form of the particle distribution function. The considered illustrative typical boundary-value problems show that the character of the change of the charged-particle energy spectrum is not a criterion that determines the change of the charged-particle energy in multiple scattering by moving magnetic-field inhomogeneities. In the general case, only an analysis of the system of the moment equations can lead to definite conclusions concerning the energy dissipation in a system consisting of charged particles and magnetic inhomogeneities.

The authors thank I. N. Toptygin for a helpful discussion of the questions considered in the paper and to the referee B. A. Tverskoĭ for a number of valuable remarks that have stimulated further work on this problem.

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Translated by J. G. Adashko

Plasma hydrodynamics in a high-current channel

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(Submitted 20 March 1980)
Zh. Eksp. Teor. Fiz. 79, 1282–1292 (October 1980)

The equilibrium configurations and low-frequency radial oscillations (ion sound) of intense charged-particle beams are considered in the approximation of the hydrodynamics of perfect electron and ion liquids interacting with each other via the electromagnetic field produced by the charges. The employed macroscopic approach is valid without any assumptions concerning the equations of state. The oscillation singularities due to the specifics of the equation of state of the medium are expressed in terms one macroscopic parameter, the speed of sound. In the intermediate low-current region, the ion-sound oscillations with wavelength exceeding a certain critical value increase exponentially. This instability is due to the tendency of the beam to split up into individual jets if the magnetic field of the current flowing through an individual channel is capable of preventing the charges from spreading radially. In a high-current beam, the instability of buildup of radial ion-sound oscillations is suppressed by the magnetic field of the current.

PACS numbers: 52.30. + r, 41.80.Gg, 52.35.Dm

1. INTRODUCTION

Research into self-compressing streams of charged particles, initiated by Bennet[1] and revived by Budker,[2] has become recently a separate branch of the physics of non-neutral plasma[3] and of strong electron-ion beams.[4] A far from complete list of the projects in which high-current devices are used—from suggestions aimed at solving the problem of controlled thermonuclear fusion[5] to the development of x-ray and gamma-ray lasers[6] and of collective accelerators for charged particles[7]—gives an idea of the degree of influence of this trend on the development of modern physics.

The study of pinch systems (see, e.g., Refs. 8–19) has shown that the most interesting experimental phenomena occur during the strong compression stage (x-ray flash, high temperature, density and multiplicity of atom ionization, acceleration of electrons and ions, the appearance of neutrons, the explosive character of the electron emission). A non-traditional approach to the pinching phenomenon,[20] based on an analysis of the equilibrium[21-24] and radiation[25,26] of a plasma in the magnetic field of the current itself, makes it possible

 0038-5646/80/100647-07$02.40

to describe from a unified point of view the phenomena that accompany the pinch effect, and fill to some degree the gaps that appear when attempts are made to explain separately individual properties of the pinch.

The analysis of the electromagnetic field produced by the charges[21-24] has shown that in addition to the well known equilibrium configuration of a plasma regarded as a classical ideal gas,[1,27-30] a much larger class is made up of structures that are compressed by collective-interaction forces to a degree that causes electron degeneracy.[21,22,24,26] The actual realization of any particular equilibrium state depends to a considerable degree on the stability of the state. Thus, once the possible equilibrium structures of the beams are analyzed, the problem of their stability comes to the forefront.

A kinetic approach to the problem of stability of pinch systems is indicated by Buneman.[28] In general form, however, this approach is not realizable in principle, both because of mathematical difficulties and because it is impossible to take into account simultaneously the entire aggregate of the phenomena that occur in the plasma in a strong-current channel.

Usually the oscillations and the stability of the beams are investigated in the collisionless-plasma limit.[3,31,32] The plasma of a maximally compressed pinch, however, cannot be regarded as collisonless. On the contrary, according to the experimental data[8,9,13,33-35] the time between the collisions of the charges during the strong-compression stage is much shorter than the characteristic lifetime of the densest possible state of the plasma, as determined from the duration of the x-ray flash. This means that with increasing compression of the current channel the plasma can rapidly reach equilibrium, and its further evolution is connected with the slow change of the equilibrium state. This makes it possible to change over to the macroscopic equations of motion in the description of slow processes in the strongly compressed beam-channel plasma. The thermodynamics approximation was in fact used in the analysis of equilibrium structures.[21-24]

