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Inhomogeneous collective oscillations of magnons

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An experimental investigation was made of oscillations excited by a radiofrequency (rf) field in a system of interacting microwave magnons. The range of existence of oscillations was determined from the magnitizing field and the rate of microwave pumping of magnons. A study was made of the behavior of the oscillation amplitude when the microwave field power and the rf signal were varied. Parametric excitation of two oscillation modes with different frequencies by a monochromatic rf signal was detected. The results obtained were in agreement with theoretical ideas on collective oscillations of magnons and on their dispersion law.

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1. INTRODUCTION

A system of magnons in a parametrically excited ferromagnet exhibits, under certain conditions, a secondary instability in the form of transient collective low-frequency oscillations of the magnon gas density. Under stable conditions these oscillations can be excited by an alternating field of appropriate frequency. Natural frequencies $\Omega_\varkappa$ of collective magnon oscillations depend on the rate of excitation of a system (i.e., on the number of magnons N) as well as on the effectiveness of the interaction of magnons and the nature of their spectrum governing the derivatives of the magnon frequency ω_k with respect to the wave number k, and particularly the magnon group velocity v. According to the calculations reported in Refs. 1 and 2, the spectrum of collective oscillations without allowance for for the damping is

$$\left.\begin{aligned}\Omega_\varkappa^2&=\left[2(T+S)N+\frac{\partial^2\omega_k}{\partial k_z^2}\frac{\varkappa^2}{2}\right]^2-(2TN)^2, \quad \varkappa\|\mathbf{M},\\ \Omega_\varkappa^2&=4S(2T+S)N^2+v^2\varkappa^2, \quad \varkappa\perp\mathbf{M}.\end{aligned}\right\} \tag{1}$$

Here, T and S are the coefficients of the Hamiltonian describing the interactions of magnons and $\varkappa$ is the wave vector of their collective oscillations.

There have been several experimental investigations of the simplest type of collective oscillations, which is a homogeneous mode Ω_0 corresponding to the gap in the spectrum (1). Earlier studies[3,4] were concerned with a system of microwave magnons in which homogeneous oscillations were induced by a resonant ($\Omega=\Omega_0$) action of a radiofrequency (rf) field. Parametric excitation of such oscillations by a field of double the frequency $\Omega=2\Omega_0$ was reported in Ref. 5. This method, known as the double paramagnetic resonance of magnons, makes it possible to generate both homogeneous and inhomogeneous collective modes with $\varkappa\neq 0$ but their identification is difficult because the signals due to the two types of oscillation are indistinguishable in a detector which records simply changes in the integrated magnetization.

Since the question of the existence of inhomogeneous collective magnon oscillations is of fundamental importance and direct observations of a signal of two such oscillations have not yet been made, we decided to detect inhomogeneous oscillations by investigating typical dependences characteristic only of these oscillations. This gave us certain positive data not only confirming the hypothesis of the existence of inhomogeneous collective magnon oscillations but also providing information on the form of their spectrum.

2. EXPERIMENTS

Collective magnon oscillations were investigated in a single crystal of yttrium iron garnet in a configuration providing the most stable conditions for spontaneous oscillations of the magnetization $\mathbf{M}\|\langle 001\rangle$. A magnon system was created parametrically by a microwave field $h\exp\{i\omega_r t\}$ of frequency 9.4 GHz by the parallel pumping method. Collective oscillations were produced by pumping additionally with an rf field $H_m\exp\{i\Omega_r t\}$ ($\Omega_r\approx 1$ MHz) polarized parallel to the static magnetic field. At a low amplitude of the rf pumping under the usual linear resonance conditions corresponding to $\Omega_r\approx\Omega_0$ the excitation produced homogeneous collective oscillations and, when a certain threshold amplitude was reached, also inhomogeneous parametric oscillations in accordance with the condition

$$\Omega_r\approx\Omega_{\varkappa_1}+\Omega_{\varkappa_2}\approx 2\Omega_\varkappa, \quad \varkappa_1\approx-\varkappa_2. \tag{2}$$

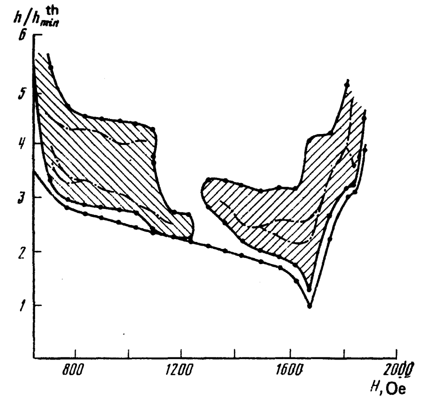

FIG. 1. Range of existence of parametric collective magnon oscillations; $\Omega_r = 1.43$ MHz, $H_m = 0.4$ Oe.

