### Lasing dynamics of Raman fiber laser

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Results are presented of an investigation of the lasing dynamics of a Raman fiber laser (RFL). It is shown that the onset of lasing in RFL is essentially statistically governed. Statistical simulation is used to calculate the principal RFL lasing regimes. The laser-system parameter range at which the RFL generates a train of ultrashort pulses with a high radiation contrast and with duration much shorter than that of the fluctuation bursts of the pump radiation is determined. The pump structure of the ultrashort pulses under conditions of Raman gain saturation is calculated. It is shown that under these conditions the pump radiation is efficiently transformed into Raman radiation. The dependence of the intensity and of the contrast of the Stokes pulses on the laser parameters is investigated. The principal means of regulating the intensity and contrast of the RFL emission are determined. An anomalously weak energy exchange between the waves of the first and second Stokes components, which leads to suppression of cascade generation of the Stokes component in such lasers, is predicted for an RFL setup with a common resonator. The effect of the dispersion of the optical fiber on the dynamics of the RFL lasing is investigated and it is shown that RFL can be used to generate trains of powerful ultrashort pulses.

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#### INTRODUCTION

The recent progress in the technology of the manufacture of low-loss optical glass fibers<sup>1</sup> has made possible the use of these fibers for the development of a new type of laser-the Raman fiber laser (RFL).<sup>2-8</sup> The optical fiber ensures effective conversion of the laser radiation (pump) into Raman radiation, since fiber optics combine a high power density with a large Ramaninteraction length of the waves. The broad lines of spontaneous Raman scattering in glasses<sup>9</sup> make is possible to tune the RFL emission frequency over a range that reaches several hundred reciprocal centimeters,<sup>7,8</sup> and can produce in principle subpicosecond light pulses. The phase locking of the radiation in fiber optics<sup>10,11</sup> can lead to broad-band sweeping of the optical frequency in the ultra-short pulses of the RFL, thereby greatly adding to the applications of such lasers.

Stimulated emission can be generated in fiber optics both in the resonator-free scheme and using a resonator with external pumping. In Refs. 6 and 7 it was shown experimentally that the efficiency of a resonator RFL is greatly increased if the optical fiber and the pump laser are combined in a single optical resonator. A theoretical analysis of the dynamics of Raman radiation in the optical resonator is usually carried out in the approximation with a given field of the external pump, either single-mode<sup>12-15</sup> or multimode.<sup>16-18</sup>

When the Raman-active medium and the pump lasers are combined in a common resonator, a complicated nonlinear wave system of two (or more) interacting lasers is produced. Such a system can be analyzed in the given-field approximation only during the initial stage of the development of the lasing, when the nonlinearity of the interaction of the waves at the fundamental and combination lasing frequencies can be neglected. This still leaves open the question of the efficiency of the Raman conversion of the radiation at high pump intensities and at large length of the Raman-active medium. Therefore the dynamics of the lasing of the considered system cannot be analyzed even in the simplest single-mode case in the given-pump-field approximation and a rigorous account must be taken of the action of the optical fields of different frequencies on one another.

In addition, it is usually assumed in the analysis of Raman conversion processes in resonantors that a lasing regime that is stationary in time is established in the system, so that the interacting fields depend only on a single spatial coordinate. When a distributed Raman-active medium is introduced, for example fiber optics of large length, an important role in the makeup of the multimode pump-laser resonator is assumed by the nonstationary character of the oscillations produced in such a laser. This nonstationarity is due primarily to the nonstationarity of the lasing regime of the pump laser (free-lasing spike or giant pulse) and the inhomogeneity of the saturation of the Raman gain in time, due to the fluctuating character of the pump radiation and to the dispersion of the Raman medium.

The foregoing distinguishing features make it difficult to apply the approach proposed and developed in Refs. 12-15 to the theory of stimulated Raman emission in an optical resonator, an approach consisting of expanding the radiation field in the resonator modes, followed by an analysis of the resultant nonlinear system of equations by the methods of oscillation theory. Development of a theory of nonstationary Raman generation in a multimode-laser resonator would call in this case for the analysis of systems of nonlinear coupled equations of high order and would make it necessary to overcome a number of difficulties in the interpretation of the results. The reason is that the mechanism considered in the present paper for the generation of ultrashort Raman-radiation pulses is statistical in character. These pulses are formed under the condition that the emission of a multimode pump laser contains ultrashort fluctuation bursts, i.e., a sufficiently large number of modes is excited within the limits of the gain-line widths of both the laser medium and the Raman-active medium.

To describe the RFL dynamics we use in the present paper a space-time approach, wherein the processes that occur in the laser system are described by partial differential equations.

The considered problem of the dynamics of RFL lasing is in fact a typical example of a large class of problems dealing with the time behavior of a laser system whose resonator contains a dispersive medium with losses that are linearly increasing functions of the instantaneous radiation intensity. In addition to the stimulated-scattering processes, a loss dependence of this kind can be ensured also by two-photon absorption and by processes of parametric frequency conversion.

The exact equations that describe the nonstationary lasing of a laser with a nonlinear medium in the resonator are quite complicated. This is apparently why there is at present no general analysis of the dynamics of such a nonlinear wave system with allowance for the variation of the fields over the resonator length and for the evolution of the fine temporal structure of the radiation over the period of the resonator.

In the present paper, on the basis of a solution of a system of nonlinear equations that describe the evolution of the generation in an RFL from the priming noise fields of the pumping and Raman radiation, we investigate the principal regularities of the RFL dynamics in a wide range of variation of the laser-system parameters. Since the investigated regularities are statistical, the system of equations is integrated for a large number of realizations of the initial noise fields in the RFL. A numerical experiment is used to describe the most critical stages of the lasing development, namely, the statistical threshold of the Raman lasing in the field of a noise pump, the separation of single ultrashort pulses in the time structure of the Raman radiation, and the stochastization of the lasing process upon saturation of the Raman gain.

