Dragging of normal component by the condensate in nonequilibrium superconductors

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If a superconductor is brought out of the equilibrium state, then dissipative fluxes proportional to the superfluid velocity may arise. These fluxes can be regarded as the result of dragging of the normal component by the condensate. The heat flux is calculated for a moving condensate when the equilibrium is disturbed by a tunnel current in an *N-S* structure. A shift in the chemical potential takes place under the influence of the temperature gradient and the superfluid motion. The shift turns out to be proportional to the momentum relaxation time. The Onsager relation for the corresponding kinetic coefficients is established. A potential difference, the analog of the Nernst effect, is produced in a plate with a magnetic field parallel to the surface. The quasielectron distribution function is introduced to describe the nonequilibrium states. A stationary kinetic equation is obtained for superconductors with arbitrary impurity concentration and with spatial inhomogeneity. A stationary diffusion equation is derived for the "dirty" state.

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1. INTRODUCTION

It is known that a superconductor can be regarded as an aggregate of a superconducting condensate and a normal component (a gas of excitations). Under equilibrium conditions the motion of the condensate exerts no influence on the excitations in the sense that the excitations retain a Fermi distribution function, expressed in terms of the total energy, and no dissipative flux arises. The purpose of the present paper is to show that if the equilibrium state is disturbed, then the motion of the condensate can change the symmetry of the nonequilibrium distribution function of the excitations and lead to the appearance of the dissipative flux proportional to the velocity of the condensate. This flux can be regarded as a result of dragging of the normal component by the condensate. The reason for this dragging is scattering with "intermixing of branches",[1] wherein exchange of particles takes place between the normal component and the moving condensate. By way of example we can cite the heat flow produced when the condensate moved, if an external action disturbs the equilibrium between the electron-like and hole-like excitations. This heat flow is calculated in this paper together with another effect having similar physical causes, namely the onset of a shift of the chemical potential under the influence of the temperature gradient and the superfluid motion.

The effects can be observed with the aid of the *N-S* film structure shown in the figure. The normal-metal film *N* is separated from the superconducting film *S* by an insulating layer, and serves to generate nonequilibrium excitations in *S*, or to measure the shift of the chemical potential.[1,2] A current *I* with a certain density *j* can be made to flow through the film *S* and this produces superfluid motion with a pair momentum $\mathbf{p}_s = m\mathbf{j}/eN_s$ (N_s is the density of the superconducting electrons). If the film thickness *S* does not exceed the London depth of penetration and the film is located over a superconducting screen, then the quantity $\mathbf{p}_s$ is constant over the film cross section.

Let a voltage *U* be applied to the *N-S* structure. As shown by Tinkham,[1] this produces in the superconductor a state with a difference between the numbers of electron and hole excitations (unbalance). To compensate for the excess charge of the excitations, a shift ϕ of the chemical potential is produced. The unbalance is proportional to the time τ_Q of transition of the excitation from one branch of the spectrum to the other ("intermixing of the branches").[1]

Let us explain the causes of the appearance of heat flow under these conditions. The heat flow is determined by the distribution function and by the excitation transport velocity. From the kinetic equation (22) it follows that the effective excitation velocity is $a^2\mathbf{v}$. The physical reason why this velocity differs from the Fermi velocity **v** is, in our opinion, the following: because of the "intermixing of the branches," the excitation no longer belongs to a definite branch of the spectrum, but spends some time, which depends on the transition frequency, also in a "foreign branch". The excitations have opposite sign in different branches, and therefore the effective transport velocity, which enters in the kinetic equation, should differ from the Fermi velocity by a factor $(1-2w_\epsilon)$, where w_ϵ is the relative time that a particle with energy ϵ stays on the "foreign" branch ($a^2 \equiv 1 - 2w_\epsilon$). If $\mathbf{p}_s \neq 0$, then w_ϵ depends on the direction of motion of the excitation and the velocity $\langle a^2\mathbf{v}\rangle$ averaged over the angle turns out to differ from zero, while the drift velocities $\langle a^2\mathbf{v}\rangle$, of the electrons and holes are shown by calculations to have opposite signs. It is the directional motion of the nonequilibrium excitation which leads to the appearance of a heat flow proportional to $U\mathbf{p}_s$. The higher the "branch intermixing" frequency τ_Q^{-1}, the larger $\langle a^2\mathbf{v}\rangle$. At the same time, the number of nonequilibrium excitations is proportional to τ_Q. As a result, the heat flux depends very weakly both on the "branch intermixing" mechanism and on the transition frequency.

The described mechanism is effective only for sufficiently "dirty" superconductors. In pure superconductors a more important role is played by the change of the nonequilibrium distribution function if $\mathbf{p}_s \neq 0$. When the condensate moves, the excitation energy contained

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in the conservation law acquires upon collision an increment $\mathbf{v}\cdot\mathbf{p}_s$. This circumstance alters the coherence factors that determine the relaxation times, and can lead to an angular dependence of the latter. It is easy to verify [see (33)] that only the time τ_Q depends on the direction of motion of the excitation. At equal total energy, the transition probability τ_Q^{-1} is larger for motion in the direction of $\mathbf{p}_s$ and smaller in the opposite direction. Under these conditions the nonequilibrium excitations, which are isotropically distrubuted at $\mathbf{p}_s=0$, become redistributed in direction and a heat flow is produced. The degree of anisotropy of the distribution function is determined by the competition between the "branch intermixing" process and the scattering within the branch. As a result, just as in "dirty" superconductors, the heat flow depends little on the time τ_Q and is determined mainly by the momentum relaxation time. We have obtained as expression for the heat flow density (48):

$$\mathbf{q}=-\zeta(T)\frac{D}{d}\frac{U}{R}\frac{\mathbf{p}_s}{e}, \tag{1}$$

where D is the diffusion coefficient, d is the thickness of the film S, U is the voltage on the contact, R is the junction resistance in the normal state per unit area, and e is the electron charge. The dimensionless coefficient $\zeta(T)$ depends on the temperature and to a small degree on the "branch intermixing" mechanism and the purity of the sample. At low temperatures $\zeta(T)$ vanishes, as does the number of excitations. Near the critical temperature, $\zeta(T)$ is of the order of unity, but the heat flow is small, since p_s by its very nature cannot exceed the critical momentum, which is equal to zero at the junction point. The value of the flux under the conditions listed in Sec. 6 can be $q/\varkappa \sim 1K/\text{cm}$ ($\varkappa$ is the electronic thermal conductivity).

We examine now the causes of the shift of the chemical potential of the condensate ϕ in the presence of a temperature gradient. If the temperature distribution in the superconductor is not uniform, then, just as in a normal metal, a drift of the electron and hole excitations towards the cooler end is produced. Owing to the difference in the signs of the group velocities of the electrons and holes along each direction in momentum space the difference between the number of electrons and holes is not zero and is proportional to $a^2\tau_\mathbf{p}\mathbf{v}\nabla T$ ($\tau_\mathbf{p}$ is the time of relaxation of a particle with momentum $\mathbf{p}$). If there is no preferred direction in the system, then the difference averaged over the angles and with it ϕ, is equal to zero. If $\mathbf{p}_s\neq 0$, then $a^2\tau_p$ acquires an anisotropic increment and an unbalance appears and relaxes with a time τ_Q. One can expect the shift ϕ to be proportional to the time τ_Q, which is large near the critical temperature. However, as already indicated, the deviation of the mean value $\langle a^2\tau_\mathbf{p}\mathbf{v}\rangle$ from zero is also connected with "branch intermixing" processes. As a result, ϕ is determined mainly by the momentum relaxation time, and all that depends on the rate of "branch intermixing" is the time of establishment of the stationary state. The expression for the potential [see (37), (38), (42), and (43)] can be written in the form1)

$$\phi=\zeta(T)Dp_s\nabla T/T, \tag{2}$$

where $\zeta(T)$ and D are the same quantities as in (1).