The macroscopic-hydrodynamics equations applicable to the description of slow motions of charged ideal electron and ion liquids are given in the next section. The stationary solutions of these equations yield hydrostatic-equilibrium configurations whose properties were explained in Refs. 21–24 on the basis of a kinetic approach. In Sec. 3, the macroscopic hydrodynamics equations of ideal charged liquids are used to analyze the radial short-wave oscillations of a plasma at equilibrium in a field of collective-interaction forces. These oscillations are ion-sound waves. It is shown in Sec. 4 that there exists an intermediate region of beam-current values in which the plasma becomes unstable to radial oscillations. This is due to the possibility of breakup of the current channel into individual jets in such a way that the energy of the magnetic field of the current flowing through an individual channel is sufficient to keep the charges from spreading radially. In the high-current region, when the Larmor radius of the electrons in the magnetic field of the current becomes less than the radius of an individual channel, the instability of the radial oscillations is suppressed by the growth of the magnetization of the electrons.

2. EQUATIONS OF MACROSCOPIC HYDROELECTRODYNAMICS

We assume that the drift velocity v_0 of the electrons relative to the ions is large compared with the velocities $v_{T\alpha}$ of the thermal spread of the charges (of species α) but is small compared with the speed of light c:

$$v_0 \gg v_{T\alpha}, \tag{2.1}$$

$$\beta = v_0/c \ll 1 \tag{2.2}$$

(the index $\alpha = i, e$ labels throughout the species of the particle).

Owing to the rapid decrease of the Coulomb cross sections with increasing relative velocity of the colliding particles, the condition (2.1) ensures faster separate relaxations of the electron and ion subsystems than the relaxation of the plasma as a whole. Of real interest are the electron-ion beams after times that are long compared with the relaxation times $1/\nu_\alpha$ of the individual subsystems, but short compared with the time $1/\nu_{ei}$ of the relaxation of the electrons with the ions. We consider beams whose length L satisfies the conditions

$$v_0/\nu_\alpha \ll L \ll v_0/\nu_{ei} \tag{2.3}$$

at frequencies ω such that

$$\nu_\alpha \gg \omega \gg \nu_{ei}. \tag{2.4}$$

The conditions (2.4) mean that within the short times $1/\nu_\alpha \ll L/v_0$ equilibrium is established in the electron and ion subsystems. The state of the subsystems then varies slowly (with frequency ω) and is locally in equilibrium at each point of space at any instant of time. If the characteristic radius a of the plasma is large compared with the mean free path l_α of the charges

$$a \gg l_\alpha, \tag{2.5}$$

then the hydrodynamics equations are applicable in the wavelength range $\lambda \gg l_\alpha$ (the relation between a and λ can be arbitrary). The collective-interaction electromagnetic field produced by the charges enters in the equations of motion as the external field.

Neglect, by virtue of conditions (2.3) and (2.4), of the collisions of the electrons with the ions excludes from consideration the electric conductivity as a source of dissipation. Disregarding viscosity and thermal conductivity, as phenomena that arise in a higher-order approximation with respect to the small parameter $l/a \ll 1$ (2.5), we arrive at the macroscopic equations of the hydrodynamics of ideal liquids of electrons and ions. The electromagnetic field of the collective interaction of the charges, expressed in terms of the potentials φ and **A**, satisfies the wave equations.

For the considered nonrelativistic motions (2.2) we can neglect the retardation in the field equations and assume that the field follows instantaneously to the slow variations of the distributions of the charges and currents. The derivatives of the field potentials with respect to time can then be omitted. This approximation means at the same time neglect of the radiation. In the absence of dissipative mechanisms, the heat-transport equations reduce, for both the electron and ion subsystems, to the continuity equations for the entropy S_α per unit volume. Thus, the equations that describe the slow changes of the state of the current channel or of the beam are given by

$$\frac{\partial n_\alpha}{\partial t}+\frac{\partial}{\partial x_k}n_\alpha v_{\alpha k}=0, \quad (2.6)$$

