Rough model of formation of a dissipative current structure in He^3–A

G. E. Volovik

L. D. Landau Institute of Theoretical Physics, USSR Academy of Sciences
(Submitted 5 February 1980)
Zh. Eksp. Teor. Fiz. **79**, 309–316 (July 1980)

A dynamic model is proposed for the description of the behavior of superfluid flow in He^3 – A. The hydrodynamic equations are approximately reduced to three ordinary differential equations for variables averaged over large space and time scales: 1) the flow density; 2) the energy of distortion of the field **l** in the flow; (3) the density of the instantons that produce the phase slippage in the flow. The trajectories of this dynamic system are investigated. It is shown that in a certain range of the model parameters both uniform flow and flow in a dissipative regime with oscillating **l** are stable. Transition from one regime to the other can be effected only by an external perturbation of a special type, as is indeed observed in experiment.

PACS numbers: 67.50.Fi

INTRODUCTION

It is known that a superfluid liquid can flow in two different ways. In one, the liquid moves without encountering any resistance. The other is a dissipative regime; to maintain the liquid flow in this case it is necessary, as in ordinary liquid, to apply a chemical-potential gradient $\nabla\mu$. The acceleration imparted to the liquid by the external energy source

$$\partial \mathbf{v}_s/\partial t = -\nabla\mu,$$

is compensated by the internal phase slippage mechanism (see Ref. 1).

The phase slippage mechanisms are different in different substances and in different geometries. In superfluid He^4 the phase slippage can be due to motion of singular vortices.[2] In the A phase of superfluid He^3 it can be produced by motion of nonsingular vortices.[3] Experiment[4] shows, however, that in the A phase it proceeds via space-time oscillations of the anisotropy factor **l**.[5,6] The distribution of the field **l** in the flow is a periodic structure in the two-dimensional (z, t) space (z is the coordinate along the flow and t is the time). One of the possible field in a cell is shown in Fig. 1. Each such cell constitutes an instanton—a topological particle in two dimensional space-time (cf. the analogous particles in field theory[7]). The field in this cell maps the cell in the sphere $\mathbf{l}\cdot\mathbf{l}=1$ with degree unity. If such a distribution of the field **l** were to be realized not in the mapped space (z, t) but in the real (x, y) space, then it would correspond to a vortex without a singularity. A lattice of vortices in (z, t) space would correspond in ordinary (x, y) space to a lattice of nonsingular vortices, which are produced when He^3 – A rotates (see Ref. 8).

A similar instanton phase-slippage mechanism exists also in thin superconductors,[9] where the instanton is the counterpart of the Abrikosov vortex in two-dimensional space-time. The dissipative current regime in thin superconductors (or the resistive state) constitutes a lattice of such instantons. Numerical solutions for the dissipative current state with the instanton phase slippage mechanism were obtained in the He^3 – A case by by Hook and Hall,[10] and for thin superconductors by Ivlev *et al.*[11]

The main problem encountered in the investigation of the flow of a superfluid is identification of the flow regime that the system prefers to choose. Numerical methods of solving the rather complicated dynamic equations can as yet not answer this question. To obtain the answer it is necessary to investigate the behavior of the trajectories of the dynamic system, a difficult task for an infinite-dimensional function space.

In this paper it is proposed to simplify the problem by reducing approximately the system of partial differential equations to a system of ordinary differential equations for some quantities that are averaged over large space and time scales. These quantities are the energy density, the superfluid flow density, and the instanton density. The topology of the trajectories in this dynamic system should indeed determine the behavior of the liquid flow. This approach yields a rough description of the flow. The flow is characterized only by the temporal and spatial scales of the oscillations and by the size of the average flow. No account is taken here, naturally, of the fact that energy barriers of topological or hydrodynamic origin can exist between different states that are close in energy and in the scales of the variation of **l**. The presence of sufficiently strong mixing that makes it easy to overcome these barriers is assumed.

This approach is equivalent to that used by Vinen[2] to investigate the vortical flow of HeII, where the variables used were the superfluid velocity averaged over the vortices and the density of the vortices (the average vortex length per unit volume).

The first section deals with the instanton mechanism

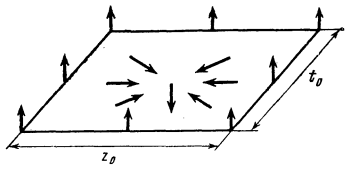

FIG. 1. One of the possible distributions of the field **l** in a cell of the two-dimensional space-time periodic structure produced in the course of liquid flow.

