Focusing and channeling of neutrons and of other uncharged particles in a ferromagnet by dynamic and static methods

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(Submitted 22 June 1979; resubmitted 18 March 1980)
Zh. Eksp. Teor. Fiz. 79, 481–496 (August 1980)

The feasibility of channeling uncharged particles that possess a magnetic moment is proved. It is shown that the interaction of such particles (particularly neutrons) with the spin wave or elastic wave of the ferromagnet leads to their trapping in a magnetic channel. The channeled particles oscillate in the transverse plane harmonically with a spatial period that depends on the phase of the accompanying spin or elastic wave. The influence of a static domain wall on the character of particle motion in the channel is investigated alongside the dynamic methods connected with wave excitation in the magnet. At certain parameters, a transition domain wall can effectively focus a beam of particles. It is observed that a system of such walls (a multidomain ferromagnet) acts as an effective focusing telescopic system.

PACS numbers: 61.80.Mk, 75.30.Ds, 75.60. – d, 75.90. + w

It is traditionally assumed that channeling in crystals can be effected only for charged particles, when the force that ensures trapping of the particles in the channel is of a purely Coulomb type. Observation of such an effect for uncharged particles (particularly neutrons) would make devices such as neutron waveguides and lenses feasible.

It is shown below that effective channeling, as well as the associated focusing, monochromatization, and polarization of the beam can be realized also for uncharged particles that possess a magnetic moment, by using magnetic crystals. The appearance of a force that keeps such particles in a channel and is directed along its axis is due to dipole-dipole interaction with the ordered inhomogeneous distribution of the magnetic moments in the crystal. Since this interaction is weaker than the Coulomb interaction, the frequency of the oscillation and the angle of capture into the channeling regime turn out to be much smaller than in ordinary channeling.

We consider for the sake of argument the motion of a particle in a rectangular channel measuring $2a \times 2b$ in the xy plane and formed by mutual intersection of systems of crystalline planes of ordered atomic magnetic moments that form a simple ferromagnetic (or oriented paramagnetic) dielectric structure with spatial periods (lattice constants $2a_0$, $2b_0$, and d_0, respectively. To simplify the solution we confine ourselves to motion of particles whose magnetic moments μ_0 are oriented in a positive or negative z direction. The secular part of the Hamiltonian of the interaction of the particle magnetic moment $\boldsymbol{\mu}_0$ with all the atomic magnetic moments $\boldsymbol{\mu}_1$ is of the form[1]

$$W=\sum_{\mathbf{n}} (A_{\mathbf{n}}+B_{\mathbf{n}})(1-3\cos^2\theta_{\mathbf{n}})r_{\mathbf{n}}^{-3},\quad A_{\mathbf{n}}=\mu_{0z}\mu_{1z\mathbf{n}},\quad B_{\mathbf{n}}=(\mu_{0z}\mu_{1z\mathbf{n}}-\mu_0\mu_{1\mathbf{n}})/2,$$

where $\mathbf{n}=\{n_x, n_y, n_z\}$ is the three-dimensional number of the magnetic-lattice point. For the system considered, the term B_n that describes the magnetic dipole-dipole interaction with the mutual flipping of the moments is equal to zero. We note also that at $a=a_0$ and $b=b_0$ the problem corresponds to motion in an elementary crystal channel made up of mutual intersection of the four nearest planes, while at $a \gg a_0$, $a \gg b$ or $b \gg b_0$, $b \gg a$ it corresponds to motion in a gap between two flat parallel ferromagnets.

1. CHANNELING OF PARTICLES INTERACTING WITH A SPIN WAVE

We assume that all the magnetic moments of the atom are oriented in the static position along the channel axis z. We consider a case when a transverse traveling spin wave (SW) is excited in the z direction, with an amplitude $\delta_s\mu_1$ (this can be either an exchange spin wave or a magnetostatic one):

$$\mu_{1\perp n_z}=\delta_s\mu_1\cos(kz_{n_z}-\omega t+\varphi),$$

$$\mu_{1zn_z}=(\mu_1{}^2-\mu_{1\perp n_z})^{1/2}.$$

In a coordinate system that moves with the particle velocity, v_z, which equals the spin-wave velocity $v_{SW}=\omega/k$, we have $z'_{nz}\approx z_{nz}-v_z t$ (see Fig. 1). If we introduce the linear density of the distribution μ_1/d_0 of the magnetic moment along z (d_0 is the distance between neighboring magnetic moments), we can change from summation over n_z to integration with respect to z. Just as in the theory of charged-particle channeling, this change is permissible if the characteristic change of the particle trajectory takes place over a distance $z \gg d_0$. For the same reason, and also because of the rapid decrease of the effectiveness of the dipole-dipole interaction with distance, we neglect edge effects at the end of the channel. As will be shown below, in a qunntum analysis the influence of the entry to and exit from the channel leads to some diffraction spreading of the particle trajectory. As a result we have

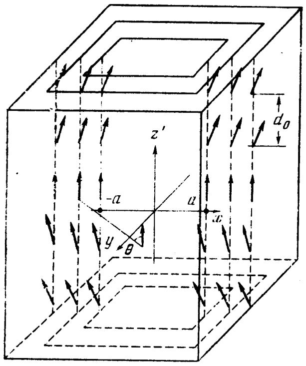

FIG. 1. Particle in magnetic channel.

$$W=(\mu_0\mu_1 g/d_0)\sum_{n_x n_y}\int_{-\infty}^{\infty} dz'[1-3\cos^2\theta(z')]$$
$$\times\{1-{}^1/_4\,\delta_s^2[1+\cos 2(kz'+\varphi)]\}(z'^2+r_\perp^2)^{-3/2}.$$

Here

$$r_\perp^2\equiv r_\perp^2(n_x,n_y)=(2n_xa_0+a+x)^2+(2n_yb_0+b+y)^2,$$

and $g=\pm1$ for parallel and antiparallel orientations of $\boldsymbol{\mu}_0$ and $\boldsymbol{\mu}_1$.