$$m_\alpha\left(\frac{\partial v_{\alpha i}}{\partial t}+v_{\alpha k}\frac{\partial v_{\alpha i}}{\partial x_k}\right)=-\frac{1}{n_\alpha}\frac{\partial p_\alpha}{\partial x_i}-e_\alpha\left[\frac{\partial \varphi}{\partial x_i}+\frac{v_{\alpha k}}{c}\left(\frac{\partial A_i}{\partial x_k}-\frac{\partial A_k}{\partial x_i}\right)\right], \quad (2.7)$$

$$\frac{\partial^2 A_i}{\partial x_k^2}=-\frac{4\pi}{c}\sum_\alpha e_\alpha n_\alpha v_{\alpha i}, \quad (2.8)$$

$$\frac{\partial^2 \varphi}{\partial x_k^2}=-4\pi\sum_\alpha e_\alpha n_\alpha, \quad (2.9)$$

$$\frac{\partial S_\alpha}{\partial t}+v_{\alpha k}\frac{\partial S_\alpha}{\partial x_k}=0. \quad (2.10)$$

In Euler's equations (2.7), the pressures p_α should be expressed by the equations of state in terms of the temperatures T_α and the charge densities n_α.

We note that the inequalities (2.3) are extremely important for the treatment of the electrons and ions as isolated equilibrium subsystems that interact with each other only via the collective-interaction electromagnetic field. In the opposite case of a long current channel $L \gg v_0/\nu_{ei}$, the collisions of the electrons with the ions would lead to deceleration of the electrons over a length on the order of v_0/ν_{ei}. In the absence of an external field along the beam, the drift velocity v_0 would vanish over this length, and the energy would go into heat. If an external electric field is applied along the beam, the drift velocity v_0 assumes a value such that the force applied by the external field and accelerating the electrons is offset by the force of the friction of the electrons against the ions. On the other hand if the current channel is so long that $L \gg v_0/\nu_{ei}\delta$ (δ is the energy fraction transferred in the collision between the electron and the ion), then the temperatures of the electrons and ions become equalized over a length $v_0/\nu_{ei}\delta$ after the stationary drift is established. A Druyvestein–Davydov distribution[36-40] is then established in the current channel at a length $L \gg v_0/\nu_{ei}\delta$. The plasma is so heated that the average thermal velocity becomes large compared with the drift velocity. If at the same time the charge mean free path remains the smallest parameter with the dimension of length, then the plasma flow is described by the equations of magnetohydrodynamics,[41] in which the essential role is played by the electric conductivity. In the opposite limiting case ($l \gtrsim a,\lambda$), and also for short beams ($L \ll v_0/\nu_\alpha$), the kinetic approach must be used.

Equations (2.7)–(2.9) contain the potentials of the average field produced by the plasma charges. The difference between the average and true microscopic value of the field is due to fluctuations. This difference is particularly substantial in a weakly inhomogeneous plasma, inasmuch as in a homogeneous equilibrium plasma the average value of the electric field is zero. In the opposite limiting case considered here, that of an essentially inhomogeneous current-channel plasma, the fluctuation fields become weaker than the average fields the stronger the current and the smaller the radius. Noting that the average field is of the order of $e^2N\beta^2/a$, and the fluctuation field is of the order of the Coulomb field at an average distance $e^2n^{2/3}$ between charges, the condition under which the field fluctuations can be neglected takes the form

$$\frac{I}{I_A} \gg \frac{\alpha}{\beta^3}\left(\frac{T_e}{m_e c^2}\right)^{1/2}.$$

Here $I_A = mc^3/e = 17$ kA is the Alfven current and $\alpha = e^2/\hbar c = 1/137$ is the fine-structure constant.

We emphasize that Eqs. (2.6), (2.7), and (2.10) are the equations of the hydrodynamics of ideal liquids and not of gases. They are valid at arbitrary ratio of the temperature T_α to the average potential energy of the paired Coulomb interaction of the charges $e_\alpha^2 n^{1/3}$: $T_\alpha/e_\alpha^2 n_\alpha^{1/3} \sim 1$. Their applicability is not restricted to the ideal-gas approximation.