We investigate the influence of the spectral composition of the pump radiation on the dynamics of the Raman lasing and determine the ranges of the values of the principal RFL parameters, under which lasing is most effective. A specific phenomenon was observed, namely anomalously weak energy exchange between the waves of the first and second Stokes components; this phenomenon suppresses the cascade generation of the Raman components in the laser under consideration. The possibility is demonstrated of using a laser whose resonator contains a Raman-active medium with a wide gain line to produce sequences of powerful ultrashort light pulses.

## 1. FORMULATION OF PROBLEM. MODEL AND INITIAL EQUATIONS

We consider the dynamics of the generation of a laser system (Fig. 1) whose resonator contains an excited active medium (the source of noise pumping) and a fiber-optics segment (a Raman-active medium with gain in a broad frequency band). We assume that waves propagating only in one direction are excited in the considered laser, and when the radiation is focused into the fiber, only one optical-fiber mode is excited. The evolution of the development of Raman lasing in the laFIG. 1. Optical diagram of RFL: 1—active medium, 2—optical fiber, 3—matching lenses.

ser can then be calculated by examining the successive passages of the priming spontaneous noise through the active element and through the optical fiber, and using the condition of cyclicity of the lasing.

We calculate the field in the active medium within the framework of the usual rate equations for the intensity  $I_{p}(x,t)$  of the pump radiation and the inversion of the populations N(t) (Ref. 19):

$$\left(\frac{\partial}{\partial x} + \frac{1}{u}\frac{\partial}{\partial t}\right)I_{p} = \sigma NI_{p}, \qquad (1.1)$$

$$\frac{\partial N}{\partial t} + (N - N_0) / T_1 = -2\sigma N I_p / \hbar \omega_p. \qquad (1.2)$$

Here u is the group velocity of the pump wave of frequency  $\omega_p$  in the active medium,  $\sigma$  is the transition cross section of the active medium,  $T_1$  is the lifetime of the laser level, and  $N_0$  is the inversion in the absence of a field and is determined by the rate of excitation of the active medium. It is assumed that the Raman-frequency waves do not interact with the active medium.

Wave propagation in the Raman-active medium (optical fiber) will be described under the assumption that the polarization of the medium manages to follow quasistatically the field variation. The validity of this assumption is limited by the time of transverse relaxation of the molecular vibrations in the glasses ( $T_2 \approx 10^{-13}$  sec, Ref. 9).

The equations for the intensities of the interacting pump waves  $I_p(x,t)$  and the Stokes components  $I_S(x,t)$ ,  $I_{SS}(x,t), \ldots$  for propagation in the optical fiber are written in the form

$$\left(\frac{\partial}{\partial x} + \frac{1}{v_{p}}\frac{\partial}{\partial t}\right)I_{p} = -g\frac{\omega_{p}}{\omega_{s}}I_{s}I_{p} - \alpha_{p}I_{p}, \qquad (1.3)$$

$$\left(\frac{\partial}{\partial x} + \frac{1}{v_{\rm s}}\frac{\partial}{\partial t}\right)I_{\rm s} = g(I_{\rm s}I_{\rm p} - I_{\rm s}I_{\rm ss}) - \alpha_{\rm s} \, {\rm s}^{\prime}, \tag{1.4}$$

$$\left(\frac{\partial}{\partial x} + \frac{1}{v_{\rm SS}}\frac{\partial}{\partial t}\right)I_{\rm SS} = g\frac{\omega_{\rm SS}}{\omega_{\rm S}}(I_{\rm SS}I_{\rm S} - I_{\rm SS}I_{\rm SSS}) - \alpha_{\rm SS}I_{\rm SS}, \qquad (1.5)$$

where  $\omega_{\rm S}, \omega_{\rm S}, \ldots$  are the frequencies of the Stokes components,  $v_{\rm p,S,SS,\ldots}$  are the group velocities of the wave propagation in the optical fiber, g is the gain of the first Stokes component (cm/W) for monochromatic pumping, are  $\alpha_{\rm p,S,SS,\ldots}$  are the linear radiation losses in the fiber.

Equations (1.3)-(1.5) describe the generation of Stokes components within the framework of the cascade (two-photon) SRS mechanism.<sup>20-22</sup> The effects connected with the parametric interaction of the components of the Raman radiation, with excitation of anti-Stokes frequencies, and with the dependence of the refractive index on the radiation intensity (which lead, in particular, to ladder-type scanning of the frequency<sup>10</sup>) are not considered in the present paper. Given the initial and boundary conditions, as well as the conditions for matching the fields when the radiations focused into the fiber, the partial differential equations (1.1)-(1.5) determined completely the dynamics of the RFL lasing within the framework of the considered approximation.

In view of the noise origin of the fields from which the laser emission evolves, the regularities of the dynamics of nonstationary RFL lasing should have an essentially statistical character.<sup>23-27</sup> Therefore, besides obtaining qualitative ideas on the time evolution of the lasing, the problem of investigating the lasing dynamics should include also an analysis of the statistical characteristics of the laser emission. Although the parameters of the priming fields in the resonator can be regarded as known (Gaussian noise), it is impossible to obtain analytic expressions for these characteristics with the aid of the nonlinear equations (1.1)-(1.5). By virtue of the circumstances, the RFL dynamics was investigated by numerical methods.