It is easy to transform the last expression into

$$W=(\mu_0\mu_1 g/d_0)\sum_{n_x n_y}\int_{-1}^{1} d\tau(1-3\tau^2)\{1-{}^1/_4\,\delta_s^2$$
$$\times[1+\cos\beta(\tau)\cos 2\varphi-\sin\beta(\tau)\sin 2\varphi]\}r_\perp^{-2},$$
$$\beta(\tau)=2kr_\perp\tau/(1-\tau^2)^{1/2},\qquad \tau=\cos\theta.$$

Let us investigate the integrand. A simple examination shows that for all atomic magnetic moments μ_1 situated within a solid angle corresponding to $|\tau|<\tau_0$ we have

$$\cos\beta(\tau)\approx1,$$

where

$$\tau_0=(1+16k^2r_\perp^2/\pi^2)^{-1/2},$$

and $\cos\beta(\tau)$ oscillates rapidly with sign reversal at $|\tau|>\tau_0$. As a result, the integral that contains $\cos\beta(\tau)$ can be calculated in the first interval of $|\tau|\leq\tau_0$ in elementary fashion, while in the second interval it is equal to zero. An investigation of the integral containing $\sin\beta(\tau)$ as a factor shows that it vanishes because the integrand is odd. Taking all this into consideration, we get

$$W\approx-8\mu_0\mu_1 g\delta_s^2k^2\cos 2\varphi(\pi^2d_0)^{-1}\sum_{n_x n_y}(1+16k^2r_\perp^2/\pi^2)^{-3/2}.$$

Calculation of the last sum yields the components of the force acting on the particle in the channel

$$F_\xi=\begin{cases}-\partial W/\partial\xi\approx256\mu_0\mu_1 g\delta_s^2\cos 2\varphi k^3\lambda_\xi\xi/\pi^3V_0 & \text{if}\quad ak,\,bk<1\\ 0 \quad \text{if}\quad ak,\,bk>1; & \xi=x,y;\quad \lambda_\xi=a,b\end{cases}$$
$$F_{z'}\approx0.$$

The solution of the classical equation of motion of a particle within the limits of one channel in the lab frame

$$md^2\mathbf{r}/dt^2=\mathbf{F},\qquad \mathbf{r}=\{x,y,z\}$$

with initial conditions

$$\xi(t=0)=\xi_0,\quad z(t=0)=0,\quad \left.\frac{d\xi}{dt}\right|_{t=0}=v_\xi,\quad \left.\frac{dz}{dt}\right|_{t=0}=v_z=v_{\text{sw}}$$

is of the form

$$\xi=(\xi_0^2+v_\xi^2/\Omega^2)^{1/2}\cos\{\Omega z/v_z-\arccos[\xi_0/(\xi_0^2+v_\xi^2/\Omega^2)^{1/2}]\},\qquad z=v_zt.$$

where

$$\Omega=\Omega_{s\xi}\approx3\{-\mu_0\mu_1 g\cos 2\varphi k\lambda_\xi/mV_0\}^{1/2}\delta_sk.$$

It is seen from the obtained solution that if $\Omega_{s\xi}$ is real the particles execute translational motion in the channel, and oscillate simultaneously about the channel axis in the transverse direction ξ. The particle trajectory in the transverse plane corresponds to bounded motion of two independent orthogonal oscillators with frequencies $\Omega_{s\xi}$ and phases determined by the system parameters and by the initial conditions. Consequently, in a system with inhomogeneous magnetization, particles having magnetic moments become channeled under certain real conditions, the angle of capture of the particle into the channel being

$$\theta_0\sim\lambda_\xi\Omega_{s\xi}/v_z.$$

We consider now in greater detail the motion in the ξz plane. Assume that the particles enter the channel along z, i.e., at $v_\xi=0$. The character of the motion depends essentially on the motion of the accompanying SW and on the orientation of the moment of the particle g. For all particles with $g=-1$, moving at a SW phase from the interval $\Delta\varphi_1$

$$-\pi/4<\varphi<\pi/4,\qquad 3\pi/4<\varphi<5\pi/4.$$

periodic focusing is observed at

$$z=\pi v_z(2s+1)/2\Omega(\varphi),\qquad s=0,1,2,\ldots$$

and defocusing at

$$z=\pi v_zs/2\Omega(\varphi).$$

We consider now the effect in the optimal case, when the channel length $z=z_0\equiv z(\varphi_0=0,\pi/2)$. If the integrated value of the homogeneous particle flux over one-half the SW at the entry into the channel is I_0, then the flux density $\rho(z_0,|\xi_1|)$ and the integral flux $I(z_0,|\xi|\leq|\xi_1|)$ in the region $|\xi|\leq|\xi_1|$ during the same time interval in the section z_0 are equal to

$$\rho(z_0,|\xi_1|)=2I_0\int_{\varphi_1(\xi_1)}^{\pi/4}d\varphi/\pi a\cos(\pi(\cos 2\varphi)^{1/2}/2).$$

$$I(z_0,|\xi|\leq|\xi_1|)=2\int_0^{|\xi_1|}\rho(z_0,|\xi|)d\xi.$$

Here $\varphi(\xi)$ is the inverse of the function $\xi(\varphi)$.

Figure 2 shows the corresponding curves obtained by numerical integration. The initial flux is focused near the channel axis, with 40% of all the particles localized in the region $|\xi|$ $0.05\lambda_\xi$ and 53% at $|\xi|\leq0.1\lambda_\xi$. The focusing is similar for particles with parallel orientation ($g=1$) and entering the channel in the interval $\Delta\varphi_2$:

$$\pi/4<\varphi<3\pi/4,\qquad 5\pi/4<\varphi<7\pi/4.$$

The particles corresponding to $\varphi=\pm\pi/4$ and $\varphi=\pm3\pi/4$ move in straight lines. The motion of the remaining

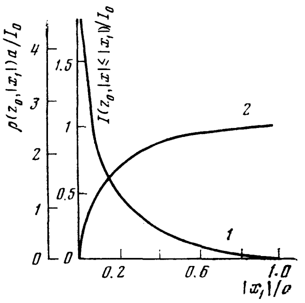

FIG. 2. Focusing characteristics at optimal channel length (curve 1 corresponds to the reduced particle-flux density $\rho(z_0 x_1)$; curve 2 corresponds to the reduced integral flux $I(z_0, |x| \leqslant |x_1|)$ in the region $|x| \leqslant |x_1|$.

particles (with $g = -1$ at $\Delta\varphi_2$ and $g = 1$ at $\Delta\varphi_1$) is different. For them we have $\mathrm{Re}\Omega = 0$, $\mathrm{Im}\Omega = |\Omega|$ and

$$\xi = \xi_0 \operatorname{ch}(|\Omega| z/v_z) + (v_\xi/|\Omega|)\operatorname{sh}(|\Omega| z/v_z).$$