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of phase slippage and with the derivation of the approximate equations for the averaged quantities. In the second section are investigated the trajectories of the obtained dynamic system. It is shown that in a certain range of model parameters the topology of the trajectories is such that both flow regimes are stable. Therefore a transition between them occurs only if the system is subjected to perturbations of a definite type, as is in fact observed in experiment.[12]

1. DERIVATION OF THE PHENOMENOLOGICAL EQUATIONS

We assume the normal component to be frozen-in, the liquid to be incompressible, and the texture to be one-dimensional, i.e., all the variables depend on the single coordinate z along the flow. In this case the A-phase hydrodynamic equations take the form[13]

$$\frac{\partial v_s}{\partial t}=-\frac{\partial \mu}{\partial z}-n, \quad n=\frac{\hbar}{2m}\mathbf{l}\left[\frac{\partial \mathbf{l}}{\partial z}\times\frac{\partial \mathbf{l}}{\partial t}\right], \tag{1}$$

$$\gamma\frac{\partial \mathbf{l}}{\partial t}=-\frac{\delta\varepsilon}{\delta \mathbf{l}}+\mathbf{l}\left(\mathbf{l}\frac{\delta\varepsilon}{\delta \mathbf{l}}\right)+\frac{\hbar}{2m}g\left[\mathbf{l}\times\frac{\partial \mathbf{l}}{\partial z}\right]. \tag{2}$$

Here v_s is the component of the velocity $\mathbf{v}_s$ along z, and g is the z-component of the flow of the superfluid component. By virtue of the continuity equation, g is independent of the coordinates.

The energy ε is a function of $\mathbf{l}$ and of $\mathbf{v}_s$;

$$2\varepsilon=\rho_s\mathbf{v}_s^2-\rho_0(\mathbf{l}\mathbf{v}_s)^2+2C\mathbf{v}_s\,\mathrm{rot}\,\mathbf{l}-2C_0(\mathbf{l}\mathbf{v}_s)(\mathbf{l}\,\mathrm{rot}\,\mathbf{l})$$
$$+K_1(\nabla\mathbf{l})^2+K_2(\mathbf{l}\,\mathrm{rot}\,\mathbf{l})^2+K_3[\mathbf{l}\times\mathrm{rot}\,\mathbf{l}]^2,\ g=\delta\varepsilon/\delta v_s. \tag{3}$$

The parameters K, C, C, ρ, and ρ_0 are of the same order of magnitude accurate to $\hbar/m$:

$$K_1\sim K_2\sim K_3\sim C\hbar/m\sim C_0\hbar/m\sim\rho_s(\hbar/m)^2\sim\rho_0(\hbar/m)^2. \tag{4}$$

Equation (1) describes the acceleration of v_s by the applied chemical-potential gradient and the deceleration of v_s by the oscillations of the vector $\mathbf{l}$.

The expression for n in (1) has a simple physical meaning. This is the density of the instanton charge. The instanton-charge quantum coincides with the circulation quantum of the superfluid velocity around a vortex without a singularity, and is equal to $2\pi\hbar/m$. Integrating the equation for n over the region ΔS occupied by one instanton (see Fig. 1), we obtain in fact the instanton charge

$$\int_{\Delta S} n\,dt\,dz=2\pi\hbar/m. \tag{5}$$

The density of the instanton charge is the counterpart of the vortex flux in the Vinen theory.[2] It is the averaged value of this variable that will be used in the rough model. If t_0 and z_0 are the time and space scales of the variation of the vector $\mathbf{l}$, then

$$|n|\sim\frac{\hbar}{m}\frac{1}{t_0 z_0}. \tag{6}$$

The variable n does not determine uniquely, however, the scales of the structure. It is therefore necessary to introduce one more variable, connected either with t_0 or with z_0. This variable is the energy.

