Anomalous passage of sound through a rough surface of a solid

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We consider the refraction of sound incident from a liquid on a rough surface of an isotropic solid. At incidence angles corresponding to excitation of Rayleigh surface waves, the transmission coefficient has a sharp maximum due to scattering of the Rayleigh waves into volume waves by the surfaces of the roughnesses. The shape of the maximum is investigated as a function of the sound impedances of the media and of the roughness parameters.

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1. INTRODUCTION

Observation of ultrasound reflected by the surface of a solid is a convenient method of investigating the surface itself.[1] Our preceding paper,[2] as well as Rollin's experiment,[1] dealt with the influence of surface roughness on a sound wave reflected at an angle equal to the incidence angle. It turns out that the plot of the reflected coefficient against the incidence angle has a sharp minimum due to excitation of Rayleigh surface waves. The depth and width of the minimum are quite sensitive to the state of the solid surface.

In this paper we investigate the influence of the roughnesses on a transmitted wave. We shall see that under certain conditions a wave incident from a liquid on the surface of a solid can pass through (with a coefficient of the order of unity), because of the surface roughnesses. The mechanism of the anomalous passage consists in the following.

It is known that the passage of a sound wave through a flat liquid-solid interface is possible only in the subcritical region of the incidence angles. The transmission coefficients of the longitudinal (D_l) and transverse (D_t) waves are given in this case by the expressions:

$$D_l=|\gamma(k)|\left|\frac{2c_t^2\omega^{-2}(k_t^3-k^2)}{c_t^4\omega^{-4}\Delta(k)+i\gamma(k)}\right|^2,\quad D_t=k_t|k_l\gamma(k)|\left|\frac{4c_t^2\omega^{-2}k}{c_t^4\omega^{-4}\Delta(k)+i\gamma(k)}\right|^2, \tag{1}$$

where

$$\Delta(k)=(k_t^2-k^2)^2+4k^2k_tk_l,\quad \gamma(k)=\rho k_l/i\rho_T k_1,$$
$$k_{t,l}=(\omega^2/c^2_{t,l}-k^2)^{1/2},\quad k_1=(\omega^2/c^2-k^2)^{1/2},$$

ω is the sound frequency, $\mathbf{k}$ is the wave-vector component parallel to the surface; c_t, c_l, and c are the velocities of the transverse sound and of the longitudinal sound in the solid and in the liquid: ρ and ρ_{sol} are the densities of the liquid and solid.

In the subcritical region, the transmission coefficients (1) are small to the extent that the ratio $\gamma\sim\rho c/\rho_T c_t$ of the sound impedances of the liquid and solid is small. Calculation, by Adamenko and Fuks,[3] of the allowance for the roughness, under the assumption that only longitudinal waves can propagate in the solid, does not change this estimate.

In the transcritical region, when k_l and k_t are imaginary and no volume waves can propagate in the solid, the coefficient of reflection from an ideal surface becomes equal to unity. This statement is valid in particular also for an incidence angle corresponding to excitation of surface Rayleigh waves. This angle is determined from the condition that tangential component of the wave vector be continuous:

$$k=(\omega/c)\sin\theta=\omega/c_R, \tag{2}$$

and the spectrum of the Rayleigh waves, i.e., their velocity c_R is obtained from the condition

$$\Delta(k)=0. \tag{3}$$

However, a Rayleigh wave can be scattered on a rough surface into a volume longitudinal or transverse wave with conservation of the frequency, but with a change of the tangential component of the wave vector. This change of k is determined by the characteristic period d^{-1} of the roughness, usually called the correlation radius or the scale dimension. Scattering of a Rayleigh wave into a volume wave becomes possible if the characteristic momentum transfer d^{-1} exceeds the "gap" $\omega/c_R-\omega/c_t$. The scattering probability, and consequently also the transmission coefficient, of sound incident at a Rayleigh angle (2) is determined by the size of the roughness. If the mean square of the roughness is designated a^2, then the magnitude of the relative roughness should be taken to be ak. It is remarkable, as will be shown below, that the transmission coefficient reaches a value of the order of unity at relatively small roughnesses $(ak)^2\sim\gamma$ if $d^{-1}\sim k$. The reason is that in the case of a flat surface the virtual Rayleigh pole [see (1)] is located at a close distance γ, and even a small roughness is sufficient to make this pole real.

We note that Zinov'era[4] has reported observation of maximum absorption of sound incident at a Rayleigh angle. The appearance of a maximum is attributed in Ref. 4 to the absorption mechanism proposed by Andreev,[5] which takes the electron viscosity into account.

