Attenuation of ultrasound in thin metal layers located in a magnetic field

V. M. Gokhfel'd, O. V. Kirichenko, and V. G. Peschanskiĭ

Donetsk Physico-technical Institute, Ukrainian Academy of Sciences and Physicotechnical Institute of Low Temperature, Ukrainian Academy of Sciences
(Submitted 21 December 1979)
Zh. Eksp. Teor. Fiz. **52**, 538–554 (August 1980)

Magnetoacoustic size effects in a thin metallic layer are studied theoretically by the kinetic equation technique, employing the integral boundary condition. The parameters of the condition, i.e., the total probability and the relative width of the scattering indicatrix, describe in greater detail the interaction between the electrons and the metal suface than does the Fuchs specularity coefficient that was employed previously [Sov. Phys. JETP. **34**, 407 (1972)]. The dependence of the damping decrement of short wavelength sound Γ on the value of the weak magnetic field turns out to be significantly different for the various models of the scattering indicatrix. Sondheimer-type oscillations of Γ are investigated in strong fields for both open and closed Fermi surfaces. The case of multichannel surface scattering of the charge carriers is analyzed. An experimental observation of the predicted effects allow us to establish the values of the parameters of the boundary condition and also the local characteristic of the Fermi surface.

PACS numbers: 43.35.Cg, 43.35.Rw, 72.55. + s

1. Magnetoacoustic phenomena in bulk samples are widely used for the study of the electron energy spectra of metals. There is no less interest in size effects, which appear in the propagation of the ultrasound in conductors that are thin in comparison with the free path lengths of the charge carriers l: they turn out to be very sensitive not only to the form of the dispersion law of the conduction electrons, but also to the character of their interaction with the surface of the conductor.

In the description of quasiclassical size effects, the boundary condition for the electron distribution function is very important. In a significant portion of theoretical researches, it is used in the form proposed by Fuchs[1]:

$$f_s(\mathbf{p}_-)=qf_s(\mathbf{p}_+)+(1-q)\ \mathrm{const}, \tag{1}$$

where the "parameter of specularity" q is the probability of specular reflections, for which the quasimomenta of the electron before and after its collision with the boundaries of the metal are connected by the relations

$$\varepsilon(\mathbf{p}_-)=\varepsilon(\mathbf{p}_-),\quad [\mathbf{p}\times\mathbf{n}]=[\mathbf{p}\times\mathbf{n}],$$
$$\mathrm{sign}\, v_n(\mathbf{p}_-)=-\mathrm{sign}\, v_n(\mathbf{p}_-)<0,$$

where $\mathbf{n}$ is the outward normal to the surface of the sample. In the specularity parameter approximation, the possibility was shown of observation in thin conductors of a number of new magnetoacoustic effects, containing information on the local characteristics of the Fermi surface.[2]

However, the applicability of this approximation is, generally speaking, limited. It is clear that in the general case, the distributions of the incident and reflected particles should be connected by the integral relation

$$f_s(\mathbf{p}_-)=\hat{W}f_s(\mathbf{p}_+)\equiv\int d\mathbf{p}_+' W(\mathbf{p}_+,\mathbf{p}_+')f_s(\mathbf{p}_+'),\quad v_n(\mathbf{p}_+')>0, \tag{3}$$

and only the very distinct form of its kernel (a sharp maximum of $W(\mathbf{p}_+,\mathbf{p}_+')$ at $\mathbf{p}_+\approx\mathbf{p}_+'$ and a smooth variation in the remaining region of integration) permits us to resort to the Fuchs condition (1). But even in this case, the specularity parameter turns out to be a functional of the distribution $f(\mathbf{p}_+)$, i.e., it describes not only the surface of the given sample, but also the conditions of the experiment (for example, it depends on the ultrasonic frequency.

What has been said is especially important for magnetoacoustic phenomena, in which the electron distribution is a rapidly changing function of the quasimomentum, and in the present work we shall use a boundary condition in the form proposed by Fal'kovskiĭ[3] and by Okulov and Ustinov[4,5]:

$$f_s(\mathbf{p}_-)=\hat{W}f_s(\mathbf{p}_+)\equiv f_s(\mathbf{p}_+)+P\int d\mathbf{p}_+' w(\mathbf{p}_+,\mathbf{p}_+')[f_s(\mathbf{p}_+)-f_s(\mathbf{p}_+')],$$
$$v_n(\mathbf{p}_+')>0. \tag{4}$$

Here P is the integral probability of electron scattering upon reflection from the boundary, $P\leqslant 1$, while $w(\mathbf{p}_+,\mathbf{p}_+')$ is the scattering indicatrix normalized to unity:

$$\int w(\mathbf{p}_+,\mathbf{p}_+')d\mathbf{p}_+'=1. \tag{5}$$

At $P=1$, the condition (4) is a completely integral one; the case $P=0$ corresponds to purely specular reflection of the particles.

It will be shown below that the oscillation magnoacoustic size effects turn out to be stable relative to the form of the boundary condition (the periods of oscillation and the very fact of their existence are not critical to the form of the scattering indicatrix) and can be used as a method of detailed study of the electron energy spectrum. Here the form of the oscillations, and the especially smooth dependence of the ultrasonic absorption on the value of the magnetic field are different for different models of the scattering indicatrix, which makes it possible to analyze the character of the interaction of the charge carriers with the boundaries of the sample by means of the experimental data.

 0038-5646/80/080271-09$02.40

2. We consider the electron absorption of a longitudinal sound wave

$$u=(u_0 \exp\{ikx-i\omega t\};\, 0;\, 0),$$

propagating along the normal to the layer of metal $0\leqslant x\leqslant d$. The thickness of the layer is assumed to be small in comparison with the volume free path length of the charge carriers l, but much greater than the sound wavelength

$$k^{-1}\ll d\ll l.$$

This condition corresponds to modern experimental possibilities and allows us to neglect distortions of the sound field near the boundaries of the sample. In a magnetic field $\mathbf{H}=(0;0;H)$ parallel to the layer, the Boltzmann equation for the nonequilibrium distribution function of the conduction electrons

$$f=f_0(\varepsilon)+\chi(p_z,\lambda,t)\,\dot{u}_{xx}\frac{\partial f_0}{\partial\varepsilon} \tag{6}$$

has the form

$$(\partial/\partial t+v_x\partial/\partial x\pm ikv_x-i\omega+\nu)\chi^{\pm}=\Lambda_{xx}\equiv\Lambda. \tag{7}$$

Here and below, t is the time of motion of the electron along the Larmor orbit, Λ_{ik} is the deviation of the deformation potential from its mean value on the Fermi surface, ν is the frequency of intravolume scattering of the charge carriers. After the next collision of the electron with the boundary of the layer (at the instant λ_n) the solution of Eq. (7) has the form

$$\chi^{+}(\lambda_n,t)=A_n\mathscr{E}_{\lambda_n}^{\,t}+\int_{\lambda_n}^{t}dt'\Lambda(t')\mathscr{E}_{t'}^{\,t}, \tag{8}$$

where

$$\mathscr{E}_{\alpha}^{\,\beta}\equiv\exp\left\{\int_{\alpha}^{\beta}(i\omega-i\mathbf{kv}-\nu)\,dt\right\}.$$

In a weak magnetic field, when the Larmor radius $r\geqslant d$, practically all the electrons inevitably collide with the surface of the layer, and the character of their collision manifests itself in significant fashion in the magnetoacoustic effects. The electrons, reflected specularly, move along open quasiperiodic orbits (Fig. 1) and if the probability of volume scattering within the period T is small,

$$\nu T\sim(dr)^{1/2}/l\ll 1,$$

then resonance sound absorption can be expected. We note that in a thin layer of the metal ($d\ll l$), this effect should be observed in weaker fields than in a bulk sample.[6,7]

