[1]E. S. Mashkova and V. A. Molchanov, Dokl. Akad. Nauk SSSR **146**, 585 (1962) [Sov. Phys. Dokl. **7**, 829 (1963)].
[2]E. S. Mashkova, V. A. Molchanov, and M. A. Faruk, Vestn. Mosk. Univ. Fiz. Astron. No. 6, 13 (1963).
[3]E. S. Mashkova, V. A. Molchanov, and Yu. G. Skripka, Dokl. Akad. Nauk SSSR **190**, 73 (1970) [Sov. Phys. Dokl. **15**, 26 (1970)].
[4]E. S. Mashkova and V. A. Molchanov, Radiat. Eff. **13**, 131 (1972).
[5]E. S. Mashkova and V. A. Molchanov, Radiat. Eff. **16**, 143 (1972).
[6]E. S. Mashkova and V. A. Molchanov, Radiat. Eff. **23**, 215 (1974).
[7]K. Morita, H. Akimune, and T. Suita, Jpn. J. Appl. Phys. **7**, 916 (1968).
[8]A. D. Marwick, M. W. Thompson, B. W. Farmery, and G. S. Harbinson, Radiat. Eff. **10**, 49 (1971).
[9]I. G. Bunin, B. M. Mamaev, V. I. Shul'ga, and V. E. Yurasova, Trudy II Vsesoyuznogo simpoziuma po vzaimodeĭstviyu atomnykh chastits s tverdym telom (Proc. Second All-Union Symposium on Interaction of Atomic Particles with Solids), Moscow State University, 1972, p. 95.
[10]V. A. Kurnaev and V. G. Tel'kovskiĭ, Trudy III Vsesoyuznoĭ konferentsii po vzaimodeĭstviyu atomnykh chastits s tverdym telom (Proc. Third All-Union Conf. on Interaction of Atomic Particles with Solids), Part I, Institute of Metal Physics, Academy of Sciences of the Ukrainian SSR, Kiev, 1974, p. 67.
[11]G. I. Zhabrev, V. A. Kurnaev, and V. G. Tel'kovskiĭ, Zh. Tekh. Fiz. **44**, 1560 (1974) [Sov. Phys. Tech. Phys. **19**, 978 (1975)].
[12]K. Miethe, U. Schiebel, A. Neufert, and G. Clausnitzer, Nucl. Instrum. Methods **164**, 183 (1979).
[13]O. B. Firsov, Dokl. Akad. Nauk SSSR **169**, 1311 (1966) [Sov. Phys. Dokl. **11**, 732 (1967)].
[14]O. B. Firsov, Fiz. Tverd. Tela (Leningrad) **9**, 2145 (1967) [Sov. Phys. Solid State **9**, 1687 (1968)].
[15]O. B. Firsov, Zh. Tekh. Fiz. **40**, 83 (1970) [Sov. Phys. Tech. Phys. **15**, 57 (1970)].
[16]O. B. Firsov, Zh. Eksp. Teor. Fiz. **61**, 1452 (1971) [Sov. Phys. JETP **34**, 773 (1972)].
[17]O. B. Firsov, E. S. Mashkova, and V. A. Molchanov, Radiat. Eff. **18**, 257 (1973).
[18]T. Ishitani and R. Shimizu, Jpn. J. Appl. Phys. **10**, 821 (1971).
[19]V. S. Remizovich and I. S. Tilinin, Trudy Vsesoyuznogo simpoziuma po vzaimodeĭstviyu atomnykh chastits s poverkhnost'yu tverdogo tela (Proc. All-Union Symposium on Interaction of Atomic Particles with Solid Surfaces), Izd. FAN, Tashkent, 1979, p. 28.
[20]É. G. Parilis and V. Kh. Ferleger, Trudy Vsesoyuznogo simpoziuma po vzaimodeĭstviyu atomnykh chastits s poverkhnost'yu tverdogo tela (Proc. All-Union Symposium on Interaction of Atomic Particles with Solid Surfaces), Izd. FAN, Tashkent, 1979, p. 44.
[21]V. I. Veksler, Izv. Akad. Nauk SSSR Ser. Fiz. **30**, 860 (1966).
[22]A. Erdélyi (ed.), Higher Transcendental Functions (California Institute of Technology H. Bateman MS Project), Vols. 1 and 2, McGraw–Hill, New York, 1953, 1955 (Russ. Transl., Nauka, M., 1972).
[23]E. P. Arkhipov and Yu. V. Gott, At. Energ. **31**, 477 (1971).
[24]V. A. Kurnaev, A. S. Savelov, V. M. Sotnikov, and V. G. Tel'kovskiĭ, in: Fizika plazmy (Plasma Physics), No. 2, Atomizdat, 1969, p. 65.
[25]L. C. Northcliffe and R. F. Schilling, Nucl. Data Sect. A **7**, 233 (1970).
[26]Yu. V. Gott, At. Energ. **27**, 543 (1969).

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Hydrodynamic stability of compression of spherical laser targets

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Hydrodynamic stability in compression of targets by laser radiation is investigated with account taken of convection, thermal conductivity, compressibility, and the spontaneous magnetic field. It is shown that the growth rate exhibits nonlinear saturation with decreasing perturbation wavelength. The conditions necessary for nearly symmetrical compression are determined, as is also the effect of the instability on the final state of the target.