#### 2. METHOD OF SOLVING THE RFL EQUATIONS

We consider the operation of an RFL in a nonstationary lasing regime, in which relaxation spikes are emitted, or in the Q-switching regime. As will be shown below, in this case a sufficiently complete description of the lasing dynamics can be obtained if the group delay of the waves in the fiber and the generation of cascade Stokes components are neglected.

Taking the boundary conditions into account, the system of partial differential equations (1.1)-(1.5) can be reduced to a system of difference equations that connect the intensities of the interacting fields at instants of time separated by an amount T equal to the time of passage of the light through the resonator:

$$=\frac{I_{p}(n+1,\tau)}{I_{p}(n,\tau)\left[J(n,\tau)+(\omega_{p}/\omega_{s})I_{s}(n,\tau)\right]},$$
  
=
$$\frac{R_{p}J_{p}(n,\tau)\left[J(n,\tau)+(\omega_{p}/\omega_{s})I_{s}(n,\tau)\exp\{G[J_{p}(n,\tau)+(\omega_{p}/\omega_{s})I_{s}(n,\tau)]\}\right]},$$
  
(2.1)

$$=\frac{I_{s}(n+1,\tau)}{\left(\omega_{p}/\omega_{s})I_{s}(n,\tau)+I_{p}(n,\tau)\exp\left\{-G\left[I_{p}(n,\tau)+\left(\omega_{p}/\omega_{s}\right)I_{s}(n,\tau)\right]\right\}}\right)}$$
  
+ $\overline{I_{sp}(n,\tau),$  (2.2)

where

$$J_{\mathbf{p}}(n,\tau) = I_{\mathbf{p}}(n,\tau)$$

$$\times \{1 - \{1 - \exp[-\sigma N(t_0) l_{\mathbf{a}}]\} \exp\left[-\eta \frac{T}{T_1} \int_0^\tau I_{\mathbf{p}}(n,\tau') d\tau'\right] \}^{-1}$$

Here  $G = g\eta \hbar \omega_p L \delta/2\sigma T_1$  is a generalized parameter that characterizes the gain at the Raman frequency, L is the effective length of the Raman medium and is expressed in terms of its geometric length l by the formula L $= [1 - \exp(-\alpha_c l)]/\alpha_c, l_a$  is the length of the active medium,  $R_{p,S} = \exp(-\alpha_{p,S}l + \ln r_{p,S})$  are the total linear losses in the resonator,  $\eta + 1$  is the ratio of the activemedium pump power to the threshold value at which lasing sets in at the frequency  $\omega_p$ ;  $I_{sp}(n,\tau)$  is a random function describing the intensity of the spontaneous Raman scattering in the resonator, and  $\delta$  is the focusing factor of the radiation into the fiber, and is determined by the ratio f the cross sections of the beam outside and inside the fiber.

In Eqs. (2.1) and (2.2) are introduced the dimensionless intensities  $I_{p,S}(n,\tau)$  obtained by normalization of the quantities  $I_{p,S}(x,t)$  to the intensity  $I_p^{st} = \eta \hbar \omega_p / 2\sigma T_1$ of the stationary lasing in the absence of Raman gain. The parameters *n* and  $\tau$  are introduced for the measurement of the time  $t = nT + \tau T$ . The quantity *n* takes on integer values and denotes the number of the pass of the radiation through the resonator, while  $0 \le \tau \le 1$ characterizes the running point in the period of the resonator. In terms of the new variables, the functions  $I_{p,S}(n,\tau)$  describe the distribution of the radiation intensity of the resonator period T in the n-th pass through the resonator. When the initial conditions  $I_{p,S}(0,\tau) = \Phi_{p,S}(\tau)$  are specified, Eqs. (2.1) and (2.2) make it possible to calculate the intensities of the fields of the principal and Stokes radiation at any pass of the radiation through the resonator.

In the absence of gain at the Raman frequency, i.e., at g=0, the system (2.1) and (2.2) describes the dynamics of unidirectional nonstationary lasing of an ordinary ring laser.<sup>28</sup> The same system, suitably simplified, can be used to analyze Raman lasing in a resonator in the given-external field approximation.

We have used (2.1) and (2.2) for a numerical simulation of a laser with neodymium-doped garnet as the active medium and with quartz optical fiber as the Ramanactive medium. The simulation consisted of the production and statistical reduction of "computer oscillograms" of the RFL lasing.

Elementary estimates show that the Raman lasing threshold is reached at reasonable values of the pump parameter  $\eta$  only during the stage of nonlinear development of the lasing spike of the pumping laser. To save computer time, the starting point of the calculations  $t_0$ was therefore chosen to be the end of the linear stage of the pump-laser lasing, determined in accordance with Ref. 28.

The pump parameter  $\eta$  and the fiber length l were varied in the course of the simulation. The remaining parameters of the RFL were the following:  $\omega_p/\omega_s = 1.05$ ;  $g = 10^{-11}$  cm/W;  $\alpha_{p,s} = 2.3 \cdot 10^{-5}$  cm<sup>-1</sup> (10 dB/km),  $T_1 = 2 \cdot 10^{-4}$  sec,  $\sigma = 3 \cdot 10^{-19}$  cm<sup>2</sup>;  $r_{p,s} = 0.7$ ;  $\delta = 400 \dots 3000$ ,  $l_a = 4$  cm.

The random distributions of the pump intensity  $\Phi_p(\tau)$ and of the Raman radiation  $\Phi_S(\tau)$ , specified on the initial pass, correspond to Gaussian noise fields with variable spectral width. The intensity of the priming radiation  $\Phi_S(\tau)$  averaged over the period *T* was chosen at the level 10<sup>-10</sup>  $I_p^{\text{st}}$ . The program for the calculation by formulas (2.1) and (2.2) ensured an accuracy not worse than 2% at lasing development times up to 100 *T*. The calculations were performed with a BÉSM-6 computer.