By repeatedly using the boundary condition

$$\chi^{+}(\lambda_n,\lambda_n)=\hat{W}\chi^{+}(\lambda_{n+1},\lambda_n), \tag{9}$$

we can write out the solution of the kinetic equation (7) in the form of a sum of perturbations undergone by the electron in the sound field on individual portions of its trajectory, separated by collisions with the boundary, with account of volume and surface scattering. Inasmuch as only electrons with $\mathbf{k}\cdot\mathbf{v}=\omega$ effectively interact with the ultrasound, one should take into account only the portions containing the turning point $v_x=0$. Taking it as the point from which the phase of the electron on each portion of the orbit is reckoned, we obtain

$$\chi^{+}(-\lambda,t)=\int_{-\lambda}^{t}dt'\Lambda\mathscr{E}_{t'}^{\,t}+\mathscr{E}_{-\lambda}^{\,t}\sum_{n=0}^{\infty}(\hat{W}\mathscr{E}_{-\lambda}^{\,\lambda})^{n}\hat{W}\int_{-\lambda}^{\lambda}dt\Lambda\mathscr{E}_{t}^{\,\lambda}. \tag{10}$$

The symbolic notation $(\hat{W})^n f(\lambda,p_z)$ indicates the n-fold application of the operation (4) in the variables λ and p_z.

By knowing the nonequilibrium increment $\chi^{\pm}\partial f_0/\partial\varepsilon$ to the electron distribution function, we can calculate the electron dissipation function and, consequently, the sound absorption coefficient

$$\Gamma(H)=\frac{4k^2eH}{cd\rho h^3}\operatorname{Re}\int_0^{p_m}dp_z\int_0^{\lambda_m}v_x(\lambda)\,d\lambda\int_{-\lambda}^{\lambda}dt\Lambda(\chi^{+}+\chi^{-}). \tag{11}$$

Here ρ is the density of the crystal, while the maximum (for a given p_z) period of the motion $T=2\lambda_m$ is determined from the conditions

$$\int_0^{\lambda_m}v_x(t)\,dt=d,\quad \lambda_m\approx(2d/v_x'(0))^{1/2}. \tag{12}$$

We note that in a not too weak magnetic field, at sufficiently small relative width of the scattering indicatrix Δ, the following inequalities are easily satisfied:

$$krs\Delta/v_F\ll 1,\quad r\Delta/l\ll 1, \tag{13}$$

and the quantity

$$\mathscr{E}_{-\lambda}^{\,\lambda}=\exp\{2\lambda(i\omega-\nu)\}$$

is practically unchanged by the surface scattering of the electron. After integration over t, the sound absorption coefficient turns out to be equal to

$$\Gamma\approx(4k^2eH/cd\rho h^3)\int_0^{p_m}dp_z\int_0^{\lambda_m}v_x(\lambda)\,d\lambda\Big\{\psi\psi^{*}+2\operatorname{Re}\sum_{n=1}^{\infty}\exp(-2n\nu\lambda)\cos(2n\omega\lambda)\,\psi^{*}(\hat{W})^{n}\psi\Big\}, \tag{14}$$

where

$$\psi(\lambda,p_z)\equiv\exp(-\lambda(i\omega-\nu))\int_{-\lambda}^{\lambda}dt\,\Lambda\mathscr{E}_t^{\,\lambda}\approx(\pi/-i\alpha)^{1/2}\exp(-i\alpha\lambda^2)\,\Phi(\lambda(-i\alpha)^{1/2}),\quad \alpha\equiv kv_x'(0)/2,$$

Φ is the probability integral, which differs significantly from unity only at small absolute values of the argument, and s is the sound speed.

We now introduce the quantity

$$Q(\lambda,p_z)\equiv 1-P\int dp_z'\int d\lambda' w(\lambda,\lambda',p_z,p_z')\,[1-\psi(\lambda',p_z')/\psi(\lambda,p_z)], \tag{15}$$

so that $W\psi=Q\psi$. As will be clear from what follows, $Q(\lambda,p_z)$ is a smooth function in comparison with the scattering indicatrix and repeated application of $\hat{W}$ reduces to multiplication by Q: $\hat{W}^n\psi\approx Q^n\psi$. This allows us to rewrite the expression (14) in the form

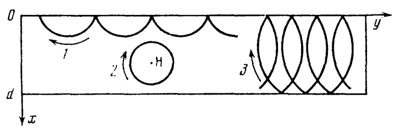

FIG. 1.

$$\Gamma(H)/\Gamma(0)=\int_0^{p_m} J(p_z)\,dp_z/p_m,$$

$$J(p_z)\approx(4/\lambda_m^2)\,\mathrm{Re}\int_0^{\lambda_m} d\lambda\lambda|\Phi|^2\sum_{n=0}^{\infty}[Q\exp(-2\nu\lambda)]^n\cos(2n\omega\lambda), \qquad (16)$$

which formally corresponds to the approximation of the specularity parameter, the role of which, however, is played by the function $Q(\lambda, p_z)$. It can be computed, however, only by specifying a definite model of the scattering indicatrix.

We shall assume that w depends only on $|p'_+ - p_+|$ and represents a Gaussian or the even simpler step distribution

$$w(|p_+'-p_+|/p_0)\equiv w(z)\sim\exp(-z^2/\Delta^2), \qquad (17)$$

$$w(z)\sim\Theta(z-\Delta), \qquad (18)$$

Θ is the Heaviside function. The normalizing factor is chosen from the condition (5); in the limit $\Delta\to 0$ the function $w(z)$ is equivalent to a δ function (purely specular reflection). Then, calculating the integral (15), we get

$$Q\approx 1-P, \quad \varkappa\gg 1,$$
$$Q\approx 1-P\xi\Delta^2[(kv_\perp p_0\lambda/p_\perp)^2+ikrp_0/p_\perp], \quad \varkappa\ll 1, \qquad (19)$$

where $p_\perp$ is the radius of curvature of the intersection of the Fermi surface with the plane p_z = const, and $v_\perp$ is the velocity of the electron in this plane at the turning point

$$r\equiv cp_0/eH=cp_{\perp\,\mathrm{extr}}/eH.$$

The choice of the model (17) or (18) for the scattering indicatrix manifests itself only in the value of the numerical factor ξ (equal respectively to $\frac{1}{4}\sqrt{\pi}$ or $\frac{1}{6}$). This is the basis for assuming that the results obtained below with the help of the expression (19) are qualitatively valid for an arbitrary central distribution w.

The parameter $\varkappa=(dr)^{1/2}k\Delta$ has a simple physical meaning: $(dr)^{1/2}\Delta$ is the mean displacement, along the wave vector k, of the turning point of the electron after its surface scattering. The smallness of this displacement in comparison with the sound wavelength makes it possible for the electron to interact in resonance fashion with the sound field; the opposite case, for our problem, is equivalent to "diffuse" scattering of the electrons with probability P.

