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Energy and angular distributions of particles reflected in glancing incidence of a beam of ions on the surface of a material

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An analytic solution is obtained of the problem of a reflection of a wide beam of charged particles incident at a glancing angle on the surface of a material. The angular and energy distributions are found for a wide range (from atomic to relativistic) of ion velocities. The theoretical results are in good agreement with experiments.

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1. INTRODUCTION

Reflection of particles from the surfaces of materials plays an important role in the problems of radiation stability of surfaces and radiation shielding. Moreover, reflection of particles from surfaces is the basis of various methods for investigating surface layers. In particular, sputtering of a surface by a glancing ion beam is used to determine the chemical composition of surface layers.

In the last two decades the problem of reflection of particles from the surfaces has been investigated intensively both experimentally[1-12] and theoretically.[13-20] The results of experimental investigations on the backscattering of ions by solid targets were summarized by Mashkova and Mochanov,[4] who formulated the following characteristic features established experimentally.

1. At low glancing angles ζ_0 the energy spectrum of reflected particles is very narrow.

2. When the scattering angle $\vartheta = \zeta_0$ is doubled (Fig. 1), the half-width of the energy spectrum changes by an order of magnitude.

3. The form of the energy spectrum is invariant under the transposition $\zeta_0 \rightleftarrows \zeta$.

4. The relative intensity of the reflected particles decreases on reduction in the nuclear charge Z_2 of the target atoms.

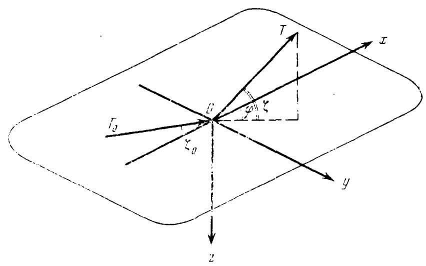

FIG. 1. Target surface coincides with the xy plane and the z axis is directed into the target. The primary beam of ions of kinetic energy T_0 is incident at an angle ζ_0 to the surface. The angular and energy distributions of the reflected particles are characterized by the angles ζ, φ and by the energy T.

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5. The energy spectrum is bell-shaped and its half-width increases on increase in Z_2.

6. Reduction in the initial energy and scattering angle ϑ produces similar changes in the energy spectra.

7. A maximum of the intensity of the reflected particles, integrated over the azimuth φ and over the energy of the reflected particles, occurs at the angle $\zeta=\zeta_0$, whereas in the plane of incidence ($\varphi=0$) the maximum corresponds to $\zeta=0.85\zeta_0$.

Some of these properties have been accounted for theoretically.[13-20] An important contribution to the theory of backscattering of charged particles from the surfaces of materials was made by O.B. Firsov. He used an original approach[13-16] to this problem and, in particular, he obtained the angular distribution of the reflected particles averaged over the azimuth without allowance for the energy losses. However, the absence of a simple analytic solution of the problem allowing for the deceleration of particles in the reflecting medium has made a complete analysis of the experimental results a fairly difficult task. For example, no studies have yet been made of such important processes as the dependence of the most probable energy and half-width of the energy spectrum of the reflected particles on the characteristics of the target, glancing angle, and initial ion energy. Moreover, the problem of the total backscattering coefficient, i.e., of the probability of emergence of a particle from a target, has not yet been solved.

Our aim is to obtain the complete distribution of the reflected particles over the angles and energies and to compare the theoretical distributions with the experimental results.

2. SOLUTION OF THE TRANSPORT EQUATION

Let us assume that a wide monoenergetic beam of ions with an initial energy T_0 is incident on a flat homogeneous target at a glancing angle $\zeta_0 \ll 1$. To be specific, we shall assume that the velocity vector of the incident particles lies in the xz plane (the z axis is directed into the target normally to the surface, see Fig. 1). We shall also assume that the effective angles of the scattering of ions by the target atoms θ_{eff} are small compared with the characteristic angles of the problem whose values are clearly of the order of ζ_0:

