Thermomechanical circulation effect in superfluid helium-II

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We observed the thermomechanical circulation effect in helium-II, predicted theoretically by V. L. Ginzburg, G. F. Zharkov, and A. A. Sobyanin [JETP Lett. 20, 97 and 69 (1974)], and have confirmed the possibility of using this effect as a new method for recording critical velocities.

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1. In Refs. 1 and 2 is described a thermomechanical circulation effect in superfluid helium-II. The authors consider an unevenly heated annular vessel consisting of two relatively broad parts (large volumes) connected by two capillaries. By virtue of the presence of a temperature difference and of the associated pressure difference, the normal and superfluid parts of the liquid circulate in the capillaries. To ensure the conditions of quantization of the circulation of the superfluid along any closed contour in the ring one introduces in the theory a countercurrent—a circulating superfluid flow having in the large volume a velocity v_{s0}:

$$v = \frac{\rho_n \rho \sigma \Delta T (s_1 - s_2)}{8\pi \eta_n l_0 \xi \rho_s} \tag{1}$$

$$\xi = 1 + \frac{s_0}{l_0}\left(\frac{l_1}{s_1} + \frac{l_2}{s_2}\right). \tag{2}$$

Here ρ, ρ_s, ρ_n are the respective densities of the liquid helium and of its superfluid and normal parts, σ is the entropy per unit mass, η_n is the viscosity of the normal component, s_1 and s_2 are the cross sections of the capillaries, l_1 and l_2 are their lengths, l_0 is the total length of the large volume, and s_0 is its cross section. At the countercurrent velocities given by (1), the total circulation of the velocity of the superfluid component is zero.

When the temperature difference is gradually increased from zero, v_{s0} increases. At the instant when v_{s0} reaches its critical value, corresponding to vortex formation in some section of the ring, a spontaneous transition to lower values of v_{s0} should take place. If the temperature difference ΔT is subsequently decreased to zero, then there remains in the ring a circulating superfluid flow with a circulation opposite in sign to the direction of the initial countercurrent.

The purpose of the present study was an experimental observation of the thermomechanical circulation effect in helium-II with the aid of the reaction of a ring suspended on an elastic suspension, and the measurement, with the aid of this effect, of the critical rate of vortex formation.

2. The thermomechanical circulation effect was generated in an annular vessel (Fig. 1) with average radius $r = 1.6$ cm and cross section 0.5×0.5 cm, made of duraluminum and filled with powder with average grain dimension $d_0 = 800$ μm. The ring was divided into two equal (large) volumes by two cylindrical capillaries with lengths $l_1 = l_2 = 0.5$ cm and diameters $D_1 = 0.4$ cm and $D_2 = 0.3$ cm. The first capillary was filled with powder with average grain dimension $d_1 = 13 \pm 2$ μm and the second with powder with average grain dimension $d_2 = 1.0 \pm 0.5$ μm. We assume that the dimensions of the resultant pores are of the order of the dimensions of the powder grains. The filling factor of the volumes by the powders was determined from the flow of kerosene through tubes filled with the corresponding powders, and amounted to $k = 0.5$. The sum of the cross sections of the pores was $\sum s_1 = k\pi D_1^2/4 = 0.64$ cm^2 in the first capillary, $\sum s_2 = 0.036$ cm^2 in the second, and $\sum s_0 = 0.125$ cm^2 in the large volume.

In one of the large volumes we placed a heater H of constantan wire with resistance $R = 10$ Ω. On the end surfaces of the second large volume there were two openings O, precisely placed on one vertical line, to fill the ring with liquid helium and for thermal contact of this half of the ring with the bath. The ring mass was 10 g.

The ring 1 (Fig. 2) was rigidly secured to a thin-wall tube 2 of stainless steel with 2 mm diameter, and suspended on an elastic filament 3 of phosphor bronze of diameter 40 μm, which served simultaneously as one of the current leads to the heater 4. The second current lead was spring 5, placed on the tube and electrically insulated from it; the second end of the spring was connected to the cap. The period of the oscillations of the system in vacuum was $\theta = 6.8$ sec.