3. We calculate the damping coefficient of the low-frequency sound. At $\omega\ll\nu$, the sound field is practically unchanged within the time of the free path of the electrons, and taking in (16) the limit as $\omega\to 0$ we obtain

$$J(p_z)=(4/\lambda_m^2)\,\mathrm{Re}\int_0^{\lambda_m} d\lambda\lambda(1-Qe^{-2\nu\lambda})^{-1}. \qquad (20)$$

In calculating this integral, it must be kept in mind that, since the characteristic angles of incidence of the "effective" electrons on the surface of the layer are small, $\vartheta\sim(d/r)^{1/2}\ll 1$, the total scattering probability P can turn out to be proportional to ϑ:

$$P(\vartheta)=C\vartheta\approx C\lambda eHv_\perp^2/cv_0p_\perp, \qquad (21)$$

(see, for example, Ref. 8), where v_0 is the velocity of the electron at the turning point. Omitting unessential factors of the order of unity, which depend on the shape of the Fermi surface, we can write down the results in the following form (Fig. 2):

1) $\varkappa\gg 1$, $P(\vartheta)$ = const. If the scattering indicatrix is a smooth function, then the dependence $\Gamma(H)$ is determined by the total scattering probability:

$$\Gamma(H)=\Gamma_{\mathrm{spec}}=\Gamma_0 l/(dr)^{1/2}, \quad Pl/(1-P)(dr)^{1/2}\ll 1,$$
$$\Gamma(H)=\Gamma_0(2-P)/P, \quad Pl/(1-P)(dr)^{1/2}\gg 1, \qquad (22)$$

and goes into saturation sooner the larger the value of P. In the case of diffuse scattering ($P\approx 1$), the sound absorption coefficient in a weak field ($r\gg d$) does not depend on H and is identical with the bulk-sample value calculated in Refs. 9 and 10:

$$\Gamma_0\sim n_0\varepsilon_F\omega/\rho s v_F; \qquad (23)$$

2) $\varkappa\gg 1$, $P(\vartheta)=C\vartheta$ Here there is a maximum on the $\Gamma(H)$ curve when $Cl\sim r$:

$$\Gamma(H)=\Gamma_{\mathrm{spec}}, \quad Cl\ll r;$$
$$\Gamma(H)=\Gamma_0(r/dC^2)^{1/2}, \quad Cl\gg r. \qquad (24)$$

Only in these two cases is the specularity-parameter approximation valid, which parameter is equal to $1-P$, as was to be expected.

If the same scattering indicatrix has a sharp maximum for angles corresponding to specular reflection of the electrons from the boundaries of the layer, then the result turns out different, i.e.,

3) $\varkappa\ll 1$, $P(\vartheta)$ = const: $\Gamma(H)=\Gamma_{\mathrm{spec}}((dr)^{1/2}/lP\varkappa^2)\ln(1+lP\varkappa^2/(dr)^{1/2})$; (25)

4) $\varkappa\ll 1$, $P(\vartheta)=C\vartheta$:

$$\Gamma(H)=R\Gamma_{\mathrm{spec}}, \quad 0<R<1, \qquad (26)$$

where the magnetic-field-independent quantity

$$R=\mathrm{Re}\int_0^1 dx(1+iu+2ukdx^2)^{-1}, \quad u=kl\xi\Delta C^2/2, \qquad (27)$$

is determined by the parameters of the scattering indicatrix model.

A common result is that the sound absorption in the conducting layer in a weak magnetic field is greater than in its absence, and at a sufficient degree of specularity of the reflection of the electrons from the boundaries of the conductor [$P\ll 1$ or $k\Delta(dr)^{1/2}\ll 1$) the specularity can be much greater than in the bulk sample. However, the dependences of $\Gamma(H)$ in cases 1)–4), shown in Fig. 2, are different. A comparison of these curves with the experimental curves obtained at various sound frequencies (in the range $\nu\gg\omega\gg s/d$) allow us to assess the character of the scattering of the

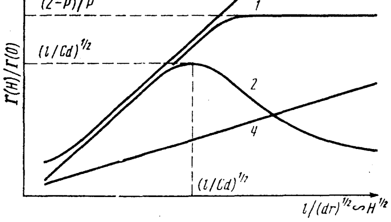

FIG. 2.

electrons by the surface of the metal and to estimate the scattering-indicatrix parameter.

We now proceed to the case of high sound frequencies, when the following inequalities are satisfied in a weak magnetic field:

$$\omega T \approx \frac{2\omega}{v_\perp}\left(\frac{2cdp_\perp}{eH}\right)^{1/2} \sim \frac{\omega}{v_F}(dr)^{1/2} \gg 1, \quad \nu T \ll 1. \qquad (28)$$

Here it is already impossible to consider the sound wave as statistical and only those electrons interact effectively with it whose period of motion in the "glancing" orbits T is a multiple of the period of the sound wave: a size cyclotron resonance is generated.

Integrating the expression (16) for the sound attenuation coefficient by the stationary phase method, we obtain

$$\frac{\Gamma(H)}{\Gamma_0} \approx 1+\eta \operatorname{Re} \sum_{n=1}^{\infty} \frac{(Q_0 \exp(-\nu T_0))^n}{(n\omega T_0)^{1/2}} \sin\left(n\omega T_0 \pm \frac{\pi}{4}\right), \qquad (29)$$

$$\eta = (8\pi T_0/|T_0''| p_m^2)^{1/2}.$$

Here T_0 is the extremal value of the period T, and T_0'' is its second derivative with respect to p_z; the sign in the argument of the sine function correspond to the sign of T_0''. Formula (29) describes the oscillations of the ultrasound absorption, which are periodic in the square root of the reciprocal field.

We can estimate the total probability of surface and volume scattering of the electron within the period of its motion, $Q_0 + \nu T_0$, from the experimental $\Gamma(H)$ dependence. Thus, if this probability is small, then the following relation is valid

$$(Q_0+\nu T_0)^{1/2} \approx \frac{\pi^{3/2}}{\zeta(3/2)} m^3\delta^2 \left|\frac{\Gamma(H_m)}{\partial\Gamma/\partial H_m}\right|, \quad \delta = \frac{\pi v_F e^{1/2}}{\omega(2cp_\perp d)^{1/2}}, \qquad (30)$$

where $\zeta(x)$ is the Riemann function, and $H_m^{-1/2} \equiv m\delta$, $m = 1, 2, 3, \ldots$. The quantity Q_0, which plays the role of the effective specularity parameter in the given case, can be expressed in terms of the relative width of the scattering indicatrix Δ and the total scattering probability $P(\vartheta)$:

$$Q_0 \approx 1 - P(\sqrt{2d/r}), \quad \varkappa \gg 1,$$
$$Q_0 \approx 1 - P(\sqrt{2d/r})\,\xi k^2\Delta^2 dr, \quad \varkappa \ll 1. \qquad (31)$$

It is easy to generalize the result obtained above to the case of arbitrary polarization of the sound waves. The distribution function of the charge carriers must be sought in the form (5) by replacing $\chi(p_z, t, x)\dot{u}_{xx}$ by the quantity $\chi_{ij}(p_z, t, \mathbf{r})\dot{u}_{ij}$ and replacing $\Lambda_{xx}\dot{u}_{xx}$ in the kinetic equation (7) by the quantity $g = \Lambda_{ij}\dot{u}_{ij} + e\mathbf{E}\cdot\mathbf{v}$. For transverse waves, we must take into account in the calculation of the damping decrement the electric field $\mathbf{E}$ produced by the crystal deformations,[10-13]; this field can be found by means of Maxwell's equations and can be considered in the form of a renormalization of the tensor Λ_{ij}:

$$g = \Lambda_{ij}\dot{u}_{ij} + e\mathbf{E}\mathbf{v} = g_{ij}(\mathbf{p})\,\dot{u}_{ij}. \qquad (32)$$

Here it is easy to verify that the magnetoacoustic effects in weak magnetic fields are little sensitive to the polarization of the sound wave.

4. In strong magnetic fields ($r \ll d$), oscillations of Γ, of the type of Sondheimer oscillations,[14] arise as functions of the magnetic field H and of the thickness of the sample d and are due to the drift of the charge carriers into interior of the plate. In the absence of open electron orbits, they are possible only when the vector $\mathbf{H}$ is inclined to the plane of the surface of the plate by an angle $\vartheta \gtrsim dv/v$.