$$\vartheta_{\text{eff}} \ll \zeta_0. \tag{1}$$

In this case a particle may emerge from the target only after experiencing a large number of collisions with the target atoms, each of such collisions reflecting it through a small angle. This circumstance makes it possible to write down the elastic part of the collision integral of the transport equation in the diffusion approximation using the angular variables ζ and φ (Ref. 13). If, moreover, the mean-square value of the scattering angle over the whole range R_0 is small:

$$\langle\theta_s^2(T_0)\rangle R_0 \ll 1 \tag{2}$$

(here, $\langle\theta_s^2(T_0)\rangle$ is the mean-square of the scattering angle per unit path), the problem can be considered in the small-angle approximation. We shall confine ourselves to the range of initial energies T_0 satisfying the condition

$$1 \text{ keV/nucleon} < T_0 \ll A\cdot 10^3 \text{ keV/nucleon}, \tag{3}$$

where A is the atomic weight of the scattering particles. At these energies the process of deceleration of particles in a medium can be described by the model of continuous slowing down.

We shall use $N(z,\zeta,\varphi,T)$ for the density of the particle flux at a depth z, when these particles are moving in the direction (ζ,φ) and their energy is T. The transport equation for the function $N(z,\zeta,\varphi,T)$ subject to allowance for the energy losses is

$$\zeta\frac{\partial N}{\partial z}=\frac{\langle\theta_s^2(T)\rangle}{4}\left\{\frac{\partial^2 N}{\partial\zeta^2}+\frac{\partial^2 N}{\partial\varphi^2}\right\}+\frac{\partial}{\partial T}\{\bar{\varepsilon}(T)N\}. \tag{4}$$

Here, $\bar{\varepsilon}(T)$ is the average energy loss per unit path. When the condition (2) is obeyed, the quantity $N(z,\zeta,\varphi,T)$ decreases rapidly on increase in the angles ζ and φ. This allows us to assume, in the adopted approximation, that the angular variables ζ, φ vary from $-\infty$ to $+\infty$ and that the particles moving into the target are characterized by angles $\zeta>0$.

Equation (4) should be supplemented by the boundary condition

$$N(z=0,\zeta,\varphi,T)=\begin{cases}N_0\delta(\zeta-\zeta_0)\delta(\varphi)\delta(T-T_0), & \zeta>0,\\ N_0S(\zeta,\varphi,T), & \zeta<0,\end{cases} \tag{5}$$

where N_0 is the density of the flux in the incident beam and the reflection function $S(\zeta,\varphi,T)$ governs the angular and energy distributions of the reflected particles. Consequently, the boundary condition (5) contains an unknown function $S(\zeta,\varphi,T)$, which should be determined in the process of solving the problem.

Since the energy T in the scattered beam may exceed the energy of the incident ions, the following condition should also be satisfied:

$$N(z,\zeta,\varphi,T)=0 \quad \text{for} \quad T>T_0. \tag{6}$$

It is difficult to solve Eq. (4) subject to the boundary condition (5) in the general case of an arbitrary dependence of the mean-square angle of the particle scattering (per unit path) on the energy T. However, this difficulty can be avoided if it is assumed that the spectrum of the reflected particles has a sharp maximum near a certain energy $T\sim T_0$. Therefore, we shall adopt the approximation

$$\langle\theta_s^2(T)\rangle\approx\langle\theta_s^2(T_0)\rangle=\text{const}. \tag{7}$$

In the model of continuous slowing down there is a single-valued relationship between the energy of a particle T and the path traveled $L(T)$:

$$L(T)=R_0-R(T)=\int_T^{T_0}\frac{dT'}{\bar{\varepsilon}(T')}, \tag{8}$$

where $R(T)$ is the residual range of particles of energy T. For convenience in further calculations, we shall introduce dimensionless variables

$$\xi=\frac{z\langle\theta_s^2(T_0)\rangle}{4\zeta_0^3},\quad \psi=\frac{\zeta}{\zeta_0},\quad \chi=\frac{\varphi}{\zeta_0},\quad s=\frac{L(T)}{R_0} \tag{9}$$