At a distance z satisfying $|\Omega| z/v_z \gg 1$ we have then

$$\xi = {}^1/_2(\xi_0 + v_\xi/|\Omega|)\exp(|\Omega| z/v_z),$$

i.e., the particles leave the channel (at $v_\xi > -|\Omega|\xi_0$ all the particles leave the channel from one side of its axis, while at $v_\xi < -|\Omega|\xi_0$ they cross the channel axis and exit through the other "wall"). An interesting situation arises when the transverse particle velocity at the entry into the channel is $v_\xi = -|\Omega|\xi_0$. In this case, in an arbitrary section z we have for the particles in question

$$\xi \sim \exp(-|\Omega| z/v_z),$$

i.e., asymptotic focusing takes place with a sharp increase of the density on the channel axis. Such a system can be produced, for example, by changing the phase of the SW by π near z_0. In this case, as can be easily verified, the necessary condition for v_ξ is satisfied and the particles will change over from periodic channeling in the first magnet to asymptotic channeling in the second section of the magnet past the phase-reversal point.

We note also that actually the degree of particle focusing can exceed the value corresponding to the de Broglie wavelength $2\pi\hbar/mn_z$. Let us make the corresponding estimates. From the dispersion equation of the longitudinal SW (Ref. 2)

$$\omega = \gamma H_0 + \eta k^2,$$

where η is the exchange parameter and H_0 is magnetic field, we can determine the required system parameters. For maximum neutron focusing it is desirable to use broad channels. Recognizing that the maximum channel dimensions $a < k^{-1}$ and $b < k^{-1}$, it is advisable to use SW with relatively small $k \sim 10^4$ cm^{-1} (these are in fact magnetostatic waves). At the realistic parameters $H_0 = 10^3$ Oe, $\delta_s \sim 0.1$, $\eta \sim 0.1$ cgs esu,[2] $V_0 \sim 10^{-23}$ cm^3 and values $\mu_1 = (2$ to $10)\,\mu_B$ we have $\omega/2\pi \sim 5 \times 10^8$ Hz, $v_{SW} \sim 3 \times 10^5$ cm/sec, $\Omega(\varphi_0) \sim (1$ to $2) \times 10^5$ Hz, and $z_0 \sim 4$ to 2 cm.

Much higher channeling frequencies and correspondingly smaller focal distances correspond to neutral atoms and molecules with uncompensated electronic moments. Recognizing that for these particles $m = 2 \times 10^{-24} A$ g (A is the atomic weight), we find that at $k = 10^5$ cm^{-1} and $\delta_s = 0.1$ we have $\Omega(\varphi_0) \sim (30$ to $130) A^{-1/2}$ MHz and $z_0 \sim (1.2$ to $0.3) \times 10^{-1} A^{1/2}$ cm. We note also that in the case of a broad channel, when $a, b \gg a_0, b_0$ and one can neglect the Coulomb interaction of the particle charge with the crystal, magnetic channeling will take place also for charged particles possessing a magnetic moment. The corresponding frequency of channeling an electron with the aid of an SW with parameters $k = 10^3$ cm^{-1}, $\delta_s = 0.1$, and $H_0 = 10^3$ Oe is $\Omega(\varphi_0) \sim 12$ to 35 MHz, and the focusing distance in a long channel is $z_0 \sim 0.3$ to 0.14 cm.

For the considered cases of a broad channel with dimensions $a, b \sim 10^{-4}$ to 10^{-5} cm, the maximum angle of capture into the channel is $\theta_0 \sim 10^{-4}$ to 10^{-5} rad for neutrons and $\theta_0 \sim (1$ to $3) \times 10^{-3}$ rad for atoms, molecules, and electrons, in the case of an elementary interplanar magnetic channel the limiting particle capture angle is $\theta_0 \sim 10^{-6}$ to 10^{-7} rad.

It is technically simpler to perform experiments with a channel of length $l \ll z_0$ but $l \gg k^{-1}$, which corresponds to a ferromagnetic film. A parallel incident particle beam is transformed at the exit from a channel of length l into a beam converging with an apex angle

$$2\psi(\varphi) = 2 \operatorname{arc\,tg}(\xi_0 \Omega^2 l \cos 2\varphi / v_z^2)$$

and is focused at the point

$$z = \tilde{z}(\varphi) = \xi_0 \operatorname{ctg} \psi(\varphi).$$

We consider now the form of the solution at $\tilde{z}_0 = \tilde{z}(\varphi_0)$. Particles with values of g and $\Delta\varphi_{1,2}$ corresponding to focusing in a long channel are focused at $\tilde{z}_0$ to values

$$\xi(\tilde{z}_0, \varphi) \leqslant \xi_0(1 - \cos 2\varphi).$$

The focusing characteristics $\rho(\tilde{z}_0, \xi_1)$ and $I(\tilde{z}_0, |\xi| \leqslant |\xi_1|)$ calculated numerically for this case hardly differ, in the entire range of variation of ξ, from the corresponding characteristics for a long channel (the difference does not exceed several percent). Consequently, a channel of length $z = l \ll z_0$ is focused just as effectively as at $z = z_0$. At the same parameters and film thickness $l = 10^{-1}$ cm the focal distance for neutrons in the case of SW with $k = 2 \times 10^4$ cm^{-1} is equal to $\tilde{z}_0 \sim 4$ to 20 cm.

The motion in the other orthogonal plane is similar, and by choosing the parameters it is easy to make the foci for both planes to coincide.

From the condition of spatial synchronism of the traveling SW and the particle channeling in a channel of length z,

$$\omega \left| \frac{1}{v_z} - \frac{1}{v_z \pm \Delta v_z} \right| z \leqslant \pi,$$

we can estimate the permissible degree of monochromaticity of the particles for which the channeling effect is realized

$$|\Delta v_z / v_z| \leqslant \pi v_z / \omega z_0.$$

In the analyzed cases, channeling is possible if

$|\Delta v_z/v_z \lesssim 10^{-4} - 10^{-1}$. We note also that when a standing spin wave is excited, the effective interaction takes place only with that of the two traveling waves which is comoving with the particle flow. It is possible, for example, to use SW with very large k at a very small thickness l, as is the case when spin-wave resonance with $k = n\pi/l$ is excited in thin films with $l \sim 10^{-5} - 10^{-6}$ cm. For such a system we have $|\Delta v_z/v_z| \leq 2/n$, where n is the number of the resonance, and the focusing distance corresponds to $\bar{z}_0 \sim 0.1$ cm.