Recent experiments by Clarke *et al.*[2] confirm both the very existence of a chemical-potential shift and its linear dependence on p_s. The theoretically predicted value of the potential $\phi(2)$ agrees quite satisfactorily with experiment (see Sec. 5).

Superfluid motion can be produced by a magnetic field applied parallel to the surface of the film. In this case the momentum p_s is of different sign on opposite surfaces of the film, and an effect reminiscent of the Nernst effect is produced, namely a potential difference $\delta\phi$ appears in the presence of a temperature gradient. This effect is discussed in Sec. 5.

The presence of identical coefficients $\zeta(T)$ in relations (1) and (2) is not accidental. Both considered effects have the same physical nature, and the corresponding kinetic coefficients are connected by the Onsager relation. In this paper, the shift of the chemical potential is calculated, and expression (1) is established, with the aid of the principal of symmetry of the kinetic coefficients (Sec. 6).

The calculation in the present paper are carried out with the aid of kinetic equation that is valid at arbitrary concentration of the impurities and in the case of spatial inhomogeneity. Its derivation is presented in Secs. 2–4 This equation differs from those obtained by Larkin and Ovchinnikov[4] and by Schmid and Schön[5] in that another method is used to introduce the distribution function. Accurate to a replacement of f by $1-f$, the introduced function goes over in pure superconductors into the distribution function of the BCS excitations. It turned out to be possible to write down a single equation both for perturbations of the heating type, which do not disturb the equilibrium between the electrons and holes, and for perturbations that do disturb the equilibrium. In this case, in our opinion, the kinetic equation, and in particular the collision integral, take on simpler forms.

2. DERIVATION OF THE KINETIC EQUATION

To derive the kinetic equation we use the method of Green's functions integrated with respect to the variable $|\mathbf{p}|$. For nonequilibrium states, this method was developed by Larkin and Ovchinnikov.[4] Just as in Ref. 4, we use the Keldysh technique.[6] In this technique is formulated a system of equations for three Green's functions, which in the case of superconductor are matrices $[x=(t,\mathbf{r})]$:

$$\begin{gathered}G_{ik}^R(x,x')=-i\Theta(t-t')\langle[\psi_i(x),\psi_k^+(x')]_+\rangle,\\ G_{ik}^A(x,x')=i\Theta(t'-t)\langle[\psi_i(x),\psi_k^+(x')]_+\rangle,\\ G_{ik}(x,x')=-i\langle[\psi_i(x),\psi_k^+(x')]_-\rangle.\end{gathered} \tag{3}$$

It is convenient to define the operators ψ_i, ψ_k^+ as follows:

$$\begin{gathered}\psi_1=\psi_\uparrow\exp(-i\chi/2),\quad \psi_1^+=\psi_\uparrow^+\exp(i\chi/2),\\ \psi_2=\psi_\downarrow^+\exp(i\chi/2),\quad \psi_2^+=-\psi_\downarrow\exp(-i\chi/2);\end{gathered} \tag{4}$$

where $\psi_\uparrow$, $\psi_\downarrow$ are the electron operators and $\chi(x)$ is a function that is for the time being arbitrary and has the meaning of the phase of the order parameter.

The Green's functions (3) have as diagonal matrix elements the symmetry properties

$$G^R(x,x')=\tau_z G^{A+}(x',x)\tau_z,\quad G(x,x')=-\tau_z G^+(x',x)\tau_z; \tag{5}$$

here the Hermitian conjugation sign and the Pauli matrix τ_z act on the indices i and k. If there is no preferred direction of the electron spin, then the mean values $\langle\psi_\alpha\psi_a^+\rangle$ do not depend on α, and the mean values $\langle\psi_\alpha\psi_\beta\rangle$ are antisymmetrical with respect to α and β (α and β are the spin indices). Using this circumstance and the definition (3) of the Green's functions, we easily show that

$$G^R(x,x')=\tau_x \tilde{G}^A(x',x)\tau_x,\quad G(x,x')=\tau_x\tilde{G}(x',x)\tau_x. \tag{6}$$

The transposition sign and the Pauli matrix τ_x act on the indices i and k. The property (6) is always satisfied for the functions $G^{R(A)}$ if the Hamiltonian is symmetrical with respect to spin. The function G is determined by the excitation distribution function. If, say by means of spin injection, the excitations become polarized, then the function G does not have the property (6).

In Ref. 4 was obtained a system of equations for the quantities R, A, g, which are connected with the initial functions G^R, G^A, G as follows:

$$R(\mathbf{n},\mathbf{r},t_1,t_2)=\frac{iv_F}{\pi}\int d^3\rho\, dp\, G^R\left(t_1,t_2,\mathbf{r}+\frac{\rho}{2},\ \mathbf{r}-\frac{\rho}{2}\right)e^{-ip\mathbf{n}\rho}, \tag{7}$$

and analogously for the remaining functions. This system is given by

$$iv\nabla R+\{H^R,R\}=0,\quad iv\nabla A+\{H^A,A\}=0, \tag{8}$$

$$(RR)_{t_1,t_2}=(AA)_{t_1,t_2}=\hat{1}\delta(t_1-t_2), \tag{8a}$$

$$iv\nabla g+H^R g-gH^A+R\Sigma-\Sigma A=0, \tag{9}$$

$$Rg+gA=0. \tag{9a}$$

In the multiplication of the matrices there is implied here also integration with respect to internal time argument:

$$XY\equiv\int_{-\infty}^{+\infty}dt'\,X(t_1,t')Y(t',t_2),\quad \{X,Y\}\equiv XY-YX.$$

The quantities $H^{R(A)}$ are given by

$$\begin{gathered}H^{R(A)}(\mathbf{n},\mathbf{r},t_1,t_2)=i\tau_z\delta'(t_1-t_2)+\delta(t_1-t_2)(\tau_z\mathbf{v}\mathbf{p}_s-i\tau_y\Delta-\phi)\\ -\Sigma_{imp}^{R(A)}(\mathbf{n},\mathbf{r},t_1,t_2)-{}^1/_2(\Sigma_{ph}^{R(A)}-\Sigma_{ph}^{A(R)})_{\mathbf{n},\mathbf{r},t_1,t_2},\\ \mathbf{p}_s=\frac{1}{2}\left(\nabla\chi-\frac{2e}{c}\mathbf{A}\right),\quad \phi=\frac{1}{2}\dot{\chi}+e\varphi.\end{gathered} \tag{10}$$