2. ENERGY DENSITY IN THE TRANSMITTED WAVE

We are interested in the normal component of the flux density of the sound energy

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$$Q_j=-\rho_{sol}[2c_t^2\dot{u}_i u_{ij}+(c_l^2-2c_t^2)\dot{u}_j u_{ii}], \quad (4)$$

where u_i are the displacement components, u_{ij} is the strain tensor, and the dots denote the derivative with respect to time.

We choose the coordinate axes in the following manner: x is directed along the normal to the surface and $\mathbf{s}$ is a two-dimensional vector in the plane of the surface. Since the frequency is preserved in scattering by roughnesses, we can confine ourselves to a monochromatic wave, which we expand in a Fourier integral with respect to $\mathbf{s}$. The dependence of the displacement on x is determined by the elasticity-theory equations, whose solution we write in the form

$$u_i(x\mathbf{k})=e_i(\gamma\mathbf{k})A_\gamma(\mathbf{k})\exp(ik_\gamma x). \quad (5)$$

For the case of a solid, the summation in (5) is over two transverse (A_{t_1}, A_{t_2}) and one longitudinal (A_l) polarizations, while for the liquid in (5) there are two terms the reflected (A_1) and incident (A_0) waves; k_γ was defined in the sequel to Eq. (1), and the polarization vectors $\mathbf{e}(\gamma\mathbf{k})$ are given in a preceding paper.[2]

The transmission and reflection amplitudes are obtained from the boundary conditions that must be satisfied on the rough liquid-solid interface. We write the equation of this boundary in the form

$$x=\xi(\mathbf{s}).$$

As noted in the Introduction, the most interesting case is when the roughness is small compared with the sound wavelength. Since the boundary conditions contain the surface-normal vector, defined by the derivaties of $\xi(\mathbf{s})$ with respect to $\mathbf{s}$, we assume them also to be small, with an aim at using the small-perturbation method.

Expanding the boundary conditions in powers of ξ and confining ourselves to first-order terms we write them in the form (see Ref. 2)

$$H_{ij}(k)A_j(k)+\int\frac{d^2q}{(2\pi)^2}\xi(\mathbf{k}-\mathbf{q})V_{ij}(kq)A_j(q)$$
$$=H_{i0}(k)A_0(k)+\int\frac{d^2q}{(2\pi)^2}\xi(\mathbf{k}-\mathbf{q})V_{i0}(kq)A_0(q). \quad (6)$$

The subscript i, which takes on four values, numbers the equations (6), the first being the continuity condition of the normal components of the displacement, and the three remaining the continuity condition of the surface forces: $\xi(\mathbf{k})$ is the Fourier component of the random function $\xi(\mathbf{s})$; the matrices H and V were given earlier.[2]

Substituting the displacements (5) and (4) and averaging over the coordinate s in the large interval S, the value of which does not enter in the final result, we obtain the energy flux density in the solid

$$Q_x=\frac{\rho_{sol}c_t^2\omega}{2S}\int\frac{d^2q}{(2\pi)^2}T_{\tau\sigma}(q)A_\tau(q)A_\sigma^*(q)$$
$$\times\exp[i(q_\tau-q_\sigma^*)x], \quad (7)$$

where T is a matrix bilinear in the polarization vectors, and the exponential factor limits the integration of the vectors of the transmitted waves to the subcritical region, in which $q_{t,l}$ are real (the contribution of the transcritical region attenuates at large differences from the surfaces). We note that (7) contains terms that are not diagonal in the two transverse polarizations.

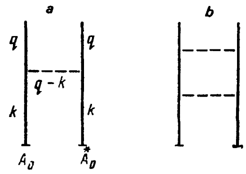

FIG. 1.

3. AVERAGING OVER THE ROUGHNESSES

To determine the transmission coefficient, we connect the amplitudes of the transmitted waves with the amplitudes of the incident wave, using the boundary conditions (6), which are integral equations. We solve them by iteration with respect to ξ.

We are interested in the case when a wave vector of the incident wave lies in the vicinity of the Rayleigh pole, and the transmitted vector lies in the subcritical region. Such transitions occur at any rate in first order in $\xi(\mathbf{k}-\mathbf{q})$. The transmission coefficient, however, does not have a first-order term, since we assume that surface to be plane in the mean, and this reduces to the condition $\langle\xi\rangle=0$.