We estimate now the influence of the always-present thermal incoherent SW on the character of the particle motion. It is obvious that the focusing and channeling can be strongly influenced by only those waves whose values of $\omega \pm \Delta\omega$ and $k \pm \Delta k$ are close to the parameters of the coherent SW that is synchronous with the particle flow. From the spatial coherence relation

$$|(\omega\pm\Delta\omega)^{1/2}-\omega^{1/2}|z/\eta^{1/2}\leqslant\pi$$

we obtain the interval of the permissible values of $\Delta\omega \lesssim 4\pi(\eta\omega)^{1/2}/z$. The total number $\bar{N}$ of magnons excited per unit volume in the range $2\Delta\omega$ is

$$\bar{N}=\sum_i \bar{n}_i=\int\limits_{2\Delta\omega} \rho(\omega)\bar{n}(\omega)\,d\omega,$$

where

$$\bar{n}(\omega)=[e^{\hbar\omega/KT}-1]^{-1}, \quad \rho(\omega)=\omega^{1/2}/8\pi^3\eta^{3/2}$$

is the frequency density of the magnon modes per unit volume.[2] Since $2\Delta\omega \ll \omega$ always, it follows that

$$\bar{N}=2\omega/\pi\eta z[e^{\hbar\omega/kT}-1].$$

We find now the upper limit of the total amplitude of the thermal SW in the interval $2\Delta\omega$

$$\delta^{max}=\mu_{1\perp}^{max}/\mu_1=\mu_1\sum \bar{n}_s/3N\mu_1=V_0\bar{N}.$$

Here $N = 1/V_0$ is the number of magnetic moments per unit volume. At $T = 300$ K and $V_0 = 10^{-22}$ cm^3 in the interval $z = z_0$ to l we get $\delta^{max} \sim 10^{-7}$ to 10^{-9}. Actually we have $\delta \ll \delta^{max}$, since the different thermal SW never add up synchronously and in phase. Comparing the calculated value of δ^{max} with the relative amplitude $\delta_s \sim 10^{-1} - 10^{-2}$ of the analyzed coherent SW we see that in all cases the effect of the thermal SW is negligibly small.

2. PARTICLE INTERACTION WITH ELASTIC WAVE IN MAGNETIC CHANNEL

We shall analyze the motion of microparticles in the case when a longitudinal elastic ultrasound wave (UW) of frequency ω and amplitude Δz is excited in the direction z of the orientation of the magnetic moments of a saturated ferromagnet. Such a wave modulates the distance between the magnetic moments in accord with the law

$$d=d_0[1+\delta_p\cos(kz_{n_z}-\omega t+\varphi)],$$
$$\mu_{1zn_z}=\mu_1, \quad \delta_p=k\Delta z\ll 1.$$

We introduce an alternating linear density of the distribution of the magnetic moment of the ferromagnet along the channel axis. In a system varying at the UW velocity

$$z_{n_z}'\approx z_{n_z}-v_z t,$$

$$W=\mu_0\mu_1 g\sum_{n_x n_y}\int\limits_{-\infty}^{\infty} dz'[1-3\cos^2\theta(z')][1-\delta_p\cos(kz'+\varphi)]/d_0(z'^2+r_\perp^2)^{3/2}.$$

Calculations in full analogy with those for the SW case yield identical solutions of the equation of motion $r(t)$, which differ from the SW case by the value

$$\Omega=\Omega_{p\xi}\approx 6\{-\mu_0\mu_1 g\delta_p\cos\varphi k\lambda_\xi/mV_0\}^{1/2}k.$$

An analysis of the characteristics of the motion and of the focusing of the particles in the UW field corresponds with the SW case when allowance is made of the fact that the focusing and the channeling correspond to the values $g = -1$, $-\pi/2 < \varphi < \pi/2$ and $g = 1$, $\pi/2 < \varphi < 3\pi/2$. Numerical estimates show that upon excitation of UW of frequency $\omega/2\pi = 10$ GHz, $v_{UW} = 2\times 10^5$ cm/sec, and amplitude $\Delta z = 5\times 10^{-9}$ cm we have for the channeling atoms $\Omega_p \sim 10^8/A^{1/2}$ Hz and $z_0 \sim 10^{-3} A^{1/2}$ cm at channel dimensions $a, b \lesssim 10^{-5}$ cm, while at $\omega/2\pi = 10^8$ Hz and $\Delta z = 2\cdot 10^{-8}$ cm we have respectively $\Omega_p \sim 10^5/A^{1/2}$ Hz, $z_0 \sim A^{1/2}$ cm, and $a, b \lesssim 10^{-5}$ cm. We note that thermal phonons, just as in the case of magnons in the SW case, do not influence the particle focusing and channeling parameters of the particles.

3. PARTICLE FOCUSING BY A DOMAIN WALL

It follows from the foregoing analysis that the appearance of an elastic force that gives rise to channeling and focusing effects is due to the alternating longitudinal component of the magnetization of the channel walls. This spatial component, alongside with excitation of SW and UW, can take place also in a Néel transition domain wall (DW).

We shall analyze the motion in a channel of magnetic moments μ_1 whose orientation change corresponds to the rotation of the magnetization within the limits of the DW. We consider a DW between two layers of a saturated ferromagnet (domains) with magnetic-moment orientations relative to the z axis φ_1 and φ_2, respectively. We assume that the wall of thickness l is located in the interval from $z = 0$ to $z = l$ and the changes of its direction $\varphi(z)$ and of its longitudinal component μ_{1zn_z} of the magnetization are described by the expressions

$$\varphi(z)=\varphi_1+\varkappa z, \quad \mu_{1zn_z}=\mu_1\cos\varphi(z), \quad \varphi(z=l)=\varphi_2.$$

Here $\varkappa = 2\pi/l_w$ is the period of the spatial rotation of the magnetization direction in the DW. Usually $l_w \sim 10^{-5} - 10^{-6}$ cm.