For calculation of $\Gamma_{\text{osc}}(H, d)$, the solution of the kinetic equation must be represented in the form

$$\chi_{ij}(p_z, t, \mathbf{r}) = \int_{\lambda_1}^{t} g_{ij}(p_z, t')\mathscr{E}_{t'}{}^{t} + \int dp_{z1}' \int d\varphi_1\, W\cdot(p_z, \lambda_1''; p_{z1}, \lambda_1')$$
$$\cdot \int_{\lambda_2}^{\lambda_1} g_{ij}(p_{z1}, t_1) G_1(t', p_{z1})\, dt'$$
$$+ \ldots + \int dp_{z1} \ldots \int dp_{zn} \int d\varphi_1 \ldots \int d\varphi_n W(p_z, \lambda_1''; p_{z1}, \lambda_1') \qquad (33)$$
$$\times W(p_{z1}, \lambda_2''; p_{z2}, \lambda_2') \ldots W(p_{zn-1}, \lambda_n''; p_{zn}, \lambda_n')$$
$$\times \int_{\lambda_{n+1}}^{\lambda_n} g_{ij}(p_{zn} t_n) G_n(t', p_{zn})\, dt' + \ldots,$$

and then the functions $g(p_z, t)$ and $W(p_z, \varphi; p_z', \varphi')$ expanded in Fourier series:

$$g(p_z, t) = \sum_m g_{ij}{}^m(p_z)\exp(im\Omega t)\,\dot{u}_{ij} = \sum_m g_m(p_z)\exp(im\Omega t),$$
$$W_m{}^{m'}(p_z, \varphi; p_z', \varphi') = \sum_{m,m'} W_m{}^{m'}(p_z, p_z')\exp(im\varphi - im'\varphi'). \qquad (34)$$

Here W is the kernel of the integral operator (3), λ_n are the moments of the collisions of electrons with the surface of the sample at the points $\mathbf{r}_n$[φ_n are the phase jumps in the trajectory of the electron upon reflection ($\lambda_{n+1} < \lambda_n$, $n = 1, 2, 3, \ldots$)]. The function $G_n(t', p_{zn})$ is determined by the following relation

$$G_n(t', p_{zn}) = \exp\{(\nu - i\omega)(t' - t) + i\mathbf{k}(\mathbf{r}_n - \mathbf{r} + \mathbf{r}(p_{zn}, t_n) - \mathbf{r}(p_{zn}, \lambda_n'))\},$$
$$\Omega_n t_n = \Omega_n t' + \varphi_1 + \varphi_2 + \ldots + \varphi_n, \quad \Omega_n\lambda_n' = \Omega_n\lambda_n + \varphi_1 + \varphi_2 + \ldots + \varphi_n,$$
$$\Omega_{n-1}\lambda_n'' = \Omega_{n-1}\lambda_n + \varphi_1 + \varphi_2 + \ldots + \varphi_{n-1},$$
$$\Omega_n \equiv \Omega(p_{zn}), \quad \mathbf{r}(p_z, t) = \int^{t} \mathbf{v}(p_z, t')\, dt'.$$

After integration over t' and φ_n with account of (34), the integrand expression for χ_{ij} will contain three types of functions

$$\exp\{id/r(p_z)\}, \quad \exp\{ikr(p_z, t)\}, \quad W_m{}^{m'}(p_{zn-1} p_{zn})$$

with a different sharpness. The first of these selects the electrons responsible for the Sondheimer oscillations. These are electrons from the vicinity of the limiting point on the Fermi surface and electrons with extremal displacements $2\pi r_e$ in the period T in the depth of the sample.

The amplitude of the Sondheimer oscillations depends significantly on the relation between the parameters d/r, kr, and the width of the scattering indicatrix Δ, and can be calculated without difficulty for an arbitrary dispersion law of the carriers and an arbitrary character of their reflection from the boundaries of the sample.

If the sound wave propagates along the plate, and drift of the charge carriers is possible only along the normal to the surface of the sample, then at $kr \ll 1$ the electrons do not "notice" the inhomogeneity of the sound wave, and the dependence of the amplitude

$\Gamma_{osc}(H,d)$ on the magnetic field and the plate thickness turns out to be the same as for Sondheimer oscillations of the transverse magnetoresistance ρ_{osc} of thin plates in a homogeneous electric field.[15] It is easy to verify this if we use formulas (33) and (34) and retain the fundamental approximation in the small parameter kr in the expression for Γ:

$$\Gamma(H,d)=\Gamma_\infty+\frac{1}{\rho\dot{u}^2 d}\operatorname{Re}\left\langle\frac{|\bar{v}_z|g_{-m}(p_z)}{i\mathbf{kv}+\alpha_m}\left[1-\exp\left\{-\frac{d}{|\bar{v}_z|}(i\mathbf{kv}+\alpha_m)\right\}\right]\right.$$
$$\times\left[-\frac{g_m(p_z)}{i\mathbf{kv}+\alpha_m}+\sum_{j=1}^{\infty}s_j\int dp_{z1}\ldots\int dp_{zj}W_m^{m_1}(p_z,p_{z1})\ldots\right.$$
$$\ldots W_{m_j}^{m_{j-1}}(p_{zj-1},p_{zj})\frac{g_{m_j}(p_{zj})}{i\mathbf{kv}_j+\alpha_{m_j}}\tag{35}$$
$$\left.\left.\times\exp\left\{-\sum_{k=1}^{j-1}\alpha_{m_k}d/|\bar{v}_{zj}|\right\}\left(1-\exp\left\{-\frac{d}{|\bar{v}_{zj}|}(i\mathbf{kv}_j+\alpha_{m_j})\right\}\right)\right]\right\rangle.$$

Here Γ_∞ is the absorption coefficient of the acoustic energy in the bulk sample, $\mathbf{v}_j=\mathbf{v}(p_{zj})$ is the electron velocity averaged over a single period,

$$\alpha_n=\nu+i(n\Omega-\omega),\quad s_j=\exp\frac{ikd\bar{v}_z}{2|\bar{v}_z|}[1+(-1)^j].$$

In the asymptotic formula (35) there are terms $\mathbf{k}\cdot\mathbf{v}_j$ in the expressions $(\alpha_{mj}+i\mathbf{k}\cdot\mathbf{v})$ inasmuch as they play the principal role in the case $\nu\ll k\nu\ll\Omega$ for $m_j=0$.

If the sound wave propagates normal to the metallic layer, and the Fermi surface possesses axial symmetry, so that v_z does not depend on t, then formula (35) is exact for any value of the magnetic field. At $P=0$, all the m_j are equal to m, and $p_{zj}=p_z(-1)^j$, and uncomplicated transformations allow us to represent Γ in the following form:

$$\Gamma=\Gamma_\infty+\frac{\Gamma_0}{kv\dot{u}^2p^3d}\operatorname{Re}\int_0^p v_z dp_z\left[\frac{g_m(p_z)}{ikv_z+\alpha_m}-\frac{g_m(-p_z)}{-ikv_z+\alpha_m}\right]$$
$$\times\left[\frac{g_{-m}(p_z)}{ikv_z+\alpha_m}-\frac{g_{-m}(-p_z)}{-ikv_z+\alpha_m}\right]\left\{2\cos kd\sum_{n=1}^{\infty}\exp\left[-(2n-1)\left(im\frac{d}{r(p_z)}+\frac{d}{l}\right)\right]\right.$$
$$\left.-2\sum_{n=1}^{\infty}\exp\left[-2n\left(im\frac{d}{r(p_z)}+\frac{d}{l}\right)\right]-1\right\}.\tag{36}$$

If $v_z(p_z)$ is a monotonic function, then $\Gamma_{osc}(H,d)$ is determined by a small neighborhood of order r/d near the upper limit of integration over p_z. Here we must keep it in mind that at $m\neq 0$, $g_m(p_z)$ goes to zero at $p_z=p$ at least like $(p-p_z)^{1/2}$. For the longitudinal wave $g_m(-p_z)=g_m(p_z)$ and for the transverse wave the odd component $g_m(p_z)$ can differ from zero.