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The hydrodynamic instability produced in low-entropy compression of spherical targets by laser radiation is the main obstacle to the attainment of the ultrahigh matter densities predicted by one-dimensional spherically symmetrical calculations.[1,2] It is known that the instability is in the main of the Rayleigh-Taylor type,[3] which has been thoroughly investigated in hydrodynamics, particularly when applied to incompressible fluid flow.[4] As a result of instability, the growth of the amplitude of the small perturbations due to variations of the intensity of the laser radiation, to deviations of the target-material density from homogeneity, and to distortion of the target shape, can lead to turbulization of the flow prior to the end of the compression process. New factors in the study of instability of a compressing plasma sphere are the electronic thermal conductivity (T_e, $T_i \sim 1$ keV), compressibility, high radial gradients of the temperature and of the velocity, and generation of magnetic fields of considerable strength (~1 MG) against the background of the fast motion of the plasma towards the symmetry center. The study of the nature and of the methods of stabilizing

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the hydrodynamic instability of the compression, and the determination of the permissible spectrum of the initial perturbations and hence of the permissible variations of the intensity of the laser radiation and of the deviations from spherical symmetry of the target, constitute one of the central problems of laser-driven thermonuclear fusion. It is obvious that the solution of this problem has a direct bearing on the general problem of limitation of spherical cumulation.

The hydrodynamic instability of target compression by laser radiation has been the subject of many studies, using either an approximation linear in the amplitude[5-8] or two-dimensional numerical experiments.[9-13] The latter are the most effective tools for the investigation of this problem, but call for a detailed physical interpretation and for a comparison with simple analytic solutions.

The purpose of the present paper is to present a complete picture of the evolution of instability in a laser target during both the linear and nonlinear hydrodynamic stages of development of the perturbations, with consistent use of analytic solutions for the interpretation of the numerical experiments with nonlinear hydrodynamics.

The results of such an approach can explain why the thermal conductivity plays an essential role, and convection a negligible one. The analysis presented below shows that when account is taken of all the indicated effects the perturbations of the interface of the media increase during the linear stage in approximately the same manner as the Taylor modes on a spherical surface that moves with variable acceleration and whose area decreases with time. We show that when the wave number $k=2\pi/\lambda$ of the perturbation increases the growth rate saturates and that this saturation is of the same type as observed by Fermi.[14] The presence of saturation suggests that the perturbations of the high harmonics are not dangerous for the compression, provided that their initial amplitude does not exceed a definite value. For the simplest target studied in present-day experiments (glass pellet filled with D_2 gas), we determine the initial-perturbation amplitude that does not lead to a substantial distortion of the spherical symmetry, and obtain quantitative data on the state of the compressed target core with allowance for the instability development; these data are used to interpret the physical experiments.

1. INSTABILITY CRITERION. INSTABILITY ZONE

We assume in the analysis of the stability of motion of laser plasma the criterion for the instability of adiabatic (constant specific entropy) flow of a heat-conducting compressible fluid; this criterion was previously obtained[15] in the linear approximation. The motion is stable in the region where the following condition is satisfied[15]:

$$\left(\frac{\partial p}{\partial \rho}\right)_s\left(\frac{\partial \rho}{\partial S}\right)_p \nabla p \nabla S<0; \tag{1}$$

here p, ρ, and S are the pressure, density, and specific entropy of the plasma. The criterion (1) was rigorously proved for all wave numbers of the perturbation at gradients constant in space, and as $k\to\infty$ for arbitrary distribution of the indicated quantities in space. If we rewrite (1) in the form

$$(\nabla p)^2-c^2\nabla p\nabla\rho>0, \tag{2}$$

then it is easily seen that at $|\nabla \ln\rho| \gg |\nabla \ln p|$ we arrive from (2) at the known stability criterion of a non-heat-conducting incompressible liquid, obtained by Rayleigh and Chandrasekhar[4]:

$$\nabla p\nabla\rho<0. \tag{3}$$

Analysis of compression of targets of various types shows that regardless of the concrete structure of the target and of its heating and compression regime, instability in the sense of (2) sets in at least during two stages of the motion[16]: in the "corona" in the reactive acceleration of the matter during the initial stage of the spherical convergence, and when the plasma is slowed down near the geometric center of the target. In contrast to the classical Rayleigh-Raylor problem,[3,4] in this case we are not dealing with instability of the interface: the flow is unstable in a certain region defined by (1), the so-called instability zone.[16] In a laser plasma near the evaporation boundary, the condition $|\nabla \ln\rho| \gg |\nabla \ln p|$ is usually satisfied, so that we can use for the definition of this zone also the criterion (3) for an incompressible fluid. It follows from numerical calculations[9,10] that the width of this zone, which is formed when the unloading wave interacts with the thermal wave, is much less than the characteristic dimensions of the target, and amounts to ~0.2–0.5 μm for glass targets ($R_0=60$–70 μm).

2. SATURATING AND STABILIZING MECHANISMS

The maximum growth rate of the perturbation amplitude is reached for a Rayleigh-Taylor instability when the interface between two nonviscous and non-heat-conducting incompressible liquids (the density discontinuity) is situated in the field of a constant acceleration g (Taylor modes[3])

$$\gamma=kg(\rho_2-\rho_1)/(\rho_2+\rho_1). \tag{4}$$

This growth rate increases without limit with increasing wave number k. However, when allowance is made for viscosity, the flow becomes stable at $k>k_\nu$.[4] In the presence of a density gradient (and in the more general case in the presence of an instability zone whose dimension L is generally speaking not equal to the characteristic scale of the density gradient $b=|\nabla \ln\rho|^{-1}$) the growth rate reaches as $k\to\infty$ a maximum value independent of the wave number

$$\gamma^2_{max}=\begin{cases} g\nabla\ln\rho, & b\gg L,\\ gL^{-1}, & L\gg b.\end{cases} \tag{5}$$

For the case of arbitrary flow of a compressible liquid[15] we have in place of (5) as $k\to\infty$

$$\gamma^2_{max}=g\rho^{-1}\left(\frac{\partial\rho_0}{\partial S}\right)_p\nabla S, \tag{6}$$

where S is the specific entropy.