The stationary solutions of Eqs. (2.6)–(2.10) describe the equilibrium configurations of beams that are homogeneous along the current and in azimuth. By virtue of conditions (2.3), each of the subsystems is at thermal equilibrium in the static field of the collective-interaction forces. Thermal equilibrium means that the charge temperatures T_α and the electron drift velocity v_0 do not depend on the coordinates. On the other hand, the dependences of the densities, pressures, and field potentials on the radius are determined by the mechanical-equilibrium conditions. The continuity equations for the entropy (2.10) at equilibrium are satisfied identically, since $\mathbf{v}_0$ is directed along the cyclic coordinate z. The Euler equation (2.7) reduces at equilibrium to equality of the sum of the forces to zero—to the macroscopic equations of hydrostatics (Ref. 42, Sec. 3):

$$\frac{1}{n_{0\alpha}}\frac{dp_{0\alpha}}{dr}+\frac{dU_{0\alpha}}{dr}=0. \quad (2.11)$$

Here $U_{0\alpha}=e_\alpha(\varphi^0-\beta_\alpha A_z^0)$ are the "potentials" of the forces applied to the charges of species α by the collective-interaction field. The subscript "zero" marks the equilibrium values of the quantities. From the field equations (2.8) and (2.9) we obtain equations for $U_{0\alpha}$:

$$\frac{1}{r}\frac{d}{dr}r\frac{dU_{0\alpha}}{dr}=-4\pi e_\alpha\sum_\beta e_\beta n_{0\beta}(1-\beta_\alpha\beta_\beta). \quad (2.12)$$

Here $\beta_\alpha = v_{0\alpha}/c$.

In Eqs. (2.11) the pressures $p_{0\alpha}$ should be expressed in terms of the temperatures and the charge densities by the equations of state, which contain the entire information on the internal properties of the medium. Taken together with the equations of state, Eqs. (2.11) and (2.12) comprise a closed system of equations that determine the equilibrium structures of the beams. Using the equations of state of ideal gases $p_{0\alpha}=n_{0\alpha}T_{\alpha}$, or of Fermi gases (Ref. 43, Sec. 56), as well as of a strongly degenerate Fermi gas (Ref. 43, Sec. 57), we can obtain with the aid of (2.11) and (2.12) all the previously considered[21-24] equilibrium configurations of high-current beams.

The macroscopic hydrostatic equations (2.11) make it also possible to investigate the equilibrium of the beams in cases when the electrons are degenerate and the ions have a Boltzmann distribution, when the electrons are hot and the ions cold, and for all other combinations, if the equations of state are known. Since the states of the charges of each species are not simply stationary but also in thermodynamic equilibrium, it is understandable that the hydrodynamic, the kinetic[21,22] approaches both lead to same beam structure.

3. SHORT-WAVE RADIAL OSCILLATIONS

We use now Eqs. (2.6)–(2.10) to investigate the short-wave low-frequency radial oscillations of the plasma in a current channel. For simplicity we assume that the small perturbations, as well as unperturbed equilibrium quantities, do not depend on z or φ. The linearized equations are

$$\frac{\partial n_\alpha^{(1)}}{\partial t}+n_{0\alpha}\frac{1}{r}\frac{\partial}{\partial r}rv_{\alpha r}^{(1)}=0,$$

$$m_\alpha\frac{\partial v_{\alpha r}^{(1)}}{\partial t}=-\frac{1}{n_{0\alpha}}\frac{\partial p_\alpha^{(1)}}{\partial r}+\frac{n_\alpha^{(1)}}{n_{0\alpha}^2}\frac{\partial p_{0\alpha}}{\partial r} \quad (3.1)$$

$$-e_\alpha\left(\frac{\partial\varphi^{(1)}}{\partial r}-\beta_\alpha\frac{\partial A_z^{(1)}}{\partial r}-\frac{v_{\alpha z}^{(1)}}{c}\frac{\partial A_z^{(0)}}{\partial r}\right), \quad (3.2)$$

$$m_\alpha\frac{\partial v_{\alpha z}^{(1)}}{\partial t}=-e_\alpha\frac{v_{\alpha r}^{(1)}}{c}\frac{\partial A_z^{(0)}}{\partial r}, \quad (3.3)$$

$$\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial A_z^{(1)}}{\partial r}=-\frac{4\pi}{c}\sum_\alpha e_\alpha(n_\alpha^{(1)}v_{0\alpha}+n_{0\alpha}v_{\alpha z}^{(1)}), \quad (3.4)$$

$$\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial\varphi^{(1)}}{\partial r}=-4\pi\sum_\alpha e_\alpha n_\alpha^{(1)}. \quad (3.5)$$