and a new dimensionless function $N(\xi,\psi,\chi,s)$, which is related to the function $N(z,\zeta,\varphi,T)$ by

$$N(z,\zeta,\varphi,T)\,d\zeta\,d\varphi\,dT=-N(\xi,\psi,\chi,s)\,d\psi\,d\chi\,ds, \tag{10}$$

so that

$$N(z,\zeta,\varphi,T)=N(\xi,\psi,\chi,s)/\zeta_0^2R_0\varepsilon(T). \tag{11}$$

A new unit of depth $\zeta_0^3(\langle\theta_s^2(T_0)\rangle)^{-1}$ has a simple physical meaning. In fact, at a depth z there are only particles which have traversed a path $L\sim z/\zeta_0$ in the target; the effective transverse displacement from the initial trajectory reaches

$$r_{\perp\,\mathrm{eff}}\,(L^2\langle\theta_s^2\rangle)^{1/2}\approx[(z/\zeta_0)^3\langle\theta_s^2\rangle]^{1/2}.$$

As soon as this displacement becomes of the order of the depth z, the particles begin to emerge from the target. This is exactly what occurs in a layer of thickness $\zeta_0^3/\langle\theta_s^2\rangle$.

In terms of these new variables, Eq. (4) and the conditions (5) and (6) become

$$\psi\frac{\partial N}{\partial\xi}=\frac{\partial^2N}{\partial\psi^2}+\frac{\partial^2N}{\partial\chi^2}-\frac{1}{\sigma}\frac{\partial N}{\partial s}, \tag{12}$$

$$N(\xi=0,\psi,\chi,s)=\begin{cases}N_0\delta(\psi-1)\delta(\chi)\delta(s), & \psi>0,\\ N_0S(\psi,\chi,s), & \psi<0,\end{cases} \tag{13}$$

$$N(\xi,\psi,\chi,s)=0 \quad \text{for} \quad s<0. \tag{14}$$

In the approximation (7), Eq. (12) contains just one dimensionless parameter

$$\sigma=\langle\theta_s^2(T_0)\rangle R_0/4\zeta_0^2. \tag{15}$$

The parameter σ again has a simple physical meaning: it is of the same order as the ratio of the mean-square value of the scattering angle over the whole path to the square of the glancing angle. The greater the value of σ, the greater the proportion of particles reflected from the surface and the narrower the energy spectrum of the reflected particles.

It is convenient to solve Eq. (12) by applying the Laplace transformation to the variable s. We multiply Eqs. (12) and (13) by $\exp\{-ik\chi-ps\}$ and integrate with respect to χ between $-\infty$ and $+\infty$ and also with respect to s between 0 and $+\infty$. Then, the function

$$N(\xi,\psi,k,p)=\int_{-\infty}^{\infty}e^{-ik\chi}d\chi\int_0^{\infty}e^{-ps}N(\xi,\psi,\chi,s)\,ds \tag{16}$$

is subject to

$$\psi\frac{\partial N}{\partial\xi}=\frac{\partial^2N}{\partial\psi^2}-\left(k^2+\frac{p}{\sigma}\right)N, \tag{17}$$

$$N(\xi=0,\psi,k,p)=\begin{cases}N_0\delta(\psi-1), & \psi>0,\\ N_0S(\psi,k,p), & \psi<0.\end{cases} \tag{18}$$

The condition (14) is satisfied automatically because of the nature of the Laplace transformation. Solving Eq. (17) by the separation of variables, we obtain the following expression for $N(\xi,\psi,k,p)$

$$N(\xi,\psi,k,p)=\int_0^{\infty}C(\lambda)\mathrm{A}\left(\lambda\psi-\frac{\varkappa^2}{\lambda^2}\right)e^{-\lambda^3\xi}\lambda\,d\lambda,\quad \left(\varkappa^2=k^2+\frac{p}{\sigma}\right), \tag{19}$$

where $A(x)$ is the Airy function

$$\mathrm{A}(x)=\frac{3^{1/3}}{2\pi}\int_{-\infty}^{\infty}\exp\left(i\frac{\omega^3}{3}-ix\omega\right)d\omega, \tag{20}$$