The setup used for automatic recording of the system oscillations is shown in Fig. 2. A vertical diaphragm 7 was placed in the path of a beam traveling from the source 6. The resultant narrow strip of light was reflected from mirror 8, passed through a triangular mask 9, and was focused by condenser 10 on the photocell 11, the signal from which passed

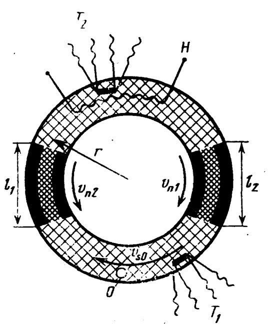

FIG. 1. Annular vessel

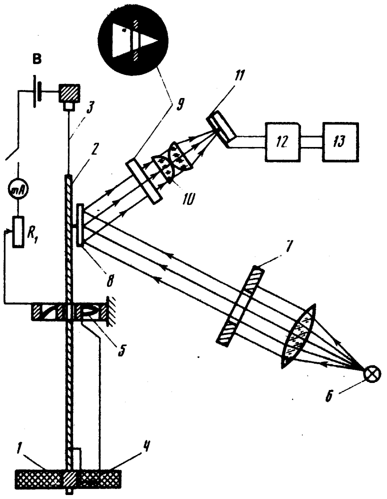

FIG. 2. The systems for suspension and automatic recording of the system oscillations.

through amplifier 12 and was fed to the automatic recorder 13.

The experiments were performed at a temperature $T=1.4$ K, which was maintained accurate to $\pm 5\cdot 10^{-4}$ K. The current in the heater was varied from 0 to 40 mA in steps of 0.2 mA, and the displacement $\Delta\varphi$ of the equilibrium point of the oscillating system as a function of the current in the heater was registered. The torque of the reaction is in this case

$$f\Delta\varphi = r\rho_n(v_{n1}^2\Sigma s_1 - v_{n2}^2\Sigma s_2) = r\rho_n\left(\frac{\rho\sigma\Delta T}{8\pi\eta_n}\right)^2\left(\frac{s_1^2\Sigma s_1}{l_1^2} - \frac{s_2^2\Sigma s_2}{l_2^2}\right),$$

where f is the elastic constant of the suspension, v_{n1} and v_{n2} are the velocities of the normal part of the liquid in the first and second capillaries, respectively.

Under the condition $s_1^2\sum s_1/l_1^2 \gg s_2^2\sum s_2/l_2^2$, which is satisfied in our ring, we have

$$v_{s0} = \frac{l_1}{l_0\xi\rho_s}\left(\frac{\rho f\Delta\varphi}{r\Sigma s_1}\right)^{1/2}. \tag{3}$$

We note that in the presence of many pores in (1) it is necessary to take s_1 and s_2 to mean the cross sections of individual pores, while (2) contains the sums of the pore cross sections and ξ takes the form

$$\xi = 1 + \frac{\Sigma s_0}{l_0}\left(\frac{l_1}{\Sigma s_1} + \frac{l_2}{\Sigma s_2}\right).$$

In a separate experiment we determined, using the carbon resistance thermometers T_1 and T_2 placed in the large volumes (Fig. 1), the dependence of the temperature difference ΔT in these volumes on the power Q released by the heater (Fig. 3).

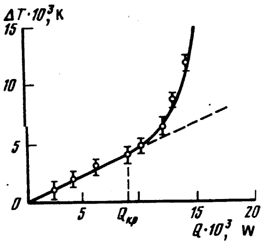

FIG. 3. Dependence of the temperature difference ΔT between the large volumes on the power Q released by the heater.

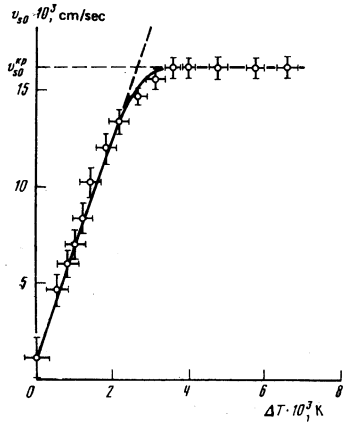

FIG. 4. Dependence of the velocity of the circulating countercurrent v_{s0} on the temperature difference ΔT between the large volumes.

Figure 4 shows plots of the values of v_{s0} obtained from (3) against the temperature difference ΔT. The linear section on this curve corresponds to an increase in the velocity of the circulating countercurrent with increasing temperature difference in full agreement with the theory: according to (1), at $T=1.4$ K we have a ratio $v_{s0}/\Delta T = 5.8$ cm-sec^{-1} deg^{-1}, whereas the slope of the linear section of the curve of Fig. 4 yields $v_{s0}/\Delta T = 6.0\pm 0.5$ cm-sec^{-1} deg^{-1}.