The continuity equation for the entropy reduces to $\partial S_\alpha^1/\partial t=0$, from which it follows that the entropy is constant. The constancy of the entropy allows us to write $p_\alpha^1=(\partial p_\alpha/\partial n_\alpha)_S n_\alpha^1=m_\alpha s_\alpha^2 n_\alpha^1$ and, as is done in the investigation of sound (Ref. 42, Sec. 63), to describe those internal properties of the medium which are connected with the specifics of the state equation in terms of a single macroscopic parameter—the speed of sound:

$$s_\alpha^2=\frac{1}{m_\alpha}\left(\frac{\partial p_\alpha}{\partial n_\alpha}\right)_s.$$

To continue the analysis, we make two simplifying assumptions

$$kv_0/\Omega_e\gg1, \quad (3.6)$$

$$ka\gg1. \quad (3.7)$$

We denote by $\Omega_\alpha=(e_\alpha/m_\alpha c)dA_z^0/dr$ the Larmor frequency of the charges of species α in the magnetic field of the current. The condition (3.6) allows us to neglect the term $n_{0\alpha}v_{\alpha z}^1$ in (3.4) compared with $n_\alpha^1 v_{0\alpha}$. The inequality (7) means that the derivatives of the unperturbed quantities with respect to the radius are much smaller than the derivatives of the perturbations. Thus, for example in (3.2) we have

$$\frac{n_\alpha^{(1)}}{n_{0\alpha}^2}\frac{dp_{0\alpha}}{dr}\sim\frac{1}{ka}\frac{1}{n_{0\alpha}}\frac{\partial p_\alpha^{(1)}}{\partial r}\ll\frac{1}{n_{0\alpha}}\frac{\partial p_\alpha^{(1)}}{\partial r},$$

and consequently the term $(n_\alpha^1/n_{\alpha0}^2)dp_{0\alpha}/dr$ can be omitted. In addition, the condition (3.7) allows us to seek the dependence of the perturbations on t and on r in the form

$$f^{(1)}(t,r)=f\exp\left\{i\omega t-i\int^r k(r)\,dr\right\}. \quad (3.8)$$

Equations (3.1)–(3.5) constitute a homogeneous linear system. At a given frequency ω this is an eigenvalue and eigenfunction problem. Under the condition (3.7) we should therefore obtain with the aid of the substitution (3.8) a system of equations for the spectrum of the possible frequencies ω_n of the oscillations and the eigenfunctions that constitute the wave numbers $k(\omega_n, r)$. The condition (3.7) is in essence the condition for the quasiclassical treatment. It follows from it that the spectrum of the natural frequencies is determined from self-consistency equations of the type[44]

$$\oint k(\omega,r)\,dr=2\pi(n+\lambda),\quad n=1,2,\ldots,$$

corresponds to the large $n\gg1$ and therefore contains so large a number of closely located eigenvalues that it can be regarded as continuous. At a given field frequency ω, the local values of the wave number $k(\omega, r)$ are determined from the equations

$$\omega n_\alpha^{(1)}-kn_{0\alpha}v_{\alpha r}^{(1)}=0,$$

$$i\omega v_{\alpha r}^{(1)}=iks_\alpha^2 n_\alpha^{(1)}/n_{0\alpha}+ik(e_\alpha/m_\alpha)U_\alpha^{(1)}+\Omega_\alpha v_{\alpha z}^{(1)},$$

$$i\omega v_{\alpha z}^{(1)}=-\Omega_\alpha v_{\alpha r}^{(1)},$$

$$k^2U_\alpha^{(1)}=4\pi e_\alpha\sum_\beta e_\beta(1-\beta_\alpha\beta_\beta)n_\beta^{(1)}. \quad (3.9)$$

Here $U_\alpha^1=e_\alpha(\varphi^1-\beta_\alpha A_z^1)$. Eliminating $v_{\alpha r}^1$, v_α^1, and U_α^1 from (3.9) we arrive at a system of equations that determine the oscillations of the relative densities $\delta n_\alpha=n_\alpha^1/n_{0\alpha}$:

$$\sum_\beta\left\{(k^2s_\alpha^2+\Omega_\alpha^2-\omega^2)\delta_{\alpha\beta}+\frac{4\pi e_\alpha e_\beta n_{0\beta}}{m_\alpha}(1-\beta_\alpha\beta_\beta)\right\}\delta n_\beta=0. \quad (3.10)$$

