Polarization of photon echo produced by standing-wave pulses

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The distinguishing features of the polarization and the laws governing the damping of photon echoes produced by two successive standing-wave pulses separated by a time interval τ and polarized in planes making an angle ψ are investigated. It is established that the even photon echoes produced at the instants 2τ, 4τ, ..., have properties greatly differing from those of the odd photon echoes that appear at the instants 3τ, 5τ, ... The even echoes are elliptically polarized, with the exception of the cases $\psi = 0$ and $\psi = \pi/2$ or of strict resonance, when the polarization becomes linear. From the damping of the intensity of the even echoes it is possible to determine experimentally the transverse relaxation time. The odd echoes, on the other hand, are always linearly polarized and their radiation intensity does not contain the characteristic narrow resonances of width $\tau/2$. The distinguishing features of the damping of the odd echoes and the law governing the rotation of their polarization plane with changing τ make it possible to determine the widths of the upper and lower resonance levels, as well as the dipole moment of the atomic transition.

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The method of separated fields, which is of great interest for nonlinear spectroscopy, has been under intensive study in the past few years. This method is being developed both for spatially separated fields[1-4] and for fields separated in time.[5-7] In the latter case, the resonant medium is subjected to the action of two successive standing-wave light pulses separated by a time interval τ. The result is coherent radiation of the excited medium in the form of photon echoes that appear at time intervals 2τ, 3τ,... after the first excited pulse. This process lasts until an equilibrium state is established by the action of the irreversible relaxation.

Experimental[5,6] and theoretical[7] investigations of the echo phenomenon in the case of standing waves were performed for identical polarizations of the exciting pulses. In the present paper we investigate, for the first time ever, the polarization of coherent radiation in the form of a photon echo produced by pulses of standing waves whose polarization planes make an angle ψ. For simplicity, we consider resonant atomic transitions with a total-angular-momentum change $1 = 0$, but the individual features of the obtained regularities remain in force also for other atomic transitions. The results are of direct interest also for the investigation of a gas medium acted upon by spatially separated fields,[1-4] inasmuch as in the c.m.s. of the moving atom the action of spatially separated fields is equivalent to the action of pulses of time-separated standing waves.

With respect to their features, a distinction is made between even and odd photon echoes, the former appearing at the respective instants of time 2τ, 4τ, ... and the latter at 3τ, 5τ, ... The appearance of even echoes is due to the phase memory of the behavior of the polarization vector of the medium in the time interval between the two exciting standing wave pulses, whereas the onset of odd echoes is due to the phase memory of the populations of the resonant intervals in the same time interval. This is also the cause of the difference between their properties. The even echoes are elliptically polarized, with the polarization ellipse dependent on ψ, τ the detuning Δ, and the phase difference $\Phi_2-\Phi_1$ between the first and second standing-wave pulses. Linear polarization appears only at $\psi=0$ and $\psi=\pi/2$ or $\Delta=\Phi_2-\Phi_1$. The damping of the even echoes is due to irreversible relaxation of the polarization vector of the medium. In contrast, the odd echoes are polarized in a plane whose position depends on ψ, τ, and the probability γ of the spontaneous emission of the atom, as well as on the widths $\hbar\gamma_b$ and $\hbar\gamma_a$ of the upper and lower levels. The relaxation constants γ_b, γ_a, and γ influence strongly the damping of the odd echoes, a fact that can be used for an experimental determination of the constants in the case $\psi\neq 0$.

The obtained polarization properties and the damping laws of the phonon echoes produced by standing-wave pulses differ in principle from the properties of photon echoes produced by traveling-wave pulses,[8,9] and are more meaningful and promising for practical applications. The use of the polarization properties of standing-wave photon echoes offer advantages over the traditional methods, in that it permits a simultaneous determination of all the sought relaxation constants: the width of the homogeneously broadened spectral line, the widths of the upper and lower levels, and the dipole moment of the atomic transition. All this adds greatly to the applicability of the echo phenomenon in nonlinear spectroscopy.

1. INITIAL EQUATIONS AND BASIC FORMULAS

In the experiments,[5,6] a short traveling-wave pulse passed through a gas medium of length L, was reflected from a perpendicular mirror, and propagated in the opposite direction, producing in this gas medium, over a short time interval, a standing wave

$$\mathbf{E}=2\mathbf{l}a\cos(kz+\varphi)e^{-i(\omega t+\Phi)}+\text{c. c.},\tag{1}$$

where $\mathbf{E}$ is the electric field intensity, $\mathbf{l}$ is a unit vector in the polarization direction, $\omega=kc$ is the wave frequency, a is the real amplitude, and φ and Φ are certain phase shifts. The time $2L/c$ of the motion of the leading front in the gas in the forward and backward directions was small compared with the characteristic times (22) and with duration of the traveling-wave pulse itself. This makes it possible to assume hereafter that the standing-wave pulse (1) is excited simultaneously over the entire length of the gas medium. Of course, there are also other methods of producing the standing-wave pulse (1).