A significant smoothing effect in the target corona can be the heat-conduction equalization of the perturbations. Temperature perturbations with wave number $k > k_T$ are smoothed out by the heat conduction

$$k_T \sim \left[\frac{Z\rho}{M_i \varkappa_0 T_e^{5/2}} \right] g^{1/2}, \qquad (7)$$

M_i, and Z are the mass and charge of the ion, ρ is the density, and $\varkappa_0$ is the heat-conduction coefficient.

The nonlinear effects become significant at $ak \gtrsim 1$ (a and k are the amplitude and wave number of the perturbation). The nonlinear interaction that causes the change from the exponential regime ($a = a_0 \exp[(kg)^{1/2}t]$) to the power-law regime $[a \sim g(t-t_0)^2]$ was investigated by Fermi,[14] who used the model problem of the evolution of a step perturbation of the interface between a heavy incompressible fluid placed on a light one in an acceleration field g. A characteristic feature of this regime is the difference between the limiting laws that govern the growth of the amplitudes of the peaks ($a \sim t^2$) and the dips ($B \sim t^{1/2}$), i.e., in essence the lowering of the peaks in the gravity force field. In the energy-based approach developed by Fermi this is the result of conversion of the potential energy of the perturbation into kinetic energy of the liquid. In spectrum terms this can be interpreted as energy transfer from the long-wave to the short-wave perturbations.

In the presence of Rayleigh-Taylor instability, magnetic fields of appreciable magnitude (1–10 MG) can be generated in the plasma by the thermoelectric currents.[17] The ponderomotive forces produced when fields of such strengths are generated react in turn on the development of the perturbations. Both effects can be quantitatively calculated in nonlinear two-dimensional numerical experiments (including those in terms of the field).[18] One other mechanism for the elimination of the instability near the evaporation wave front in the target corona can be the flow of plasma through the instability zone (convective stabilization). This effect was investigated both analytically[6,10] and with the aid of numerical calculations.[5,7,9,10] The influence of the convection in the growth rate can be determined from the simple problem discussed below.

3. CONVECTIVE STABILIZATION

Consider the stability of a spherically symmetrical flow of an incompressible liquid with gradients of the density, velocity, and pressure. Let the initial flow be known and described by the functions $\mathbf{v} = \{v_{0r}, 0, 0\}$, ρ_{0r}, v_{0r}, P_0 of r and t. We represent the arbitrary perturbations in the form of an expansion in spherical harmonics:

$$\{u, v, w, \delta\rho, p\} \sim \sum_n f_n(r,t) Y_n(\theta, \varphi). \qquad (8)$$

We linearize in the usual manner the hydrodynamic equations, using the incompressibility condition ($\operatorname{div}\mathbf{v} = 0$). In addition, we change to a coordinate frame that moves the velocity v_{0r}, so as to eliminate effects connected with drift, and assume that the velocity gradients are small:

$$\int \frac{\partial v_{0r}}{\partial r} dt \ll 1.$$

In this approximation we can reduce the problem to one equation for the coefficient of expansion of the perturbation of the radial velocity component $u_n(r)$ in spherical harmonics:

$$u_n\left(\gamma_n + \frac{\partial v_{0r}}{\partial r}\right) = \frac{\gamma_n}{n(n+1)} \frac{1}{\rho_0} \frac{\partial}{\partial r} \rho_0 \frac{\partial r^2 u_n}{\partial r} + \frac{u_n}{\gamma_n} g \frac{\partial \ln \rho_0}{\partial r}, \qquad (9)$$

assuming that the time dependence is exponential:

$$u_n(r, t) = u_n(r) \exp(\gamma_n t).$$

We have put here for simplicity $g = -\rho_0^{-1}\partial P_0/\partial r$. As $n \to \infty$, we easily obtain from (9)

$$\gamma_{n\to\infty} = \left[g \frac{\partial \ln \rho_0}{\partial r} \right]^{1/2} - \frac{\partial v_0}{\partial r}. \qquad (10)$$

The stabilizing role of the convection is thus connected, as expected, with the presence of the velocity gradient. It follows from (10) that even as $n \to \infty$ the convection does not lead to stable flow ($\gamma < 0$), in contradiction to Bodner's results.[8] Comparison of the first and second terms in (10) for conditions typical of a plasma target shows that convection cannot play an appreciable role.

4. INSTABILITY OF SPHERICAL INTERFACE BETWEEN A COMPRESSIBLE AND AN INCOMPRESSIBLE MEDIUM

Problems closely related to the instability of compression a laser target are those of the stability of a cavitation bubble in an incompressible liquid. Such problem were solved both for a spherical interface between two incompressible liquids[19] and for the surface of a vacuum bubble in an infinite liquid.[20]

We consider the stability of the boundary of a spherical gas bubble located in an infinite incompressible liquid. As will be shown below, this problem has all the main features of the instability on a contracting spherical interface between two media. The unperturbed motion is the compression of the gas under the influence of spherically symmetrical pressure that decreases at infinity in such a way that the total energy remains finite.

The law governing the unperturbed motion of the interface can be obtained from the energy conservation condition, in analogy with the Rayleigh problem.[21] Such a solution was obtained by Zababakhin[22] under the assumption that the kinetic energy of the gas is much less than its internal energy and the kinetic energy of surrounding liquid, and that the gas is compressed adiabatically:

$$v_0 = \frac{dR}{dt} = \left(\frac{E_0}{2\pi\rho_0} \right)^{1/2} \frac{(R^2 - R_{min}^2)^{1/2}}{R^{5/2}}; \qquad (11)$$

here E_0 is the total energy, R and R_{min} are respectively the running and minimal radii of the cavity, and ρ_0 is the density of the density of the liquid.