#### 3. PRINCIPAL LAWS OF RFL LASING DYNAMICS

At small values of  $\eta$ , the pump intensity is insufficient to excite resonator Raman lasing, and all that is emitted is an ordinary relaxation spike at the frequency  $\omega_{\rm p}$ . The time structure of the pump spike radiation is a quasiperiodic sequence of fluctuation intensity bursts, which are gradually amplified in the active medium. Starting with a certain threshold value  $\eta_{\rm thr}$ , the largest of the fluctuation bursts reaches in the course of its development, with high probability, a level  $I_{\rm p}^{\rm thr}$ =  $(\ln R_{\rm s}^{-1})/gL$ , at which the condition for the self-excitation of Raman generation are satisfied:

$$\eta_{\rm thr} = \frac{2\sigma T_{\rm s} \ln R_{\rm s}^{-1}}{gL\delta \ln m_{\rm p} \ln (I_{\rm p}^{\rm st}/I_{\rm sp})}, \qquad (3.1)$$

where  $m_p$  is the average number of pump-intensity fluctuation bursts within the resonator period T; this number is determined by the width of the pump spectrum.

Since the pump is of noise origin, the transition from spontaneous scattering to resonator Raman lasing has a statistical character. This transition consists of predominant amplification of those spontaneous-scattering temporal-field-profile points that pass through the resonator, together with the maximum fluctuation bursts in the noise structure of the pump. The temporal structure of the Raman emission  $I_{\rm S}(\tau)$  in one period of the resonator is determined according to (2.2)by the value of the generalized gain G, and depends on the ratio of the durations of the fluctuation bursts of the pump and the priming Raman radiation. If the widths of the pump-radiation spectra and of the Stokes component differ significantly  $(m_s \gg m_p)$ , then individual groups of pulses are singled out in the temporal structure over the period T of the resonator in the course of the development of the Raman generation; these groups correspond to the maximum radiation bursts in the noise structure of the pump (Fig. 2). In the case when the spectral width of the priming Raman radiation is comparable with or much narrower than the pump spectrum, individual pulses are separated and correspond to the maximum bursts in the pump (Fig. 3a).

So long as the intensity of the Raman radiation re-



FIG. 2. Computer oscillograms of generation at  $m_{\rm S} \gg m_{\rm p}$ . The numbers next to the maxima of each curve indicates the value of this maximum in units of  $I_{\rm p}^{\rm st}$ . The numbers next to the maxima of each curve indicates the value of this maximum in units of  $I_{\rm p}^{\rm st}$ . The laser parameters are:  $\eta = 7$ , l = 70 m,  $\delta = 400$ ,  $m_{\rm B} = 60$ ,  $m_{\rm p} = 6$ .



FIG. 3. Linear regime of RFL lasing. The structure of the pump radiation is shown only for n = 0. The laser parameters are:  $\eta = 4$ , l = 70 m,  $\delta = 400$ ,  $m_{\rm S} = m_{\rm P} = 35$ .

mains weak compared with the intensity of the pumping burst, i.e., at

$$I_{\rm S}(n,\tau) \leq (\omega_{\rm s}/\omega_{\rm p}) (I_{\rm sp}/I_{\rm p}^{\rm st})^{\rm G},$$

little of the energy of this burst is converted into Raman radiation and all the pump bursts develop just as in the absence of the Raman medium. This Ramangeneration regime can be called linear.

In this regime, the change of the intensity of the Raman radiation with time is described by

$$I_{s}(n+1,\tau) = I_{s}(n,\tau) \exp[GJ_{n}(n,\tau)]R_{s}.$$
(3.2)

The exponential character of the dependence of the Raman gain on the instantaneous pump intensity leads to a substantial ordering of the Raman radiation during the linear state of development of the RFL lasing. In accordance with (3.2), in the most probable case the gain of the group of pulses corresponding to the maximum pump burst turns out to be, during each pass through the resonator, larger by at least  $\exp[G\langle I_p(n)\rangle \ln 2]$ times than the gain of any other group. As a result, only one group of pulses is separated over the period Tafter several passes. On the computer oscillograms of Fig. 2, a strong discrimination of pulse groups is reached at 5 < n < 10. Only in individual oscillograms obtained by us did we observe a practically simultaneous development of two (or more) groups corresponding to the presence of powerful pump bursts that are very close in intensity. This agrees with the experimental data given in Refs. 6 and 7 on the dynamics of RFL.

Within each group, the first to develop are those corresponding to the maximum of the fluctuation pump burst. This leads to a gradual reduction of the duration of the group and to discrimination of the pulses within the group. The condition for separation of a single pulse with contrast<sup>1</sup> M in a group during the *n*th pass (from instant that the threshold  $I_p^{thr}$  is reached) can be obtained by using (3.2):

$$G\sum_{i=1}^{n} I_{p}^{(i)} \geq \frac{1}{4} \frac{m_{S}^{2}}{m_{p}^{2}} \ln M, \qquad (3.3)$$

where  $I_{p}^{(i)}$  is the intensity of the pumping fluctuation burst on the *i*-th pass. The duration  $\Delta t$  of the separated pulse is determined by the relation

$$\Delta t(n) \approx \frac{T}{m_{\rm p}} \left( G \sum_{i=1}^{n} I_{\rm p}^{(i)} + \frac{m_{\rm s}^2}{m_{\rm p}^2} \right)^{-\gamma_{\rm h}} . \tag{3.4}$$

If  $m_{\rm S} \leq 2m_{\rm p}(\ln m_{\rm p}/\ln 2)^{1/2}$ , then the discrimination of the pulses in the group takes place simultaneously or else more rapidly than the discrimination between groups. In this case, separation of single ultrashort pulses with high contrast takes place during the linear stage of development of the RFL lasing. For the case of a monochromatic Stokes primer and a specified noise pump field, the separation of individual pulses was first considered theoretically in Ref. 29. In spectrum language this phenomenon can be interpreted as the effect of phasing of the spectrum of the Raman radiation in parametric interaction of the modes of the fundamental and first Stokes component in the optical resonator.<sup>13</sup>

The complete picture of Raman generation excited in a spike of free generation of the pumping laser is determined by the value of the pump parameter  $\eta$ .