The interaction of the pulse (1) with the resonant atoms is described by the equations

$$\left(\frac{\partial^2}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{E}=\frac{4\pi}{c^2}\frac{\partial^2}{\partial t^2}\int \mathbf{p}\,dv, \tag{2}$$

$$\left(\frac{\partial}{\partial t}+v\frac{\partial}{\partial z}\right)\rho=\frac{i}{\hbar}[\rho(H-\mathbf{Ed})-(H-\mathbf{Ed})\rho]+\left(\frac{\partial\rho}{\partial t}\right)_{st}, \tag{3}$$

in which $\mathbf{p}=\mathrm{Tr}\rho\mathbf{d}$ is the polarization vector of the medium, ρ is the density matrix, $\mathbf{d}$ is the dipole-moment operator, H is the hamiltonian of the atom in the c.m.s., and v is the atom velocity along the Z axis. The last term in the right-hand side of (3) takes into account the irreversible relaxation, pumping, and arrival of the atoms at the lower level on account of spontaneous emission on the upper level, as is seen from the formulas

$$\left(\frac{\partial\rho_{\mu\mu'}}{\partial t}\right)_{st}=\frac{\gamma_b N_b}{2j_b+1}f(v)\delta_{\mu\mu'}-\gamma_b\rho_{\mu\mu'},\quad \left(\frac{\partial\rho_{\mu m}}{\partial t}\right)_{st}=-\gamma_0\rho_{\mu m},$$

$$\left(\frac{\partial\rho_{mm'}}{\partial t}\right)_{st}=\frac{\gamma_a N_a-\gamma N_b}{2j_a+1}f(v)\delta_{mm'}-\gamma_a\rho_{mm'}+\gamma\frac{2j_b+1}{|d|^2}d_{m\mu}{}^{\alpha}\rho_{\mu\mu'}d^{\alpha}_{\mu' m'},$$

$$f(v)=\frac{\exp(-v^2/u^2)}{\pi^{1/2}u}.$$

Here $\rho_{\mu\mu'}$ and $\rho_{mm'}$ are the density matrices that describe the atom on the upper and lower levels with energies E_b and E_a and total angular momenta j_b and j_a, with $(E_b-E_a)/\hbar=\omega_0$ and $|\omega-\omega_0|\ll\omega_0$. The matrix $\rho_{\mu m}$ describes transitions between these levels, whose degeneracy is due to the projections μ and m of the total angular momenta. Next, $d^{\alpha}_{m\mu}$ is the matrix element of the angular-momentum projection, N_b and N_a are the stationary populations of the upper and lower degenerate levels in the absence of the external field (1), u is the average thermal velocity, $1/\gamma_0$, $1/\gamma_a$ and $1/\gamma_b$ are the irreversible-relaxation times due to collisions and radiative decay, $\gamma=4|d|^2\omega_0^3/3(2j_b+1)\hbar c^3$ is the probability of the spontaneous emission of a quantum $\hbar\omega_0$ by an isolated atom, and d is the reduced dipole moment.[10] Summation over all the repeated matrix and tensor indices is implied throughout. The described procedure is valid also for molecular transitions whose degeneracy is connected with the projections of the total angular momenta.

In the resonance approximation we obtain from (3) the following system of equations:

$$\left(\frac{\partial}{\partial t}+v\frac{\partial}{\partial z}-i\Delta+\gamma_0\right)Q^{\alpha}_{\mu\mu'}=\frac{2ia|d|^2}{\hbar}\cos(kz+\varphi)l_\beta(N^{\beta\alpha}_{\mu\mu'}-M^{\beta\alpha}_{\mu\mu'}), \tag{4}$$

$$\left(\frac{\partial}{\partial t}+v\frac{\partial}{\partial z}+\gamma_a\right)N^{\alpha\beta}_{\mu\mu'}=\frac{\gamma_a N_a-\gamma N_b}{2j_a+1}f(v)\Pi^{\alpha\beta}_{\mu\mu'}+(2j_b+1)\gamma\Pi^{\alpha\sigma}_{\mu\mu''}M^{\sigma\beta}_{\mu''\mu'}$$
$$+\frac{2ia}{\hbar}\cos(kz+\varphi)l_\sigma(\Pi^{\alpha\sigma}_{\mu\mu''}Q^{\beta}_{\mu''\mu'}-Q^{\alpha+}_{\mu\mu''}\Pi^{\sigma\beta}_{\mu''\mu'}), \tag{5}$$