Using the previously analyzed method of calculating the expression for W, we obtain

$$W=\mu_0\mu_1 g\sum_n (1-3\cos^2\theta_n)\cos(\varphi_1+\varkappa z_{n_z})[(z-z_{n_z})^2+r_\perp^2]^{-3/2},$$

where $g = \pm 1$ for parallel and antiparallel orientation of μ_0 relative to z. We change to a coordinate system connected with the moving neutron (or other particle) $z - z_{nz} = z_{nz}'$. Proceeding as in the case of the SW, i.e., replacing the summation over n_z by integration, we obtain expressions for the transverse and longitudinal components of the force acting on the particle in the channel:

$$F_\xi \approx -10^3 \mu_0 \mu_1 g \cos(\varphi_1+\varkappa z)\varkappa^3\lambda_\xi \xi / \pi^3 V_0,$$
$$F_z \approx \pi\mu_0\mu_1 g \sin(\varphi_1+\varkappa z)\varkappa / V_0, \quad a, b<\varkappa^{-1}.$$

The longitudinal-motion equation

$$m\frac{d^2z}{dt^2}=F_z(z)=F_z\sin(\varphi_1+\varkappa z)$$

can be easily transformed into

$$\frac{1}{2}\frac{d}{dt}\left(\frac{dz}{dt}\right)^2=\frac{F_z}{m}\frac{dz}{dt},$$

the solution of which takes the form

$$z+F_z\{[\sin(\varphi_1+\varkappa z)-\sin\varphi_1]/\varkappa-z\cos\varphi_1\}/m\varkappa v_z^2=v_zt.$$

Regarding F_z as a small parameter we obtain by successive approximations

$$z\approx v_zt-F_z\{[\sin(\varphi_1+\varkappa v_zt)-\sin\varphi_1]/\varkappa-v_zt\cos\varphi_1\}/m\varkappa v_z^2,$$
$$v_z(t)\approx v_z-F_z[\cos(\varphi_1+\varkappa v_zt)-\cos\varphi_1]/m\varkappa v_z=v_z-\Delta v_z(t).$$

Estimates show that at $v_z \gtrsim 10^5$ cm/sec we have $|\Delta v_z^{\max}| \lesssim 10^{-3}$ cm/sec.

The equation of motion in the transverse plane is that of an oscillator with variable frequency $\Omega(t)$. To find the solution in the transverse ξz plane we use in first approximation the zeroth approximation of the longitudinal motion $z=v_zt$, as a result of which we have

$$d^2\xi/dt^2=-\Omega_0^2\cos(\varphi_1+\varkappa v_zt)\xi,$$

where

$$\Omega_0\approx 6\{\mu_0\mu_1 g\varkappa\lambda_\xi/mV_0\}^{1/2}\varkappa.$$

The form of the solution depends significantly on the relation between Ω_0 and $\omega_0=\varkappa v_z$. In the case when $|\Omega_0| \gg \omega_0$, the system corresponds to an oscillator with slowly varying frequency and the solution can be sought in the form

$$\xi=\tilde{\xi}(t)\exp\left\{\pm i\int_0^t\Omega(\tau)d\tau\right\},\quad \Omega(\tau)=\Omega_0\cos(\varphi_1+\varkappa v_z\tau),$$

where the amplitude $\tilde{\xi}(t)$ varies slowly compared with the phase factor. Changing over in the usual manner to the abbreviated equation, we obtain

$$\xi=\left[\xi_0^2+\frac{v_\xi^2}{\Omega^2(t)}\right]^{1/2}\cos\left\{\int_0^z\frac{\Omega(t(z))}{v_z}dz-\arccos\frac{\xi_0}{(\xi_0^2+v_\xi^2/\Omega^2(t))^{1/2}}\right\},$$

which corresponds qualitatively to the channeling for SW and UW.

In the other limiting case of very rapid variation of the instantaneous frequency $\Omega(t)$, i.e., $|\Omega_0| \ll \omega_0$, the system cannot be interpreted as an oscillator even approximately, and the solution can be found by using Ω_0/ω_0 in the small-parameter method. The solution for this case is

$$\xi=\xi_0+v_\xi z/v_z+\Omega_0^2\left\{(\xi_0/v_z\varkappa)\left[\frac{\cos(\varphi_1+\varkappa z)-\cos\varphi_1}{\varkappa}+z\sin\varphi_1\right]\right.$$
$$\left.+(v_\xi/v_z^2\varkappa^2)[z(\cos\varphi_1+\cos(\varphi_1+\varkappa z))+2(\sin\varphi_1-\sin(\varphi_1+\varkappa z))/\varkappa]\right\}.$$

To choose the type of solution, we shall make numerical estimates. For typical values $l_w \sim 6\times 10^{-6}$ cm we have $\varkappa \sim 10^6$ cm^{-1} and for neutrons and atoms we have respectively $|\Omega_0| \sim 10^9$ Hz and $|\Omega_0| \sim 3\cdot 10^{10}A^{1/2}$ Hz, where A is the atomic weight. For all particles, at velocities $v_z \gtrsim 10^5$ cm we have $\omega_0 \geqslant 10^{11}$ Hz and $\omega_0 \gg |\Omega_0|$, i.e., the second case is usually realized, and it is this which will be considered in greater detail.

For a parallel incident beam, the convergence angle and the focusing distance are respectively

$$\psi\approx\operatorname{tg}\psi=-\frac{d\xi}{dz}\approx\frac{\Omega_0^2\xi_0}{v_z^2\varkappa}[\sin\varphi_2-\sin\varphi_1],$$

$$\tilde{z}_0=\xi_0\operatorname{ctg}\psi\approx\frac{\xi_0}{\psi}.$$

Let us clarify the character of the focusing. In the case $g=+1$ and for $\sin\varphi_2>\sin\varphi_1$ (or at $g=-1$ and $\sin\varphi_1>\sin\varphi_2$) we have $\psi>0$ and $\tilde{z}_0>0$, corresponding to a thin converging lens with a real focus at $\tilde{z}_0$. It is seen that the largest beam convergence angle and accordingly the smallest focal length $\tilde{z}_0$ are realized for a 180° wall joining two domains with magnetization orientations $\varphi_1=-\pi/2$, $\varphi_2=\pi/2$ at $g=1$ and $\varphi_1=\pi/2$, $\varphi_2=3\pi/2$ at $g=-1$. For this optimal system the focal length for neutrons is $\tilde{z}\sim 0.04$ cm at $v_z=2\times 10^5$ cm/sec and $\tilde{z}_0\sim 1$ cm at $v_z=10^6$ cm/sec, while for light atoms $\tilde{z}_0\sim 2\times 10$ cm at $v_z=10^6$ cm/sec.

For the other set of parameters, i.e., $g=-1$, $\sin\varphi_2>\sin\varphi_1$ and $g=1$, $\sin\varphi_2<\sin\varphi_1$ we have $\psi<0$, and the situation is analogous to a thin diverging lens with a virtual focus at a point $\tilde{z}_0<0$.