At relatively small  $\eta$ , the Raman gain is small and the pump spike attenuates before the combination radiation manages to reach a level comparable with the intensity of the pumping fluctuation burst. In this case, the linear Raman-gain regime is realized during the entire spike. This takes place if the following condition is satisfied:

$$0 < \frac{\eta}{\eta_{\text{thr}}} - 1 < \frac{T \ln (I_p^{\text{st}}/I_{\text{sp}})}{T_{\text{spike}} \ln R_s^{-1}}, \qquad (3.5)$$

where  $T_{spite}$  is the duration of the free-lasing spike.<sup>28</sup> In this regime, the RFL generates a train of ultrashort pulses with high contrast and with duration much shorter than the duration of the pump spikes (Fig. 3a). Figure 3b shows how the contrast and the duration of the pulses changes with developing train. It shows also the change, from pass to pass, of the intensities of the maximum fluctuation pump bursts and of the Raman pulses. In accordance with (3.3) and (3.4), the contrast and duration of the Raman pulses on the trailing edge of the pump spike reach a quasistationary level.

With increasing  $\eta$ , the Raman gain increases. This leads, first, to an abrupt increase of the contrast and to a shortening of the pulses, and second, to the fact that during the time of development of the pump spike the Raman pulse reaches a level comparable with the intensity of the pumping burst (Fig. 4). An important role is then assumed by the transfer of the energy from the pumping spike into the Raman radiation, and a narrow dip appears in the central part of the pump spike; this deep becomes deeper and broadens from pass to pass (Fig. 4a, n = 14). The gain at the central part of the burst decreases practically to zero, but remains high enough on its edges. As a result, the central part



FIG. 4. Saturation of Raman gain. In the upper plot on the right the dashed curve shows the intensities of the maximum fluctuation bursts of the pump in the absence of Raman lasing (g=0). The laser parameters are  $\eta=5$ , l=70 m,  $\delta=400$ ,  $m_{\rm S}=m_{\rm p}=35$ .

of the Raman pulse attenuates rapidly, and its edges continue to be amplified—the pulse breaks up into two parts (Fig. 4a, n=17). As seen from the plots of Fig. 4b, the instant of breakup is characterized by a sharp decrease of the contrast to a value ~1.

Thus, in the regime of saturation of the Raman gain, the developing Raman pulse causes breakup on the pump burst, and this in turn leads to a breakup of the Raman pulse itself.

In the succeeding passes through the resonator, each part of the pulse burns out its own section of the pumping burst, and the dip in the burst gradually broadens. By the time the maximum pump burst burns out, additional Raman-radiation pulses manage to develop and correspond to smaller pump bursts (Fig. 4a, n=25), i.e., randomization of the Raman generation takes place. Starting with this instant, the character of the RFL lasing differs little from that of ordinary Raman lasers. All the fluctuation bursts with intensity higher than  $I_p^{\text{thr}}$  are gradually transformed into mutually superimposed trains of Raman pulses that are in the earlier stages of their development. Ultimately, at sufficiently large  $\eta$ , highly effective conversion of the pump into Stokes radiation having a random temporal structure is realized in the RFL (see Fig. 7 below).

Thus, by varying  $\eta$  we can control the peak power and the contrast of the RFL lasing. From the train of pulses generated in the Raman-gain saturation regime it is easy to separate a single pulse with ultrahigh contrast (more than 10<sup>5</sup>) and with intensity close to the peak intensity of the pump. In the linear regime are generated low intensity pulses, that maintain in the course of the entire train (~10 passes) practically a stationary duration and a contrast that exceeds 10<sup>2</sup>.

#### 4. DEPENDENCE OF THE INTENSITY AND CONTRAST OF THE RAMAN RADIATION ON THE LASER PARAMETERS

The maximum intensity  $I_{s}^{max}$  and contrast M at the maximum of a Raman-lasing train are determined by the linear interaction of two random fields and vary from laser flash to laser flash. Figure 5 shows histo-



FIG. 5. Histograms of the distributions of  $l_{\rm S}^{\rm max}$  and of M at different  $\eta$ : a)  $m_{\rm S} = m_{\rm p} = 35$ , b)  $m_{\rm S} = 0$ ,  $m_{\rm p} = 35$ . The remaining laser parameters are l = 70 m and  $\delta = 400$ .

grams of the distributions of these quantities at the maximum of a Raman-lasing train at different values of the pump parameter  $\eta$ . Each histogram was obtained by reducing N = 100 computer oscillograms of RFL lasing with a fiber 70 m long. For the histograms of Fig. 5a, the parameters of the fluctuation structure of the pumping and priming Raman radiations were chosen equal:  $m_{\rm p} = m_{\rm S} = 35$ , thereby guaranteeing separation of single Raman bursts during the linear stage.