Representing the function $g(p_z)$ in the form

$$g_m(p_z)=pvk\dot{u}(1-p_z^2/p^2)^{1/2}[f_m(p_z/p)+f_m'(p_z/p)],$$
$$f_m(-x)=f_m(x),\quad f_m'(-x)=-f_m'(x),\quad f_m(x)\sim 1,\quad f_m'(x)\sim 1,$$

and integrating over p_z in the formula (36), we obtain the following expression for Γ_{osc}

$$\Gamma_{osc}(H,d)=\begin{cases}\Gamma_0\dfrac{kr^4}{d^3}(|f_m'|^2+(kr)^2|f_m|^2)J_m, & kr\ll 1,\\[2mm] \Gamma_0\dfrac{1}{(kd)^3}(|f_m'|^2(kr)^2+|f_m|^2)J_m, & kr\gg 1,\quad kr\neq m.\end{cases}$$

where

$$J_m=\sum_{n=1}^{\infty}\left[-\frac{\cos kd}{(2n-1)^2}\exp(-(2n-1)\beta_m)+\exp(-2n\beta_m)\frac{1}{4n^2}\right],$$
$$\beta_m=im\frac{d}{r}+\frac{d}{l}.\tag{38}$$

It is easy to note that the factor J_m, which oscillates with H and d, tends to a finite limit as $l=v/\nu\to\infty$; however, its derivative dJ_m/dH has a logarithmic singularity $dJ_m/dH\approx\ln(l/d)$ if the electron at the limiting point of the Fermi surface executes an integral number of revolutions on a path equal to d.

The amplitude of the Sondheimer oscillations Γ increases significantly if the condition $kr=m$ is satisfied for a given magnetic field. For the mth component in formula (36), the denominators in the square brackets are much steeper functions than $e^{id/r}$ near $p_z=p$ and the result for Γ_{osc} turns out to be completely different:

$$\Gamma_{osc}=\frac{\Gamma_0}{kd}\sum_{n=1}^{\infty}\exp(-2n(ikd+d/l))\left\{2\int_{2n\phi}^{\infty}\frac{dx\exp(-(i+\gamma)x)}{x-2in\delta}\right.$$
$$\left.-\int_{(2n-1)\phi}^{\infty}\frac{dx\exp(-(i+\gamma)x)}{x-(2n-1)i\delta}-\int_{(2n+1)\phi}^{\infty}\frac{dx\exp(-(i+\gamma)x)}{x-(2n+1)i\delta}\right\},\tag{39}$$

where $\phi=md/r-kd$, $\delta=d/l$, $\gamma=r/l$. Formula (39) is valid only at small ϕn and simple analysis shows that at $\phi=0$ the amplitude of the Sondheimer oscillations is equal to

$$\Gamma_{osc}^{max}=(\Gamma_0/kd)\ln(l/d).\tag{40}$$

If the function $v_z(p_z)$ has an extremum at some point $p_z=p_e$, then the factor J_m, which oscillates with H and d, has different form:

$$J_m=\left(\frac{d}{r}\right)^{1/2}\sum_{n=1}^{\infty}\left\{-\frac{\cos kd}{(2n-1)^{1/2}}\exp(-(2n-1)\beta_m')+\frac{1}{(2n)^{1/2}}\exp(-2n\beta_m')\right\},\tag{41}$$

where $\beta_m'=imd/r_e+d/l$. Since the numerical factors of order unity are omitted, in the presence of an extremal displacement of the electron in the interior of the sample we can use formula (37) for Γ_{osc}. However, J_m must be taken in the form (41). It is easy to show that the amplitude of Γ_{osc} increases with increase in l as $(l/d)^{1/2}$ if $kr_e\neq m$, and at $kr_e=m$ we shall have $\Gamma=\Gamma_0(kl)^{1/2}l/d$.

Simple analysis shows that for a description of the oscillatory dependence of Γ on H and d for arbitrary character of the reflection, it is quite appropriate to use the specularity-parameter approximation and we easily obtain the following formula for Γ_{osc}:

$$\Gamma_{osc}=\Gamma_{osc}^{spec}(\nu_{eff}),\tag{42}$$

i.e., ν in formulas (39)–(41) must be replaced by the effective collision frequency $\nu_{eff}=\nu-(v/d)\ln(1-P)$, which takes into account the dissipative character of reflections of the electrons from the boundaries of the sample.

In metals with open Fermi surfaces, oscillations of the sound absorption of the Sondheimer type take place even in a parallel magnetic field, thanks to the drift of the electrons with open orbits into the interior of the sample. Their displacement along the normal to the plate within the period of motion is identical for the whole layer of open cross sections of the Fermi surface; therefore the amplitude of the oscillations does not contain small factors of the type $(r/d)^{1/2}$ or $(r/d)^2$, which determine the fraction of the effective electrons in the case of closed orbits. At not too low temperatures, when the classical consideration of magnetoacoustic effects is valid, it is easy to calculate $\Gamma_{osc}(H,d)$ with the help of formulas (35)–(37).

In thin metallic films ($d \ll l$) one must, generally speaking, take into account the quantization of the transverse motion of the electrons.

Let a layer of metal $0 \le x \le d$ have open sections of the Fermi surface along p_y:

$$\pm p_x = p_x(\varepsilon_F, p_y, p_z) = p_x(\varepsilon_F, p_y + P_0, p_z). \quad (43)$$

The electrons of these sections in an arbitrarily strong magnetic field $H=(0,0,H)$ inevitably collide with the boundaries of the layer and in the case of specular reflection their spectrum turns out to be discrete:[16]

$$\sigma(\varepsilon, p_{y0}, p_z) \equiv \frac{2c}{eHh} \int_{p_{y0}-eHd/2c}^{p_{y0}+eHd/2c} p_x(\varepsilon, p_z, p_y)\, dp_y = n \quad (44)$$

(n is a positive integer). Consequently, the density of states has singularities at the points $\varepsilon_n(H,d)$ at which the condition (44) is satisfied for the extremal values of p_z and the position of the "center of orbit" p_{y0} in p space. At temperatures comparable with the distance between levels $\varepsilon_{n+1}(H,d) - \varepsilon_n(H,d)$, this should lead to a new type of oscillations of the thermodynamic and kinetic quantities, which has the same period as the classical Sondheimer oscillations.

Using the quantization rule (44), we can calculate the contribution of open sections of the Fermi surface to the ultrasonic damping coefficient from perturbation theory:

$$\Gamma = \frac{\Gamma_0}{2} \sum_{\pm} \sum_{m=-\infty}^{\infty} \frac{m\beta_0}{\mathrm{sh}(m\beta_0)} \int_{-P_0/2}^{P_0/2} \frac{dp_{y0}}{P_0} \cos[2\pi m \sigma(\varepsilon \pm \mu H), p_{y0}, 0]. \quad (45)$$

Here P_0 is the period of the open Fermi surface, $\beta_0 \equiv 2\pi^2 T/\Delta\varepsilon_F$,

$$\Delta\varepsilon_F \approx \frac{h}{2d} \left| \frac{\partial \bar{p}_x}{\partial \varepsilon_F} \right|^{-1} = \frac{h}{2d} \left| \frac{\partial}{\partial \varepsilon_F} \int_0^{P_0} \frac{dp_y}{P_0} p_x \right|^{-1}$$

is the distance between size-quantized levels of the electron energy, and $2\mu H$ is their spin splitting. As a consequence of the laws of conservation in the case of interaction of the electron with the acoustical phonon, all the quantities in (45) are taken on the central section of the Fermi surface, which is also assumed to be open.

At low temperatures, when $\beta_0 \ll 1$, the expression (45) describes gigantic oscillations of the sound absorption coefficient as a function of the magnetic field (and layer thickness) in the case $\mathbf{k} \| \mathbf{H}$.