We simplify the problem by putting the parameters $\rho_0=C_0=0$ in the expression (3) for the energy. The energy of the A phase then becomes, just as for a nematic liquid crystal,

$$\varepsilon=\tfrac{1}{2}\rho_s v_s^2+\bar{\varepsilon}\{\mathbf{l}\},$$
$$\bar{\varepsilon}=\tfrac{1}{2}K_1(\nabla\mathbf{l})^2+\tfrac{1}{2}K_2(\mathbf{l}\,\mathrm{rot}\,\mathbf{l})^2+\tfrac{1}{2}K_3[\mathbf{l}\times\mathrm{rot}\,\mathbf{l}]^2. \tag{7}$$

With this choice of energy, the velocity

$$v_s=g/\rho_s$$

is constant in space, and the energy of distortion of the field of the vector $\mathbf{l}$ is connected in the following manner with the spatial scale z_0:

$$\bar{\varepsilon}\sim K/z_0^2. \tag{8}$$

The variables n, $\bar{\varepsilon}$, and v_s, averaged over scales that exceed the characteristic scales t_0 and z_0 of the structure, thus determine the size of the flow and the scales of the structure produced in this flow. Our task is to write down a system of ordinary differential equations for these large-scale variables.

The equation for v_s is obtained by averaging Eq. (1), i.e., by replacing all the variables in this equation by their mean values. The quantity $\partial\mu/\partial z$ is either specified, if the difference between the chemical potentials at the ends of the channel is maintained, or determined from this equation if the current is specified. In the latter case

$$\partial\mu/\partial z=-n. \tag{9}$$

The equation for $\bar{\varepsilon}$ is obtained from (2) in the following manner:

$$\frac{\partial\bar{\varepsilon}}{\partial t}=\frac{\partial\bar{\varepsilon}}{\delta \mathbf{l}}\frac{\partial \mathbf{l}}{\partial t}=-\gamma\left(\frac{d\mathbf{l}}{\partial t}\right)^2+\frac{\hbar}{2m}\rho_s v_s\mathbf{l}\left[\frac{\partial \mathbf{l}}{\partial z}\times\frac{\partial \mathbf{l}}{\partial t}\right]. \tag{10}$$

The first term in the right-hand side of this equation can be easily estimated by using (6), (8), and (4):

$$\left(\frac{\partial \mathbf{l}}{\partial t}\right)^2\sim t_0^{-2}\sim\frac{n^2}{\bar{\varepsilon}}\frac{Km^2}{\hbar^2}\sim\rho_s\frac{n^2}{\bar{\varepsilon}}.$$

As a result we have

$$\frac{\partial\bar{\varepsilon}}{\partial t}=\rho_s v_s n-a_1\rho_s\gamma\frac{n^2}{\bar{\varepsilon}}. \tag{11}$$

Here $a_1>0$ is a numerical coefficient of the order of unity. This coefficient cannot be determined exactly; it must be regarded as a phenomenological parameter of the model.

We call attention to form of the time derivative of the total energy:

$$\frac{\partial\varepsilon}{\partial t}=\frac{\partial\bar{\varepsilon}}{\partial t}+\rho_s v_s\frac{\partial v_s}{\partial t}=-\rho_s v_s\frac{\partial\mu}{\partial z}-a_1\rho_s\gamma\frac{n^2}{\bar{\varepsilon}}. \tag{12}$$

The second term in the right-hand side of (12) is the dissipation function

$$R=\gamma\left(\frac{\partial \mathbf{l}}{\partial t}\right)^2\sim\gamma\rho_s\frac{n^2}{\bar{\varepsilon}}, \tag{13}$$

and the first term is the work of the external forces. Dissipation of the flow is due precisely to the instantons (there is no dissipation at $n=0$); in exactly the same manner, dissipation in the Vinen model is due to the vortices.

To find the equation for n, we consider the time derivative of the dissipative function:

$$\frac{\partial}{\partial t}\left(\gamma\rho_s\frac{n^2}{\bar{\varepsilon}}\right)\sim\frac{1}{2}\frac{\partial R}{\partial t}=\frac{\partial \mathbf{l}}{\partial t}\frac{\partial}{\partial t}\left(-\frac{\delta\bar{\varepsilon}}{\delta \mathbf{l}}+\mathbf{l}\left(\mathbf{l}\frac{\delta\bar{\varepsilon}}{\delta \mathbf{l}}\right)+\frac{\hbar}{2m}\rho_s v_s\left[\mathbf{l}\times\frac{\partial \mathbf{l}}{\partial z}\right]\right)$$
$$=-\frac{\partial \mathbf{l}}{\partial t}\frac{\partial}{\partial t}\frac{\delta\bar{\varepsilon}}{\partial \mathbf{l}}+\left(\frac{\partial \mathbf{l}}{\partial t}\right)^2\left(\mathbf{l}\frac{\delta\bar{\varepsilon}}{\delta \mathbf{l}}\right)+\frac{\hbar}{2m}\rho_s v_s\frac{\partial \mathbf{l}}{\partial t}\left[\mathbf{l}\frac{\partial}{\partial t}\times\frac{\partial \mathbf{l}}{\partial z}\right]+n\rho_s\frac{\partial v_s}{\partial t}. \tag{14}$$