Here $\mathbf{A}$ and φ are the vector and scalar potentials, and the self-energy parts are expressed in terms of the Green's functions in accordance with the formulas

$$\begin{gathered}\Sigma_{imp}(\mathbf{n},\mathbf{r},t_1,t_2)=\int\frac{d\Omega'}{4\pi}\left[\frac{W_{\mathbf{nn}'}}{2i\tau}g(\mathbf{n}')+\frac{W^{(s)}_{\mathbf{nn}'}}{2i\tau_s}\tau_z\{g(\mathbf{n}'),\tau_z\}\right],\\ \Sigma_{ph}(\mathbf{n},\mathbf{r},t_1,t_2)=\frac{\lambda\pi}{2}\int\frac{d\Omega'}{4\pi}[D(p_F\mathbf{n}-p_F\mathbf{n}',t_1,t_2)(R-A)_{\mathbf{n}'}\\ +(D^R-D^A)_{p_F\mathbf{n}-p_F\mathbf{n}',t_1,t_2}g(\mathbf{n}')].\end{gathered} \tag{11}$$

The quantities $\Sigma^{R(A)}_{\text{imp}}$ are obtained from Σ_{imp} by the substitution $g\rightarrow R(A)$, and $\Sigma^R_{\text{ph}}-\Sigma^A_{\text{ph}}$ is obtained from Σ_{ph} by the subsitution $D\rightleftarrows(D^R-D^A)$. The Green's-function arguments not written out in (11) coincide with the arguments of Σ in the left-hand side. The following notation is introduced in (11): τ is the free path time without spin flip, $W_{nn'}$ is the normalized scattering probability, τ_s is the spin relaxation time, $W^{(s)}_{\mathbf{nn}'}$ is the normalized scattering probability (in the case of isotropic scattering $W_{\mathbf{nn}'}=W^{(s)}_{\mathbf{nn}}=1$)[2], D, $D^{R(A)}$ are the phonon Green's functions,[4] and λ is the dimensionless electron-phonon interaction constant.

In expression (10) are separated the real phonons which are described by the term $\Sigma^R_{\text{ph}}-\Sigma^A_{\text{ph}}$, while the virtual phonons lead to a renormalization of the Fermi velocity into the appearance of a self-consistent potential Δ:

$$\Delta(\mathbf{r},t)=\frac{\pi\lambda i}{4}\,\mathrm{Sp}\int\frac{d\Omega}{4\pi}\tau_y g(\mathbf{n},\mathbf{r},t,t). \tag{12}$$

The self-consistency equation (12) was written with allowance for the fact that the phase χ in (4) can always be chosen equal to the phase of the order parameter; the potential Δ is then a real quantity, while $\mathbf{p}_s$ and ϕ are gauge-invariant.

In the expression for the self-energy parts (11), all the functions have the same coordinate, i.e., the scattering is assumed local. The approximations used in the derivation of (11) correspond to the following condition: the dimension δ of the scatterer is smaller than the length over which the change of the excitation wave function takes place. For impurities and virtual phonons $\delta\sim\hbar/p_F$, and this requirement is always satisfied. For real phonons $\delta\sim\hbar s/T$ (s is the speed of sound) and the condition for the applicability of expressions (11) takes the form

$$\frac{v_F}{s}\min\left\{\frac{\tau T}{\hbar},\ \frac{T}{(\varepsilon^2-\Delta^2)^{1/2}}\right\}\gg 1, \tag{13}$$

where ε is the characteristic electron energy.

Equations (8) and (8a) make it possible to obtain the effective wave functions of the electron in the superconductor, while the equation for g serves to determine the occupation numbers of these states, i.e., the distribution function. The evolution of the function g is determined both by the behavior of the wave functions and by the dependence of the distribution function on the energy, coordinates, and the time. To obtain the kinetic equation it is necessary to separate the dependence on the wave functions.

As noted in Ref. 4, condition (9) makes it possible to express two matrix elements of the matrix g in terms of two others. To separate independent component of matrix elements, we introduce the "distribution functions" $F_1(\mathbf{n},\mathbf{r},t_1,t_2)$, $F_2(\mathbf{n},\mathbf{r},t_1,t_2)$ and express g in the form

$$g={}^1/_2P_1{}^RF_1P_1{}^A+{}^1/_2P_2{}^RF_2P_2{}^A, \tag{14}$$

$$\begin{gathered}P_i{}^R(t_1,t_2)=\hat{1}\delta(t_1-t_2)+\eta_iR(t_1,t_2),\quad i=1,2,\\ P_i{}^A(t_1,t_2)=\hat{1}\delta(t_1-t_2)-\eta_iA(t_1,t_2),\quad \eta_1=1,\quad \eta_2=-1;\end{gathered} \tag{15}$$

certain arguments of the functions have been left out for brevity. Comparing (14) with the corresponding expression for a pure superconductor or a normal metal, we note that the functions $F_1(F_2)$ are connected with the distribution function of quasielectrons with up (down) spin.

Taking the properties (8a) into account, it is easy to show that the matrix g in the form (14) satisfies identically Eq. (9a) at all F_1 and F_2. The quantities $P_i^{R(A)}$ have the following properties which follow from (8a):

$$P_i^{R(A)}P_i^{R(A)}=2P_i^{R(A)},\quad P_1^{R(A)}P_2^{R(A)}=P_2^{R(A)}P_1^{R(A)}=0. \tag{16}$$

These properties make it possible to obtain from (9) separate equations for F_1 and F_2. For this purpose we need multiply Eq. (9) by $P_1^R(P_2^R)$ from the left and by $P_1^A(P_2^A)$ from the right and taking (8), (14), and (16) into account. As a result we obtain

$$\mathrm{Sp}\,P_1{}^R(iv\nabla F_1+H^RF_1-F_1H^A+\Sigma)P_1{}^A=0, \tag{17}$$

$$\mathrm{Sp}\,P_2{}^R(iv\nabla F_2+H^RF_2-F_2H^A-\Sigma)P_1{}^A=0. \tag{17a}$$

Each of the matrix equations obtained by the additional multiplication of (9) has, by virtue of (16), only one independent component. It is this which explains the appearance of the trace operation in Eq. (17) and (17a).

From the symmetry relations (5) and (6) as well as from (14) we get

$$R(\mathbf{n},t_1,t_2)=-\tau_zA^+(\mathbf{n},t_2,t_1)\tau_z,\quad R(\mathbf{n},t_1,t_2)=\tau_x\tilde{A}(-\mathbf{n},t_2,t_1)\tau_x, \tag{18}$$

$$g(\mathbf{n},t_1,t_2)=\tau_zg^+(\mathbf{n},t_2,t_1)\tau_z,\quad F_{1,2}(\mathbf{n},t_1,t_2)=F_{1,2}^*(\mathbf{n},t_2,t_1), \tag{19}$$

$$g(\mathbf{n},t_1,t_2)=\tau_x\tilde{g}(-\mathbf{n},t_2,t_1)\tau_x,\quad F_2(\mathbf{n},t_1,t_2)=F_1(-\mathbf{n},t_2,t_1). \tag{20}$$

We recall that relations (20) denote that states that differ only in the spin orientation are equally populated. This is the only case considered hereafter. The functions F_1 and $_2$ are then connected by relation (20) and it suffices to find only one of them.