Under real experimental conditions it is difficult to produce such a two-domain magnetic structure with a single wall.[3] Usually an unsaturated ferromagnet of even small thickness consists of a large number of domains with dimensions $L_0 \gtrsim 10^{-4}$ cm. We consider the motion of particles in such a system of domains with alternating magnetization directions $\varphi_1, \varphi_2, \varphi_1, \varphi_2, \ldots$ and with allowance for the fact that usually $L_0 \ll \tilde{z}_0$. Assume the first and second (along the particle-motion direction) domains are characterized by values of g and by magnetization orientations φ_1 and φ_2 such that their DW becomes focusing, i.e., $\psi>0$, $\tilde{z}_0>0$. On passing through the second domain towards the second wall the aperture of the entering beam decreases by a factor $(1-2L_0\psi/\xi_0)$. The second DW, when the substitutions $\varphi_1\to\varphi_2$, $\varphi_2\to\varphi_1$ are made, is similar to a diverging lens with divergence angle $-\psi<0$. Since the beam incident on this wall was convergent, with $\psi>0$, it leaves the wall as a parallel beam similar to the initial one, but with a decreased aperture.

Repeating thus beam-narrowing cycle in each successive pair of walls (Fig. 3) we find that after passage through a channel of length z an entering parallel beam of particles remains parallel (after an even number of domains) or convergent (after an odd number), and its transverse dimension decreases by a factor K_0:

$$K_0^{-1}=(1-2L_0\psi/\xi_0)^{z/2L_0}\approx\exp(-z/\tilde{z}_0).$$

For a magnet of thickness $z=0.2$ cm we have $K_0=150$ at $\tilde{z}_0=0.04$ cm. This situation is qualitatively similar to the already considered asymptotic focusing of particles for SW and UW, but is reached by a simpler method in a natural multidomain structure.

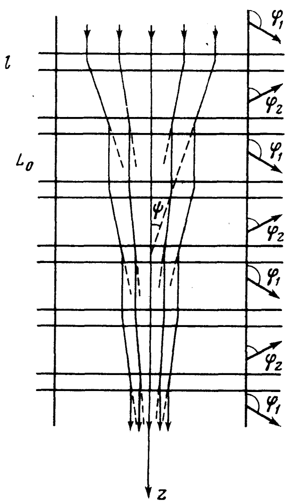

FIG. 3. Asymptotic focusing of particle flux by a multidomain structure with alternating magnetization directions.

In the other case when the first and second domains have values of φ_1, φ_2, and g such that after going through the first we have $\psi < 0$, the beam broadens and the particles leave the channel. This process corresponds to

$$1/K_0(-|\psi|)=\exp(z/|\tilde{z}_0|).$$

It is possible to analyze similarly the situation with domains of arbitrary thickness and nonrepeating magnetization directions.

4. ALLOWANCE FOR DIFFUSE SCATTERING IN THE PROBLEM OF CHANNELING IN A DIELECTRIC

A dielectric channel whose walls are made up to ordered polarized atomic moments can contain a certain amount of disordered free electrons. This situation corresponds to an intracrystal interplanar channel or to a macroscopic nonempty channel. It is useful to examine the diffusion due to scattering by such electrons, a process that competes with focusing (and leads to irreversible defocusing). Let us consider the probability aspects of this process.

The total cross section for magnetic scattering of a thermal neutrons by an unpolarized electron is[4]

$$\sigma_p=2\pi(r_0\gamma_0)^2,$$

where $r_0=e^2/2m_0c^2$ is the classical radius of the electron and $\gamma_0=-1.91$. The corresponding value of σ_p for paramagnetic atoms is $(\mu_B/\mu_N)^2\sim 10^6$ times larger.

Usually the characteristics of a stream of particles in a scattering medium are described by kinetic equations or by the diffusion equation that follows from them.[5] One of the conditions for the applicability of such equations is that the particle mean free path $\lambda_p=1/\sigma_p n_0$ relative to scattering by free electrons with concentration n_0 be much less than the thickness z of the medium in the direction of the initial motion, i.e., $\lambda_p \ll z$. In ordinary dielectrics (including ferromagnets) n_0 is very small. Thus, for microwave iron garnets $n_0\sim 10^4-10^7$ cm^{-3} and for spinels $n_0\sim 10^6-10^8$ cm^{-3}.

Recognizing that $\sigma_p\sim 10^{-24}-10^{-18}$ cm^{-2}, we have $\lambda_p\sim 10^{10}-10^{20}$ cm, i.e., the inverse inequality $\lambda_p \gg z$ is satisfied for this case. Consequently only single scattering is possible, with a probability $w=\sigma_p n_0 z$. The characteristics of such a process can be expressed in terms of the mean squared transverse deviation $[\overline{(\Delta\xi)^2}]^{1/2}$ of the particle trajectory from the initial direction over the sample thickness.

In a coordinate system moving with velocity v_z, the average kinetic energy $\overline{\Delta E}$ and the mean squared transverse velocity $[\overline{\Delta v_\xi)^2}]^{1/2}$ acquired by a neutron or atom passing through a layer of thickness z on account of collisions with electrons in the course of elastic scattering are equal to

$$\overline{\Delta E}\approx 2m_0^2\overline{v^2}\sigma_p n_0 z/m,\quad [\overline{(\Delta v_\xi)^2}]^{1/2}\approx m_0({}^1/{}_3\overline{v^2}\sigma_p n_0 z)^{1/2}/m,$$

where $(\overline{v^2})^{1/2}=(3kT/m_0)^{1/2}$ is the mean squared velocity of the electron.

From the obtained expression it is easy to estimate the sample thickness (the channel length) z_p, for which the diffusion smearing of the trajectory $\Delta\xi=[\overline{(\Delta v_\xi)^2}]^{1/2}z_p/v_z$ becomes comparable with the particle wavelength $\lambda=2\pi\hbar/mv_z$, corresponding to the minimum region of localization of the particle flux at which the results of the classical analysis are compatible with the quantum theory. It is clear that at $z>z_p$ the analysis of the focusing must be carried out simultaneously with the allowance for the diffuse scattering, which has a negligible effect at $z\lesssim z_p$.

From the condition $\Delta\xi=\lambda$ we find

$$z_p=(4\pi^2\hbar^2/m_0kT\sigma_p n_0)^{1/3}.$$

Then at $T=300$ K and $n_0\sim 10^4-10^8$ cm^{-3} we have $z_p\sim 20-500$ cm for neutrons and $z_p\sim 0.5-5$ cm for atoms, and at the usual values $z<1$ cm the influence of the scattering on the channeling and focusing is disregarded.