The equations for the coefficients of the expansion of an arbitrarily small perturbation $\Delta_n(r, t)$ of the surface in spherical harmonics can be obtained by assuming that the perturbations in the gas become equalized instantaneously; the boundary condition for the perturbed velocity is its continuity condition:

$$u_n = \left(\frac{d\Delta_n}{dt} + \Delta_n \frac{\partial v_0}{\partial r} \right)_R, \qquad (12)$$

and the perturbation of the pressure p_n on the boundary is balanced by the difference between the bydrostatic pressures of the liquid and the gas in the field of the acceleration dv_0/dt:

$$p_n|_R=(\rho_0-\rho_{gas})\Delta_n\frac{dv_0}{dt};\quad \rho_{gas}=\rho_{gas}\left(\frac{R_0}{R}\right)^3. \tag{13}$$

Using these two conditions and the equation for the perturbations in the liquid,[15] we can obtain the sought equation for the amplitude of the boundary perturbation[8]:

$$R\frac{d^2\Delta_n}{dt^2}+3v_0\frac{d\Delta_n}{dt}+\left[2-\frac{\rho_0-\rho_{gas}}{\rho_0}(n+1)\right]\frac{dv_0}{dt}\Delta_n=0. \tag{14}$$

Equation (14) can be easily integrated numerically for any law of motion, and the eomplete solution the problem can be obtained. We confine ourselves to an analytic solution that describes two characteristic stages of the development of the instability of the considered surface. For short-wave perturbations ($n\gg 1$) we can obtain from (14) an analytic solution without specifying in detail the law of the motion:

$$\Delta_n=\Delta_n(R)\frac{R_0}{R}\left[\frac{v_0(R_0)}{v_0}\right]^{1/2}\left[\frac{d\ln v_0}{d\ln R}\right]^{1/2}_{R=R_0}\left[\frac{d\ln v_0}{d\ln R}\right]^{1/2}$$
$$\times\cos\left\{n^{1/2}\int\limits_{R_0}^{R}-\frac{(\rho_0-\rho_{gas})}{\rho_0}\frac{d\ln v_0}{d\ln R}\frac{dR}{R}\right\}. \tag{15}$$

During the initial stage, when the gas pressure does not affect the motion of the liquid, the law of motion of the interface is the same as in the Rayleigh problem[21]: $v_0\sim R^{-3/2}$, and the solution (15) coincides with the solution for the boundary of a vacuum bubble in an incompressible liquid.[20] In this case $\Delta_n\sim R^{-1/4}$, and the interface oscillates at a frequency that depends on the number n of the spherical harmonic of the perturbation. As soon as the gas pressure begins to slow down the liquid ($dv/dt>0; d\ln v/d\ln R>0$) the growth of the perturbations becomes exponential. If we change in (15) to the variable t ($R\to t$), then expression (15) becomes

$$\Delta_n\sim\exp\left\{\int\limits_0^t\left[\frac{\rho_0-\rho_{gas}}{\rho_0}\frac{n}{R}\left|\frac{dv}{dt}\right|\right]^{1/2}dt\right\}. \tag{16}$$

It is easily seen that the last equation is a generalization of the Taylor formula[3] to include the case of a time-varying gas density ρ_{gas} and acceleration dv/dt, when the instability develops on the contracting surface of radius R.

Thus, the development of the perturbations of the boundary between a compressing as sphere and an incompressible gas liquid goes through two stages. In accelerated motion, the boundary executes quasiperiodic oscillations with variable phase and weakly increasing amplitude. The main increase of the amplitude takes place during the stage of the exponential growth, which sets in after the start of the deceleration.

5. GASDYNAMIC INSTABILITY IN COMPRESSION OF A GAS-FILLED GLASS SHELL(TWO-DIMENSIONAL NUMERICAL EXPERIMENT)

A self-consistent treatment of a number of the effects indicated above is possible only by numerical simulation of the phenomenon. The physical model on which the numerical experiment is based includes a system of two-dimensional equations of two-temperature single-fluid magnetohydrodynamics, and of the electronic thermal conductivity in the axial-symmetry approximation.[9,10] Account is taken of the ion viscosity and of the Spitzer thermal conductivity; the energy of the laser radiation is released in the vicinity of the critical density. Generation of a magnetic field as a result of the thermoelectric currents due to the instability is calculated in the linear approximation without allowance for the reaction of the field on the hydrodynamics, but with allowance for convection and for the finite conductivity. The procedure and the mathematical program for the numerical solution of the problem indicated above were developed at the Institute of Applied Mechanics of the USSR Academy of Sciences.[23]

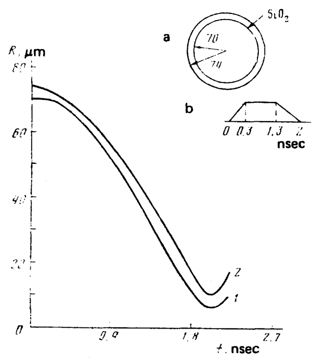

FIG. 1. R—t diagram of motion of the characteristic boundaries (1—inner boundary, 2—evaporation boundary) and typical parameters of the investigated target (a) and of the laser pulse (b).

A check of the procedure using a system of test problems has shown that the numerical solution duplicates qualitatively and quantitatively the dynamics of the evolution of the perturbations in known analytic solutions.[24]

Typical parameters of the investigated target and of the acting laser pulse, and a diagram of the motion of the shell-gas interface and of the evaporation boundary are shown in Fig. 1. We investigated the perturbations of the shape of the target surface $R(\theta,t)$ and of the homogeneity of the laser flux $q(\theta,t)$:

$$f=f_0\left(1+\sum_n\Delta_n\cos n\theta\right);\quad f\to R_0(\theta,t), q(\theta,t). \tag{17}$$

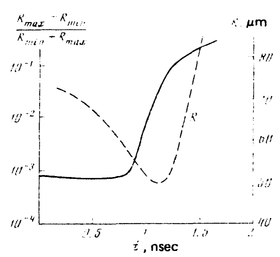

FIG. 2. Angle-averaged trajectory of Lagrangian particle near the evaporation boundary and relative amplitude of perturbation on it.