The approximate nature of the equality (2) was due to the low Q factor of the system of magnons interacting with one another. However, the equality was always exact for the actual frequencies of the oscillations participating in the resonance.

The signal due to collective magnon oscillations was detected by an induction coil located near the sample and a selective detector tuned to the frequency $\Omega_\varkappa$. The details of the technique of double paramagnetic resonance was described in Ref. 5. The potentialities of this method were limited to the detection of the oscillations for which $\varkappa^{-1}$ was comparable with the dimensions of the sample. The appropriate value of $\varkappa$ for a sphere 1.6 mm in diameter, used in our experiments, was $\varkappa_{max} \sim 10^3$. However, even this narrow range of $\varkappa$ was sufficient (as shown below) to identify the laws characteristic only of inhomogeneous oscillations.

Figure 1 shows the range of existence of parametric collective magnon oscillations on the scale of the static field H and of the amplitude of the microwave pumping, expressed in units of the minimum threshold. This figure gives also the threshold curve for microwave magnons (lowest curve).

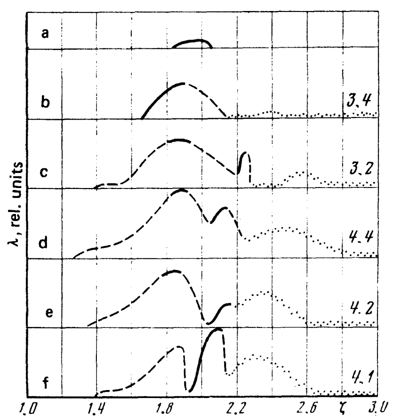

FIG. 2. Dependences of the amplitude of the signal due to parametric collective oscillations on the supercriticality of the magnon system for $H = 1670$ Oe, $\Omega_r = 1.43$ MHz. Curves a–f correspond to $H_m = 0.18$, 0.24, 0.30, 0.36, 0.40, and 0.44 Oe, respectively.

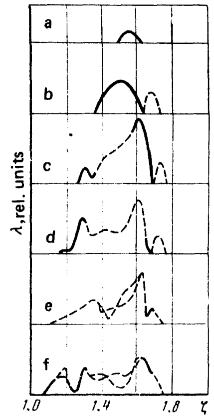

FIG. 3. Same as Fig. 2, but for $H = 780$ Oe, $\Omega_r = 1.43$ MHz. Curves a–f correspond to $H_m = 0.12$, 0.18, 0.24, 0.30, 0.34, and 0.40 Oe, respectively.

The nature of the diagram obtained is an indication of the complexity of the oscillation spectrum. In fact, the natural frequencies of the collective oscillations described by Eq. (1) were governed by the number of magnons, i.e., by the excess of the microwave pumping above the threshold called the super criticality $\zeta = h/h_{th}$, whereas the width of the supercritical range in Fig. 1 depended in a complex manner on the static field. There was a characteristic division of the whole range of fields into two zones divided by $H_{cr} = (1.2 \pm 0.1) \times 10^3$ Oe. Inside each zone the integrated signal of parametric collective magnon oscillations reached maxima repeatedly and decreased practically to the noise level when the supercriticality was increased monotonically. The oscillation amplitude maxima fitted smooth dependences represented by chain curves in Fig. 1.

Figures 2 and 3 demonstrate the development of a parametric collective instability of magnons when the microwave field amplitude was varied and the rf pumping amplitude was kept constant. The curves in Figs. 2(e) and 3(f) represent "vertical sections" through the region of existence of oscillations in Fig. 1 at the points corresponding to $H = 1670$ Oe and $H = 780$ Oe, respectively. The curve taken as a whole indicated that an increase in H_m increased the complexity of the dependence of the amplitude of the collective oscillation signal λ on the supercriticality of the microwave magnon system ζ. Since ζ determined the natural frequency of the collective oscillations, the recorded curves represented effectively the frequency spectrum of the excited oscillations multiplied by the coefficient representing the coupling of the oscillations to the signal detector.