It is easily noted that in fields $H_N = NcP_0/ed$, $N = 1, 2, 3, \ldots$, ($|H - H_N| < \hbar H_N/dp_F$) the quantity σ does not depend on p_{y0} and is equal to

$$\sigma_{\pm}(H_N) = 2dp_x(\varepsilon_F \pm \mu H)/h.$$

At these values of the field, the sound attentuation

$$\Gamma(H_N) \approx \Gamma_0 \frac{\pi^2}{4\beta_0} \sum_{\pm} \mathrm{ch}^{-2} \left\{ \frac{\pi^2}{\beta_0} [\sigma_{\pm}(H_N) - n] \right\}$$

is exponentially small (except for cases of integer values of $\sigma_{\pm}(H_N)$. Outside the vicinity of the points H_N, integration over p_{y0} is carried out by the stationary-phase method:

$$\Gamma(H) = \Gamma_0 \left\{ 1 + \frac{1}{2\pi P_0} \sum_{e,\pm} |\sigma''_{p_{ye}}|^{-1/2} \sum_{m=1}^{\infty} \frac{m^{1/2}\beta_0}{\mathrm{sh}(m\beta_0)} \cos\left(2\pi m \sigma_{e,\pm}(H) - \xi_e \frac{\pi}{4} \right) \right\},$$

$$\xi_e = \mathrm{sign}\, \sigma''_{p_{ye}}. \quad (46)$$

On the curve $\Gamma(H)$, a sharp spike appears of height

$$\Gamma_{max} = \Gamma(H_n) \approx \Gamma_0 |\beta_0 P_0^2 \sigma''_{p_{ye}}(H_n)|^{-1/2}, \quad (47)$$

when one of the extremal (in p_y) values $\sigma_{e,\pm}(H) \equiv \sigma(\varepsilon_F \pm \mu H, p_{ye}, 0)$ is equal to the quantum number n. The location of the spikes H_n and the shape of their envelope $\Gamma(H_N)$ are determined by the specific dependence $p_x = p_x(\varepsilon, p_y, 0)$ which makes it possible to improve the precision of the existing model of open Fermi surfaces from the experimental data.

The case of "weakly corrugated" open Fermi surfaces also deserves attention. If $|p_x - \bar{p}_x| \ll \bar{p}_x$ on the central cross section, then in strong fields, when $(p_x - \bar{p}_x) \ll eH\hbar/cP_0$, observation of the oscillations of $\Gamma(H)$ due to spin splitting of the size-quantized electrons energy levels is possible:

$$\Gamma(H) \approx \frac{\Gamma_0}{2} \sum_{\pm} \sum_{m=-\infty}^{+\infty} \frac{m\beta_0}{\mathrm{sh}(m\beta_0)} \cos\left[2\pi m \left(\frac{2d\bar{p}_x(\varepsilon_F)}{h} \pm \frac{\mu H}{\Delta\varepsilon_F} \right) \right]. \quad (48)$$

The condition for observation of these effects is, as is usual for quantum oscillations, the smallness of the temperature in comparison with the distance between the levels $\Delta\varepsilon_F \approx h\bar{v}_\perp/d$. For films of thickness $d \sim 10^{-3} - 10^{-4}$ cm, the required temperature range ($T \sim 1$ K) turns out to be quite realistic.

5. Up to the present time, we have assumed that the surface scattering of the electrons is "single-channel," i.e., in the case of a given $\mathbf{p}_-$, the equations (2) have a single solution $\mathbf{p}_+$, to which corresponds a sharp maximum in the kernel of the integral boundary condition (3). However, in the general case, the coupling between the quasi-momenta of the incident ($\mathbf{p}_+$) and reflected ($\mathbf{p}_-$) of the electrons is not single-valued in one case of purely specular reflection: if the line $\mathbf{p} \times \mathbf{n} = \text{const}$ intersects the Fermi surface at several non-equivalent points with $v_n(p_i) < 0$, then at the instant of collision with the boundaries of the layer, the electron can be thrown, for example, to another cavity of the Fermi surface. Such "surface hopping" significantly change the character of the electron orbits in the magnetic field and, consequently, the conditions of interaction of electrons with the sound wave propagating along the normal to the layer.[17]

Let there be two convex cavities in the Fermi surface, consistent with the parallel transfer

$$\varepsilon(\mathbf{p}_1) = \varepsilon(\mathbf{p}_2) = \varepsilon_F, \quad \mathbf{p}_2 - \mathbf{p}_1 = 2\pi\hbar\mathbf{b},$$

where the vector $\mathbf{b}$, which in particular can simply coincide with the period of the reciprocal lattice of the crystal, is located in the plane (p_x, p_y) and makes an angle φ with the p_x axis.

Then, in a magnetic field $\mathbf{H} = (0, 0, H)$ parallel to the layer, the electrons colliding with one of its boundaries

$x=0$, d will move along open trajectories which, as a consequence of surface hopping, consist of sections of the complete orbit, displaced along the normal by a distance

$$cB/eH=2\pi\hbar b|\sin\varphi|c/eH,$$

as shown in Fig. 3. It is clear that in the static case, when $\omega\ll\nu$, the contribution of such electrons to the acoustic absorption depends on the phase difference of the sound wave over a length cB/eH, and this leads to oscillations of the absorption with change in H.

We find the electron distribution in the layer. The solution of the kinetic equation (7) in the given case is a sum of functions of type (8) for particles located at the given instant on the i-th cavity of the Fermi surface:

$$\chi=\sum_i\chi(0,t_i)=\sum_i\left(A_i\mathscr{E}_0^{t_i}+\int_0^{t_i}dt\,\Lambda_i\mathscr{E}_i^{t_i}\right).$$

Here the time is measured from the last instant of collision with the boundaries $t_i=t-\lambda_i$. The boundary condition for finding A_i should be written down with account of the fact that at the moment λ_i a hop of the electron from the k-th cavity of the Fermi surface can occur with probability a_i $(i\neq k)$:

$$\chi(\tau_i,\tau_i)=a_i\chi(0,\tau_k)+(1-a_i)\chi(0,\tau_i),\qquad\int_{\lambda_i}^{\lambda_i+\tau_i}v_x\,dt=\int_0^{\tau_i}v_x\,dt_i=0,\tag{49}$$

where τ_i is the time of passage of the i-th section of orbit. At the same time, the total distribution of nonequilibrium particles should not change in the case of specular reflection:

$$\sum_i\chi(\tau_i,\tau_i)=\sum_i\chi(0,\tau_i).$$

Consequently, $a_1=a_2\equiv a$. Solving Eq. (49), we find

$$A_i=\frac{a(\psi_1+\psi_2)+\psi_i(1-2a)(1-\exp(-\nu\tau_k))}{a(1-\exp(-\nu(\tau_1+\tau_2)))+(1-a)(1-\exp(-\nu\tau_1))(1-\exp(-\nu\tau_2))}\tag{50}$$

$$\psi_i=\int_0^{\tau_i}dt_i\,\Lambda_i\mathscr{E}_{t_i}^{t_i}\approx\Lambda_{i0}(2\pi/ikv_0')^{1/2}\exp(-ikx_{i0}),$$

where $x_{i0}-x$ is the coordinate of the turning point of the electron; the remaining quantities with index 0 also are taken at this point. We are interested in the case in which the hopping probability a exceeds the probability of intravolume scattering of the electron in the time τ_i: $1>a\gg\nu\tau_i\sim r/l$. Here the quantities A_i do not depend on a and are equal to

$$A\approx\frac{(2\pi/ikv_0)^{1/2}}{\nu(\tau_1+\tau_2)}\sum_i\Lambda_{i0}\exp(-ikx_{i0}).\tag{51}$$

Thus, the electron distribution in the layer is known, and the sound damping coefficient can be calculated from the formula

$$\Gamma(H)=\frac{2k^2eH}{cd\rho h^3}\operatorname{Re}\int_{p_{z\,min}}^{p_{z\,max}}dp_z\int\frac{d\tau_1}{v_x^{-1}(\tau_1)-v_x^{-1}(0)}\sum_i\int_0^{\tau_i}dt_i\,\Lambda_i\chi(0,t_i).\tag{52}$$

The limits of integration with respect to τ_i are so chosen that the turning points on the orbits 1 and 2, at which the electron effectively interacts with the sound field, are located simultaneously in the layer.