The principal second-order term is shown in Fig. 1a. The dashed line shows the Fourier component of the correlator $w(\mathrm{s}-\mathrm{s}')=\langle\xi(\mathrm{s})\xi(\mathrm{s}')\rangle$. The solid lines correspond to the matrices H_{ij}^{-1}, the vertices to V_{ij}, and the stubbed ends correspond to the factors $H_{i0}A_0$. On the solid line terminating in A_0^*, all the quantities are complex conjugate. Summation is carried out over the matrix indices and integration over the internal momentum.

The diagram shown in Fig. 1a makes a substantial contribution if the wave vector $\mathbf{k}$ of the incident wave satisfies the condition (2) for excitation of the Rayleigh wave. In this case the inverse matrix $H^{-1}(k)$ has a denominator proportional to the denominator of expression (1), and is small because $\Delta(k)=0$ and the corresponding value is

$$\gamma=\frac{\rho c}{\rho_\tau c_R}\left(\frac{1-c_R^2/c_l^2}{1-c^2/c_R^2}\right)^{1/2}\sim\frac{\rho c}{\rho_\tau c_t}\ll 1.$$

The roughness influence due to V_{i0} in the right-hand side of (6), and also to the emission from (6) of second-order terms in ξ, is small compared with the diagram of Fig. 1a, since they do not contain the small denominator of the matrix $H^{-1}(k)$.

As is customary in problems of this kind, the terms of higher order in ξ contain additional factors H^{-1}. This makes it necessary, in particular to replace the free Green's function H^{-1} by the complete functions (with allowance for the roughnesses):

$$\mathscr{H}^{-1}(k)=\left[H(k)-\int w(\mathbf{k}-\mathbf{q})V(kq)H^{-1}(q)V(qk)\frac{d^2q}{(2\pi)^2}\right]^{-1}. \quad (8)$$

The influence of the roughness is most substantial near the Rayleigh pole and the matrix $\mathscr{H}^{-1}$ can be represented in the form

$$\mathscr{H}_{ij}^{-1}(k)=\frac{h_{ij}(k)}{c_t^4\omega^{-4}\Delta(k)+i(\tau+\gamma)},$$

where the minors h do not depend on ξ,

$$\tau=-\mathrm{Im}\int\frac{d^2q}{(2\pi)^2}w(k-q)\frac{f(kq)}{\Delta(q)+i\gamma\omega^4c_t^{-4}}$$

and the function f is given in the preceding paper.[2]

For a flat monochromatic incidence wave with wave vector $k_x = k_1$, $\mathbf{k}_s = \mathbf{k}$ Fig. 1a corresponds to the following transmission coefficient:

$$D=\frac{\rho_T c_t^2/\rho\omega^2 k_1}{c_t^8\omega^{-8}\Delta^2(k)+(\tau+\gamma)^2}\int\frac{d^2q}{(2\pi)^2}w(\mathbf{q}-\mathbf{k})T_{\gamma\sigma}(q)a_\gamma(q)a_\sigma^*(q), \qquad (9)$$

where

$$a_\gamma(q)=h_{\gamma i}(q)V_{ij}(qk)h_{jk}(k)H_{k0}(k)\omega^4 c_t^{-4}/\Delta(q),$$

and in the normalization used by us the energy flux in the incident wave is $Q_x^0 = \rho\omega^2 k_1 |A_0|^2/2$.

Using the explicit forms of the employed matrices, we obtain

$$D=\frac{\gamma\mu}{c_t^8\omega^{-8}\Delta^2(k)+(\tau+\gamma)^2},$$
$$\mu=\sum_{\gamma=l,t}\int\frac{d^2q}{(2\pi)^2}w(\mathbf{q}-\mathbf{k})\varphi_\gamma(qk), \qquad (10)$$

where the summation is over the longitudinal and transverse polarizations, and the integration over the corresponding subcritical regions,

$$\varphi_l(qk)=\frac{4c_t^4 q_l}{\omega^4|k_t||\Delta(q)|^2}\left|\frac{\omega^4}{c_t^4}k_l(q^2-q_t^2)+q_t(k^2-k_t^2)\left(\frac{\omega^2}{c_t^2k^2}\alpha\mathbf{kq}-2\beta q^2\right)\right|^2,$$