The form of the signal displayed on an oscilloscope screen varied along the curves. The signal could be harmonic (continuous curves), in the form of beats (dashed curves), or it could be irregular with a randomly varying amplitude of the noise type (dashed curve). This was found for the curves in Fig. 2, where the numbers of the right-hand side of each curve give the final value on the supercriticality ζ_c at which the noise-type oscillations disappeared completely.

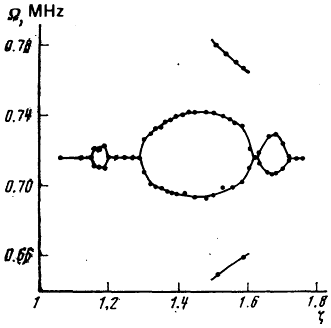

FIG. 4. Frequencies of collective oscillations excited parametrically by an rf field of frequency $\Omega_r = 1.43$ MHz and amplitude $H_m = 0.4$ Oe; $H = 780$ Oe.

Splitting of the frequencies of collective oscillations into doublets was of special interest. Monochromatic rf pumping excited simultaneously two collective oscillation modes with different frequencies. When the frequency difference was small, this effect appeared in the form of low-frequency (~10 kHz) beats of a signal identified in Figs. 2 and 3. Two dashed lines in the same region indicated that the amplitudes of the excited modes were different. The amplitudes were measured with a selective detector and a spectrum analyzer.

In some cases the single-mode parametric pumping of collective oscillations alternated several times with double-mode pumping when the supercriticality was increased. Occasionally the excitation produced simultaneously four modes with frequencies which were pairwise symmetric relative to half the rf pump frequency. Figure 4 shows the oscillation frequencies recorded by a selective detector when the parameter ζ was varied from the threshold value to ζ_c.

3. DISCUSSION OF RESULTS

The main features of the experimental results can be explained readily using a theory[1,2] predicting the existence of inhomogeneous collective magnon oscillations. The most important result of the theory is the dispersion law of the oscillations (1). This law is shown schematically in Fig. 5 for the case $T+S>0$ applicable to yttrium iron garnet at room temperature.

In particular, the law (1) predicts a dependence of the natural frequencies of inhomogeneous magnon oscillations on a static field, governing the derivatives $\partial\omega_k/\partial k$ and $\partial^2\omega_k/\partial k_z^2$, whereas the frequency of homogeneous oscillations, i.e., the gap of the spectrum (1)

$$\Omega_0=2[S(2T+S)]^{1/2}N \tag{3}$$

is independent of the magnetizing field. Therefore, the complex dependence of the interval ζ where the oscillations are observed on the static field H (Fig. 1) can be interpreted as confirming the existence of spatial inhomogeneity of the oscillations.

A more detailed analysis of the dispersion law (1) shows that there is a certain critical field H_{cr} in which the form of the spectrum of the $\varkappa \parallel \mathbf{M}$ vibrations changes qualitatively. In fact, writing down the magnon spectrum in the usual form

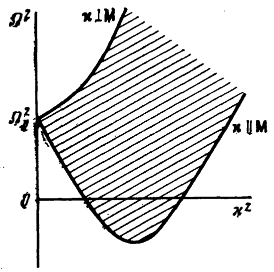

FIG. 5. Spectrum of collective magnon oscillations for $T+S>0$ and $\partial^2\omega_k/\partial k_z^2<0$.

$$\omega_k^2=g^2(H+H_{ex}a^2k^2)\left(H+H_{ex}a^2k^2+4\pi M\frac{k_\perp^2}{k^2}\right), \quad k^2=k_\perp^2+k_z^2,$$

where a is the lattice constant and H_{ex} is the exchange field, we obtain

$$\frac{\partial^2\omega_k}{\partial k_z^2}=4a^2g^2\frac{H_{ex}H_c}{\omega_p}\left[1-\frac{2\pi MH}{H_c(H_c-H)}\right]. \tag{4}$$

Here, H_c is the field corresponding to the minimum of the threshold curve in Fig. 1. The sign of the derivative (4) determines this form of the spectrum (1) and the condition $\partial^2\omega_k/\partial k_z^2$ gives the critical field

$$H_{cr}=H_c^2/(H_c+2\pi M). \tag{5}$$

In the case of yttrium iron garnet this field is 1100 Oe.

The behavior of the branch $\varkappa \parallel \mathbf{M}$ of the spectrum of the collective magnon oscillations is shown in Fig. 6 for various values of the magnetizing field.