Equation (17) [or (17a)] is a convenient starting point for the derivation of the kinetic equation. It is apparently impossible to obtain a simple and universal kinetic equation for a superconductor, unlike a normal metal, because of the fact that the spectrum is complicated and different in the different problems. If the relaxation times are long, so that the scattering has little effect on the spectrum, and the potentials vary slowly in space and time, then we can obtain from (17) the classical Boltzmann equation derived for a superconductor by Aronov and Gurevich,[7] and Eq. (17) can be useful in this case for the determination of the quantum corrections.

We use hereafter (17) to obtain a kinetic equation for two cases: nonequilibrium stationary state, without any limitations (except $\tau\varepsilon_F\gg\hbar$, $\hbar\nabla\ll p_F$) on the impurity concentration and on the magnitudes of the spatial gradients, and a dirty superconductor ($\tau\Delta\ll\hbar$) in a field that varies slowly with time. A kinetic equation was derived under the same assumptions in Refs. 4 and 5. The two distribution functions introduced in these references are certain linear combinations of the functions F_1 and F_2. The coefficients in these combinations are quantities made up of R and A matrix elements (the connection is given below for the stationary case). These two functions do not go over into each other by the symmetry transformation (20), therefore even in the absence of spin polarization the equations for each of them are independent. The property (20) of the matrix g leads to symmetry of the distribution functions $\varphi(\mathbf{n},t_1,t_2)$, $\varphi_1(\mathbf{n},t_1,t_2)$ introduced in Ref. 4:

$$\varphi(\mathbf{n},t_1,t_2)=-\varphi(-\mathbf{n},t_2,t_1),\quad \varphi_1(\mathbf{n},t_1,t_2)=\varphi_1(-\mathbf{n},t_2,t_1).$$

For this reason, the two equations for φ and φ_1 are equivalent to one equation for F_1 or F_2, which do not have this symmetry. The main advantage of introducing distribution of functions defined by (14) is the simpler form of the collision integral.

3. STATIONARY NONEQUILIBRIUM STATE

Nonequilibrium stationary states arise in many physical situations: in thermal perturbations, when current flows in the boundary between normal and superconducting metals, in microwave pumping, or in tunnel injection of excitations. In many cases the kinetic equation obtained below should be supplemented by an excitation source whose form is determined by the specifics of the problem.

In the stationary state, the Green's function depends only on the difference between its time arguments. We take the Fourier transform with respect to the difference t_1-t_2 in Eq. (17), express g, R, A in terms of (10), (11), (14), and (20), and put

$$F_1(\varepsilon,\mathbf{n},\mathbf{r})=1-2f_{\boldsymbol{\varepsilon}}(\mathbf{r}),\quad F_2(\varepsilon,\mathbf{n},\mathbf{r})=1-2f_{-\boldsymbol{\varepsilon}}(\mathbf{r}),\quad \boldsymbol{\varepsilon}=(\varepsilon,\mathbf{n}). \tag{21}$$

As a result we obtain

$$a^2\mathbf{v}\nabla f_{\boldsymbol{\varepsilon}}+I_{imp}+I_{ph}=0, \tag{22}$$

$$I_{imp}=\frac{1}{\tau}\int\frac{d\Omega'}{4\pi}W_{\mathbf{nn}'}[K_1(\boldsymbol{\varepsilon},\boldsymbol{\varepsilon}')(f_{\boldsymbol{\varepsilon}}-f_{\boldsymbol{\varepsilon}'})+K_2(\boldsymbol{\varepsilon},\boldsymbol{\varepsilon}')(f_{\boldsymbol{\varepsilon}}+f_{-\boldsymbol{\varepsilon}'}-1)]\big|_{\varepsilon'=\varepsilon}$$
$$+\frac{1}{\tau_s}\int\frac{d\Omega'}{4\pi}W_{\mathbf{nn}'}^{(s)}[K_3(\boldsymbol{\varepsilon},\boldsymbol{\varepsilon}')(f_{\boldsymbol{\varepsilon}}-f_{\boldsymbol{\varepsilon}'})+K_4(\boldsymbol{\varepsilon},\boldsymbol{\varepsilon}')(f_{\boldsymbol{\varepsilon}}+f_{-\boldsymbol{\varepsilon}'}-1)]\big|_{\varepsilon'=\varepsilon},$$

$$I_{ph}=\frac{\pi\lambda}{2(sp_F)^2}\int\limits_{-\infty}^{+\infty}d\varepsilon'\int\limits_0^{\infty}d\omega\omega^2[K_1(\boldsymbol{\varepsilon},\boldsymbol{\varepsilon}')\Pi_1(\boldsymbol{\varepsilon},\boldsymbol{\varepsilon}',\omega)$$
$$+K_2(\boldsymbol{\varepsilon},\boldsymbol{\varepsilon}')\Pi_2(\boldsymbol{\varepsilon},\boldsymbol{\varepsilon}',\omega)]\big|_{\mathbf{n}'=\mathbf{n}},$$

$$\Pi_1(\boldsymbol{\varepsilon},\boldsymbol{\varepsilon}',\omega)=[f_{\boldsymbol{\varepsilon}}(1-f_{\boldsymbol{\varepsilon}'})(N_\omega+1)-(1-f_{\boldsymbol{\varepsilon}})f_{\boldsymbol{\varepsilon}'}N_\omega]\delta(\varepsilon-\varepsilon'-\omega)$$
$$+[f_{\boldsymbol{\varepsilon}}(1-f_{\boldsymbol{\varepsilon}'})N_\omega-(1-f_{\boldsymbol{\varepsilon}})f_{\boldsymbol{\varepsilon}'}(N_\omega+1)]\delta(\varepsilon-\varepsilon'+\omega),$$

$$\Pi_2(\boldsymbol{\varepsilon},-\boldsymbol{\varepsilon}',\omega)=[f_{\boldsymbol{\varepsilon}}f_{\boldsymbol{\varepsilon}'}(N_\omega+1)-(1-f_{\boldsymbol{\varepsilon}})(1-f_{\boldsymbol{\varepsilon}'})N_\omega]\delta(\varepsilon+\varepsilon'-\omega)$$
$$+[f_{\boldsymbol{\varepsilon}}f_{\boldsymbol{\varepsilon}'}N_\omega-(1-f_{\boldsymbol{\varepsilon}})(1-f_{\boldsymbol{\varepsilon}'})(N_\omega+1)]\delta(\varepsilon+\varepsilon'+\omega),$$

where $\mathbf{v}=\mathbf{n}v_F$, N_ω is the phonon distribution function, and a^2 and the coherence factors K_1, K_2, K_3, and K_4 are expressed in terms of the matrices R and A:

$$\begin{aligned}
K_1(\boldsymbol{\varepsilon},\boldsymbol{\varepsilon}')&={}^1/_{16}\mathrm{Sp}\,P_1{}^RP_1{}^{R'}P_1{}^{A'}P_1{}^A={}^1/_2(a^2a^{2\prime}+\mathbf{a}_1\mathbf{a}_2{}'),\\
K_2(\boldsymbol{\varepsilon},\boldsymbol{\varepsilon}')&=-{}^1/_{16}\mathrm{Sp}\,P_1{}^RP_2{}^{R'}P_2{}^{A'}P_1{}^A={}^1/_2(\mathbf{a}_1\mathbf{a}_1{}'-a^2a^{2\prime}),\\
K_3(\boldsymbol{\varepsilon},\boldsymbol{\varepsilon}')&={}^1/_{16}\mathrm{Sp}\,P_1{}^R\tau_z\{P_1{}^{R'}P_1{}^{A'},\tau_z\}P_1{}^A=a_{1z}a_{2z}{}'-\mathbf{a}_1\mathbf{a}_2{}',\\
K_4(\boldsymbol{\varepsilon},\boldsymbol{\varepsilon}')&=-{}^1/_{16}\mathrm{Sp}\,P_1{}^R\tau_z\{P_2{}^{R'}P_2{}^{A'},\tau_z\}P_1{}^A=a_{1z}a_{1z}{}'-\mathbf{a}_1\mathbf{a}_1{}'.
\end{aligned} \tag{23}$$