$$\varphi_t(qk)=\frac{4c_t^6 q_t}{\omega^6|k_t||\Delta(q)|^2}\Big\{q^2\frac{\omega^2}{c_t^2}|\beta(k_t^2-k^2)(q_t^2-q^2)+2k_lq_l\omega^4c_t^{-4}|^2$$
$$+2\frac{\omega^2}{c_t^2}\alpha\mathbf{kq}(q_t^2-q^2)(k^2-k_t^2+2k_tk_l)\left[\beta(k_t^2-k^2)(q_t^2-q^2)+2\frac{\omega^4}{c_t^4}\operatorname{Re}k_lq_l\right]$$
$$+\alpha^2(k^2-k_t^2+2k_tk_l)^2(k_yq_z-k_zq_y)^2[q^2q_t^{-2}|q^2-q_t^2+4q_tq_l|^2$$
$$+2(q^2-q_t^2+4\operatorname{Re}q_tq_l)]+\alpha^2(q_t^2-q^2)(k^2-k_t^2+2k_tk_l)^2[k^2q_t^2+(\mathbf{kq})^2]\Big\},$$

$$\alpha=\omega^2/c_t^2-2\mathbf{kq},\qquad \beta=\omega^2/c_t^2-2\omega^2/c_l^2.$$

We note that k_t and k_l are imaginary in the vicinity of the Rayleigh angle of interest to us, q_t and q_l are real in the term for the longitudinal scattered wave while q_t is real and q_l can be either real or imaginary in the term for the transverse wave.

In addition to replacement of the free Green's function by the total functions, it is necessary to sum all the diagrams of the ladder type, analogous to that shown in Fig. 1b. This summation is effected by the equation

$$Y_{ij}(k)=H_{i0}(k)H_{j0}^*(k)$$
$$+\int\frac{d^2q}{(2\pi)^2}w(\mathbf{k}-\mathbf{q})V_{ik}(kq)V_{jl}^*(kq)\mathscr{H}_{km}^{-1}(q)\mathscr{H}_{ln}^{*-1}(q)Y_{mn}(q). \qquad (11)$$

The total transmission coefficient is determined by formula (9) in which $H_{i0}H_{j0}^*$ must be replaced by the matrix $Y_{ij}(k)$, followed by integration with respect to $\mathbf{k}$.

4. DISCUSSION OF RESULT

The transmission coefficient (10) has a sharp maximum if the incident wave propagates at an angle corresponding to the condition of excitation of Rayleigh waves $\Delta(k) = 0$. The depth of the maximum is determined by the relation between the roughness and the ratio γ of the acoustic impedances. The roughness is taken into account in (10) both in the denominator via τ, and in the numerator. Let us estimate the magnitude of the numerator.

Figure 2 shows the plane of the vectors $\mathbf{q}$ of the re-

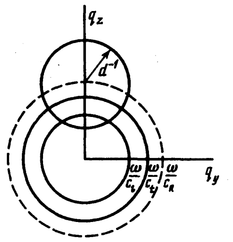

FIG. 2.

fracted waves. The circles of radius ω/c_l and ω/c_t show the regions corresponding to propagation of volume waves in the solid. Rayleigh waves correspond to the dashed circle. The circle of radius d^{-1} shows the region where the Fourier component of the roughness correlator $w(\mathbf{s}-\mathbf{s}')$ differs from zero. The value of $w(\mathbf{s}-\mathbf{s}')$ at $\mathbf{s}=\mathbf{s}'$ will be denoted a^2, where a has the meaning of the average height of the roughness.

Let the wave vector of the incident wave correspond to a maximum of the transmission coefficient. Then the center of the circle of radius d^{-1} lies on the Rayleigh circle. If d^{-1} is smaller than the gap $\omega/c_R - \omega/c_t$ separating the surface and the volume oscillations, then the transmission coefficient vanishes—the roughness is too gently sloping for the Rayleigh wave to be able to be scattered into the subcritical region.

If the overlap $p = d^{-1} - (\omega/c_R - \omega/c_t)$ of the circles with radii d^{-1} and ω/c_t is small, then the excited Rayleigh waves is scattered into a transverse volume wave propagating mainly in the same direction as the initial Rayleigh wave. Estimating the integral with respect to q, we obtain

$$\mu\sim(ap)^2(kd)^{1/2},\qquad 0<p\ll\omega/c_t. \qquad (12)$$

With further increase of p there appear also longitudinal volume waves, and for a maximally diffuse surface, when $d^{-1} \gtrsim \omega/c_t$, we obtain

$$\mu\sim(adk^2)^2,\qquad d^{-1}\gg\omega/c_t. \qquad (13)$$

In the region where the estimate (13) we have $\mu \sim \tau$ (see formula (20) of Ref. 2). Therefore the transmission coefficient (10) as a function of the propagation direction of the incident wave has a sharp maximum with height of the order of