At $\Delta T = 3\cdot 10^{-3}$ K the critical rate of vortex formation in the circulating countercurrent $v_{s0}^{cr} = (1.6\pm 0.2)\cdot 10^{-2}$ cm/sec is reached. Starting with $\Delta T = 3\cdot 10^{-3}$ K, the system oscillates about a certain value $\Delta\varphi_{av}$, which does not change with increasing ΔT. This is the cause of the plateau on the plot of $v_{s0} = f(\Delta T)$ (Fig. 4).

The critical power $Q_{cr} = 9$ mW, which we determine from the deviation of the $\Delta T = f(Q)$ curve from linearity (Fig. 3) makes it also possible to estimate the critical velocity of vortex formation

$$v_{s0}^{cr} = \frac{Q_{cr}\rho_n l_1}{\rho_s\rho\sigma T l_0\xi\,\Sigma s_1} = (1.5\pm 0.2)\cdot 10^2 \text{ cm/sec}$$

(where it is taken into account that $v_{n1}\sum s_1 \gg v_{n2}\sum s_2$).

The good agreement between the values of the critical velocities of vortex formation, estimated above by two independent methods, is evidence of the possibility of using the thermomechanical circulation effect as a new method of recording critical velocities.

We note that the obtained values of v_{s0}^{cr} agree with the estimate of the critical velocity of vortex formation v_s^{cr} in channels with dimension d by the formula[3]

$$v_s^{cr} = \frac{h}{4\pi m d}\left(\ln\frac{8d}{a} - \frac{3}{4}\right),$$

where the radius of the core of the vortex is $a = 3\times 10^{-8}$ cm. For channels with dimension $d = 8\times 10^{-2}$ cm we have

$$v_s^{cr} = 1.6\cdot 10^{-2} \text{ cm/sec}.$$

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[2] G. F. Zharkov and A. A. Sobyanin, ibid. 20, 163 (1974) [20, 69 (1974)].
[3] Masakazu Ichiyanagi, J. Low Temp. Phys. 23, 351 (1976)

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Nonlinear polarizability tensor of an anisotropic medium

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Inversion of the retarded Green's function of the electromagnetic field of an anisotropic medium yielded a correct solution of the problem of the limiting transition in the nonlinear polarizability tensor (NPT) from the independent variables $\{\mathbf{k}, \omega\}$ to variables $\{\mathbf{k}, \omega_\rho(\mathbf{k})\}$ that satisfy the dispersion law of normal electromagnetic waves; this corresponds to the conditions of resonance between the extraneous sources and the natural oscillations of this system. An exact limiting expression having no poles over the entire interval of variation of its frequency arguments is obtained for the total NPT of a medium of arbitrary symmetry. The intensity of generation of the summary harmonic is calculated in the given-field approximation.

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I. INTRODUCTION

A microscopic calculation of the total nonlinear polarizability tensor (NPT) for concrete models of a crystal medium remains one of the vital problems of nonlinear optics. This is due to the need for knowing the NPT to solve Maxwell's equations for a nonlinear medium, as well as to obtain the probabilities (intensities) of various nonlinear processes.

It is known that normal electromagnetic waves in an anisotropic medium are in the general case neither longitudinal nor transverse,[1] and that this division is quite arbitrary. Therefore the choice of such models as optically isotropic cubic crystals[2] or a weakly anisotropic medium,[3] in which account is taken of the contribution of only transverse waves, is not always satisfactory for the calculation of the limiting value of the total NPT. First, the transverse-wave approximation leads not to a total but to a transverse NPT $\varepsilon^{\perp}_{ijl}$, having poles at the frequencies of the Coulomb exciton, i.e., in those frequency regions where the transverse-wave approximation itself cannot be used because the amplitudes of the electric fields of the corresponding transverse waves vanish. Second, in addition to the contribution of the transverse waves it is necessary to take into account also the contribution of the longitudinal waves, which turns to exert a substantial influence on the dispersion properties of the NPT ε_{ijl} even in the case of low anisotropy.