Here $\mathbf{a}$, $\mathbf{a}_1$, and $\mathbf{a}_2$ are "vectors" [the primes in the right-hand sides of (23) pertain to the argument], which are connected with the Green's functions R and A expanded in terms of Pauli matrices in the following manner[3)]:

$$R=\boldsymbol{\tau}\mathbf{R},\quad A=\boldsymbol{\tau}\mathbf{A},\quad \mathbf{a}=(\mathbf{R}-\mathbf{A})/2,\quad \mathbf{b}=(\mathbf{R}+\mathbf{A})/2,$$
$$a^2=\mathbf{aa},\quad b^2=\mathbf{bb},\quad \mathbf{a}_1=\mathbf{a}+i[\mathbf{ab}],\quad \mathbf{a}_2=\mathbf{a}-i[\mathbf{ab}],\quad a^2+b^2=1,\quad \mathbf{ab}=0. \tag{24}$$

The connection between the vectors $\mathbf{a}$ and $\mathbf{b}$ follows from the normalization conditions (6a). The coherence factors K_1 to K_4 can be represented in the form of the squared modulus of the matrix element of the corresponding transition, and are therefore negative. The quantity f_ε is real by virtue of (19).

Account was taken in the expression for the phonon collision operator that at low temperatures the phonon momentum is small compared with the Fermi momentum, so that the change of the electron momentum in the collision can be disregarded. If the phonon distribution function depends on the direction, then the collision integral (22) contains the phonon function averaged over the directions perpendicular to the direction of the electron momentum.

As seen from (22) the scattering processes with coherence factors K_1 and K_3 leads to establishment of an isotropic Fermi distribution function with an arbitrary chemical potential, whereas processes with coherence factors K_2 and K_4 establish a zero chemical potential, i.e., describe the process of "branch intermixing."

The quantities a and b are obtained with the aid of stationary equations (8). In the case of infrequent collisions and slowly varying potentials we get

$$\begin{gathered}\mathbf{a}=\xi_\varepsilon^{-1}[(\varepsilon-\mathbf{v}\mathbf{p}_s)\mathbf{e}_z-i\Delta\mathbf{e}_y]\Theta(|\varepsilon-\mathbf{v}\mathbf{p}_s|-\Delta),\\ a^2=\Theta(|\varepsilon-\mathbf{v}\mathbf{p}_s|-\Delta),\quad a^2\mathbf{b}=0,\\ \xi_\varepsilon=\operatorname{sign}\varepsilon[(\varepsilon-\mathbf{v}\mathbf{p}_s)^2-\Delta^2]^{1/2}.\end{gathered}\tag{25}$$

Then (22) goes over into the classical Boltzmann equation[7] if we change over from the independent variable ε to the momentum $\mathbf{p}$ using the formula

$$\phi+v_F(p-p_F)=\xi_\varepsilon,\qquad \mathbf{p}=p\mathbf{n}\operatorname{sign}\varepsilon.$$

In addition, it must be recognized that f_ε corresponds to the quasielectron description, and the distribution function of the BCS quasiparticles is equal to f at $\varepsilon>0$ and to $1-f$ at $\varepsilon<0$. However, the obtained equations do not contain certain effects that are usually small, namely the increment $\mathbf{p}_s/m$ to the group velocity of the excitations, and the twisting of the trajectories by the magnetic field. The accuracy of the initial equations (8) and (9), which are valid only in the principal approximation in Δ/ε_F, is insufficient for the description of these phenomena.

The expressions for the density and the current, and the self-consistency equation, take the form

$$\delta N=\frac{mp_F}{\pi^2}\left[-\phi+\frac{1}{2}\int_{-\infty}^{+\infty}d\varepsilon\int\frac{d\Omega}{4\pi}a^2(2f_\varepsilon-1)\right],\tag{26}$$

$$\mathbf{j}=e\frac{mp_F}{\pi^2}\int_{-\infty}^{+\infty}d\varepsilon\int\frac{d\Omega}{4\pi}\mathbf{v}\left[a_zf_\varepsilon+(1-2f_\varepsilon)\frac{i[\mathbf{a}\times\mathbf{b}]_z}{2}\right],\tag{27}$$

$$\Delta=\lambda\int_{-\infty}^{+\infty}d\varepsilon\int\frac{d\Omega}{4\pi i}\left[a_yf_\varepsilon+(1-2f_\varepsilon)\frac{i[\mathbf{a}\times\mathbf{b}]_y}{2}\right].\tag{28}$$

The phase of the order parameter is determined from the continuity equation, which coincides with the imaginary part of the self-consistency equation.[8]

4. THE DIFFUSION EQUATION

As shown in many papers, in the case of a small mean free path ($\tau\Delta\ll\hbar$) it is possible to determine the angular dependence of the Green's functions, and for their isotropic part it is possible to obtain the diffusion equation. These equations take the form[4]

$$-iD[\tilde{\nabla},R[\tilde{\nabla},R]]+\{H^R,R\}=0,\tag{29}$$

$$-iD([\tilde{\nabla},R[\tilde{\nabla},g]]+[\tilde{\nabla},g[\tilde{\nabla},A]])+H^Rg-gH^A+R\Sigma-\Sigma A=0,\tag{30}$$

$$[\tilde{\nabla},X]\equiv\nabla X(\mathbf{r},t_1,t_2)+i\mathbf{p}_s(t_1,\mathbf{r})\tau_zX(\mathbf{r},t_1,t_2)-i\mathbf{p}_s(t_2,\mathbf{r})X(\mathbf{r},t_1,t_2)\tau_z,$$

where D is the diffusion coefficient, and the self-energy parts no longer contain scattering by nonmagnetic impurities, and that part $\mathbf{n}\cdot\mathbf{g}'$ of the Green's function g which depends on the direction $\mathbf{n}$ is given by

$$\mathbf{g}'/\tau v_F=-R[\tilde{\nabla},g]-g[\tilde{\nabla},A].\tag{31}$$

Using the representation (14) and Eqs. (29) and (30), we obtain an equation for the distribution function F_1:

$$\begin{aligned}&\operatorname{Sp}P_1^R\{-iD([\tilde{\nabla},[\tilde{\nabla},F_1]]+[\tilde{\nabla},P_1^R][\tilde{\nabla},F_1]+[\tilde{\nabla},F_1][\tilde{\nabla},P_1^A]\\&\quad+{}^1/_2[\tilde{\nabla},P_2^R](F_1+F_2)[\tilde{\nabla},P_2^A])+H^RF_1-F_1H^A+\Sigma\}P_1^A=0.\end{aligned}\tag{32}$$