As seen from Fig. 5a, with increasing  $\eta$  the scatter of the values of  $I_{\rm s}^{\rm max}$  and M decreases. This continues up to values  $\eta \approx 5$ . With further increase of  $\eta$ , an ever increasing role is assumed by the rapid depletion of the pump and by the excitation of additional bursts within the period of the resonator, and this leads to randomization of the temporal structure of the radiation. The large spread of the values of  $I_{\rm s}^{\rm max}$  and M at  $\eta = 3$  characterizes the statistical character of the threshold of the Raman generation and is determined by a superposition of the fluctuations of the intensity of the pumping radiation and the priming Raman radiation.

The histograms shown in Fig. 5b were obtained for the case of nonfluctuating, i.e., strongly monochromatic priming Raman radiation ( $m_s = 0$ ,  $m_p = 35$ ). These histograms show that the narrowing the priming-radiation spectrum contributes to stabilization of the lasing process, i.e., decreases the spreads of the values of  $I_s^{max}$  and M, although it does not change significantly their most probable values. Physically this is explained by the fact that in the case of monochromatic Raman priming the fluctuations of the principal characteristics of the lasing are connected only with the fluctuations in the pump.

With increasing length of the fiber, the Raman gain in each pass over the resonator increases. At the same time, an increase takes place in the linear losses of the resonator and in its period T, thus leading to a substantial change in the dynamics of the pump laser, and influences in turn the Raman gain. To establish the optimal parameters of the RFL, we calculated the dependences of  $I_S^{max}$  and M on the length of the fiber at different values of  $\eta$ . The results of the calculations are shown in the form of a three-dimensional projection in Fig. 6.

As seen from Fig. 6a, the largest values of  $I_{\rm S}^{\rm max}$  are reached only in a definite fiber-length interval that broadens with increasing  $\eta$ . A plot drawn in the  $(I_{\rm S}^{\rm max}, \eta)$  plane illustrates the effect of saturation of the peak power of the Raman radiation.

The dependence of the contrast on l and  $\eta$  (Fig. 6b) is more complicated. At small  $\eta$  there is one peak of M. With increasing  $\eta$ , the peak splits, and a deep dip appears in the contrast. A comparison of Figs. 6a and 6b shows that the requirement of simultaneously having high intensity and contrast of the Raman radiation is not incompatible only in a rather narrow region of variation of the parameters l and  $\eta$ . The reason is that to obtain high contrast it is necessary that the Raman lasing progress linearly as long as possible. At the same time, saturation of the gain is mandatory if high intensity is to be obtained.

We note that the appearance of a second peak of M, corresponding to large l, is due to the decrease of the pump spike as a result of the increase in the resonator losses. The intensity of the spike becomes in this case insufficient to allow the Raman gain to saturate before the Raman-lasing train reaches a maximum.

#### 5. SOME SPECIFIC FEATURES OF THE GENERATION OF CASCADED STOKES COMPONENTS IN RFL

When the intensity of the Raman generation on the first Stokes component becomes strong enough (com-



FIG. 6. Plots of  $I_{\rm S}^{\rm max}$  and of *M* against the laser parameters at  $\delta = 400$  and  $m_{\rm g} = m_{\rm p} = 35$ . The regions of variation of the paraemters at which close to maximum values of  $I_{\rm S}^{\rm max}$  and *M* are reached are shown shaded in the  $(l, \eta)$  planes.

parable with the intensity of the pump), the field of this component begins to serve as a pump for the second Stokes component. Cascaded generation of many Stokes components can thus take place in the laser. We have assumed above that cascade processes are forbidden, e.g., as a result of appropriate selective losses introduced into the resonator.

We have considered also the more general problem of the generation of many Stokes components in RFL in the field of a noise pump. Integration of the complete system of equations (1.1)-(1.5) that describe the generation of two Stokes components in the fiber has shown that the considered model of the RFL is characterized by a distinguishing phenomenon—anomalously weak energy exchange between the first and second Stokes components.

Thus, even at pump-parameter values corresponding to Q-switching of the laser resonator ( $\eta = 50$ ), the second Stokes component did not reach intensities higher than 10% of the intensity of the first Stokes component. A typical picture of the evolution of the RFL lasing in this case is shown in Fig. 7.

The causes of the suppression of the lasing of the cascade Stokes components in RFL with a common resonator are the following. The lasing threshold of the second Stokes component is exceeded only when the intensity of the first component becomes comparable with the intensity of the pump, i.e., when the gain in the first component is saturated (Fig. 7, n=5). As already noted, a dip is then burned and the pumping burst and leads, in the successive process of the radiation through the resonator, to the appearance of a dip in the pulse of the first Stokes component. As a result, the intensity of the first component is preserved at the level above the threshold of the generation of the second only during a limited number of passes, which is insufficient for the second Stokes component to enter into the gain saturation regime (Fig. 7, n=6, n=10).



The described situation is typical of intra-cavity excitation of Raman lasing, and differs radically from the situation that takes place in Raman amplifiers, i.e., in single-pass devices. In the case of an amplifier with sufficiently long Raman medium having low linear losses, the noise-pump intensity burst is at first almost completely transformed into the pulse of the first Stokes component. This pulse, despite the absence of pumping, retains a high intensity as it propagates further in the low-loss medium. It is therefore transformed into a second Stokes component pulse, etc.<sup>20, 22</sup>

When Raman lasing is excited in the common resonator, an intense Stokes pulse can exist only until its exciting pump burst burns out. Extraction of part of the radiation from the resonator causes the intensity of the Stokes pulse to drop rapidly below the threshold of the Raman generation after the burning out of the pump after several passes, in practice. As a result, the second Stokes component does not manage to reach the level of the first Stokes component.

Thus, in the considered RFL model, the generation of cascade Stokes components is difficult. This conclusion agrees with the experimental results of Ref. 6.