and the unknown function $C(\lambda)$ can be determined from the boundary conditions (18). The functions $\mathrm{A}(\lambda\psi-\varkappa^2/\lambda^2)$ satisfy the normalization condition

$$\int_{-\infty}^{\infty}\mathrm{A}\left(\lambda\psi-\frac{\varkappa^2}{\lambda^2}\right)\mathrm{A}\left(\lambda_1\psi-\frac{\varkappa^2}{\lambda_1^2}\right)\psi\,d\psi=\frac{\delta(\lambda-\lambda_1)}{\lambda_1}. \tag{21}$$

We shall substitute $\xi=0$, in Eq. (19); then multiplying the right- and left-parts of Eq. (19) by $\psi\mathrm{A}(\lambda_1\psi-\varkappa^2/\lambda_1^2)$, $\lambda_1<0$ and integrating with respect to ψ between $-\infty$ and $+\infty$ subject to Eqs. (18) and (21), we obtain

$$\mathrm{A}\left(\lambda_1-\frac{\varkappa^2}{\lambda_1^2}\right)+\int_{-\infty}^{0}\mathrm{A}\left(\lambda_1\psi_1-\frac{\varkappa^2}{\lambda_1^2}\right)S(\psi_1,k,p)\psi_1\,d\psi_1=0. \tag{22}$$

Multiplying both parts of Eq. (22) by $\lambda_1\mathrm{A}(\lambda_1\psi-\varkappa^2/\lambda_1^2)$, $\psi<0$ and integrating with respect to λ_1 between $-\infty$ and 0, we obtain the final integral equation for the reflection function $S(\psi,k,p)$:

$$\int_0^{\infty}\frac{S(\psi_1,k,p)K_1(2\varkappa(\psi^2-\psi\psi_1+\psi_1^2)^{1/2})}{(\psi^2-\psi\psi_1+\psi_1^2)^{1/2}}\psi_1\,d\psi_1=\frac{K_1(2\varkappa(\psi^2+\psi+1)^{1/2})}{(\psi^2+\psi+1)^{1/2}}, \tag{23}$$

where for convenience we have made the substitutions $\psi\to-\psi$ and $\psi_1\to-\psi_1$. In the resultant equation, $K_1(x)$ denotes the Macdonald function. It follows from Eq. (23) that the kernel and the right-hand side of the integral equation are described by functions $\rho^{-1}K_1(\rho)$, where $\rho^2=x^2+y^2-2xy\cos\gamma$. This suggests the following way of solving Eq. (23). Using the Gegenbauer addition theorem:[22]

$$\rho^{-1}K_1(\rho)=\frac{2}{xy}\sum_{n=1}^{\infty}\frac{n\sin(n\gamma)}{\sin\gamma}\Phi_n(x,y), \tag{24}$$

where

$$\Phi_n(x,y)=\begin{cases}I_n(x)K_n(y), & x<y,\\ I_n(y)K_n(x), & x>y\end{cases} \tag{25}$$

[$I_n(x)$ are modified Bessel functions], we expand the kernel and the right-hand side of Eq. (23) as a series. We seek the unknown function $S(\psi,k,p)$ in the form of a series

$$S(\psi,k,p)=\frac{1}{\psi}\sum_{m=0}^{\infty}A_m\Phi_{\alpha_m}(2\varkappa,2\varkappa\psi), \tag{26}$$

where the coefficients A_m and the function indices α_m have to be determined. Substituting the expansions (24) and (26) in Eq. (23) and bearing in mind that

$$\int_0^{\infty}\Phi_\alpha(x,y)\Phi_\beta(y,z)\frac{dy}{y}=\frac{\Phi_\beta(x,z)-\Phi_\alpha(x,z)}{\alpha^2-\beta^2}, \tag{27}$$

we find that

$$\alpha_m=3m+\frac{3}{2},\quad A_m=\frac{9}{2\pi}(-1)^m(2m+1). \tag{28}$$

Consequently, the function $S(\psi,k,p)$ is of the form

$$S(\psi,k,p)=\frac{9}{2\pi\psi}\sum_{m=0}^{\infty}(-1)^m(2m+1)\begin{cases}I_{3m+3/2}(2\psi\varkappa)K_{3m+3/2}(2\varkappa), & \psi<1,\\ I_{3m+3/2}(2\varkappa)K_{3m+3/2}(2\psi\varkappa), & \psi>1.\end{cases} \tag{29}$$