1. Dynamics of instability development in the corona near the evaporation boundary

A Lagrangian particle moving initially with the "cold" part of the shell towards the center of the target is subsequently captured by the thermal wave, is heated, and is carried away into the corona. Instability develops when the particle is stopped and the sign of the velocity is reversed. Figure 2 shows the angle-averaged trajectory of such a particle and the relative amplitude of the perturbation on it

$$\frac{R_{max}-R_{min}}{R_{max}+R_{min}}=\frac{a}{\bar{R}};\quad \bar{R}={}^1/_2(R_{max}+R_{min})$$

for the case when at the initial instant of time there are specified on the outer and inner shell boundaries perturbation of equal amplitude (10^{-2} of the initial shell thickness) and of the wavelength ($\lambda=2\pi R_0/n$; $n=10$). The growth rate of the perturbations is maximal during the slowing-down time and is practically zero in the region of the supersonic flow. The instability zone is narrow in this case, 0.2–0.3 μm, therefore the saturation of the growth rate with increasing wave number ($k=n/R$) is determined by the scale of the density gradient, so that the growth rate becomes independent of n at $n\simeq 10$–20.

At low amplitudes (exponential growth) the growth rate is somewhat smaller than the increment of the Taylor modes, because of the convective stabilization and heat-conduction equalization. During the final stage of the compression the amplitude of the perturbation increases by more than 10^2 times, so that the condition $ak\gtrsim 1$ is satisfied even for long waves ($n=10$). This means that the nonlinear interaction connected with energy transfer into the short-wave part of the spectrum can also lead to effective stabilization. By the instant of maximum compression, the amplitude of the perturbation becomes comparable with the shell thickness. This, however is no evidence that the shell is destroyed, since the perturbations are concentrated near the evaporation boundary and have no time to penetrate inside. This is clearly demonstrated by the degree of bending of the Lagrangian lines in different parts of the shell (Fig. 3).

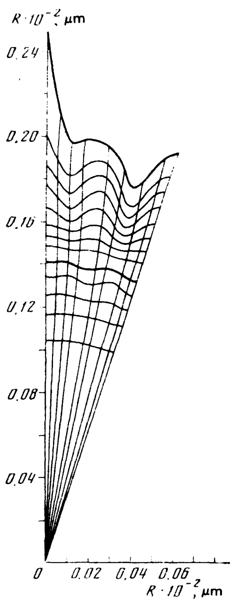

FIG. 3. Shape of perturbation near the evaporation boundary ($n=40$, $a=10^{-2}$, $t=1.725$).

The smoothing effect of the heat conduction can also be easily seen by comparing the development of the perturbation of the laser flux (the inhomogeneity of the target heating) with the perturbation of the shape of the surface of the same wavelength and relative amplitude. As expected, the temperature inhomogeneities lead to smaller final amplitudes of the perturbation during the concluding phase of the compression (smaller by approximately a factor of two in this case). This result agrees qualitatively with the data of Refs. 5, 11, and 13.

2. Transfer of perturbations from the corona to the inner shell boundary

Since the speed of sound in the unevaporated part of the shell is low [$(2-3)\cdot 10^5$ cm/sec] compared with the characteristic velocities [$(3-5)\cdot 10^6$ cm/sec], the perturbations are transported from the outer shell boundary to the inner one mainly by the first shock wave. This has been demonstrated both for the case of density perturbations[11] and for shell-shape perturbations.[10]

During the free-flight stage the shell–gas boundary begins to execute quasiperiodic oscillations with an amplitude that increases weakly with time.[10] These results agree well with the analytic solutions obtained for the oscillations of the cavity of a vacuum[20] or a gas-filled[8] bubble in an incompressible liquid. The thermal conductivity and the compressibility lead only to small quantitative changes.

3. Instability of glass-gas interface. Nonlinear saturation of growth rate

Instability sets in on this interface when the contained gas begins to slow the shell down. The behavior of the perturbations of the shape, with different wavelengths and with different initial amplitudes, were investigated.

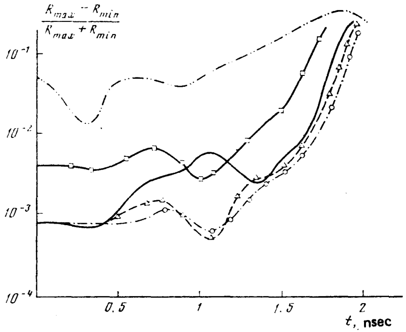

FIG. 4. Relative amplitude of perturbation on the inner shell boundary as a function of the time for different perturbation wavelengths: □—$n=10$, $a=5\cdot 10^{-2}$; △—$n=10$, $a=10^{-2}$; ○—$n=6$, $a=10^{-2}$; dashed—$n=40$, $a=10^{-2}$; dash and two dots—relative thickness δ/R of evaporated part of shell.

The relative amplitude of the perturbation

$$\frac{R_{max}-R_{min}}{R_{max}+R_{min}}=\frac{a}{\bar{R}}$$

is shown in Fig. 4 as a function of the time. As expected, an appreciable growth of the amplitude begins at the instant of slowing down ($t\simeq 1.5$ nsec).