$$\left(\frac{\partial}{\partial t}+v\frac{\partial}{\partial z}+\gamma_b\right)M^{\alpha\beta}_{\mu\mu'}=\frac{\gamma_b N_b}{2j_b+1}f(v)\Pi^{\alpha\beta}_{\mu\mu'}$$
$$+\frac{2ia}{\hbar}\cos(kz+\varphi)l_\sigma(Q^{\sigma+}_{\mu\mu''}-Q^{\sigma}_{\mu\mu''})\Pi^{\alpha\beta}_{\mu''\mu'}, \tag{6}$$

where $\Delta=\omega-\omega_0$ is the detuning and the remaining symbols stand for

$$Q^{\alpha}_{\mu\mu'}=\rho_{\mu m}d^{\alpha}_{m\mu'}e^{i(\omega t+\Phi)},\quad \Pi^{\alpha\beta}_{\mu\mu'}=d_{\mu m}{}^{\alpha}d^{\beta}_{m\mu'}/|d|^2,$$

$$N^{\alpha\beta}_{\mu\mu'}=d_{\mu m}{}^{\alpha}\rho_{mm'}d^{\beta}_{m'\mu'}/|d|^2,\quad M^{\alpha\beta}_{\mu\mu'}=\rho_{\mu\mu''}\Pi^{\alpha\beta}_{\mu''\mu'}.$$

It is seen from (4)–(6) that the degeneracy of the levels complicates the initial equations and the subsequent solution compared with the nondegenerate case.[7,11] However, allowance for the degeneracy is needed in principle for the description of the polarization effects.

We shall investigate the coherent phenomena in the given-field approximation, when the reaction of the resonant medium on this specified external field is negligibly small. For this purpose, the product of the absorption coefficient by the length of the gas medium must be small [see (20) and (21)].

In the given-field approximation, Eqs. (4)–(6) are linear, since the amplitude a and the phase shifts φ and Φ of the standing wave (1) are assumed to be known. The initial conditions for these equations at the instant of time $t=0$ prior to excitation of the standing wave (1) are written in the form

$$Q^{\alpha}_{\mu\mu'}(z,t)|_{t=0}=0,\quad N^{\alpha\beta}_{\mu\mu'}(z,t)|_{t=0}=\frac{N_a f(v)}{2j_a+1}\Pi^{\alpha\beta}_{\mu\mu'},$$
$$M^{\alpha\beta}_{\mu\mu'}(z,t)|_{t=0}=\frac{N_b f(v)}{2j_b+1}\Pi^{\alpha\beta}_{\mu\mu'}. \tag{7}$$

We consider below for simplicity the atomic transition $j_b=0\to j_a=1$. Then $\Pi^{\alpha\beta}_{00}=\delta_{\alpha\beta}/3$, and the zero subscripts $\mu=\mu'=0$ of the sought quantities will be dropped

$$Q_{00}{}^{\alpha}=Q_\alpha,\quad N_{00}{}^{\alpha\beta}=N_{\alpha\beta},\quad M_{00}{}^{\alpha\beta}=\rho_{00}\delta_{\alpha\beta}/3=M\delta_{\alpha\beta}. \tag{8}$$

We change over in (4)–(6) from the variables z and to two independent variables ξ and t', where $\xi=z-vt$ and $t'=t$. At the initial instant $t=0$ the sought functions (8) do not depend on ξ, and in the region $0<t$ we have

$$Q_\alpha=Q_\alpha(\xi,t),\quad N_{\alpha\beta}=N_{\alpha\beta}(\xi,t),\quad M=M(\xi,t),$$

where α and β run through the values 1, 2, and 3 and label the projections on the axes X, Y, and Z, and t will no longer be primed ($t'\to t$).

We choose the X axis along the vector $\mathbf{l}$. Then the general solution of Eqs. (4)–(6) for $\gamma=\gamma_0=\gamma_a=\gamma_b=\Delta=0$ in the presence of a degenerate lower level takes the form

$$Q_1(\xi,t)=Q_1'(\xi,t_0)+iQ_1''(\xi,t_0)\cos\Delta F+\frac{3^{1/2}}{2}i|d|[N_{11}(\xi,t_0)-M(\xi,t_0)]\sin\Delta F, \tag{9}$$

$$Q_2(\xi,t)=Q_2(\xi,t_0)\cos\frac{\Delta F}{2}+3^{1/2}i|d|N_{12}(\xi,t_0)\sin\frac{\Delta F}{2}, \tag{10}$$