Lugovoi and Strel'tsov<sup>15</sup>, in an investigation of the stationary regimes of single-mode generation in a laser with a Raman-active medium in the resonator, have shown that under conditions of strong saturation of the transition line of the laser medium, suppression of the generation of Stokes components of higher order is possible at the threshold of generation of the first Stokes component. The principal difference of the effect considered here, of the anomalously weak energy exchange between the first and second Stokes components in a multimode laser, lies in its dynamic character. This effect takes place in the absence of saturation of the gain of the active medium. As shown by numerical calculations, the strongest influence on the dynamics of the generation of the higher Stokes components is

FIG. 7. Generation of the second Stokes component. Randomization of the time structure of the first Stokes component. The laser parameters are:  $\eta = 10$ , l = 70 m,  $\delta = 3000$ ,  $m_{88} = \eta_8 = m_p = 35$ .

exerted by the dispersion of the refractive index of the Raman-active medium. In particular, it can lead both to an enhancement of the energy exchange between the Stokes components and generation of components with intensity higher than the pump intensity, and to a complete suppression of the generation of the Stokes components of higher order. Simultaneous phasing of the spectrum of the Raman radiation at both the first and the succeeding Raman frequencies is also possible here.

#### 6. INFLUENCE OF DISPERSION OF THE FIBER ON THE DYNAMICS OF THE RFL LASING

An important advantage of Raman laser using fiber optics is the wide gain line, and consequently the possibility of generation by these lasers of picosecond and even subpicosecond light pulses. If this is to be accomplished, however, account must be taken of the fiber dispersion, which leads to a detuning of the group velocities of the interacting waves.

With each pass of the radiation through the resonator, the influence of the detuning of the group velocities is cumulative in the course of the development of the lasing. Therefore the dispersion in our model can be neglected only up to a certain definite number q of passes. It is easy to show that this number is connected with the parameter  $m_p$  of the noise pumping by the following approximate relation:

$$qm_{\mathbf{p}} \leq [k'' v_{\mathbf{p}}(\omega_{\mathbf{p}} - \omega_{\mathbf{s}})]^{-1}, \tag{6.1}$$

where  $k'' = \partial^2 k / \partial \omega^2$  is the dispersion of the Raman-active medium. For a quartz fiber  $k_0'' = 2 \times 10^{-28} \sec^2 \cdot \text{cm}^{-1}$ (Ref. 30),  $(\omega_p - \omega_s)/2\pi c = 460 \text{ cm}^{-1}$ , and in accordance with (6.1)  $qm_p \leq 3000$ .

The main regularities of the RFL dynamics manifest themselves, as shown by our calculations, even during the first 10 passes of the radiation through the resonator. Therefore neglect of the dispersion in our laser model is justified up to values  $m_p \approx 300$  (the pump spectra used in our calculations were characterized by values  $m_p \leq 100$ ). At  $m_p = 300$ , in a fiber 60 m long, the duration of the fluctuation pump bursts is of the order of 1 nsec, and the duration of the ultrashort Ramanradiation pulses is 100-150 psec.

To simulate the influence of the dispersion on going to short pulses, we have integrated the system (1.1)-(1.5) at different values of the parameter k'' that characterizes the spreading of the wave packets at the fundamental and Raman frequencies. As expected,<sup>27</sup> allowance for the dispersion leads to a certain slowing down of the development of the Raman generation. The most substantial, however, turned out to be the influence of the dispersion on the dynamics of the Raman generation in the saturated regime.

Since the interacting wave packets move with different group velocities, the Raman pulse, burning out the pump, gradually climbs over to the unburned sections of the pump burst. As a result, the leading front of the pulse (at  $v_s > v_p$ ) is amplified more strongly and becomes steeper than the trailing edge. Starting with certain values of the dispersion, the characteristic dip at the maximum of the pulse vanishes (Fig. 8).

The shift of the pulse to the unburned sections of the pump makes it impossible, in an RFL with a dispersive medium, to generate Raman pulses with intensity higher than the peak intensity of the pump. This phenomenon is qualitatively similar to the formation of highpower pulses in Raman amplifiers operating with opposing waves. In this case, however, the efficiency of conversion of the pump energy into coherent-pulse energy turns out to be much higher than in the case of opposing SRS.

It is possible to make the developing Raman pulse to move over in each pass through the resonator to an unburned section of the pump also by another method, by making the lengths of the pump and Raman resonators somewhat different with the aid of selective mirrors or with a strongly dispersing element. Our calculations have the dynamics of such an RFL with a complex resonator have shown that, at a definite optimal difference between the resonator lengths and at a definite length of the Raman-active medium, a train of identical pulses is generated during the entire lasing spike; the intensity of these pulses is several times larger than the peak intensity of the pump. The results of these calculations will be presented in greater detail elsewhere.

#### 7. CONCLUSION

The foregoing investigations of the dynamics of the generation of a Raman fiber laser (RFL) have revealed



FIG. 8. Oscillograms of lasing at different values of the dispersion and at n=25. The laser parameters and the realizations of the initial noise fields are the same as in Fig. 4. the principal regularities of the generation of a laser of this type. Calculations and numerical experiments have shown that the RFL can be used to generate sequences of powerful ultrashort light pulses at the Raman frequency. The operating regime of the RFL must be chosen in accordance with the required parameters of the Raman pulses, and in particular with the required intensities and contrast of the radiation.