Let us compare these results with the development of Taylor modes under the assumption that the density differential across the considered boundary is large, and the size of the gradient is small $((|\nabla \ln \rho|)^{-1} \ll \lambda)$. Then, taking into account the time variation of the radius and of the acceleration, we have[8]

$$\frac{a}{a_0}\simeq \exp\left(n^{1/2}\int \frac{g}{\bar{R}}\,dt\right). \tag{18}$$

We refer the amplitude to its value at the instant $t=1.5$. The values of $g(t)$ and $R(t)$ for (18) were determined from the results of the two-dimensional calculation. A comparison of the time dependence of the growth of the amplitude for the Taylor modes (18) and that obtained from the two-dimensional calculation (Fig. 4) is shown in Fig. 5. It follows from Fig. 5 that for a long-wave perturbation ($n=10$) the amplitude increases somewhat more slowly than for the Taylor modes, but the two dependences are close so long as $ak<1$. In the nonlinear stage ($ak>1$) the two-dimensional calculation predicts already a substantial decrease of the growth rate compared with (18). The amplitude of the short-wave perturbation ($n=40$) increases more slowly than for the Taylor modes from the very outset, and the nonlinear stage sets in prior to the maximum acceleration, after which the growth of the amplitude slows down substantially. It is easily seen that this is the transition to nonlinearity which was investigated by Fermi.[14] In fact, the dependence of the relative amplitude on the time during the nonlinear stage (at $ak>1$) is close to the dependence

$$\frac{1}{a_{1.5}}\int_{1.5}^{t} dt' \int_{1.5}^{t'} g(t'')\,dt'',$$

which corresponds to the asymptotic law for the fall-off of the humps in the field of a time-alternating acceleration. Also favoring this interpretation is an analysis of the shape of the shell boundary during the nonlinear stage. Even for the long-wave perturbation ($n=10$) it is seen that the humps and dips develop asymmetrically, thus attesting to the transition to the indicated asymptotic regime (see Fig. 6).

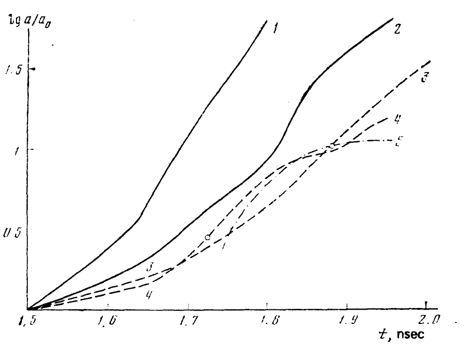

FIG. 5. Comparison of the time dependence of the amplitude of the Taylor modes with the analogous dependence obtained from the two-dimensional calculation. Solid curves—calculation by formula (18): 1—$n=40$, 2—$n=10$; dashed curves—two-dimensional calculation: 3—$n=10$, 4—$n=40$; dash-dot—free fall of the hump $a\sim gt^2$; the points on curves 3 and 4 correspond to satsifaction of the condition $ak=1$; the maximum acceleration occurs at the instant $t=1.8$ nsec.

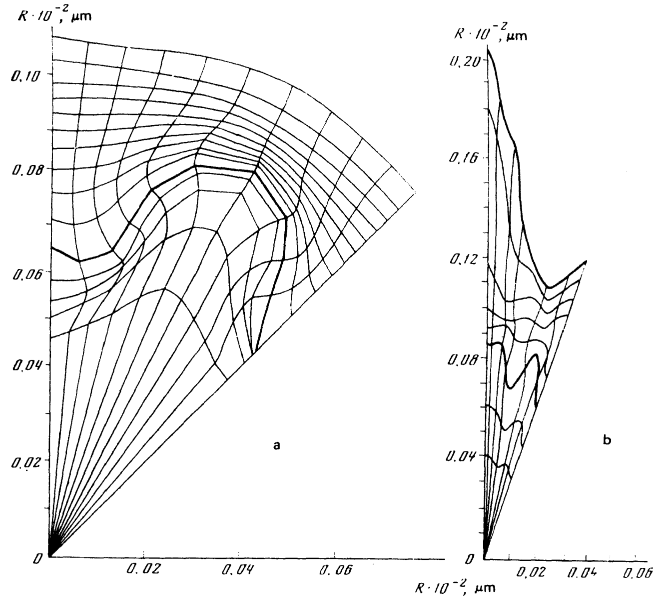

FIG. 6. Boundary between shell at gas at the instant of maximum compression: a—$n=10$, $a_0=10^{-2}\Delta R_0$; b—$n=40$, $a_0=10^{-2}\Delta R_0$.

It is seen even from Fig. 5 that the growth rates for the short-wave ($n=40$) and long-wave ($n=10$) perturbations are close. This circumstance can be more clearly revealed by plotting the dependence of $(R_{max}-R_{min})/(R_{max}+R_{min})$ on the number of the harmonic for several instants of time near the maximum compression (Fig. 7).

Thus, at $n\gtrsim 15$ the finite amplitude does not depend on the number of the spherical harmonic (or the wavelength) of the perturbation. All that depends on the wavelength is the time of transition into the nonlinear regime $[t_{tr}\sim(\lambda/g)^{1/2}]$. The practical conclusion of the foregoing results is that for targets with an aspect ratio $R_0/\Delta R_0 \lesssim 20$ the high-frequency perturbations

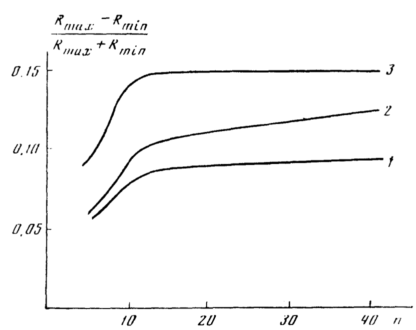

FIG. 7. Plot of $(R_{max}-R_{min})/(R_{max}+R_{min})$ against the number of the spherical harmonic for different instants of time: 1—$t=1.9125$; 2—$t=1.95$; 3—$t=1.968$.

present no special danger; it is merely necessary that the initial relative amplitude of the perturbation for all the wavelengths not exceed 10^{-2} (the ratio of the initial amplitude to the thickness of the shell).