Applying the inverse Fourier transformation to the variable k and the inverse Laplace transformation to the variable p, we find that Eq. (29) readily yields the reflection function $S(\psi,\chi,s)$:

$$S(\psi,\chi,s)=\frac{3^{1/2}}{2\pi^2}\frac{\exp\{-[4(\psi^2-\psi+1)+\chi^2]/4\sigma s\}}{\sigma^{1/2}s^{5/2}}\mathrm{Erf}\left(\left(\frac{3\psi}{s\sigma}\right)^{1/2}\right). \tag{30}$$

In Eq. (30), $\mathrm{Erf}(x)$ is the probability integral. The knowledge of the explicit form of the function $S(\psi,k,p)$ makes it possible to find the function $C(\lambda)$ in Eq. (19) and thus solve the transport equation.

The density of the particle flux at a depth ξ traveling in the direction (ψ,χ) after traversing a distance s is given by

$$N(\xi,\psi,\chi,s)=\frac{1}{4\pi^2 i}\int_{-\infty}^{\infty}e^{-ik\chi}dk\int_{c-i\infty}^{c+i\infty}dpe^{ps}\int_0^{\infty}e^{-\lambda^2\xi}\lambda d\lambda A\left(\lambda\psi-\frac{\varkappa^2}{\lambda^2}\right)$$

$$\times\left\{A\left(\lambda-\frac{\varkappa^2}{\lambda^2}\right)-\int_0^{\infty}\psi_1 S(\psi_1,k,p)A\left(-\lambda\psi_1-\frac{\varkappa^2}{\lambda^2}\right)d\psi_1\right\}. \quad (31)$$

We can easily show that the above solution of Eq. (12) satisfies the boundary conditions (13).

3. ANGULAR AND ENERGY DISTRIBUTIONS OF REFLECTED PARTICLES

We shall now concentrate on the angular and energy distributions of the particles emerging from a target. It is convenient to describe this distribution by the differential backscattering coefficient $W(\psi,\chi,u)$ ($u=T/T_0$ is the dimensionless energy variable).

The quantity $W(\psi,\chi,u)$ represents the ratio of the number of particles reflected in a given direction from a unit surface area per unit time to the number of particles incident on the same unit area per unit time. Since the number of particles emerging from a unit area of the target per unit time in the direction (ψ,χ) is $N_0\psi S(\psi,\chi,u)$, the differential backscattering coefficient is

$$W(\psi,\ \chi,\ u)=\psi S(\psi,\ \chi,\ u). \quad (32)$$

From Eqs. (30) and (32), we obtain

$$W(\psi,\chi,u)=\frac{3^{1/2}}{2\pi^2}\frac{T_0\psi}{R_0\bar\varepsilon(u)}\frac{\exp\{-[4(\psi^2-\psi+1)+\chi^2]/4\sigma s(u)\}}{\sigma^{3/2}[s(u)]^{5/2}}$$

$$\times\operatorname{Erf}\left(\left(\frac{3\psi}{\sigma s(u)}\right)^{1/2}\right). \quad (33)$$

The total backscattering coefficient

$$w=\int W(\psi,\chi,u)\,d\chi d\psi du\leqslant 1 \quad (34)$$

represents the fraction of the particles reflected from the target, whereas $1-w$ is the probability that a particle remains trapped in the target.