The second term in the right-hand side of (14) is estimated with the aid of (7):

$$|\delta\tilde{\varepsilon}/\delta\mathbf{l}|\sim\tilde{\varepsilon}. \tag{15}$$

The third term is of the order of

$$\frac{\hbar}{2m}\rho_s v_s \frac{1}{t_0}\frac{\partial}{\partial t}\frac{1}{z_0}\sim\frac{\rho_s v_s n}{\tilde{\varepsilon}}\frac{\partial\tilde{\varepsilon}}{\partial t}\sim\frac{\rho_s^2 v_s^2 n^2}{\tilde{\varepsilon}}-\tilde{a}_1\frac{\rho_s^2 v_s\gamma n^3}{\tilde{\varepsilon}^2}. \tag{16}$$

In this estimate the coefficient $\tilde{a}_1$ need not necessarily coincide with a_1 in (11).

Finally, the first term can be estimated by putting for simplicity $\tilde{\varepsilon}=\frac{1}{2}K(\partial\mathbf{l}/\partial z)^2$, and then

$$\left\langle -\frac{\partial\mathbf{l}}{\partial t}\frac{\partial}{\partial t}\frac{\delta\tilde{\varepsilon}}{\delta\mathbf{l}}\right\rangle = K\left\langle\frac{\partial\mathbf{l}}{\partial t}\frac{\partial}{\partial t}\frac{\partial^2}{\partial z^2}\mathbf{l}\right\rangle = -K\left\langle\left(\frac{\partial}{\partial t}\frac{\partial\mathbf{l}}{\partial z}\right)^2\right\rangle$$
$$\sim -K\left(\frac{\partial}{\partial t}\frac{1}{z_0}\right)^2\sim-\frac{1}{\tilde{\varepsilon}}\left(\frac{\partial\tilde{\varepsilon}}{\partial t}\right)^2. \tag{17}$$

Substituting (15), (16), and (17) in (14) we obtain the equation for n:

$$\frac{\partial n}{\partial t}=\frac{a_2}{\gamma}n\tilde{\varepsilon}+\frac{a_3\rho_s}{\gamma}nv_s^2+a_4\rho_s\frac{n^2v_s}{\tilde{\varepsilon}}-a_5\rho_s\gamma\frac{n^3}{\tilde{\varepsilon}^2}+\frac{\rho_s}{\gamma}\tilde{\varepsilon}\frac{\partial v_s}{\partial t}. \tag{18}$$

Here $a_2>0$, $a_5>0$, a_3 and a_4 are phenomenological numerical parameters of the order of unity.

We have thus obtained a closed system of equations (1), (11), and (18) for the large-scale variables v_s, $\tilde{\varepsilon}$, and n. The system depends on five dimensionless parameters, but effectively it depends only on four, since one of them can be set equal to unity by a gauge transformation. On the other hand if we consider flow with a specified current, then only three parameters remain. The equations with the current given are written by changing to a system of units in which $\rho_s=\gamma=1$:

$$\dot{\tilde{\varepsilon}}/\alpha_1=nv_s-n^2/\tilde{\varepsilon},\quad \alpha_1>0, \tag{19}$$
$$\dot{n}=n\tilde{\varepsilon}-n^3/\tilde{\varepsilon}^2+\alpha_2 n^2v_s/\tilde{\varepsilon}-\alpha_3 nv_s^2. \tag{20}$$

2. INVESTIGATION OF THE PHENOMENOLOGICAL EQUATIONS

The trajectories of the dynamic system (19), (20) depend on the parameters α_1, α_2, and α_3 of the model, and these depend in turn on the coefficients in the expression (3) for the energy, and can consequently be functions of the temperature. The three-dimensional space of the parameters α_i can be broken up into domains. The trajectories within each domain have a definite topology, which is the same for each point of this domain. When the temperature is changed, a transition from one parameter domain to another is possible, and this is accompanied by bifurcation, i.e., by a change of the topology of the trajectories. We clearly ascertain first which stationary points of the equations exist and in which parameter domains they are stable.