$$N_{11}(\xi,t)=\frac{1}{2}[N_{11}(\xi,t_0)+M(\xi,t_0)]+\frac{1}{2}[N_{11}(\xi,t_0)-M(\xi,t_0)]\cos\Delta F$$
$$-\frac{1}{3^{1/2}|d|}Q_1''(\xi,t_0)\sin\Delta F, \tag{11}$$

$$M(\xi,t)=\frac{1}{2}[N_{11}(\xi,t_0)+M(\xi,t_0)]-\frac{1}{2}[N_{11}(\xi,t_0)-M(\xi,t_0)]\cos\Delta F$$
$$+\frac{1}{3^{1/2}|d|}Q_1''(\xi,t_0)\sin\Delta F, \tag{12}$$

$$N_{12}(\xi,t)=N_{12}(\xi,t_0)\cos\frac{\Delta F}{2}+\frac{i}{3^{1/2}|d|}Q_2(\xi,t_0)\sin\frac{\Delta F}{2}, \tag{13}$$

$$N_{22}(\xi,t)=N_{22}(\xi,t_0),\quad N_{23}(\xi,t)=N_{23}(\xi,t_0),\quad N_{33}(\xi,t)=N_{33}(\xi,t_0),$$
$$F=\frac{2}{kv}\sin\frac{kv(t-t_0)}{2}\cos\left[k\xi+\varphi+\frac{1}{2}kv(t+t_0)\right], \tag{14}$$

$$\Lambda=\frac{4a|d|}{3^{1/2}\hbar}=2a\left(\frac{\gamma c^3}{\hbar\omega_0^3}\right)^{1/2},$$

where t_0 is the initial instant of time, Λ is the characteristic Rabi frequency,[12] and the prime and double prime denote respectively the real and imaginary parts of the function $Q_1(\xi, t_0)$. We obtain $Q_3(\xi, t)$ and $N_{13}(\xi, t)$ from (10) and (13) by replacing the subscripts 2 by 3.

Once $\mathbf{Q}=\mathbf{Q}(z-vt, t)$ is found, the polarization vector of the medium is written in Eq. (2) in the form

$$\mathbf{p}=\mathbf{Q}e^{-i(\omega t+\Phi)}+\text{c. c.}$$

To find the field E_i induced by the excited medium, we use the Fourier expansion

$$\mathbf{Q}(z-vt,t)=\sum_{n=-\infty}^{\infty}\mathbf{q}_n e^{inkz},$$

and seek the solution of Eq. (2) in the interior of the medium in the form

$$\mathbf{E}_i=\left[\boldsymbol{\varepsilon}_0+\sum_{n=1}^{\infty}(\boldsymbol{\varepsilon}_n^{(+)}e^{inkz}+\boldsymbol{\varepsilon}_n^{(-)}e^{-inkz})\right]e^{-i(\omega t+\Phi)}+\text{c. c.},$$

where $\boldsymbol{\varepsilon}_0=\boldsymbol{\varepsilon}_0(z,t)$ and $\boldsymbol{\varepsilon}_n^{(\pm)}=\boldsymbol{\varepsilon}_n^{(\pm)}(z,t)$ are slow functions compared with the exponential $\exp[i(nkz\pm\omega t)]$. Equating the factors of the equal exponentials in both sides of (2), we get

$$\left[\frac{1}{c}\frac{\partial}{\partial t}\pm n\frac{\partial}{\partial z}+\frac{i\omega}{2c}(n^2-1)\right]\boldsymbol{\varepsilon}_n^{(\pm)}=i\frac{2\pi\omega}{c}\int\mathbf{q}_{\pm n}\,dv.$$

As a result of the integration of this equation we find that at $n>1$ the quantities $\boldsymbol{\varepsilon}_0$ and $\boldsymbol{\varepsilon}_n^{(\pm)}$ are small compared with $\boldsymbol{\varepsilon}_1^{(\pm)}\equiv\boldsymbol{\varepsilon}^{(\pm)}$ in a ratio $c/\omega L|n^2-1|\ll 1$, and can therefore be discarded. Thus, the induced field E_i is a superposition of two waves of frequency ω, propagating in opposite directions:

$$\mathbf{E}_i=\boldsymbol{\varepsilon}^{(+)}e^{i(kz-\omega t-\Phi)}+\boldsymbol{\varepsilon}^{(-)}e^{-i(kz+\omega t+\Phi)}+\text{c.c.} \tag{15}$$

This means that the excited medium radiates in the positive and negative directions of the Z axis. The amplitudes $\boldsymbol{\varepsilon}^{(+)}$ and $\boldsymbol{\varepsilon}^{(-)}$ of the forward and backward waves that have emerged to the outside of the resonant medium are given by

$$\boldsymbol{\varepsilon}^{(\pm)}=iL\frac{\omega^2}{c^2}\int_{-\infty}^{\infty}dv\,e^{\mp ikv(t\mp z/c)}\int_{-\pi c/\omega}^{\pi c/\omega}\mathbf{Q}(\xi,t\mp z/c)e^{\mp ik\xi}d\xi. \tag{16}$$

The derived equations (9)–(16) are needed to determine the echo produced after successive excitation of a medium by several pulses of standing waves with different polarizations.