The dispersion equation that follows from (3.10) and determines the function $k(\omega, r)$ is

$$\det\left\{(k^2s_\alpha^2+\Omega_\alpha^2-\omega^2)\delta_{\alpha\beta}+\frac{e_\alpha}{e_\beta}\frac{m_\beta}{m_\alpha}\omega_\beta^2(1-\beta_\alpha\beta_\beta)\right\}=0,$$

where $\omega_\beta^2=4\pi e_\beta^2 n_{0\beta}$ and ω_β is the frequency of the Larmor oscillations of the particles of species β. In the reference frame in which the ions are as a whole immobile ($\beta_i=0$, $\beta_e=\beta$), we have

$$\left[\left(\frac{ks_e}{\omega_e}\right)^2+\frac{\Omega_e^2-\omega^2}{\omega_e^2}+1-\beta^2\right]\left[\left(\frac{ks_i}{\omega_i}\right)^2+\frac{\Omega_i^2-\omega^2}{\omega_i^2}+1\right]-1=0. \quad (3.11)$$

Given the values of the radius r and of the wave number k, the dispersion equation determines the frequency ω of the local oscillations.

In the low frequency region $\omega \lesssim \omega_i \ll \omega_e$ at nonrelativistic drift velocity, the dispersion equation (3.11) is simpler and yields

$$\omega^2=\omega_i^2\left\{k^2r_d^2+\frac{\Omega_e^2}{\omega_e^2}-\beta^2\right\}. \tag{3.12}$$

Here $r_d^2=\sum_\alpha(s_\alpha/\omega_\alpha)^2$, and r_d is of the order of the Debye radius, inasmuch as the speed of sound s_α is of the order of the thermal velocity $v_{\alpha T}$ of the charges. The ratio of the Larmor frequency of the electrons to the frequency of their Langmuir oscillations is of the order of

$$(\Omega_e/\omega_e)^2\sim\beta(I/I_A).$$

In a weak current, when $(\Omega_e/\omega_e)^2\sim\beta^2\ll(r_d/a)^2$, we obtain from (3.12) theusual ion-sound spectrum:

$$\omega=ks,\quad s^2=\omega_i^2\sum_\alpha(s_\alpha/\omega_\alpha)^2,\quad \beta,\ \Omega_e/\omega_e\ll r_d/a.$$

At $T_i\gg T_e$ the speed of sound s is of the order of the ion thermal velocity, and at $T_e\gg T_i$ it increases by a factor $(T_e/T_i)^{1/2}$.

4. INSTABILITY OF RADIAL OSCILLATIONS

If the current in the beam is so weak that

$$I\ll\beta I_A,$$

the term $(\Omega_e/\omega_e)^2$ in (3.12) can be neglected, and we obtain

$$\omega=\omega_i(k^2r_d^2-\beta^2)^{1/2}. \tag{4.1}$$

It follows from (4.1) that for waves longer than the critical wavelength

$$\lambda>\lambda_{cr}=r_d/\beta, \tag{4.2}$$

the frequencies become imaginary:

$$\omega=\pm i\gamma,\qquad \gamma=\omega_i(\beta^2-k^2r_d^2)^{1/2}. \tag{4.3}$$

This means that oscillations with $\lambda>\lambda_{cr}$ (4.2) increase at a rate (4.3), i.e., the beam is unstable to radial ion-sound oscillations.

The physical meaning of the ensuing instability can be easily understood by noting that the number $\mathcal{N}_e$ of the electrons per unit length of the beam, passing through a cross section $\gtrsim\lambda_{cr}^2$, is of the order of

$$\mathcal{N}_e\gtrsim n_e\lambda_{cr}^2\sim T_e/e^2\beta^2. \tag{4.4}$$

It follows from (4.4) that the magnetic-compression energy $e^2\mathcal{N}_e\beta^2$ of these $\mathcal{N}_e$ particles, becomes larger than the energy $e^2\mathcal{N}_e\beta^2\gtrsim T_e$ of the thermal spread in the radial direction. This means that the energies of the magnetic compression of electrons passing through a cross section of the order of λ_{cr}^2 become sufficient for radial self-containment. Under these conditions the total electron stream has a tendency to split up into individual channels, and it is this which causes, in the considered geometry, the buildup (during the linear stage of the instability) of the oscillations with wavelengths $\lambda>\lambda_{cr}$. Radial oscillations of the considered symmetry (homogeneous in z and φ) would lead to a subdivision into tubes, which in turn would be unstable to oscillations that are inhomogeneous in φ and would breakup into separate jets.