We take the Fourier transforms with respect to the time difference t_1-t_2 in Eqs. (29) and (32). If the fields vary sufficiently slowly, then the dependence of the functions R and A on the time $t=(t_1+t_2)/2$ is determined mainly by the instantaneous value of the potentials. The nonadiabatic corrections are small if the frequency of the external field is small compared with the characteristic electron energy. In this case the matrix $R(\varepsilon,r,t)$ can be obtained from the static equation (29). Taking into account the normalization condition (8a) and (29), we write down the equation for R in the form [the equations for the function $A(\varepsilon,r,t)$ are similar]

$$\begin{gathered}-\frac{i}{2}D[\tilde{\nabla},[\tilde{\nabla},R]]+\xi^RR=H^R,\\ \xi^R\equiv\frac{1}{2}\operatorname{Sp}R\left(H^R+\frac{i}{2}D[\tilde{\nabla},[\tilde{\nabla},R]]\right),\\ H^R=\varepsilon\tau_z-i\Delta\tau_y+\frac{i}{2\tau_s}\tau_z\{\tau_z,R\}-\frac{1}{2}(\Sigma_{ph}{}^R-\Sigma_{ph}{}^A).\end{gathered}\tag{33}$$

From (32), taking into account only the terms proportional to the first power of the external-field frequency, we obtain ($F_1=1-2f_\varepsilon$)

$$\begin{gathered}a_z\frac{\partial f_\varepsilon}{\partial t}+a^2\dot{\phi}+ia_y\dot{\Delta}\frac{\partial f_\varepsilon}{\partial\varepsilon}-Da_z(\operatorname{div}\dot{\mathbf{p}}_s+2\dot{\mathbf{p}}_s\nabla)\frac{\partial f_\varepsilon}{\partial\varepsilon}-2D\dot{\mathbf{p}}_sa^2(\nabla a_z)\frac{\partial f_\varepsilon}{\partial\varepsilon}\\ -D\nabla a^2\nabla f_\varepsilon+\mathbf{d}\nabla f_\varepsilon+\frac{1}{2\tau_Q}(f_\varepsilon+f_{-\varepsilon}-1)+I_{ph}=0,\\ \mathbf{d}=\frac{4}{i}D\mathbf{p}_sb_za^2-\frac{D}{8}\operatorname{Sp}\{R,A\}\nabla(R-A),\\ \frac{1}{2\tau_Q}=\left(\frac{1}{\tau_s}+2Dp_s^2\right)(a_{1z}{}^2-(a^2)^2)+\frac{D}{8}\operatorname{Sp}P_1^R(\nabla R)(\nabla A)P_1^A\\ +\frac{iD}{8}\mathbf{p}_s\operatorname{Sp}P_1^R((\nabla R)\{\tau_z,A\}+\{\tau_z,R\}(\nabla A))P_1^A\end{gathered}\tag{34}$$

and the integral of the collisions with the phonons is defined by (22) and (23). In the coefficients of the terms containing time derivatives we have discarded terms proportional to $[\mathbf{a}\times\mathbf{b}]_{y,z}$, which are usually small: it can be concluded from (33) that they are of the order of $D(\mathbf{p}_s\nabla\Delta)/|\xi^R|^2$. For the case of a spatially homogeneous gap and small p_s, equations of type (34) were obtained by Ivlev and Bulyzhenkov.[9]

The potential Δ and the density are defined by (26) and (28), in which the integration with respect to the angles can be left out. The expression for the current has in the general case a rather complicated form and will not be needed here, but can be calculated with the aid of the relation

$$\mathbf{j}(t)=-eD\frac{mp_F}{4\pi}\frac{\operatorname{Sp}\tau_z\mathbf{g}'(t,t)}{\tau v_F},$$

where the current part of the Green's function $\mathbf{g}'$ is determined by (14), (20), and (31).

5. SHIFT OF CHEMICAL POTENTIAL

We proceed now to calculate the shift produced in the chemical potential under the influence of the temperature gradient when the condensate moves. We note first certain papers in which the Boltzmann equation[7] was solved under these conditions ($\mathbf{p}_s\neq0$, $\nabla T\neq0$).

The correction to the distribution function was calculated in connection with the problem of the thermoelectric current in Refs. 10 and 11, and was found to be proportional to the energy relaxation time τ_{ph} and to the quasiparticle velocity averaged over the equal-energy surface

$$\langle \partial \varepsilon_p/\partial \mathbf{p}\rangle|_{\tilde{\tau}_p=\text{const}}, \quad \varepsilon_p=(\xi_p^2+\Delta^2)^{1/2}+\mathbf{p}_s\mathbf{p}/m.$$

At first glance this mean value is equal to $\mathbf{p}_s/m$. The small corrections to the velocity, of the order of $p_s/m \sim v_F\Delta/\varepsilon_F$, are beyond the limits of the accuracy of Eq. (22), but might be substantial, since the associated deviation from equilibrium is determined by the slow processes of energy relaxation. However, allowance for the dependence of $|\mathbf{p}|$ on the position on the equal-energy surface leads to a zero average velocity, and with it to corrections proportional to the long time[4] τ_{ph} for the distribution function and for the thermoelectric current.

The shift of the chemical potential ϕ calculated in Refs. 13 and 14 is proportional to the "branch intermixing" time. It seems to us that the expansion in terms of p_s in these papers is not correct enough, namely, no due account was taken of the terms linear in p_s contained in the integral of the collisions with the impurities. The change produced in the result by allowance for these terms is exceedingly significant: the time τ_Q is replaced by the momentum relaxation time.

The correction to the distribution function, due to the temperature gradient, can be obtained from the kinetic equation (22). As will be shown below, the cause of the appearance of the potential ϕ is the angular dependence of the "branch mixing" time, and the effect is not sensitive to the form of the mechanism that leads to the "branch mixing." For this reason we confine ourselves to elastic relaxation processes. Then the solution of the kinetic equation (22) can be obtained at arbitrary impurity concentration. The formula for the obtained solution is most cumbersome, and we confine ourselves therefore to two limiting cases relative to the mean free path.

We consider first the case of a pure superconductor. The solution of the kinetic equation is sought in the form

$$f=f_0(\varepsilon, T(\mathbf{r}))+f_1.$$

Here f_0 is the Fermi distribution. The function f_1 turns out to be even in ε, and Eq. (22) for f_1 takes the form (we assume for simplicity that the scattering is isotropic, $W_{\mathbf{nn}'}=1$)

$$\begin{gathered} a^2\mathbf{v}\nabla f_0+\frac{1}{\tau}\int\frac{d\Omega'}{4\pi}a^2a^{2\prime}(f_1-f_1')+\frac{f_1}{\tau_Q}=0,\\ \frac{1}{2\tau_Q}=\int\frac{d\Omega'}{4\pi}\left[\frac{1}{\tau}K_2(\varepsilon,\varepsilon')+\frac{1}{\tau_s}K_4(\varepsilon,\varepsilon')\right]\Bigg|_{\varepsilon'=-\varepsilon}, \end{gathered} \tag{35}$$

where the quantities a^2, K_2, and K_4 are defined by relations (23)–(25). Solving Eq. (35) and using (26) and the neutrality condition $\delta N=0$, we get

$$\begin{gathered} \phi=\int_{-\infty}^{+\infty}d\varepsilon\frac{\varepsilon}{T}\frac{\partial f_0}{\partial\varepsilon}\left\langle\frac{a^2(\mathbf{v}\nabla T)}{\tau\tau^{-1}+\langle a^2\rangle a^2}\right\rangle\left\langle\frac{\tau_Q^{-1}+\tau^{-1}(1-\langle a^2\rangle)a^2}{\tau\tau^{-1}+a^2\langle a^2\rangle}\right\rangle^{-1},\\ \langle(\ldots)\rangle=\int(\ldots)d\Omega/4\pi. \end{gathered} \tag{36}$$