Using (9) and (16), we write down in explicit form the amplitude $\boldsymbol{\varepsilon}^{(\pm)}$ of the induced field (15), so as to estimate the reaction of the resonant medium on the standing wave (1) excited at the instant of time $t=0$ under the initial conditions (7)

$$\boldsymbol{\varepsilon}^{(\pm)}=-12\left(\frac{\pi}{3}\right)^{1/2}\frac{\omega}{c}|d|N_0Le^{\pm i\varphi}$$
$$\times\int_0^{\infty}\cos\left[\frac{s}{2T_0}\left(t\mp\frac{z}{c}\right)\right]J_1\left(\frac{2\Lambda T_0}{s}\sin\frac{s(t\mp z/c)}{2T_0}\right)e^{-s^2}ds, \tag{17}$$

where $N_0=N_a/3-N_b$ is the stationary excess population of the Zeeman sublevels in the absence of the external field (1), $T_0=1/ku$ is the time of the reversible Doppler relaxation, and $J_1(\theta)$ is a Bessel function of first order and of argument θ.

The integral in (17) can be easily estimated in the limiting case of low-energy irradiation of the medium

$$\Lambda T_0\ll 1 \tag{18}$$

and in the high-energy regime

$$\Lambda T_0\gg 1. \tag{19}$$

In case (18) the reaction of the resonant medium on the standing wave (1) is small if the following inequality holds

$$T_0|N_0|L|d|^2\omega/\hbar c\ll 1. \tag{20}$$

For the high-energy regime (19), the condition (20) is replaced by

$$|N_0|L|d|\omega/ac\ll 1. \tag{21}$$

The characteristic time intervals, after which the functions (17) differ noticeably in cases of (18) and (19), are respectively of the order T_0 and $1/\Lambda$. Therefore the retardation of the light within the limits of the resonant medium can be neglected in the boundary conditions for Eqs. (4)–(6) if

$$L/c\ll T_0,\quad L/c\ll 1/\Lambda. \tag{22}$$

2. PHOTON-ECHO POLARIZATION

Let two linearly polarized standing-wave pulses

$$\mathbf{E}=2\mathbf{l}_1a_1\cos(kz+\varphi_1)e^{-i(\omega t+\Phi_1)}+\text{c. c. at } 0\leqslant t\leqslant\tau_1,$$
$$\mathbf{E}=2\mathbf{l}_2a_2\cos(kz+\varphi_2)e^{-i(\omega t+\Phi_2)}+\text{c. c. at } \tau+\tau_1\leqslant t\leqslant\tau+\tau_1+\tau_2.$$

be successively excited in the medium. Here a_1, a_2, φ_1, φ_2, Φ_1 and Φ_2 are real constants, and the unit vectors $\mathbf{l}_1$ and $\mathbf{l}_2$ make an angle ψ. The durations τ_1 and τ_2 of the pulses are so short that the irreversible relaxation and detuning during these time intervals can be neglected in (4)–(6):

$$\gamma_0\tau_n\ll 1,\quad \gamma_a\tau_n\ll 1,\quad \gamma_b\tau_n\ll 1,\quad |\Delta|\tau_n\ll 1,$$

where $n=1,2$. The interval τ between the pulses, however, is large, $T_0\ll\tau$, $\tau_1\ll\tau$ and $\tau_2\ll\tau$, so that the irreversible relaxation and detuning during an interval of the order τ must be taken into account.

When the first or second standing-wave pulse is turned on, the state of the medium is described by expressions (9)–(14) with the appropriate boundary conditions, while in the absence of these pulses we solve Eqs. (4)–(6) again with allowance for relaxation and pumping at $a=0$. In the region $\tau+\tau_1+\tau_2\leqslant t$ we determine only $\mathbf{Q}$. Next, using (16), we obtain the amplitudes of the forward and backward waves of the induced field (15). We retain only the terms that contribute to the photon echo. Finally, in the coordinate frame XYZ with X directed along the vector $\mathbf{l}_2$, we obtain

$$\mathbf{\varepsilon}^{(\pm)}=\varepsilon_0T_0\int\limits_0^\infty e^{-(\eta T_0)^2}\sum_{m=2}^\infty G_m^{(\pm)}\left[\frac{1+(-1)^m}{2}\mathbf{I}_m+\frac{1-(-1)^m}{2}\mathbf{J}_m\right]d\eta,$$