Integration of Eq. (33) over the azimuthal angle χ gives the energy and polar distribution of the scattered particles irrespective of the azimuth:

$$W(\psi,u)=\frac{3^{1/2}}{\pi^{3/2}}\frac{T_0\psi}{R_0\bar\varepsilon(u)}\frac{\exp\{-(\psi^2-\psi+1)/\sigma s(u)\}}{\sigma[s(u)]^2}\operatorname{Erf}\left(\left(\frac{3\psi}{\sigma s(u)}\right)^{1/2}\right). \quad (34')$$

If we now integrate Eq. (34) with respect to the energy u between zero and unity, we clearly obtain the distribution of the emerging particles with respect to the polar angle ψ irrespective of the energy u and of the azimuthal angle χ:

$$W(\psi)=\frac{3^{1/2}}{\pi^{3/2}}\frac{\psi\exp\{-(\psi^2-\psi+1)/\sigma\}}{(\psi^2-\psi+1)}\operatorname{Erf}\left(\left(\frac{3\psi}{\sigma}\right)^{1/2}\right)$$

$$+\frac{3\psi^{3/2}}{\pi^{1/2}(1+\psi^3)}\operatorname{Erfc}\left(\frac{\psi+1}{\sigma^{1/2}}\right). \quad (35)$$

Firsov[13] found the polar distribution of the reflected particles in the case when the deceleration can be ignored. It is interesting to see how this special case is obtained from Eq. (35). The deceleration of particles in a medium becomes of little importance when the mean-square value of the scattering angle is much greater, over the whole path, than the square of the glancing angle, because in this case the particles emerge from the target before a significant loss in their velocity. Thus, the almost elastic reflection of particles corresponds to large values of the parameter $\sigma\gg 1$. Going in Eq. (35) to the limit $\sigma\to\infty$, we obtain

$$W_{el}(\psi)=\frac{3}{2\pi}\frac{\psi^{3/2}}{1+\psi^3}, \quad (36)$$

which is exactly identical with the results given in Ref. 13. The complete angular distribution in the elastic scattering case can easily be obtained from Eq. (33):

$$W_{el}(\psi,\chi)=\frac{1}{12\pi^2\psi^{1/2}}\left[\frac{\omega^4}{1+\omega^2}+\omega^3\operatorname{arctg}\omega\right], \quad (37)$$

$$\omega=\left\{\frac{3\psi}{\psi^2-\psi+1+(\chi/2)^2}\right\}^{1/2}. \quad (38)$$

As expected, the distribution (37) is normalized to unity:

$$\int_{-\infty}^{\infty}d\chi\int_0^{\infty}W_{el}(\psi,\chi)\,d\psi=1. \quad (39)$$

The mean square of the azimuthal angle in the elastic reflection case is given by the expression

$$\langle\chi^2(\psi)\rangle_{el}=\frac{1}{W_{el}(\psi)}\int_{-\infty}^{\infty}\chi^2 W_{el}(\psi,\chi)\,d\chi$$

$$=\frac{2}{(3\psi)^{1/2}}(1+\psi^3)\ln\left\{\frac{1+\psi+(3\psi)^{1/2}}{1+\psi-(3\psi)^{1/2}}\right\}. \quad (40)$$

The energy distribution of the scattered particles irrespective of the direction of emergence from the target can be obtained by averaging Eq. (33) over the angles:

$$W(u)=\frac{3}{8\cdot 2^{1/2}\pi^{1/2}}\frac{T_0\sigma}{R_0\bar\varepsilon(u)}F(r), \quad (41)$$

$$F(r)=\exp\{-r/4\}\left[r^{5/4}D_{-1/2}(r^{1/2})+\frac{1}{2}\sum_{n=0}^{\infty}\left(\frac{3}{2}\right)^n r^{(n+7/2)/2}D_{-n-3/2}(r^{1/2})\right],$$

$$r=\frac{2}{\sigma s(u)}. \quad (42)$$

Here, $D_\nu(x)$ is a parabolic cylindrical function. The function on the right-hand side of Eq. (41) decreases exponentially on increase in the argument r. This means that a particle may emerge from a target only when the mean-square value of the angle of deflection from the initial direction over a path s exceeds the square of the incidence angle, i.e., $L\langle\theta_s^2\rangle\gtrsim\xi_0^2$ or $r<1$. In the case of small values it tends to zero proportionally to $r^{5/4}$. For $r=r_m\approx 1.7$, the function $F(r)$ reaches a maximum.