5. PARTICLE CAPTURE INTO THE CHANNELING REGIME

The foregoing analysis corresponds to motion of particles whose trajectory is confined to a single channel, so that the solution is valid at

$$(\xi_0^2+v_\xi^2/\Omega^2)^{1/2}<\lambda_\xi.$$

The limiting capture angle of such particles

$$\theta_0\approx v_\xi/v_z\lesssim\Omega(\lambda_\xi^2-\xi_0^2)^{1/2}/v_z$$

is very small and amounts to a fraction of a second of angle for an elementary crystal magnetic channel. Nonetheless, as will be seen from the analysis that follows, capture into the channeling regime is possible also for particles that are incident on the crystal within an angle interval $\theta\gg\theta_0$ relative to the channel axis.

To solve the macroscopic capture problem we shall disregard the singularities of the microscopic variations of the trajectories of the uncaptured particles within each elementary channel, and consider macroscopically averaged motion, wherein the actual crystal lattice is replaced by a continuous medium. Obviously, this replacement is valid for particles whose transverse

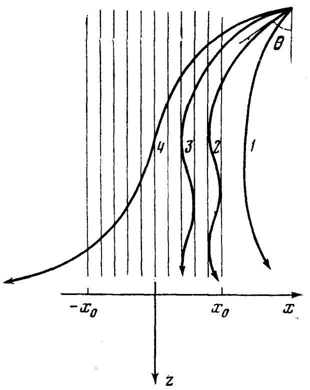

FIG. 4. Capture of particles into a magnetic channel: 1—reflection of slow particles; 2, 3—capture of particles with initial velocities $|v_x|_2 < |v_x|_3 < |\Delta v_x^{\max}|$ into the elementary channel; 4—motion of uncaptured particle with $|v_x|_4 > |v_x^{\max}|$.

velocities v_ξ exceed the velocity interval within which capture is possible.

We consider the macroscopic motion of a particle incident at an angle $\theta = \tan^{-1}(v_x/v_z)$ on a magnet in the form of a plane-parallel plate of thickness $2x_0$ (Fig. 4). Using the expression for the two-dimensional density of the distribution of the atomic magnetic moments in the continuous-medium approximation, $w = 1/4a_0b_0$, we reduce the expression for the averaged energy of the interaction of the particle with the magnet to a form that permits direct calculation of $\overline{W}$ at arbitrary x:

$$\overline{W} = -(8\mu_0\mu_1 gPk^2/\pi^2V_0)\int_{-x_0}^{x_0} d\eta \int_{-\infty}^{\infty} dy \left\{1+\frac{16k^2}{\pi^2}[(\eta+x)^2+y^2]\right\}^{-3/2},$$

where $P = \delta_s^2\cos 2\varphi$ for SW and $P = 4\delta_p\cos\varphi$ for UW.

Integrating directly, we obtain

$$\overline{W} = -\frac{2\mu_0\mu_1 gP}{V_0}\left(\operatorname{arctg}\frac{8k(x_0+x)}{\pi}+\operatorname{arctg}\frac{8k(x_0-x)}{\pi}\right).$$

The solution of the averaged equation of motion is of the form

$$v_x^2 = v_x^2(x=\pm\infty)+\Delta v_x^2(x),$$

where

$$\Delta v_x^2 = \frac{8\mu_0\mu_1 gP}{mV_0}\left(\operatorname{arctg}\frac{8k(x_0+x)}{\pi}+\operatorname{arctg}\frac{8k(x_0-x)}{\pi}\right).$$

It is seen from this solution that a particle having an orientation g and moving with such a phase of the SW or UW that $g\cos\varphi_{SW} > 0$ or $g\cos\varphi_{UW} > 0$, is accelerated as it approaches the magnet, and its velocity reaches a maximum value in the plane $x = 0$. With further motion it slows down and the asymptotic value $v_x^2(\pm\infty)$ reaches the initial value $v_x^2(\mp\infty)$. Obviously, if the initial velocity exceeds the value $v_{0x} = \Omega a$ needed for channeling, then the considered macroscopic acceleration will *a fortiori* not lead to channeling.

The situation is different for particles whose motion satisfies the relation $g\cos 2\varphi_{SW} < 0$ or $g\cos\varphi_{UW} < 0$. The approach of these particles to the magnet leads to a decrease of their transverse velocity. The particle slowdown is physically due to the interaction of its magnetic moment with the macroscopic field of the traveling SW or UW. In particular, if $|\Delta v_x^{\max}| \geq |v_x(\pm\infty)|$, then total stopping of the particle is possible (in transverse-velocity space). If the last-mentioned relation takes place at $|x| > |x_0|$, then the particle will not land at all inside the magnet, and will reverse motion after stopping (Fig. 3). On the other hand if the equality $|\Delta v_x(x)| = |v_x(\pm\infty)|$ holds in some section inside the magnet, i.e., at $0 < |x| < |x_0|$, the transverse velocity $|v_x|$ of the particle will decrease to a value $|v_{0x}|$ at which the conditions for trapping in the microscopic channel begin to be satisfied. Since the channeling phase conditions $g\cos 2\varphi_{SW} < 0$, $g\cos\varphi_{UW} < 0$ obtained above agree with the slowing-down conditions, these particles are trapped into the channel. We note that in all these interactions the longitudinal velocity of the particle remains unchanged.

The foregoing analysis leads to the conditions for macroscopic trapping of a particle into a microscopic channel:

$$g\cos 2\varphi_{sw}<0, \quad g\cos\varphi_{uw}<0,$$
$$|\Delta v_x^{\max}|^2 \geq v_x^2(\pm\infty)-\Omega^2a^2.$$

We estimate now the magnitude of the slowdown effect. For a magnet with the usual parameters we have $|\Delta v_x^{\max}| \sim 3\cdot 10^2\delta_s$ cm/sec and $|\Delta v_x^{\max}| \sim 6\cdot 10^2\delta_p^{1/2}$ cm/sec for SW and UW respectively in the case of neutrons, and $|\Delta v_x^{\max}| \sim 5\cdot 10^3\delta_s$ and $|\Delta v_x^{\max}| \sim 10^4\delta_p^{1/2}$ cm/sec for paramagnetic atoms and molecules. Consequently the considered mechanism makes possible spatial monochromatization of particles with an initial transverse-velocity scatter in the interval 10^1–10^2 cm/sec for neutrons and 10^2–10^3 cm/sec for atoms and molecules with uncompensated magnetic moments.