The oscillating part of the attenuation coefficient turns out to be equal to

$$\begin{gathered}\Delta\Gamma_{osc}(H)\approx\Gamma_0(l/d)\varphi(H)\cos(kcB/eH);\\ \varphi(H)=0,\quad H<H_1,\\ \varphi(H)\sim H/H_1-1,\quad 0<H-H_1\ll H_1,\\ \varphi(H)=\text{const}\sim1,\quad H_2<H,\\ H_1=cB/ed,\quad H_2=c\Delta p_{y\,max}/ed,\end{gathered}\tag{53}$$

where Δp_y is the size of the section of the Fermi surface $p_z=$ constant. The amplitude of these oscillations increases monotonically from zero in fields $H_1<H<H_2$ and does not depend on the magnetic field at $H>H_2$. In fields $H\sim H_2$, it reaches the order of the total attenuation coefficient and by far exceeds $[\sim(kr)^{1/2}H_2/(H-H_2)\gg1]$ the amplitude of the ordinary Pippard resonance, which is possible at $H>H_2$.[18]

In contrast with the well known magnetoacoustic effects, the period of oscillations $\delta(H^{-1})=2\pi e/kcB$ is determined not by the shape but by the mutual location of the individual cavities of the Fermi surface in p space, i.e., by the vector $\mathbf{b}$. If $\mathbf{b}$ is identical with the basis vector of the reciprocal lattice of the crystal, then the considered example with $i=1,2$ is, of course, a special case of the more general situation, when there is a set of nonequivalent intersections

$$[\mathbf{p}_i\mathbf{n}]=\text{const},\quad\varepsilon(\mathbf{p}_i)=\varepsilon_F,\quad v_n(\mathbf{p}_i)<0$$

in the scheme of repeating bands.

Let the cell of the reciprocal lattice $\mathbf{b}_1\times\mathbf{b}_2$ in the plane perpendicular to the magnetic field contain the convex curve $\varepsilon(\mathbf{p})=\varepsilon_F$: $p_z=$ const, and the p_x axis is parallel to the direction $\{m_1,m_2\}$ ($m_1\neq m_2$ are positive integers which do not have a common divisor). Then on the orbit of the electron colliding with one and the same boundary of the layer there can be no more than m_1+m_2 nonequivalent portions, shifted relative to one another along the normal (x axis) by a distance

$$m\frac{cB'}{eH}=m\frac{2\pi\hbar c[b_1^2b_2^2-(\mathbf{b}_1\mathbf{b}_2)^2]^{1/2}}{eH|m_1\mathbf{b}_1+m_2\mathbf{b}_2|},\qquad 0<m<m_1+m_2.\tag{54}$$

Generalizing the previous consideration, it is not difficult to show that even in this case, in fields $H>cB'/ed$ the function $\Gamma(H)$ experiences oscillations with period $2\pi e/kcB'$, the amplitude of which at $H>H_2$ does not depend on the magnetic field and in order of magnitude is equal to $\Gamma_0 l/d$.

We now discuss another important case, when two

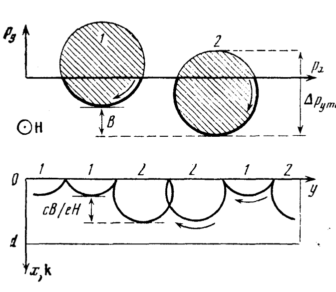

FIG. 3.

different cavities of the Fermi surface are located along the p_x axis, parallel to the normal to the layer of the metal. For simplicity, we assume that the surfaces $\varepsilon_1(\mathbf{p}) = \varepsilon_F$ and $\varepsilon_2(\mathbf{p}) = \varepsilon_F$ are convex, and their projections on the plane (p_y, p_z) are symmetric relative to the p_y and p_z axes. Then the calculation differs from that given above only in that after the surface hopping the turning point of the electron is shifted by an amount $x_{20} - x_{10}$, which depends on p_z. Integration over p_z by the stationary phase method leads to the appearance of a factor $(kr)^{-1/2}$ in the amplitude of oscillations of the sound absorption:

$$\begin{gathered}\Delta\Gamma_{osc}(H) \approx \Gamma_0 (l/d\sqrt{kr})\varphi'(H)\cos(kdH_-/H - \pi/4),\\ \varphi'(H) = 0, \quad H < H_-,\\ \varphi'(H) \sim H/H_- - 1, \quad 0 < H - H_- \ll H_-,\\ \varphi'(H) = \text{const} \sim 1, \quad H_+ < H,\\ H_\pm = \frac{c}{2ed}|\Delta p_{y1} - \Delta p_{y2}|.\end{gathered} \tag{55}$$

Experimentally, this case can be realized in bismuth, the Fermi surface of which is shown schematically in Fig. 4: the bisector axis intersects the electron and hole ellipsoids, the dimensions of which along the trigonal axis are essentially different: $\Delta p_{y2}/\Delta p_{y1} = 0.15$.[19]

If the magnetic field is directed along the binary axis, then at $H > H_-$, as a consequence of the surface hopping of the charge carriers on the "glancing" orbits, both electron and hole segments are possible, whose turning points are located simultaneously in the sample. Here the following $\Gamma(H)$ dependence should be observed in the experiment: at the points $H < H_-$ only electron orbits are completely contained in the layer and $\Gamma(H)$ undergoes Pippard oscillations with a relatively large period $\delta_1(H^{-1}) = 2\pi e/kc\Delta p_{y1}$. Then at $H_- < H < c\Delta p_{y2}/ed$ the fact oscillations (55) are superposed on them $[\delta_2 = 4\pi e/kc(\Delta p_{y2} - \Delta_{y1}])$ while at $H > c\Delta p_{y2}/ed$ there appears also the period $\delta_3 = 2\pi e/kc\Delta p_{y2}$.

Such an experimental result, when there are three periods of magnetoacoustic oscillations in the case of two convex cavities of the Fermi surface, would be indicative of a two-channel surface scattering of the charge carriers. For observation of the effect, a high quality of the surface of the sample is necessary, and its thickness should not exceed $2\pi/k\delta_1 H_0$, where H_0 is the field, starting at which the Pippard oscillations on the electron ellipsoid in the bulk sample are seen.

6. The analysis of the most characteristic special cases that has been given shows that in thin conducting layers one can observe new magnetoacoustic effects that do not occur in bulk conductors. These effects arise as the result of a change in the dynamics of the electrons that are multiply reflected from the surfaces of the layer. Here a calculation of the dependence of the sound attenuation coefficient Γ on the applied magnetic field H can be carried out without simplifying assumptions on the specularity parameter, and the results are formulated in terms of an integral boundary condition for the charge carrier distribution function in the layer.

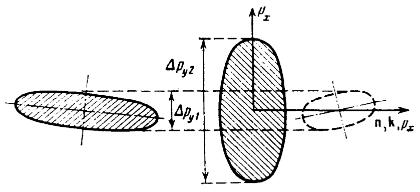

FIG. 4.