II. CONNECTION OF NPT WITH THE ANHARMONICITY COEFFICIENTS

With the aid of a Fourier transformation, Maxwell's equations for the electric field in a nonlinear homogeneous medium, with both the quadratic and linear polarizabilities taken into account in the material relations, can be represented in the form

$$\Delta_{ij}(\mathbf{k},\omega)E_j(\mathbf{k},\omega)=\frac{\omega^2}{c^2}\int\varepsilon_{ijl}(\mathbf{k},\omega;\mathbf{k}_1,\omega_1;\mathbf{k}_2,\omega_2)E_j(\mathbf{k}_1,\omega_1)\times E_l(\mathbf{k}_2,\omega_2)\delta(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\delta(\omega-\omega_1-\omega_2)d\mathbf{k}_1\,d\mathbf{k}_2\,d\omega_1\,d\omega_2, \tag{1}$$

where $\Delta_{ij}(\mathbf{k},\omega)=k^2\delta_{ij}-k_ik_j-\omega^2\varepsilon_{ij}(\mathbf{k},\omega)/c^2$, while $\varepsilon_{ij}(\mathbf{k},\omega)$ and $\varepsilon_{ijl}(\mathbf{k},\omega;\mathbf{k}_1\omega_1;\mathbf{k}_2,\omega_2)$ are the permittivity and nonlinear polarizability tensors.

The method of extraneous currents[4] was used in Refs. 3, 5, and 6 to establish the connection between the NPT and the anharmonicity coefficients in a polariton system:

$$\varepsilon_{ijl}(\mathbf{k},\omega;\mathbf{k}_1,\omega_1;\mathbf{k}_2,\omega_2)=\frac{V^2c^6\Delta_{im}(\mathbf{k},\omega)\Delta_{nj}(\mathbf{k}_1,\omega_1)\Delta_{pl}(\mathbf{k}_2,\omega_2)}{(4\pi)^2\hbar^3\omega_1\omega_2\omega}\times\sum_{\rho_1\rho_2\rho_3}\left\{\frac{S^m_{\rho_3}(\mathbf{k})S^{*n}_{\rho_1}(\mathbf{k}_1)S^{*p}_{\rho_2}(\mathbf{k}_2)[W(\mathbf{k}_1\rho_1;\mathbf{k}_2\rho_2;\mathbf{k}\rho_3)+W(\mathbf{k}_2\rho_2;\mathbf{k}_1\rho_1;\mathbf{k}\rho_3)]}{\omega_{\rho_3}(\mathbf{k})\omega_{\rho_1}(\mathbf{k}_1)\omega_{\rho_2}(\mathbf{k}_2)[\omega-\omega_{\rho_1}(\mathbf{k}_1)-\omega_{\rho_2}(\mathbf{k}_2)][\omega_2-\omega_{\rho_2}(\mathbf{k}_2)]}\times\frac{1}{[\omega-\omega_{\rho_3}(\mathbf{k})]}+\ldots\right\}, \tag{2}$$

where $\mathbf{k}=\mathbf{k}_1+\mathbf{k}_2$, $\omega=\omega_1+\omega_2$, the three dots denote five additional terms of similar structure, and $S\rho^i(\mathbf{k})$ is the amplitude of the electric field intensity of the polariton mode:

$$S_\rho^i(\mathbf{k})=S_\rho(\mathbf{k})l_i^{\mathbf{k}\rho}=-\left(\frac{2\pi\hbar\omega_\rho^2(\mathbf{k})}{Vkc^2}\right)^{1/2}\left(\frac{\mathbf{v}_{\rho\,\mathrm{gr}}(\mathbf{k})\mathbf{s}}{c}\right)^{1/2}g_{\mathbf{k}\rho}l_i^{\mathbf{k}\rho}. \tag{3}$$

Here V is the cyclicity volume, $\mathbf{s}=\mathbf{k}/|\mathbf{k}|$; $g_{\mathbf{k}\rho}$, $l_i^{\mathbf{k}\rho}$ and $\mathbf{v}_{\rho\mathrm{gr}}(\mathbf{k})=\partial\omega_\rho(\mathbf{k})/\partial\mathbf{k}$ are respectively the normalization factor, the unit polarization vector, and the group velocity of the ρ-the normal mode, the explicit forms of which are given in the Appendix.

The relation (2) is not accidental, since the use, on the one hand, of phenomenological equations (1) and, on the other, the Hamiltonian of a crystal accurate to three-particle interactions

$$H=\sum_{k\rho}\hbar\omega_\rho(\mathbf{k})\xi_\rho^+(\mathbf{k})\xi_p(\mathbf{k})+\sum_{\substack{\mathbf{k}_1\mathbf{k}_2\\ \rho_1\rho_2\rho_3}}\{W(\mathbf{k}_1\rho_1;\mathbf{k}_2\rho_2;\mathbf{k}_1+\mathbf{k}_2\rho_3)\times\xi_{\rho_3}^+(\mathbf{k}_1+\mathbf{k}_2)\xi_{\rho_2}(\mathbf{k}_2)\xi_{\rho_1}(\mathbf{k}_1)+\mathrm{H.c.}\} \tag{4}$$

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