4. COMPARISON OF THE THEORY WITH EXPERIMENT

In comparing the theoretical expressions for the differential reflection coefficient obtained in the preceding section with the available experimental results we have to know the actual form of the dependence $\bar\varepsilon(u)$ and also the value of the parameter σ. The fullest experimental study has been made of the reflection of heavy charged particles with an initial energy of several tens of kiloelectron volts per nucleon or less. In this case the velocity of the incident particles is usually less than the atomic velocity and the stopping power of the medium can be described by the Lindhard formula

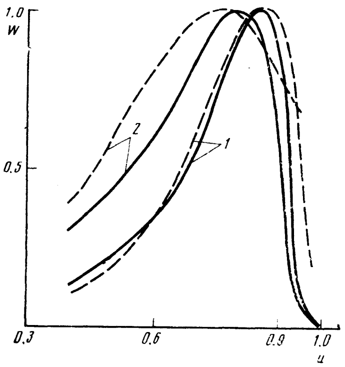

FIG. 2. Energy spectra of protons reflected by a nickel target ($T_0 = 20$ keV) at various angles ζ: 1) $\zeta = (1/3)\zeta_0$; 2) $\zeta = \zeta_0$. The dashed curves are the experimental results from Ref. 11. The continuous curves are calculated using Eq. (33) of the present paper.

$$\varepsilon(u) = 2T_0 u^{1/2}/R_0, \tag{43}$$

$$s(u) = 1 - u^{1/2}. \tag{44}$$

If particles bombarding a target are ions of charge Z_1 and mass M_1, the mean-square value of the scattering angle per unit path is described by[23,24]

$$\langle \theta_s^2(T) \rangle = \frac{1.82 a_0 e^2 n_0 Z_1 Z_2 (M_1 + M_2)}{(Z_1^{2/3} + Z_2^{2/3}) M_2 T}, \tag{45}$$

where a_0 is the Bohr radius; e is the electron charge; Z_2, M_2, and n_0 are, respectively, the nuclear charge, the atomic mass, and the density of atoms in the target. The range R_0 for these particles is given by[7]

$$R_0 = \frac{(Z_1^{2/3} + Z_2^{2/3})^{1/2} (T_0 T_B)^{1/2}}{4\pi \xi_e n_0 a_0 e^2 Z_2 Z_1^{1/6}}. \tag{46}$$

Here, ξ_e is a certain constant of the order of unity found experimentally, and

$$T_B = M_1 Z_1^{4/3} v_B^2/2, \quad v_B = e^2/\hbar.$$

Tel'kovskii *et al.*[10,11] determined the energy spectra of protons of initial energy $T_0 = 20$ keV scattered by a nickel target ($Z_2 = 28$). The glancing angle was $\zeta_0 = 15°$. Moreover, measurements were made of the stopping power of nickel and of the mean-square value of the scattering angle per unit path, corresponding to the experimental conditions in Ref. 24. It was found that $\xi_e = 1.5$ and the value of $\langle \theta_s^2 \rangle$ was in good agreement with Eq. (45). In this case the parameter σ was 4.1. It should be stressed that although $\langle \theta_s^2(T_0) \rangle R_0 \sim 1$, the theory developed above is applicable in the case of particles emerging with an energy T close to the initial energy. In fact, such particles travel a distance $L \ll R_0$ before emerging from the target and the quantity $\langle \theta_s^2 \rangle L$ still remains less than unity, so that the small-angle approximation holds. Figure 2 shows the energy spectra of the backscattered protons in the plane of incidence of the primary beam ($\chi = 0$) and those calculated from Eq. (33), and the spectra obtained in Ref. 11. For convenience of comparison the spectra are normalized, as in Ref. 11, to a unit intensity at the maximum.

It follows from Fig. 2 that the theory is in satisfactory agreement with the experimental results. The greatest discrepancy between the experimental and theoretical curves occurs in the "tails" of the distribution where the contribution of the singly scattered particles to the spectrum becomes important.