In the region $|\varepsilon|>\Delta+\mathbf{vp}_s$, which is decisive near the critical temperature, the quantity a^2 is independent of angle. For this reason the first factor in (36), which contains averaging over the angles, differs from zero only because of the angular dependence of τ_Q, which is due to the shifts of the energies by an amount $\mathbf{v}\cdot\mathbf{p}_s$ (25). This factor is proportional to $\tau\mathbf{vp}_s\partial\tau_Q^{-1}/\partial\varepsilon(\tau\ll\tau_Q)$. The second factor is approximately equal to τ_Q. Thus, the potential ϕ is proportional to the momentum relaxation time, and not to the "branch mixing" time. In elastic scattering, the energy dependence of τ_Q is usually a power function, therefore, apart from a numerical factor, the effect is independent of the "branch mixing" mechanism.

Using (36) for the calculation, we obtain near the critical temperature

$$\phi=\zeta D\mathbf{p}_s\nabla T/T, \tag{37}$$

where ζ is a numerical coefficient that depends on the relaxation mechanism. If the momentum p_s is small and the "branch mixing" is due, e.g., to scattering by magnetic impurities, then $\zeta=1$. If $(vp_s)>T(\tau/\tau_s)^{1/2}$, then the "branch mixing" takes place in scattering by nonmagnetic impurities and $=4/5$.

At low temperatures, $(T\ll\Delta)$ and sufficiently large currents $(vp_s\gg T)$ we obtain from (36) for superconductors without magnetic impurities

$$\phi=2(3\sqrt{3}+\pi)(2\pi-3\sqrt{3})^{-1}l(\mathbf{p}_s\nabla T)\Delta\exp[-(\Delta-vp_s)/T]/T|\mathbf{p}_s|, \tag{38}$$

where l is the mean free path.

If the free path time is short, then it is more convenient to use the diffusion equation (34):

$$-D\nabla a^2\nabla f_\varepsilon+\mathbf{d}\nabla f_\varepsilon+\frac{1}{2\tau_Q}(f_\varepsilon+f_{-\varepsilon}-1)=0, \tag{39}$$

where in this case $\mathbf{d}=4D\mathbf{p}_s b_\varepsilon a^2/i$, and the inelastic scattering is disregarded as before.

If $\mathbf{p}_s$ and ∇T are uniformly distributed, we get from (39) and (26)

$$\phi=-4D(\mathbf{p}_s\nabla T/T)\int_{-\infty}^{+\infty}d\varepsilon(a^2)^2 ib_\varepsilon\tau_Q\varepsilon\,\partial f_0/\partial\varepsilon. \tag{40}$$

If the temperature is close to critical, then an important role is played in the integral (40) by energies $\varepsilon\sim T$. Let the concentration of the magnetic impurities and Dp_s^2 be not too large, then the quantity $b_\varepsilon=i\,\text{Im}\,R_\varepsilon$ can be obtained from Eqs. (33) by iterating with respect to the parameter $\Gamma/\xi^R\ll 1$, $\Gamma=2Dp_s^2+\tau_s^{-1}$. The time τ_Q is given by (34). Calculations in the principal approximation in T/ξ^R yield

$$\begin{gathered} ib_\varepsilon\approx\Gamma\varepsilon\Delta^2/(\varepsilon^2-\Delta^2)^2, \quad \tau_Q\approx(\varepsilon^2-\Delta^2)/2\Gamma\Delta^2,\\ a^2\approx\Theta(|\varepsilon|-\Delta), \end{gathered} \tag{41}$$

and the potential ϕ turns out to be

$$\phi\approx 2D\frac{\mathbf{p}_s\nabla T}{T}\left(1+\frac{\Delta}{2T}\ln\frac{\Delta}{\Gamma}\right). \tag{42}$$

If $\Gamma\gg\Delta$, then the superconductor is in the gapless state. The suppression of the gap can be due to a sufficiently strong magnetic field or by current through the film and by frequent scattering by magnetic impurities or phonons. We then obtain in the entire region of existence of the gapless superconductivity, from (33) and (38),

$$ib_z=\Delta^2\varepsilon\Gamma/(\varepsilon^2+\Gamma^2)^2,\quad \tau_Q=(\varepsilon^2+\Gamma^2)/2\Gamma\Delta^2,\quad a^2\approx 1,$$

$$\phi=2D\frac{\mathbf{p}\,\nabla T}{T}\left[1-\frac{\Gamma}{2\pi T}\psi'\left(\frac{1}{2}+\frac{\Gamma}{2\pi T}\right)\right]. \tag{43}$$

The experiment of Clarke *et al.*[2] confirms the linear dependence of the potential ϕ on p_s and ∇T. In Ref. 2 is given the experimental value of

$$\gamma=\frac{\phi}{e}\left(1-\frac{T}{T_c}\right)\Big/ j\frac{\nabla T}{T}, \tag{44}$$

where j is the density of the current flowing through the film. The measurements were made with dirty tin ($l\sim 6\times 10^{-6}$ cm). If p_s is expressed in terms of the currents using the formulas for a "dirty" superconductor,[14] then (42) and (43) yield (assuming that $\Gamma \ll T$)

$$\gamma=\frac{4}{3\pi^2}\frac{\Delta(0)}{T_c}\rho l\xi_0, \tag{45}$$

where ρ is the resistivity and $\xi_0=\hbar v_F/\pi\Delta(0)$ is the coherence length. This formula contains no adjustment parameters and γ is determined only by the chemical composition of the sample. For tin, $\rho l=1.5\cdot 10^{-11}$ $\Omega\cdot$cm^2, $\xi_0=2.3\cdot 10^{-5}$ cm[15], $\Delta(0)/T_c\approx 1.76$, and γ turns out to equal $0.8\times 10^{-16}\,\Omega\cdot$cm^3. The experimental value of γ for different samples fluctuates between 0.8 and $3.8\times 10^{-16}\,\Omega\cdot$cm^3. The reason for this spread is not quite clear, since almost all the samples had very close parameters. Under these conditions the quantitative agreement between the theory and experiment can be regarded as perfectly satisfactory.

Let us dwell finally on the effect reminiscent to the Nernst effect. If a superconducting plate is situated in a magnetic field parallel to its surface, then the Meissner screening results in a superconducting current near the surface of the plate. If the temperature is not uniform along the plate, then a transverse potential difference $\delta\phi=\phi_1-\phi_2$ is produced, where ϕ_1 and ϕ_2 are the nonequilibrium potentials on the opposite surfaces of the plate.