$$\begin{aligned}
G_m^{(\pm)}&=J_{m-1}\left(\frac{2\Lambda_1}{\eta}\sin\frac{\eta\tau_1}{2}\right)\cos\left[\eta\left(t-\frac{2\tau+\tau_1+\tau_2}{2}m-\frac{\tau_1}{2}\right)\right]\\
&\quad\times\exp\{(i\Delta-\gamma_0)(t-\tau)\pm i[m(\varphi_2-\varphi_1)+\varphi_1]\},\\
\mathbf{I}_m&=\left\{\mathbf{e}_xJ_m\left(\frac{2\Lambda_2}{\eta}\sin\frac{\eta\tau_2}{2}\right)\cos\psi\cos(\Delta\tau+\Phi_2-\Phi_1)\right.\\
&\quad\left.+\mathbf{e}_yJ_m\left(\frac{\Lambda_2}{\eta}\sin\frac{\eta\tau_2}{2}\right)\sin\psi\exp[i(\Delta\tau+\Phi_2-\Phi_1)]\right\}e^{-\gamma_0\tau},\\
\mathbf{J}_m&=-\left\{\frac{\mathbf{e}_x}{2}J_m\left(\frac{2\Lambda_2}{\eta}\sin\frac{\eta\tau_2}{2}\right)\left[\cos^2\psi+e^{\gamma_{ab}\tau}+\frac{\gamma}{3\gamma_{ab}}(1-e^{\gamma_{ab}\tau})\right]\right.\\
&\quad\left.+\mathbf{e}_yJ_m\left(\frac{\Lambda_2}{\eta}\sin\frac{\eta\tau_2}{2}\right)\sin\psi\cos\psi\right\}e^{-\gamma_a\tau},\\
\varepsilon_0&=(4\pi/3)^{1/2}LN_0|d|\omega/c,\quad \gamma_{ab}=\gamma_a-\gamma_b,\quad \Lambda_n=4a_n|d|/3^{1/2}\hbar,\quad n=1,\ 2,
\end{aligned}\tag{23}$$

where $\mathbf{e}_x$ and $\mathbf{e}_y$ are the unit vectors of the Cartesian axes, $J_m(\theta)$ is a Bessel function of order m and of argument θ, and the terms $\mp z/c$, which take into account the delay of the signals of the forward and backward waves, are left out for brevity ($t\mp z/c\to t$). When (23) is substituted in (15), it is necessary to make the reverse substitution $t\to t\mp z/c$, and we must put $\Phi=\Phi_2$ in (15). The angle ψ is positive when the vector $\mathbf{l}_2\times\mathbf{l}_1$ is directed along the $\mathbf{Z}$ axis.

The basic formula (23) is valid for both a broad spectral line

$$1/T_0\gg1/\tau_1,\quad 1/T_0\gg1/\tau_2,\tag{24}$$

and a narrow one

$$1/T_0\ll1/\tau_1,\quad 1/T_0\ll1/\tau_2.\tag{25}$$

In the case of a narrow spectral line (25), the pulses of the given standing waves interact with all the atoms within the Doppler contour. This enhances the coherent phenomena and simplifies greatly the principal formula

$$\begin{aligned}
\mathbf{\varepsilon}^{(\pm)}&=\varepsilon_0\frac{\pi^{1/2}}{2}\sum_{m=2}^\infty g_m^{(\pm)}\left[\frac{1+(-1)^m}{2}\mathbf{i}_m+\frac{1-(-1)^m}{2}\mathbf{j}_m\right]\\
&\quad\times\exp\left\{-\frac{[t-m(\tau+(\tau_1+\tau_2)/2)]^2}{4T_0^2}+(i\Delta-\gamma_0)(t-\tau)\right\},\\
g_m^{(\pm)}&=J_{m-1}(\Lambda_1\tau_1)\exp\{\pm i[m(\varphi_2-\varphi_1)+\varphi_1]\},\\
\mathbf{i}_m&=\{\mathbf{e}_xJ_m(\Lambda_2\tau_2)\cos\psi\cos(\Delta\tau+\Phi_2-\Phi_1)\\
&\quad+\mathbf{e}_yJ_m(\Lambda_2\tau_2/2)\sin\psi\exp[i(\Delta\tau+\Phi_2-\Phi_1)]\}e^{-\gamma_0\tau},\\
\mathbf{j}_m&=-\left\{\frac{\mathbf{e}_x}{2}J_m(\Lambda_2\tau_2)\left[\cos^2\psi+e^{\gamma_{ab}\tau}+\frac{\gamma}{3\gamma_{ab}}(1-e^{\gamma_{ab}\tau})\right]\right.\\
&\quad\left.+\mathbf{e}_yJ_m(\Lambda_2\tau_2/2)\sin\psi\cos\psi\right\}e^{-\gamma_a\tau}.
\end{aligned}\tag{26}$$