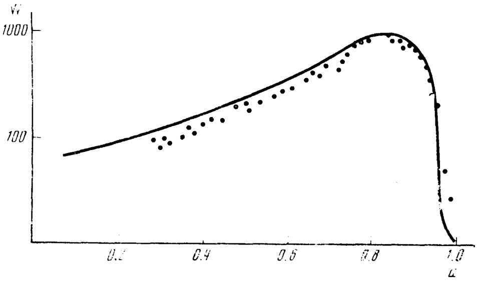

FIG. 3. Energy spectrum of protons ($T_0 = 275$ keV) reflected by an amorphous WO_3 target. The points are the experimental results from Ref. 8. The continuous curve is calculated from Eq. (33). The glancing angle is $\zeta_0 = 1.7°$ and the reflection angle is $\zeta = 2.7°$.

The small-angle approximation holds much better for particles whose initial velocity is greater than atomic. In the experiments of Marwick *et al.*[8] the energy spectra were obtained for protons reflected by a very thick amorphous tungsten oxide (WO_3) film evaporated on a metal substrate. The initial proton energy was 275 keV ($\zeta_0 = 1.7°$). In this range of energies the stopping power of the target for these protons was practically constant[8,25,26] i.e.,

$$\varepsilon(u) = T_0/R_0, \tag{47}$$

$$s(u) = 1 - u. \tag{48}$$

The mean-square value of the scattering angle per unit path can be described by

$$\langle \theta_s^2(T) \rangle = 2\pi n_0 Z_1^2 Z_2^2 L_k r_e^2/T^2, \tag{49}$$

where L_k is the Coulomb logarithm; r_e is the classical electron radius; and the energy T is in units of $m_e c^2$. Employing the published data on the range of protons in various media,[25] we can calculate R_0 for tungsten oxide. Having found $\langle \theta_s^2(T_0) \rangle$ from Eq. (49), we obtain the value of σ:

$$\sigma \approx 5.6. \tag{50}$$

Figure 3 shows a theoretical curve describing the energy distribution of the protons reflected in the plane of incidence of the primary beam ($\chi = 0$). The calculations were carried out using Eq. (3) and the relationships (48), (49), and (50). For convenience of comparison with the experimental data of Ref. 8, the scale is logarithmic along the ordinate. As expected, the greatest discrepancy between the theory and experiment is observed far from the maximum of the energy distribution. This is primarily due to neglect of the dependence of the mean-square value of the scattering angle per unit path on the particle energy. Nevertheless, the theoretical curve as a whole describes well the behavior of the spectrum.

The above comparison with the experimental results confirms that the proposed theory of the reflection of particles incident at a glancing angle on the surface of a target is in good agreement with the experimental data.

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Hydrodynamic stability of compression of spherical laser targets

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Hydrodynamic stability in compression of targets by laser radiation is investigated with account taken of convection, thermal conductivity, compressibility, and the spontaneous magnetic field. It is shown that the growth rate exhibits nonlinear saturation with decreasing perturbation wavelength. The conditions necessary for nearly symmetrical compression are determined, as is also the effect of the instability on the final state of the target.

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The hydrodynamic instability produced in low-entropy compression of spherical targets by laser radiation is the main obstacle to the attainment of the ultrahigh matter densities predicted by one-dimensional spherically symmetrical calculations.[1,2] It is known that the instability is in the main of the Rayleigh-Taylor type,[3] which has been thoroughly investigated in hydrodynamics, particularly when applied to incompressible fluid flow.[4] As a result of instability, the growth of the amplitude of the small perturbations due to variations of the intensity of the laser radiation, to deviations of the target-material density from homogeneity, and to distortion of the target shape, can lead to turbulization of the flow prior to the end of the compression process. New factors in the study of instability of a compressing plasma sphere are the electronic thermal conductivity (T_e, $T_i \sim 1$ keV), compressibility, high radial gradients of the temperature and of the velocity, and generation of magnetic fields of considerable strength (~1 MG) against the background of the fast motion of the plasma towards the symmetry center. The study of the nature and of the methods of stabilizing

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