4. Effect of instability on the characteristics of the gas in the maximum compression stage

To determine the maximum permissible and initial-perturbation amplitude that still does not lead to a substantial difference from the spherically symmetrical stage, we investigated perturbations with initial amplitudes amounting to 10^{-2}, $3\cdot 10^{-2}$ and $5\cdot 10^{-2}$ of the initial thickness, and with $n=12$. At an initial amplitude $5\cdot 10^{-2}$ the rupture of the shell occurs prior to the maximum compression. In the case $a_0=3\cdot 10^{-2}$ the final amplitude is comparable with the smallest average radius of the compressed gas, and the entire mass of the shell is involved in the perturbation (all the Lagrangian lines are bent). The closest to spherical symmetry are the results of shell compression with initial amplitude 10^{-2}. The amplitude of the perturbation of the inner boundary of the shell at the instant of the maximum compression is comparable with its thickness, but this still does not mean rupture of the shell, only that the glass has penetrated into the gas and these two substances can become intermixed as a result of the Kelvin-Helmholtz instability.

At a given initial perturbation amplitude (10^{-2}), the average density and average gas temperature over the nonspherical volume decrease with increasing number of the harmonic. The decrease of the neutron yield can in this case be larger than the value estimated from the average temperature since the neutron yield, especially at low temperatures (0.3–0.5 keV) depends substantially also on its spatial distribution.

CONCLUSION

The reported results show that the influence of convection on the instability in the case of spherical compression is not great. More significant is the role of the heat-conduction equalization, which is incidentally physically obvious. The effect of compressibility is small. It is important that the modified Taylor formula (16) yields the correct description of the development of the perturbations during the linear stage in the case of sufficiently complicated motion of the compressing spherical target, i.e., at arbitrary $R(t)$ and $\ddot{R}(t)$ dependences. The most important, in our opinion, is the effect of nonlinear saturation of the growth rate with decreasing wavelength of the perturbation, an effect having a clear physical interpretation stemming from Fermi's work.[14] The presence of saturation eliminates the danger faced by compression because of the short-wave perturbations that exist when pure Taylor modes ($\gamma^2=gk$) develop. After the completion of the present study, we learned of the results of Bodner and Boris,[12] who also observed nonlinear saturation, in qualitative agreement with our results. The cited paper[12] does not contain the here-presented analysis of the nonlinear stage.

The described picture of the development of instabilities and the results of the numerical experiment attest, in our opinion, to the feasibility of stable compression of spherical shell targets with a sufficiently large aspect ratio $R_0/\Delta R_0$. When assessing the feasibility of stable compression it must be remembered that a number of the factors discussed above (the symmetry of the irradiation and of the preparation of the target, the decrease of the density gradients, the equalization due to heat conduction and mixing, nonlinear saturation, change of the shell thickness during the compressions, oscillations of the shell surface, low sound velocity in the shell) contribute to the stability, and the problem is to find an acceleration regime, a target preparation scheme, or some other way of using these possibilities of decreasing the influence of hydrodynamic instability.

[1] Yu. V. Afanas'ev, N. G. Basov, P. P. Volosevich, E. G. Gamaliĭ, S. P. Kurdyumov, O. N. Krokhin, E. I. Levanov, V. B. Rozanov, A. A. Samarskiĭ, and A. N. Tikhonov, Pis'ma Zh. Eksp. Teor. Fiz. **21**, 150 (1975) [JETP Lett. **21**, 68 (1975)].
[2] J. Nuckolls, L. Wood, A. Thiessen, and G. Zimmerman, Nature **239**, 139 (1972).
[3] G. Taylor, Proc. Roy. Soc. Ser. A **201**, 192 (1950).
[4] S. Chandrasekhar, Hydrodynamic and Hydromagnetic Stability, Clarendon Press, Oxford, 1961.
[5] D. B. Henderson, R. L. McCrory, and R. L. Morse, Phys. Rev. Lett. 33, 105 (1974).
[6] S. Bodner, Phys. Rev. Lett. **33**, 761 (1974).
[7] K. Brueckner, S. Jorna, and R. Janda, Phys. Fluids **17**, 1554 (1974).
[8] E. G. Gamaliĭ, Kr. soobshch. po fizike (FIAN) No. 5, 23 (1976).
[9] P. P. Volosevich, E. G. Gamaliĭ, A. V. Gulin, V. B. Rozanov, A. A. Samarskiĭ, N. N. Tyurina, and A. P. Favorskiĭ, Pis'ma Zh. Eksp. Teor. Fiz. **24**, 283 (1976) [JETP Lett. **24**, 252 (1976)].
[10] E. G. Gamaliĭ, V. B. Rozanov, A. A. Samarskiĭ, V. F. Tishkin, N. N. Tyurina, and A. P. Favorskiĭ, Hydrodynamic Stability of Compression of Spherical Laser Targets, Preprint No. 117, Inst. Appl. Math. USSR Acad. Sci., 1978.
[11] N. N. Bokov, A. A. Bunatyan, V. A. Lykov, V. E. Nuvazhaev, L. P. Strotseva, and V. D. Frolov, Pis'ma Zh. Eksp. Teor. Fiz. **26**, 630 (1977) [JETP Lett. **26**, 478 (1977)].
[12] S. E. Bodner, J. P. Boris, G. Cooperstein, *et al.*, Report at 7th Internat. Conf. on Plasma Physics and Contr. Fusion Research, Insbruck, Austria, 23–30 August, 1978.
[13] W. C. Mead and J. D. Lindl, Preprint UCRL-78459, 1976; Preprint UCRL-77057, 1975.
[14] E. Fermi, Taylor Instabilities (Russ. transl.), Collected Works, Vol. 2, Nauka, 1972, p. 490.
[15] E. G. Gamaliĭ, V. E. Nuvazhaev, and L. I. Shibarshov, in: Chislennye metody mekhaniki sploshnoĭ sredy (Numerical Methods of Fluid Mechanics), ed. by Acad. N. N. Yanenko, Vol. 7, Novosibirsk, 1976, p. 47.
[16] Yu. V. Afanas'ev, N. G. Basov, E. G. Gamaliĭ, O. N. Krokhin, and V. B. Rozanov, Pis'ma v Zh. Eksp. Teor. Fiz. **23**, 617 (1976) [JETP Lett. **23**, 566 (1976)].
[17] Yu. V. Afanas'ev, E. G. Gamaliĭ, I. G. Lebo, and V. B. Rozanov, Zh. Eksp. Teor. Fiz. **74**, 516 (1978) [Sov. Phys. JETP **47**, 271 (1978)].
[18] E. G. Gamaliĭ, V. A. Gasilov, I. G. Lebo, V. B. Rozanov, V. F. Tishkin, and A. P. Favorskiĭ, Generation and Dissipation of Magnetic Fields in a Laser Plasma, Preprint No. 57, Phys. Inst. USSR Acad. Sci., 1979.
[19] M. S. Plesset, J. Appl. Phys. **25**, 96 (1954).
[20] A. D. Pernik, Problemy kavitatsii (Problems of Cavitation),