At all values of the parameters α_i the equations have stationary points corresponding to stationary current states without dissipation, $n=0$ and $\tilde{\varepsilon}=\varepsilon_0$, where ε_0 is arbitrary, i.e., there exists an entire line of such states. A state with $\varepsilon_0\neq 0$ means flow with an inhomogeneous field $\mathbf{l}$, for example a spiral structure (see Ref. 11). At $\alpha_3<0$ an inhomogeneous current state with arbitrary ε_0 is unstable. At $\alpha_3>0$ the inhomogeneous states that are stable are those for which

$$0<\varepsilon_0<\alpha_3 v_s^2.$$

Under the condition

$$1+\alpha_3>\alpha_2 \tag{21}$$

the equations have a stationary point corresponding to a dissipative oscillatory flow regime:

$$\tilde{\varepsilon}=(1+\alpha_3-\alpha_2)v_s^2,\quad n=\tilde{\varepsilon}v_s. \tag{22}$$

The temporal and spatial scales of the oscillations in this regime are

$$t_0\sim\gamma/\rho_s v_s^2,\quad z_0\sim\hbar/mv_s. \tag{23}$$

This flow regime is easily observed in experiment because the vector $\mathbf{l}$ oscillates at a frequency of the order of t_0^{-1}.[4,12] These oscillations are observed with the aid of ultrasound, the damping of which is sensitive to the orientation of the vector $\mathbf{l}$. The work expended by the external source to maintain this flow is

$$R\sim\gamma t_0^{-2}\sim\rho_s^2 v_s^4/\gamma, \tag{24}$$

and the chemical-potential gradient, equal to the friction force exerted by the normal component on the superfluid one via the instanton, is of the order of

$$\partial\mu/\partial z\sim\hbar/mt_0z_0\sim\rho_s v_s^3/\gamma. \tag{25}$$

The cubic dependence of $\partial\mu/\partial z$ on v_s is the same as in the Vinen theory.[2]

The obtained dissipative structure is stable only under the condition

$$\alpha_1+\alpha_2<2, \tag{26}$$

while under the condition

$$4\alpha_1(1+\alpha_3-\alpha_2)>(2-\alpha_1-\alpha_2)^2 \tag{27}$$

the stationary point is a stable focus, whereas for the inverse inequality it is a stable break.

A uniform current state without dissipation ($n=0$, $\tilde{\varepsilon}=0$) is a complex saddle-node. To assess the stability of this stationary point it is necessary to investigate Eqs. (19) and (20) in which the first term of the right-hand side of (20) can be neglected compared with the fourth. Putting $v_s=\gamma=\rho_s=1$ we have

$$\dot{\tilde{\varepsilon}}/\alpha_1=n(1-n/\tilde{\varepsilon}),\quad \dot{n}=n(-\alpha_3+\alpha_2 n/\tilde{\varepsilon}-(n/\tilde{\varepsilon})^2).$$

We set $y=n/\tilde{\varepsilon}$ and change over to the variables $\tilde{\varepsilon}$ and y:

$$\alpha_1^{-1}\dot{\tilde{\varepsilon}}/\tilde{\varepsilon}=y(1-y), \tag{28}$$
$$\dot{y}=y((\alpha_1-1)y^2+(\alpha_2-\alpha_1)y-\alpha_3). \tag{29}$$

The trajectories of the dynamic system (28), (29) depend on the coefficients of the quadratic trinomial

$$(\alpha_1-1)y^2+(\alpha_2-\alpha_1)y-\alpha_3. \tag{30}$$

We consider directly the following range of parameters:

$$\alpha_1>1,\quad \alpha_3>0. \tag{31}$$

In this case the stationary points of Eq. (29) are $y=0$ as well as $y=y_1$ and $y=y_2$, which are roots of the trinomial (30), with $y_1<0$ and $y_2>0$. Of these three stationary points, only $y=0$ is stable. Therefore as $t\to\infty$, depending on the initial conditions, y tends either to $\pm\infty$ or to zero. It follows from (28) that in this case $\tilde{\varepsilon}$ tends either to zero or to arbitrary ε_0, while n tends to zero independently of the initial conditions. Consequently, in the domain of the parameters (31) the homo-