It follows from (23) and (26) that following the action of two standing-wave pulses the excited medium emits spontaneously, at approximate instants of time $t=m\tau$, where $m=2,3,\ldots$, electromagnetic pulses in the form of two traveling waves that propagate in opposite directions (photon echoes). In the particular case of like polarization of the excited pulses, $\psi=0$, the principal formulas (23) and (26) coincide with the results of Ref. 7, where it was assumed that $\gamma_a=\gamma_b$, $\varphi_1=\varphi_2$, $\Phi_1=\Phi_2$, and $\gamma=0$. For exciting pulses polarized at an angle $\psi\neq0$, however, formulas (23) and (26) contain a number of essentially new effects.

We note first that the polarization properties and the law of damping of the amplitude of photon echoes with even numbers $m=2,4,\ldots$ (even echoes) differs substantially from the case of odd echoes with numbers $3,5,\ldots$. In final analysis this is due to the fact that the even echoes are the result of the phase memory concerning the state of the medium in the interval $\tau_1\leqslant t\leqslant\tau_1+\tau$, a memory stored in the matrix elements that make up the medium polarization vector $\mathbf{p}$. At the same time, the odd echoes are the result of the phase memory stored in the matrix elements M and $N_{\alpha\beta}$, which describe the populations of the upper and lower levels in the indicated time interval.

Even echoes are in general elliptically polarized, and the semi-axes of the polarization ellipse and their inclination depend on ψ, on the time interval τ, on the detuning Δ, and on the phase difference $\Phi_2-\Phi_1$. The projection of the amplitude (23) on the direction of the vector $\mathbf{l}_2$ for the even echo contains a characteristic factor $\cos(\Delta\tau+\Phi_2-\Phi_1)$ that causes narrow resonances of width $1/2\tau$ to appear in the radiation intensity when the frequency ω is scanned, if $\Phi_2-\Phi_1$ is constant.[5,6]

In the particular case $\Delta=0$ and $\Phi_2=\Phi_1$ or $\Delta\tau+\Phi_2-\Phi_1=n\pi$ ($n=0,\pm1,\ldots$), the polarization ellipse degenerates into a segment, and the even echoes become linearly polarized. The angle ψ_{me} between the direction of the amplitude of the electric field of the even echo and the vector $\mathbf{l}_2$ is then determined in the case (24) by numerical methods (Figs. 1 and 2), and for the narrow spectral line (25) it is given by the simple relation $\tan\psi_{me}=i_{my}/i_{mx}$ or

$$\operatorname{tg}\psi_{me}=\frac{J_m(\Lambda_2\tau_2/2)}{J_m(\Lambda_2\tau_2)}\operatorname{tg}\psi,\quad m=2,4,\ldots.\tag{27}$$

It follows from (27) that the polarization plane of photon echoes with different values of m do not coincide. This makes their identification easier.

At $\psi=\pi/2$, even echoes are linearly polarized along the vector $\mathbf{l}_1$, and their intensity at the maximum depends on τ only via the factor $\exp(-2m\gamma_0\tau)$, where $m=2,4,\ldots$. If a set of measurements of this intensity is made at different values of τ and with the other parameters a_1, τ_1, a_2, τ_2, m, and $\psi=\pi/2$ fixed, then the experimental values of the intensity can determine

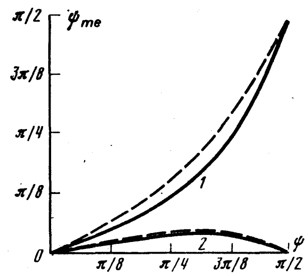

FIG. 1. Dependence of the angle ψ_{me} of the maximum-echo polarization plane on the angle ψ between the polarization planes of the exciting pulses. It is assumed that $t=m[\tau+(\tau_1+\tau_2)/2]+\tau_1/2$, $\Lambda_1\tau_1=\Lambda_2\tau_2=\pi$, and $|\gamma_{ab}|\tau\ll1$. Curves 1 and 2 correspond to even ($m=2$) and odd ($m=3$) echo respectively in the case of a broad spectral line $\tau_1/T_0=\tau_2/T_0=31.4$. The dashed curves corresponding to the same parameters and to the narrow spectral lines (25) are shown for comparison.