We calculate this potential difference under conditions when relations (41) are valid. The distribution functions satisfies the equation (39). The source of the disequilibrium $\mathbf{d}\nabla f_0$ is concentrated at low energies, at which the diffusion length $(D\tau_Q)^{1/2}$, and the term with the second derivatives can be neglected in Eq. (39). At high energies, at the same time, the effective diffusion mechanism of relaxation comes into play, and the distribution function is close to equilibrium. For this reason the potentials ϕ_1 and ϕ_2 can be obtained from Eq. (40), in which the integration limits are determined from the condition $(D\tau_Q)^{1/2}=\Lambda$, $\Lambda=\min(\lambda, d)$, where λ is the London depth of penetration and d is the thickness of the plate. Simple calculations yield

$$\frac{\delta\phi}{ed}=N[\mathbf{n}\times\mathbf{H}]\nabla T,\quad N=\frac{1}{c}\frac{D}{T}\frac{\lambda}{d}\,\mathrm{th}\frac{d}{2\lambda}\frac{\Delta}{T}\ln\left(\frac{\Delta^2\Lambda}{\Gamma D}\right). \tag{46}$$

The superfluid momentum $\mathbf{p}_s$ is expressed here in terms of the applied magnetic field $\mathbf{H}$ with the aid of the Londons' equations, and $\mathbf{n}$ is the normal to the surface of the plate. If $d\sim\lambda$, $T\sim T_c$, then N differs by a factor $(\varepsilon_F/T_c)^2$ from the Nernst constant in the normal metal.

The unbalance in the potential ϕ in the presence of ∇T is due to the drift term $\mathbf{d}\cdot\nabla f$ in (39). This equation can be derived directly from (22) by the Chapman-Enskog procedure. In the course of the derivation it can be verified that the vector $\mathbf{d}$ in (34) and (39) is none other than the average, over the equal-energy surface, of the effective excitation velocity $\mathbf{d}=\langle a^2\mathbf{v}\rangle$. The physical interpretation of the difference between the effective velocity $a^2\mathbf{v}$ and the Fermi velocity $\mathbf{v}$ was discussed in the introduction. The value of the vector $\mathbf{d}$, as expected, is determined by the "branch mixing." This can be verified from relations (34) and (41). The presence of the drift term is not confined to dirty superconductors. A diffusion equation of the type (39) can be obtained also for pure superconductors and for perturbations that are weakly inhomogeneous in space. The corrections connected with the angular dependence of a^2 are in this case small in terms of the parameter $(vp_s/\Delta)(\tau\tau_Q\Delta^2)^{-1/2}$, and the term $\mathbf{d}\cdot\nabla f$ stems from the angular dependence of the time τ_Q (35).

A drift of excitations with a velocity proportional to p_s means in a certain sense drgagging of the excitations by the condensate, and leads under nonequilibrium conditions to the onset of dissipative currents.

6. HEAT FLUX

Assume that a voltage U is applied to an N-S junction (see the figure). An injection of excitations leads to a deviation from equilibrium in the superconductor and to the appearance of a potential ϕ. If we pass a current through S, then the dragging of the nonequilibrium excitations by the condensate leads to the appearance of the heat flux for the reasons indicated in Sec. 1. Understandably, this effect and the appearance of the potential ϕ under the influence of the temperature gradient and the superfluid motion are closely connected. This connection can be established with the aid of the Onsager relation. It is important that a current smaller than critical does not take the superconductor out of the equilibrium state, and the principle of symmetry of the kinetic coefficients is applicable in this case.

Using the expressions for the entropy production (see, e.g., Ref. 7) and the tunnel source of excitations,[9,16] as well as the kinetic equation (22), we obtain

$$\dot{\sigma}=\frac{1}{T}\frac{U}{Rd}\frac{\phi}{e}, \tag{47}$$

where $\dot{\sigma}$ is the density of the entropy production due to the "branch mixing" processes in the superconductor, and ϕ is the value of the nonequilibrium potential at a

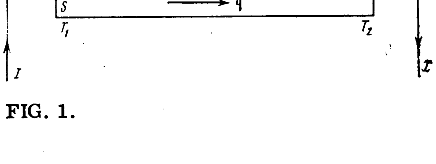

FIG. 1.

voltage U on the contact. This relation can be verified for elastic "branch mixing" mechanisms at any temperature, and for scattering by phonons at least near the critical temperature.

The expressions obtained in the preceding section for the potential ϕ can be written in the form (2). The quantity $\zeta(T)$ is obtained by comparing (2) with formulas (37), (38), (42) and (43). From the Onsager principle, (47), and from the fact that $\mathbf{p}_s$ reverses sign when time is reversed, we obtain an expression for the heat flux $\mathbf{q}$:

$$\mathbf{q}=-\varkappa\nabla T-\zeta(T)\frac{D}{d}\frac{U}{R}\frac{\mathbf{p}_s}{e}, \tag{48}$$

where $\varkappa$ is the electronic thermal conductivity. To estimate the effect, we represent (48) in the form $q=-\varkappa[\nabla T-(\nabla T)']$, where $(\nabla T)'$ has the meaning of the temperature gradient established in the sample under adiabatic conditions ($q=0$) if only the electrons of the film S take part in the heat-conduction process. We express $(\nabla T)'$ in the form

$$(\nabla T)'=\frac{\zeta(T)}{\pi^2}\frac{\varkappa(T_c)}{\varkappa(T)}\frac{eU}{T_c}\frac{v_F p_s}{T_c}\frac{\rho l}{R}\frac{T_c}{d}. \tag{49}$$

As expected, the quantity $(\nabla T)'$ vanishes both as $T\to 0$ and at the transition point T_c. In the former case this is due to the behavior of $\zeta(T)$ and $\varkappa(T)$ at low temperatures, and in the latter to the vanishing of the critical momentum. The maximum value of $(\nabla T)'$ takes place at $T\sim T_c$. In this case all the dimensionless factors in (49), except for the last one, can be of the order of unity. If $R\sim 10^{-6}\,\Omega\cdot\mathrm{cm}^2$, $\rho l\sim 10^{-11}\,\mathrm{cm}^2$, and $d\sim 10^{-5}$ cm, then $(\nabla T)'\sim 1$ K/cm. Naturally, it is extremely difficult to obtain such a temperature gradient in a real situation, for example because of the heat fluxes in the substrate.

We note in conclusion that the described mechanism should contribute to the Ettingshausen effect for superconductors in the intermediate or in the mixed state.

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[1] After this work was completed, an article by Schmid and Schön appeared,[3] in which a similar relation was obtained.

[2] The part of Σ_{imp} connected with scattering by magnetic impurities was written down for the case when there is no spin polarization.

[3] The distribution functions φ and φ_1 introduced in Ref. 4 are connected with f by the relations

$$\varphi(\varepsilon)=f_{-\varepsilon}-f_\varepsilon-[\mathbf{a}\times\mathbf{b}]_z(1-f_\varepsilon-f_{-\varepsilon})/a_z,$$

$$\varphi_1(\varepsilon)=a^2(1-f_\varepsilon-f_{-\varepsilon})/a_z.$$

[4] This circumstance is indicated in the review by A. G. Aronov, Yu. M. Gal'perin and V. L. Gurevich and V. I. Kozub (in press). The author thanks Aronov and Kozub for a discussion of this question.

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