A most interesting result of the numerical experiments is the potential feasibility of generating Raman pulses with peak intensity larger than or comparable with the pump intensity. It was assumed up to now that when a noise pump is used a single Raman pulse can be isolated only in the linear regime of Raman gain, i.e., at intensities  $I_{\rm S} \ll I_{\rm p}$  (Ref. 29). Our numerical experiment has shown that isolation of a single ultrashort Raman pulse takes place up to intensities  $I_{\rm S} \approx I_{\rm p}$ . This regime can be observed also in the case of saturation of Raman gain in an RFL with an additional linear dispersing element.

It should be noted that certain features of the dynamics of generation of ultrashort pulses in the considered Raman laser recall the dynamics of generation of the widely used lasers with saturable filters.<sup>31</sup> Thus, the generation of isolated pulses has a statistical character: a single pulse can be separated only with a certain probability determined by the laser parameters. Just as in a laser with a filter, the narrowing of the spectrum of the priming radiation leads to an increase of the probability of separating single pulses.

At the present time, pulses with approximate duration 1 psec were obtained by the method of passive mode locking in solid-state lasers. Raman fiber lasers, having a broad gain line, make it possible to obtain shorter pulses. To realize this possibility we must use the fact that broadband frequency modulation of the optical carrier takes place in the course of propagation of the pulses through the fiber.<sup>10,11</sup> Therefore, by placing in the RFL resonator a suitable device for compressing the FM pulses and a dispersive element that compensates for the detuning of the group velocities of the pump waves and of the Raman radiation, it is possible to attain generation of pulses with limiting duration down to 100 fs.

With further development of the theory of generation of ultrashort pulses in a Raman fiber laser, it will be necessary to take into account the joint action of the cascade (two-photon) and parametric (i.e., those dependent on the phase relations) mechanisms of excitation of the higher Stokes components. It is known that the parametric interaction between the different components of stimulated Raman emission can lead to their mutual synchronization and to formation of a periodic sequence of ultrashort pulses.<sup>32</sup> In solids and liquids, parametric interaction between different components is usually absent because of the sufficiently strong dispersion of a refractive index. In fiber optics, however, the joint action of the dispersion and of the nonlinearity of the refractive index of the fiber material can lead under definite conditions (e.g., when a suitable choice is made of a reflection coefficients of the RFL

resonator mirrors at different lasing frequencies) to an equality of the phase velocities of the interacting waves and to an increase of the efficiency of the parametric mechanism of generation of higher Stokes components, and also to the possibility of self-synchronization of these components and formation of femtosecond radiation pulses.

In this paper we have considered the mechanism of spectrum phasing of only individual Raman-radiation components (i.e., the formation of ultrashort pulses within the limits of the spectral widths of individual lines of the Raman radiation). The possibility of simultaneous phasing of the spectrum within each component and of individual components with one another calls for additional research.

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# Narrowing of a Rayleigh line wing in stratifying solutions of various concentrations

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Measurements were made of the half-width of a Rayleigh line wing as a function of the concentration of nitrobenzene-*n*-hexane, nitrobenzene-*n*-dodecane, and aniline-cyclohexane solutions at various temperatures. It was found that the wing became narrower on approach to the stratification point. A special feature of the experimental curves was a maximum at temperatures far from critical and a minimum at temperatures close to critical. An explanation of this property is proposed.

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Narrowing of a Rayleigh ling wing on approach to the critical temperature was first observed in the homogeneous phase of stratifying binary nitrobenzene-nhexane and aniline-cyclohexane solutions of critical concentration.<sup>1</sup> Further investigations of this effect<sup>2,3</sup> showed that the wing narrowing was nonmonotonic in most solutions: the half-width of the wing depended in a steplike manner on temperature. Moreover, it was later found that the depolarized Raman scattering lines, whose width (after subtraction of the temperature-independent terms) was governed by the same mechanism as the width as the Rayleigh line wing, also became narrower on approach to the critical temperature in solutions of critical concentration.<sup>3</sup> This observation was not only of independent interest but it also confirmed the existence of the narrowing of Rayleigh line wings.

Narrowing of various depolarized Raman lines was observed in various temperature ranges, indicating the existence of several anisotropy relaxation times and thus yielding information on the reason for the appearance of steps in the temperature dependences of the wing widths. These steps were attributed to the existence of several anisotropy relaxation times.<sup>4</sup> The narrowing of a wing was found to vary considerably from solution to solution. Moreover, according to several authors,<sup>5,6</sup> this effect was not observed at all in nitroethane-isooctane and nitroethane-3-methylpentene solutions. Consequently, Phillies *et al.*<sup>7</sup> compared and discussed the experimental methods and the methods of analysis of the results used in Refs. 5, 6, and 1-3.

One of us proposed earlier<sup>8</sup> an explanation of the narrowing of Rayleigh line wings in stratifying solutions of critical concentration based on allowance for the fact that the difference between the energies of interaction of two identical and two different molecules, which governs the critical temperature, is a function of the mutual orientation of these molecules, i.e., it is a function of the square of the anisotropy tensor. This dependence gives rise to a fourth-order term in the expression for the free energy and this term is proportional to the square of the anisotropy tensor and to the square of the concentration fluctuations. A special feature of the concentration fluctuations near the critical point of stratification is manifested as wing narrowing. This theory was used in Ref. 9 to discuss steps in the temperature dependence of a wing.

In all the cited investigations the narrowing of Rayleigh line wings was investigated in mixtures of critical concentration. The interpretation of the effect given in Ref. 8 also applies to such mixtures. However, additional information on the narrowing effect can be obtained also by investigating mixtures with concentrations other than critical. The present paper reports such an investigation of the narrowing of Rayleigh line wings of mixtures of different concentration on approach to the stratification point.

#### 1. THEORY

In accordance with the overall aim of the present investigation, we shall generalize the theory of narrow-