Sudostroenie, 1966.
[21]Ya. B. Zel'dovich and Yu. P. Raĭzer, Fizika udarnykh voln i vysokotemperaturnykh gidrodinamicheskikh yavleniĭ (Physics of Shock Waves and High-Temperature Hydrodynamic Phenomena), Fizmatgiz, 1966 [Academic, Chap. I only].
[22]E. I. Zababakhin, Prik. Mekh. Mat. 24, 1129 (1960).
[23]V. M. Goloviznin, V. F. Tishkin, and A. P. Favorskiĭ, Variational Approach to the Construction of Difference Schemes for Hydrodynamic Equations in Spherical Coordinates, Preprint, Inst. Appl. Math. USSR Acad. Sci., No. 16, 1977.
[24]V. A. Gasilov, V. M. Goloviznin, V. F. Tishkin, and A. P. Favorskiĭ, Numerical Solution of one Model Problem on Rayleigh-Taylor Instability, Preprint No. 119, Inst. of Appl. Math. USSR Acad. Sci., 1977.

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Propagation of electromagnetic solitons in nonequilibrium dispersive media

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We establish the possibility of the existence of stationary solitary electromagnetic waves in nonequilibrium plasma-beam systems. We show that the propagation of wave packets in resonance with the plasma (the retarding medium) or the beam is not accompanied by a dispersive smearing of the plasma. We determine the conditions for the existence of solitons, find their characteristics, and give a physical interpretation of the solutions obtained. As a concrete example we consider a helicon soliton in a plasma penetrated by an ion beam.

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It is well known that small monochromatic electromagnetic perturbations are unstable in plasma-beam systems and grow exponentially with time.[1] The non-linear stage of the interaction of mono-energetic beams with a plasma (the retarding medium) is accompanied with the appearance of oscillations in time of the field amplitude which are caused by the trapping of the beam particles by the field of the wave and their periodic shift from decelerating to accelerating phases.[2-8]

In the present paper we wish to call attention to the possibility of the existence of stationary non-linear waves in non-equilibrium media, which are retarding systems which are penetrated by charged particle beams. In comparison with the above cited papers[2-8] which assume the wave number fixed and the initial amplitude of the perturbation to be constant along the beam, we obtain a solution of a solitary-wave type for the field amplitude. Since the carrier frequency and wave number in this case satisfy resonance conditions, the wave energy in each point of space is replenished from the translational energy of the beam particles. At the same time, however, the energy of each beam particle remains unchanged after passing through the region where the field is a maximum due to the non-linear effect of getting out of phase with the wave which leads to a shift of the particle from a decelerating to an accelerating phase and an increase in its energy up to its initial value. This nature of the interaction between the beam particles and the field explains the possibility of the existence of resonance solitons in non-equilibrium plasma-beam systems.

As an example of an actual model of a non-equilibrium medium we consider an arbitrary dispersive retarding medium through which a charged particle beam propagates along a constant external magnetic field $\mathbf{H}_0$. Since practically the whole information about the conditions for the existence of non-linear waves of the kind

$$f(t,x)=\mathrm{Re}\,A(\xi)\exp(i\omega_0 t-ik_0 x) \tag{1}$$

($\xi=x-ut$, u is the group velocity) is contained in the linear dispersion equation and allowance for the non-linearity only enables one to determine the maximum amplitude and to find the shape of the wave pulse, we consider first the problem in the linear approximation.

The dispersion equation connecting the frequency ω and the wave number k of an electromagnetic perturbation propagating along the magnetic field in a medium with refractive index $n(\omega)$ has the form[2,6]

$$\frac{c^2k^2}{\omega^2}=n^2(\omega)-\frac{\omega_b^2(\omega-kv_0)}{\omega^2(\omega-kv_0+\omega_H)}, \tag{2}$$

where $\omega_b^2=4\pi e^2\rho_b/M$, $\omega_H=eH_0/Mc$, while ω_b, v_0, and M are the density, initial velocity, and mass of the beam.

We look for a solution of (2) in the form $\omega=\omega_0+\Delta\omega$ and $k=k_0+\Delta k$, assuming that ω_0 and k_0 satisfy the resonance conditions:

$$ck_0=\omega_0 n(\omega_0),\quad \omega_0-kv_0=-\omega_H. \tag{3}$$

The small corrections to the frequency and wave number are then connected by the relation

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