With increasing current, the critical wavelength increases with increasing term Ω_e^2/ω_e^2 in (3.12):

$$\lambda_{cr}=r_d(\beta^2-\Omega_e^2/\omega_e^2)^{-1/2},\qquad \Omega_e/\omega_e<\beta,$$

and at $\Omega_e/\omega_e>\beta$ the instability is completely suppressed by the self-magnetic-field of the current. Thus, at a current $I\gtrsim\beta I_A$ the radial oscillations of the ion sound are stable, and a gap appears in their spectrum:

$$\omega=(\omega_0^2+k^2s^2)^{1/2},\quad \omega_0^2=\omega_i^2\left(\frac{\Omega_e^2}{\omega_e^2}-\beta^2\right).$$

At $\Omega_e/\omega_e>\beta$ the Larmor radius of the electrons become smaller than the critical wavelength. Under these conditions the suppression of the instability with increasing current can be attributed to the increased magnetization of the electrons. An analysis of the electron trajectories in pinches as functions of the current is carried out in Refs. 20, 45, and 46. We emphasize that neither the equilibrium conditions nor the magnetization of the electrons lead to direct limitations on the total beam current. The total current is equal to $I=eN_ev_0$ and, depending on the values of N_e and v_0, can be arbitrarily large. In the limiting case of strong magnetization, the electron motion of the electrons reduces to uniform rotation in azimuth (along the magnetic field of the current), small oscillations in the rz plane (perpendicular to the magnetic field), and to drift along the beam under the influence of the mutually perpendicular collective-interaction electric and magnetic fields. The summary drift of all the electrons gives the total current of the beam.

In the approximation of macroscopic hydrodynamics of of ideal liquids of electrons and ions, the oscillations of the ion sound turned out, naturally, to be undamped. Their real damping is due to failure to take into account, in this approach, the deviations of the liquids from ideal (viscosity, the presence of finite conductivity due to the friction of the electrons against the ions, and the conversion of energy into radiation). However, over a time $\Delta t\ll\nu_{ei}^{-1}$ or over a length $L\ll v_0/\nu_{ei}$ the damping of the oscillations is small and can be neglected in first-order approximation.

In the most interesting frequency region $\omega\sim\omega_i\beta$ the condition $\omega\ll\nu_i$ takes the form

$$\frac{\omega}{\nu_i}\sim\frac{\beta}{\Lambda}\left(\frac{T_i}{e_i^2n_i^{1/3}}\right)^{3/2}\ll1, \tag{4.5}$$

where Λ is the Coulomb logarithm. The hydrodynamics equations are valid at an arbitrary ratio $T_i/e^2n_i^{1/3}\sim1$. Therefore the condition $\omega\ll\nu_i$ is satisfied at any rate on account of (2.2). The condition $kl_\alpha \ll 1$ coincides with (4.5), inasmuch as $kl_i \sim \omega/v_i$ in the region $\omega \sim kv_i$ of interest to us.

The condition (3.6) used in the derivation of the dispersion equation can be represented in the form

$$\frac{\Omega_e}{kv_0} \sim \frac{v_T}{v_0}\left(\frac{\Omega_e}{\beta\omega_e}\right) \ll 1. \tag{4.6}$$

In the region $\Omega_e/\omega_e \sim \beta$ which is of greatest interest from the point of view of stability analysis, the condition (4.6) is satisfied because the thermal velocity is small compared with the drift velocity (2.1).