As mentioned earlier, the experiments reveal also the existence of a characteristic field $H_{cr} \approx 1200$ Oe which separates the two zones in Fig. 1. A comparison of Figs. 2 and 3 shows that the difference between the zones is basic. The dependence of the amplitude of the oscillation signal on the microwave and rf pumping at $H=1670$ Oe and $H=780$ Oe corresponds to the difference between the form of the oscillation spectrum in two zones.

In the former case we have $H>H_{cr}$ and the spectrum corresponding to small $\varkappa$ has a falling branch $\varkappa \parallel \mathbf{M}$. The gap in the spectrum increases on increase in the supercriticality. Consequently, Fig. 2 shows the excitation of first one mode (clearly, homogeneous) and then in higher fields H_m the "width" of the signal measured along ζ increases. This is equivalent to a tran-

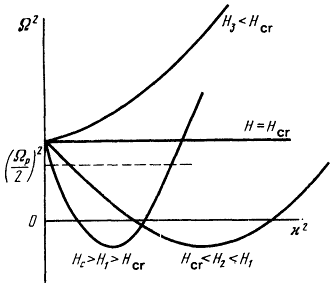

FIG. 6. Branch $\varkappa \parallel \mathbf{M}$ of the spectrum of collective magnon oscillations obtained for various values of the static field intensity.

sition to the wider (in respect of Ω) "inhomogeneous" part of the spectrum in Fig. 5. The natural assumption that the effective oscillation threshold increases on increase in $\varkappa$ accounts for the increase in the amplitude of the rf pumping necessary for this transition.

The increase in the width of the signal with respect to ζ on increase in H_m is observed also in the range $H < H_{cr}$ (Fig. 3). However, a distinguishing feature of the second range of fields is that, at a certain value of $\zeta = \zeta_c$, the point $\Omega_r/2$ which defines the resonance frequency emerges beyond the lower boundary of the spectrum (curve for H_3 in Fig. 6) and the oscillations disappear. The experiments also showed that in a field of $H = 780$ Oe the collective oscillation signal abruptly disappears as soon as the supercriticality reaches $\zeta_c = 1.76$. As expected, with the exception of a narrow interval $H_m = 0.12 - 0.18$ Oe (when only one mode is excited), this effect is independent of the rf pump amplitude.

The existence of several peaks in the curves of Figs. 2 and 3 can also be explained in a natural manner by the excitation of the corresponding new inhomogeneous collective oscillation modes, whose number increases on increase in the field H_m which crossed successively the thresholds of these modes.

Splitting of the frequency of parametric collective oscillations can also be regarded as indicating the existence of a multitude of modes differing in respect of Ω and $\varkappa$. For $\varkappa \lesssim 10^3$ the nature of the excitation of these modes is affected by the discrete nature of the spectrum due to the influence of the boundaries of the sample. When the supercriticality of the system is varied, the spectrum shifts relative to the point $\Omega_r/2$; this is accompanied by changes in the modes which are in resonance with the rf pumping. Intermediate states are also possible and in this case two modes with different frequencies are excited simultaneously. The sum of the frequencies of the excited oscillations in Fig. 4 is always equal to the pump frequencies (within the limits of the experimental error); consequently, a parametric resonance takes place in such cases in accordance with the first equality in Eq. (2).

The frequency splitting is also of intrinsic interest as a manifestation of the three-frequency parametric resonance. This raises an interesting possibility of simple modification of the oscillation frequencies participating in a resonance by altering easily controlled parameters, such as the static magnetic field or the supercriticality of the magnon system.

We shall conclude by identifying the main observations supporting spatial inhomogeneity of collective oscillations and corresponding ideas on the dispersion law of these oscillations:

1) the field dependence of the range of existence of collective magnon oscillations in respect of the supercriticality, which is not exhibited by the homogeneous modes;

2) the different nature of the behavior of the amplitude of collective magnon oscillations when the supercriticality is varied in the range $H > H_{cr}$ and $H < H_{cr}$;

3) the agreement between the theoretical estimates of the critical field and the experimental value of H_{cr};

4) the increase in the number of maxima of the oscillation amplitude (depending on the supercriticality) on increase in the rf pump power;

5) successive alternation of one- and two-mode regimes of parametric excitation of collective magnon oscillations when the supercriticality is varied.

We can thus see that all the experimental results can be explained in a natural manner in the framework of theoretical ideas postulating the existence of a wide spectrum of collective magnon oscillations, thus confirming the main conclusions of the theory.

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