Static resonance in a weak magnetic field (see Sec. 3) turns out to be very sensitive to the shape of the scattering indicatrix, and its investigation at different acoustic frequencies ω allows us to determine the width of the scattering indicatrix Δ, the total probability of surface scattering P, and also to analyze the dependence of P on the angle of incidence of the electron on the surface of the sample.

The investigation of periodic changes of Γ in the region of strong magnetic fields of the type of Sondheimer oscillations makes it possible to determine the degree of specular reflection of the charge carriers from the surface of the sample in independent fashion. The study of the dependence of Γ on H at low temperatures, when account of the quantization of the transverse motion of the electrons in the plate is important, allows us to obtain information on the form of the open electron orbits.

The oscillation size effects provide the possibility, lacking in bulk samples, of determining the local characteristics (velocity of the electrons and radius of curvature of the Fermi surface) of the electron energy spectrum. One can investigate multichannel reflection of electrons from the boundaries of the sample with the help of magneto-acoustic measurements. This effect (see Sec. 5) makes it possible to make precise the mutual arrangement of the individual cavities of the Fermi surface, which is very important for its establishment from experimental data.

The authors thank K. B. Tolpygo for useful discussions, stimulating the present research.

[1]K. Fuchs, Proc. Cambridge Phil. Soc. 34, 100 (1938).
[2]V. M. Gokhfel'd and V. G. Peschanskiĭ, Zh. Eksp. Teor. Fiz. 61, 762 (1971) [Sov. Phys. JETP 34, 407 (1972)].
[3]L. A. Fal'kovskiĭ, Zh. Eksp. Teor. Fiz. 58, 1830 (1970) [Sov. Phys. JETP 31, 981 (1970)].
[4]V. I. Okulov and V. V. Ustinov, Zh. Eksp. Teor. Fiz. 67, 1176 (1974) [Sov. Phys. JETP 40, 584 (1975)].
[5]V. I. Okulov and V. V. Ustinov, Fiz. Nizk. Temp. 5, 213 (1979) [Sov. J. Low Temp. Phys. 5, 101 (1979)].
[6]A. B. Pippard, Phil. Mag. 2, 1147 (1957).
[7]V. L. Gurevich, Zh. Eksp. Teor. Fiz. 37, 71 (1959) [Sov. Phys. JETP 10, 51 (1960)].
[8]A. F. Andreev, Usp. Fiz. Nauk. 105, 113 (1971) [Sov. Phys. Uspekhi 14, 609 (1972)].
[9]A. B. Pippard, Phil. Mag. 46, 1104 (1955).
[10]A. I. Akhiezer, M. I. Kaganov and G. Ya. Lyubarskiĭ, Zh. Fksp. Teor. Fiz. 32, 837 (1957) [Sov. Phys. JETP 5, 685 (1957)].
[11]V. P. Silin, Zh. Fksp. Teor. Fiz. 38, 977 (1960) [Sov. Phys. JETP 11, 703 (1960)].
[12]V. M. Kontorovich, Zh. Eksp. Teor. Fiz. 45, 1638 (1963) [Sov. Phys. JETP 18, 1125 (1964)].
[13]V. L. Gurevich, O. G. Lang and S. T. Pavlov, Zh. Eksp. Teor. Fiz. 59, 1679 [Sov. Phys. JETP 32, 914 (1971)].
[14]E. H. Sondheimer, Phys. Rev. 80, 401 (1950).
[15]O. V. Kirichenko, V. G. Peschanskiĭ and S. N. Savel'eva, Zh.

Eksp. Teor. Fiz. **67**, 1451 (1974); 77, 2045 (1979) [Sov. Phys. JETP **40**, 722 (1975); **50**, 976 (1979)].
[16]A. M. Kosevich and I. M. Lifshitz, Zh. Eksp. Teor. Fiz. **29**, 743 (1955) [Sov. Phys. JETP **2**, 646 (1956)].
[17]V. M. Gokhfel'd and V. G. Peschanskiĭ, Pis'ma Zh. Eksp. Teor. Fiz. **30**, 613 (1979) [JETP Lett. **30**, 557 (1979)].
[18]É. A. Kaner and V. L. Fal'ko, Zh. Eksp. Teor. Fiz. 1344 (1964) [Sov. Phys. JETP **19**, 910 (1964)].
[19]D. Shoenberg, transl. in: Fizika metallov (Physics of Metals), Mir, 1972, p. 75.

Translated by R. T. Beyer

Instabilities of two-dimensional plasma waves

M. V. Krasheninnikov

Institute of Semiconductor Physics, Siberian section, Academy of Sciences of the USSR

A. V. Chaplik

Siberian State Scientific-Research Institute of Metrology
(Submitted 28 December 1979)
Zh. Eksp. Teor. Fiz. **79**, 555–560 (August 1980)

The main types of plasma instabilities in two-dimensional electron systems are investigated from the point of view of amplifying two-dimensional plasma waves. Structures consisting of two plasma layers or of a plasma layer above a conducting half-space are considered. The conditions for the onset of two-stream, kinetic, and dissipative instabilities are found. Under certain conditions the instability criteria differ qualitatively from their three-dimensional analogs. The critical drift velocities and oscillation growth factors are calculated.

PACS numbers: 52.35.Py

Experimental studies of plasma waves in two-dimensional electron systems carried through during the last three years[1-3] have confirmed the principal theoretical conclusions concerning the dispersion law for such oscillations. Theis's report[4] includes a detailed review of these studies. Certain specific characteristics of two-dimensional plasmons—their gapless spectrum, complicated dispersion law, and relatively low group velocity—make them very attractive objects for physical research and open up prospects for interesting applications. From this point of view it would certainly be desirable to work with two-dimensional plasmons as with "ordinary" traveling waves (e.g. ultrasonic waves), i.e., to modulate them, amplify them, etc. We note that the experimental results now available relate to the case of standing plasma waves, whose presence was detected either by a change in the Q factor of a resonator (in the case of electrons above a liquid helium surface) or by the resonant absorption of radiation in the far infrared (in the case of the inversion layer in a metal insulator-semiconductor structure).

In this paper we examine the principal types of plasma instabilities in two-dimensional systems as they relate to the problem of amplifying two-dimensional plasma waves. As in the three-dimensional problem, a wave may become unstable as a result of the drift of one part of the plasma with respect to another (see, e.g., Ref. 5).

A specific feature of the case we are considering is that the two parts of the plasma are spatially separated: for example, they may be two parallel thin plasma layers, or a plasma layer above a conductive half-space. In such systems coupled waves arise and amplification can be achieved at a certain drift velocity. The coupling coefficient depends on the distance between the layers and on the plasmon momentum; this considerably complicates the dispersion law for the waves. In addition, the criterion for instability may differ substantially from its three-dimensional analog. In particular, it turns out that the beam instability is characterized by a threshold drift velocity that depends on the distance between the two plasma layers. At below-threshold drift velocities, instability can arise only for plasmons whose wave number k lies within a certain interval: $k_{min} < k < k_{max}$ ($k_{min} = 0$ in the three-dimensional case).

What was said above is valid for the beam instability of a cold plasma. When the thermal motion of the particles is taken into account, wave amplification as a result of kinetic instability becomes possible. The most favorable case for the development of this instability is realized when the effective masses of the particles of the moving and stationary plasmas differ greatly. Finally, when electrons are strongly scattered in one of the plasmas and the drift velocity is low enough the situation is reminiscent of the amplification of sound by an electric current in a piezoelectric medium.

Estimates show that in the case of electrons above a helium film on a conductive backing, amplification begins at comparatively low drift velocities of the carriers in the backing. At present, such a system is the most promising for obtaining amplification of two-dimensional plasma waves.

The problem of oscillations in spatially nonuniform plasma streams has been discussed in the literature. For example, Mikhaĭlovskii and Pashitskiĭ[6] investigated the stability of two neighboring electron streams separa-

 0038-5646/80/080279-04$02.40