Finally, the condition (3.7) for oscillations with $k \sim \beta/r_d$ is equivalent to the inequality

$$e^2N_e\beta^2 \gg T_e. \tag{4.7}$$

This means that the energy of the magnetic attraction of the electron is large compared with the temperature. With respect to equilibrium, the condition (4.7) is satisfied in the approximation of classical statistics of ideal gases if the magnetic-attraction energy is offset mainly by the space-charge electrostatic repulsion energy, compared with which the temperature terms are small. It is precisely to beams that are at equilibrium and are "cold" in this sense

$$T/e^2N \ll \beta^2 \ll 1$$

that the foregoing analysis of the stability to short-wave radio oscillations applies.

In another formulation, as applied to the passage of a beam through a dense plasma, the question of the instabilities that lead to the breakup of the current into individual filaments was considered in Refs. 47 and 48. It was shown that the beam is unstable in a tenuous plasma and stable in a dense one. It should be noted that while the instability investigated in Ref. 48 and the instability of the ion sound described above lead to the same result, breakup of the current into individual strands, they differ nevertheless in their physical nature.

The breakup of the current channel into individual jets was observed in experiment.[49] However, it is difficult at present to carry out a complete quantitative comparison of the theory with experiment, owing to the limits imposed by the spatial and temporal resolution. In particular, it is not clear to what extent the observed subdivision into jets is connected with the jet-like escape of the electrons from the cathode during the course of development of explosive emission,[50,51] and to what extent it is connected with the instabilities that break up the beam as it travels. It appears that further progress in the understanding of the physics of high-power electron-ion beam calls for both an improvement of the old diagnostic methods and development of fundamentally new ones (see, e.g.,[52]) for dense nonstationary plasma formations.

The author thanks A. F. Andreev, S. V. Iordanskii, I. M. Liftshitz, and L. P. Pitaevskii for a helpful discussion.

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Translated by J. G. Adashko

Self-limiting of a wave field following supersonic dispersal of a plasma

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(Submitted 16 April 1980)
Zh. Eksp. Teor. Fiz. 79, 1293–1302 (October 1980)

It is shown that in a collisionless dispersing plasma moving at supersonic speed, striction monlinearity imposes an upper limit on the field intensity of the electromagnetic wave propagating in the plasma. This limit is due to the fact that in the supersonic stream the plasma is swept into the region of the strong electric field, and this decreases the refractive index and hinders the wave propagation. The field of s- and p-polarized radiation in a plasma with cubic nonlinearity is considered, where the plasma inhomogeneity is determined entirely by the influence of the ponderomotive forces. Self-confinement of the field is demonstrated in the case of s-polarization also for a plane-inhomogeneous linear plasma layer.

PACS numbers: 52.35.Hr, 52.35.Mw, 52.40.Db

One of the pressing problems of nonlinear theory of plasma is that of the behavior of a strong electromagnetic field in a moving dispersing plasma. A certain group of questions raised in this problem can be explained by numerically solving the equations of nonlinear electrodynamic and hydrodynamics (see, e.g., Ref. 1). At the same time, the numerous restrictions on the use of numerical methods leave many aspects of this problem uninvestigated. The present communication reports an attempt to investigate the electromagnetic field in a stationary inhomogeneous plasma stream. Recognizing that the case of subsonic flow is similar in many respects to the situation considered in a paper by one of us,[2] and striving to obtain qualitative results of interest for modern experiments, we focus our attention in this article on the case of supersonic plasma flow. With an aim also to make the results as conclusive as possible, we consider in detail the picture corresponding to a maximum deviation of the plasma density from the critical value, when the "striction" increment to the dielectric constant of the plasma can be accounted for by a cubic nonlinearity only.

Both a qualitative investigation of the equations of the nonlinear electrodynamics, and the analytic solutions obtained by us for the field, show that the supersonic flow in a classically transparent (in accord with the linear theory) plasma the striction nonlinearity imposes an upper limit on the electric field intensity. The reason is that in supersonic flow, in contrast to subsonic flow, the plasma is not expelled from the strong-field region but, on the contrary, becomes denser there. In the relatively weak field region the plasma becomes more tenuous. The increased density of the plasma in the strong-field region hinders the wave propagation. This effect takes place for both standing and for traveling waves, and for both s- and p-polarized radiation. What changes qualitatively in the supersonic flow is the structure of the electromagnetic soliton, in which the decrease of the plasma density is accompanied by a weakening of the electric field.

 0038-5646/80/100653-06$02.40