In the case of thermal particles with $v_z \gtrsim 10^3$ cm/sec, the magnetic deceleration makes possible trapping into the channel at an angle interval up to several degrees, which exceeds by 3–5 orders of magnitude the critical angle θ_0 of trapping into an individual independent microscopic channel, and is of the same order of magnitude as the trapping angles in Coulomb interaction.

The existence of large angles for macroscopic trapping into a microscopic channel and the possibility of spatial monochromatization of a diverging neutron wave make it possible to realize experiments using conventional neutron generators of moderate intensity, producing a thermal-neutron flux 10^7–10^9 sec^{-1} cm^{-2}. Taking into account the obtained spatial aperture factor $4\pi/(\theta_{\rm capt})^2 \sim 10^{-4}$–$10^{-5}$ we find that the intensity of the flux of trapped particles corresponds to a value 10^3–10^4 sec^{-1} cm^{-2}, which satisfies the experimental requirements.

We note in conclusion that within the limits of elementary channels that do not lie near the plate axis we have $\overline{W} \neq 0$ and the form of the potential well for them differs insignificantly from a parabola; it is easy to verify, however, that this difference changes little all the characteristics of the channeled particle motion.

6. LIMITS OF APPLICABILITY OF THE CLASSICAL TREATMENT OF MAGNETIC CHANNELING

The foregoing analysis of the peculiarities of the motion of particles in a magnetic channel does not take into account the quantum nature of these particles (except for the impossibility of localizing quantum objects in a region smaller than their wavelength). The use of this single restriction is justified only at certain relations between the channeling frequency, the particle mass, and the channel width; this imposes additional limitations on the ranges of variation of these parameters.

In fact, a quantum treatment of particle motion in a parabolic potential field $\hat{H}=m\Omega^2\hat{\xi}^2/2$ leads to the known problem of the quantum oscillation with energy eigenvalues E_n:

$$\hat{H}\Psi_n=E_n\Psi_n,\quad E_n=\hbar\Omega(n+{}^1/_2),\quad n=0, 1, 2, \ldots$$

It follows from the correspondence principle that the classical and quantum treatments agree at $n\gg 1$. Only in this case can we neglect the tunneling through the channel walls, and all the channeling directions form in this case a quasi-uninterrupted continuum, which corresponds in fact to classical trapping into a channel at all angles smaller than critical. Equating the maximum height of the potential barrier $W^{\max}=m\Omega^2\lambda_\xi^2/2$ to E_n, we easily obtain a criterion for the validity of the classical solution

$$n^{\max}=m\Omega\lambda_\xi^2/2\hbar\gg 1.$$

We see from this condition that the requirement that the channel dimensions be increased to values bounded only by the condition ak, bk, $a\varkappa$, $b\varkappa<1$ in order to obtain maximum focusing is compatible with the quantum-mechanical criterion. Let us estimate the minimal channel dimensions for which the motion corresponds to classical channeling. Substituting actual parameters we find that for the case of the maximum k when using DW and helicons and at spin-wave resonance the maximum channel dimensions $2a$ and $2b$ for neutrons and atoms are $\sim 5\times 10^{-7}$ and 5×10^{-8} cm, respectively. The last figure gives grounds for hoping to be able to realize the effect in an intracrystal channel. At still smaller channel dimensions, the results of the theory provide only a qualitative description of the motion of microparticles in a magnetic channel.

Notice must be taken of one more circumstance that can lead to a discrepancy between the theoretical and observed results. It is connected with the possibility of spatial defocusing at the exit from the channel, due to diffraction of particles by a slit with dimension $2\lambda_\xi$ within the limits of the angle maximum

$$\Delta\theta\leqslant\pi\hbar/mv_z\lambda_\xi.$$

The last relation is identical with the condition of impossibility of localization within a region smaller than the particle wavelength.

CONCLUSION

In conclusion we examine the possibility of verifying and using the effects of channeling and focusing uncharged particles by using magnets.

In the simplest case it is possible to observe induced transparency in magnetic media by channeling. If intracrystal channeling is realized, experiments corresponding to three-dimensional x-ray interferometry can be performed.[6] Using one magnetic crystal as a focusing device and placing at its focus a second identical crystal having the same crystallographic position, it is possible to vary the transparency of the latter by rotating it or moving it in parallel with itself. If the crystal structures are spatially perfectly identical, the particles are focused in the interstices and the transparency is maximal. Displacement, rotation, or deformation of one of the crystals leads to a decrease in intensity or to a three-dimensional moiré pattern[6] as a result of the trapping of the particles or their scattering.

In view of the considerable difficulties faced by realization of intracrystal channeling, it is apparently advisable to organize experiments on neutron focusing in a macroscopic channel measuring 10^{-3}–10^{-5} cm. The use of asymptotic or periodic focusing makes it possible in a number of cases to increase the density of the neutron flux in the foucs of the system by several orders of magnitude. It is of interest to use neutrons focusing in laboratory research and simulation of an active medium for a gamma laser via neutron capture.[7] In fact, if a working medium consisting of Mössbauer nuclei with large thermal-neutron capture cross section is placed along the focusing axis, it becomes possible to produce an excited region in the form of a needle with transverse dimension less than 0.1 μm and length 0.1–0.01 cm, and with axis along the beam motion. This shape is most convenient for a γ laser.[7] Increasing the density of the neutron beam by K_0^2 times by focusing with the aid of a system of natural or artificial DW, when the focusing effect takes place for the entire energy spectrum of the thermal neutrons, makes possible a four- or fivefold lowering of the threshold of neutron pumping for the γ laser and the use of laboratory neutron sources more realistic than heretofore proposed. If a two-dimensional system of such channels (of the focusing-mask type) is produced, excitation of a system of periodically disposed needles becomes possible, and this can lead to coherent γ superradiance. Further research in this direction is necessary, and considerable difficulties will apparently be encountered when it comes to produce first a weakly diverging beam of thermal neutrons. We note that the estimates presented above for trapping into a channel point to a rather high effectiveness of the use of even isotropic streams.

Obviously, it is much simpler to perform experiments on magnetic focusing and channeling of paramagnetic atoms and ions in a broad channel by using an ultrasound wave of moderate amplitude. Such experiments can be performed even with ordinary molecular beams.

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Translated by J. G. Adashko