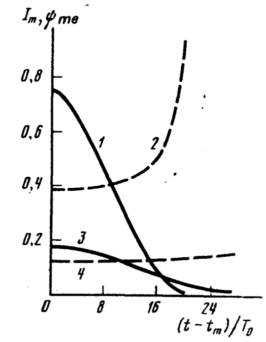

FIG. 2. Intensity I_m and angle ψ_{me} of the polarization plane of the m-th photon echo vs $t-t_m$, where $t_m = m[\tau + (\tau_1+\tau_2)/2] + \tau_1/2$ is the instant of time of the echo maximum. The solid curves described I_m, and the dashed ψ_{me}. Curves 1 and 2 correspond to even echo $m=2$, while curves 3 and 4 correspond to odd echo $m=3$. The indicated curves are symmetrical about the ordinate axis. It is assumed that $\psi = \pi/4$, $\Lambda_1\tau_1 = \Lambda_2\tau_2 = \pi$, and $\tau_1/T_0 = \tau_2/T_0 = 31.4$. Unity intensity on the ordinate axis corresponds to $c(\mathcal{E}_0\Lambda_1 T_0)^2/40\pi$.

the time $1/\gamma_0$ of the transverse relaxation. The angle $\psi = \pi/2$ is singled out here because in this case the measured even-echo intensity does not depend on the phase shifts φ_1, Φ_1, φ_2, and Φ_2.

The odd echoes are always linearly polarized. Their amplitudes have a different dependence on the angle τ and do not contain the characteristic factor $\cos(\Delta\tau + \Phi_2 - \Phi_1)$. Moreover, the intensity of the odd echoes depends under no condition on the phase shifts φ_1, Φ_1, φ_2, and Φ_2. The angle ψ_{me} between the direction of the amplitude of the electric field of the odd echo and the vector $\mathbf{l}_2$ is calculated numerically (Figs. 1 and 2) in the case of the broad spectral line (24), and is determined in the case of the narrow line (25) by the relation $\tan\psi_{me} = j_{my}/j_{mx}$ or

$$\operatorname{tg}\psi_{me} = \frac{J_m(\Lambda_2\tau_2/2)}{J_m(\Lambda_2\tau_2)} \frac{2\sin\psi\cos\psi}{\cos^2\psi + e^{\gamma_{ab}\tau} + (1-e^{\gamma_{ab}\tau})\gamma/3\gamma_{ab}}, \qquad (28)$$

where $m = 3, 5, \ldots$. According to (28), the plane of polarization of the odd echo is rotated when τ changes. In the case $\gamma_{ab}\tau \gg 1$ the dependence on τ reduces to the factor $\exp(-\gamma_{ab}\tau)$ in the numerator of (28), whereas for $|\gamma_{ab}| \ll 1$ the numerator of (28) takes the simple form $\cos^2\psi + 1 + (\gamma_{ab} - \gamma/3)\tau$.

The projections of the amplitude $\varepsilon^{(\pm)}$ of the odd echoes on the direction of the vector $\mathbf{l}_2$ (the X axis) and on the perpendicular direction (Y axis) as τ is increased, have different damping rates, so that the relaxation constants γ_a, γ_b, and γ can be determined. In fact, let $\psi \neq 0$ and $\psi \neq \pi/2$. Then the projections of the amplitude $\varepsilon^{(\pm)}$ of the odd echoes on the Y axis at the instants of time $t = m\tau$ depend on τ on account of the factor $\exp[-\gamma_a\tau - (m-1)\gamma_0\tau]$, where $m = 2, 5, \ldots$, and γ_0 is known. This makes it possible to determine experimentally the width γ_a of the lower level. At low values $\gamma_a\tau \ll 1$ and $\gamma_b \ll 1$ and at $\psi = 0$ the amplitudes of the odd echoes are proportional to the simple factor $2 - (2\gamma_a - \gamma_{ab} + \lambda/3)\tau$. The experimental determination of these amplitudes as functions of τ, together with the determination of the rotation of the polarization plane (28) as a function of τ, makes it possible to determine γ_{ab} and γ, since the constant γ_a has already been determined above. This means that all the sought constants γ_0, γ_a, γ_b, and γ can be determined experimentally from the polarization properties of the photon echoes produced by standing-wave pulses.

For the atomic transition $j_b = 1 \rightarrow j_a = 0$. The calculations are similar, and the basic formulas are obtained from the (23) and (26) above by making the formal substitution

$$\gamma \to 3\gamma,\ N_0 \to N_a - N_b/3,\ \gamma_a \to \gamma_b,\ \gamma_b \to \gamma_a,$$

which is not reflected in the polarization properties of the photon echo.

We note for comparison that after the passage of two exciting traveling-wave pulses, a photon echo with polarization of the second exciting pulse, is produced on the atomic transitions $1 \rightleftarrows 0$ and its amplitude attenuates like $\exp(-2\gamma_0\tau)$.[8,9] Thus, the excitation of the photon echoes by standing wave pulses is accompanied by qualitatively new effects.

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