$$D_{max}\sim\gamma\tau/(\gamma+\tau)^2$$

and with angle width

$$|\theta-\theta_R|/\theta_R\sim\gamma+\tau.$$

It is seen therefore that the maximum value of the transmission coefficient turns out to be of the order of unity at relatively small roughnesses when $\tau \sim \gamma$. Since the width of this maximum is proportional to γ, the contribution of the Rayleigh maximum to the transmission coefficient integrated over the incidence angle, for the case of isotropic incident radiation, turns out to be of the same order $\gamma c^2/c_t^2$ as the contribution of the subcritical region; the factor c^2/c_t^2 is the measure of the solid angle that determines the subcritical region.

Thus, this integral estimate coincides with that obtained by Khalatnikov[6] for the Rayleigh contribution to the resistance. According to measurements by Zinov'eva,[4] 50% of the subcritical absorption goes to the Rayleigh maximum.

At a small overlap, when the estimate (12) is valid, the transmission coefficient (10) calculated in second order in ξ may turn out to be larger than unity, as is seen from the limiting expressions for τ (see (18) and (19) of Ref. 2). In this case we must consider Eq. (11) for Y. The integral term in this equation is of the order of $\mu/(\gamma+\tau)$. We then obtain for the order of magnitude of the transmission coefficient expression (10), in which μ is replaced by the smaller of μ and $\gamma+\tau$.

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Occurrence of noncollinear magnetic structures in $DyCo_{5.3}$ near the compensation temperature in strong magnetic fields

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The influence of hexagonal anisotropy on the occurrence of noncollinear magnetic structures, induced by an external magnetic field, is investigated in the intermetallic compound $DyCo_{5.3}$ near the compensation temperature (124 K). Experimental data on the magnetization and magnetostriction, obtained in strong pulsed magnetic fields up to 280 kOe, are compared with the theoretical results of A. K. Zvezdin and A. F. Popkov, [Fiz. Met. Metalloved. **49**, No. 8 (1980)]. It is shown that at low temperatures, the hexagonal anisotropy strongly influences the form of the field dependences of the magnetization and magnetostriction, leading to the appearance of wavelike singularities in the noncollinear phase and also to the occurrence of phase transitions of the first kind. Experimental and theoretical magnetic phase diagrams are constructed for fields applied along the easy and hard directions in the basal plane. The causes of qualitative differences between theory and experiment, such as the presence of appreciable hysteresis in the experimental field dependence of the magnetostriction, are discussed. The molecular field exerted on the Dy sublattice by the Co sublattice is determined experimentally to be $H_M = (950\pm 50)$kOe.

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Noncollinear magnetic structures induced by an external field in ferrimagnets have been well studied, theoretically and experimentally, principally on crystals that have cubic or uniaxial magnetocrystalline anisotropy.[1] It has been shown that the anisotropy may strongly influence the phase state of a ferrimagnet placed in an external field, and that this leads in a number of cases to qualitative differences from the isotropic model, such as the occurrence of phase transitions of the first kind, the appearance of new lines on the magnetic phase diagrams, etc. Singularities due to the occurrence of noncollinear magnetic structures and to the presence of crystalline anisotropy express themselves in the field and temperature dependences of the magnetization, the magnetostriction, and other magnetic characteristics of the ferrimagnet.

At present, for ferrimagnets that possess hexagonal anisotropy in a plane of easy magnetization, there is only the theoretical analysis given in the paper of Zvezdin and Popkov[2] for the noncollinear magnetic structures that originate in an external field. In this paper, magnetic phase diagrams are constructed for an external field directed along the easy and hard axes in the basal plane, together with the temperature dependences of the angles between the magnetic moments of the sublattices and the crystallographic axes for various values of the external field.

Suitable objects for experimental investigation of field induced noncollinear magnetic structures in hexagonal ferrimagnets are the intermetallic compounds $DyCo_5$ and $TbCo_3$, which possess anisotropy of the "easy plane" type at temperatures below room temperature. The presence in these compounds of rare-earth atoms with nonvanishing orbital moments leads to appreciable anisotropy in the basal plane at low temperatures, including the magnetic compensation temperatures (120–150 K). This leads us to expect in them a strong influence of the hexagonal anisotropy on the occurrence of field-induced noncollinear magnetic structures.

The compound $DyCo_{5.3}$, which is the subject of study in

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