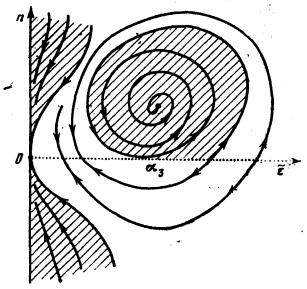

FIG. 2. Behavior of the trajectories of the dynamic model in the case when a homogeneous superfluid flow and a dissipative flow are simultaneously stable.

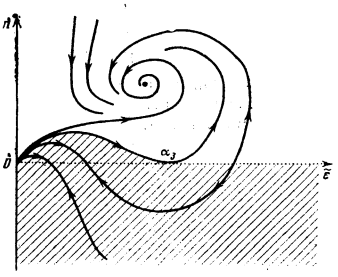

FIG. 3. Behavior of the trajectories in the case when the homogeneous flow is unstable and the dissipative one is stable.

geneous current state is stable.

In addition to the domain of the parameters (31) there are also other stability regions of the homogeneous current state. We shall not consider them here. We call attention to the fact that the domain of parameters (31), in which the homogeneous current state is stable, and the domain of parameters (21) and (26), where the dissipative flow regime is stable, overlap. The behavior in the overlap region, defined by the inequalities (21), (26), and (31), is shown in Fig. 2. We have chosen here that part of the overlap region in which the condition (27) is also satisfied, i.e., the dissipative current state is a stable focus.

It is seen that, depending on the initial conditions, the liquid flow becomes either uniformly superfluid or dissipative. If the initial state of the flow is located in one of the shaded regions of Fig. 2, then the flow reaches either the regime with $n=0$ and $\bar{\varepsilon}=0$, or the regime (22). On the other hand if the initial state lies outside the shaded regions, then the glow goes over into a nondissipative spiral structure (the segment $0<\bar{\varepsilon}<\alpha_3 v_s^2$ on the abscissa axis). The diffusive motion of the system along this segment takes the flow out to one of the two regimes. If, however, the system is in one of these regimes, then a transition to the other regime is possible only via a special perturbation of the flow. It appears that it is this behavior of the flow which is observed in an experiment[12] in which the transition from a homogeneous current state into an oscillatory regime is attained by using a definite scheme of turning the magnetic field on and off.

There are also other ranges of the parameters α_i in which both regimes are also stable. Therefore, to establish uniquely which topology of the trajectories corresponds to the real situation, a more detailed comparison with the existing experiment is necessary, as well as a more detailed experimental investigation of the regimes of turning on the oscillatory state.

Of particular interest is an experimental investigation of the possible bifurcations that occur when the temperature or the external fields change, and the parameters α_i change. We cite here only one example of what can happen when the parameters α_i go outside the domain (21), (26), (31). We consider the case when only one of the conditions of (31) is violated, namely, let $\alpha_i<1$, i.e., the homogeneous current state becomes unstable. Let the departure from the domain $\alpha_i>1$ occurs in the following ranges of parameters:

$$\alpha_1+\alpha_2<2,\quad \alpha_3>0,\quad \alpha_1<1,$$
$$4(1-\alpha_1)\alpha_3<(\alpha_2-\alpha_1)^2,$$
$$4\alpha_1(1+\alpha_3-\alpha_2)>(2-\alpha_1-\alpha_2)^2.$$

The behavior of the trajectories in this case is shown in Fig. 3. If the initial state lies in the unshaded region, then the flow goes over immediately into the dissipative oscillatory regime. If, however, the flow is in the shaded region at the initial instant, then the flow goes over first to the spiral structure (the segment $0<\bar{\varepsilon}<\alpha_3$) and then diffuses gradually to the ends of the segment and ultimately also goes over into the dissipative regime. This process should proceed much more slowly than the departure from the unshaded region. Therefore the existence of such a topological trajectory structure may also be experimentally verifiable.

In conclusion, the author thanks B. I. Ivlev, S. V. Iordanskiĭ, N. B. Kopnin, and I. A. Fomin for helpful discussions.

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Translated by J. G. Adashko